

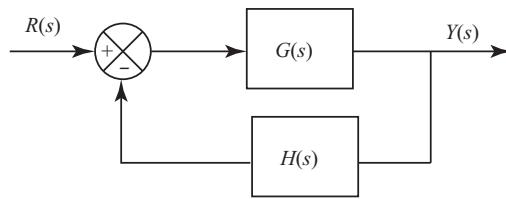
ME 6202 Control Systems I

Formula Sheet for Final Exam

Fall 2022

- Closed-loop transfer function of a negative feedback system:

$$\frac{Y(s)}{R(s)} = \frac{G(s)}{1 + G(s)H(s)}$$



- Transfer function of a general second-order transfer function:

$$G(s) = \frac{\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2}$$

- Transient-response specifications of the second-order system:

1. Rise time t_r :

$$t_r = \frac{\pi - \beta}{\omega_d}, \quad \text{where } \omega_d = \omega_n \sqrt{1 - \xi^2}, \quad \cos \beta = \xi$$

2. Peak time t_p :

$$t_p = \frac{\pi}{\omega_d}$$

3. Maximum overshoot M_p :

$$M_p = e^{-(\xi/\sqrt{1-\xi^2})\pi}$$

4. Settling time t_s :

$$t_s = \frac{4}{\xi\omega_n} \quad (2\% \text{ criterion})$$

$$t_s = \frac{3}{\xi\omega_n} \quad (5\% \text{ criterion})$$

- Final value theorem:

$$\lim_{t \rightarrow \infty} f(t) = \lim_{s \rightarrow 0} sF(s)$$

where $F(s)$ is the Laplace transform of $f(t)$.

- **The Routh-Hurwitz criterion:**

The Routh-Hurwitz criterion is based on ordering the coefficients of the characteristic equation

$$a_n s^n + a_{n-1} s^{n-1} + a_{n-2} s^{n-2} + \cdots + a_1 s + a_0 = 0$$

into an array or schedule as follows

$$\begin{array}{c|cccc} s^n & a_n & a_{n-2} & a_{n-4} & \dots \\ s^{n-1} & a_{n-1} & a_{n-3} & a_{n-5} & \dots \end{array}$$

Further rows of the schedule are then completed as

$$\begin{array}{c|cccc} s^n & a_n & a_{n-2} & a_{n-4} & \dots \\ s^{n-1} & a_{n-1} & a_{n-3} & a_{n-5} & \dots \\ s^{n-2} & b_{n-1} & b_{n-3} & b_{n-5} & \dots \\ s^{n-3} & c_{n-1} & c_{n-3} & c_{n-5} & \dots \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ s^0 & h_{n-1} & & & \end{array}$$

where

$$\begin{aligned} b_{n-1} &= \frac{a_{n-1}a_{n-2} - a_n a_{n-3}}{a_{n-1}} = \frac{-1}{a_{n-1}} \begin{vmatrix} a_n & a_{n-2} \\ a_{n-1} & a_{n-3} \end{vmatrix}, \\ b_{n-3} &= -\frac{1}{a_{n-1}} \begin{vmatrix} a_n & a_{n-4} \\ a_{n-1} & a_{n-5} \end{vmatrix}, \dots \\ c_{n-1} &= \frac{-1}{b_{n-1}} \begin{vmatrix} a_{n-1} & a_{n-3} \\ b_{n-1} & b_{n-3} \end{vmatrix}, \dots \end{aligned}$$

and so on. The algorithm for calculating the entries in the array can be followed on a determinant basis or by using the form of the equation for b_{n-1} .

- **System sensitivity:**

System sensitivity is the ratio of the change in the system transfer function $T(s)$ to the change of a process transfer function $G(s)$ (or parameter) for a small incremental change:

$$S = \frac{\partial T/T}{\partial G/G} = \frac{\partial \ln T}{\partial \ln G}$$

- Block diagram transformation:

Transformation	Original Diagram	Equivalent Diagram
1. Combining blocks in cascade		 or
2. Moving a summing point behind a block		
3. Moving a pickoff point ahead of a block		
4. Moving a pickoff point behind a block		
5. Moving a summing point ahead of a block		
6. Eliminating a feedback loop		

- Root-locus procedure:

1. Locate the poles and zeros of $G(s)H(s)$ on the s plane. The root-locus branches start from open-loop poles and terminate at zeros (finite zeros or zeros at infinity).
2. Determine the root loci on the real axis. If the total number of real poles and real zeros to the right of this test point is odd, then this point lies on a root locus.
3. Determine the asymptotes of root loci.

$$\text{Angles of asymptotes} = \frac{\pm 180^\circ(2k+1)}{n-m} \quad (k = 0, 1, 2, \dots)$$

where n = number of finite poles of $G(s)H(s)$, m = number of finite zeros of $G(s)H(s)$.

The asymptotes intersection point on the real axis is given by

$$s = \frac{\sum \text{real parts of poles of } G(s)H(s) - \sum \text{real parts of zeros of } G(s)H(s)}{n-m}$$

4. Find the breakaway and break-in points. Suppose that the characteristic equation is given by

$$B(s) + KA(s) = 0$$

The breakaway or break-in points can be determined from the roots of

$$\frac{dK}{ds} = -\frac{B'(s)A(s) - B(s)A'(s)}{A^2(s)} = 0$$

where the prime indicates differentiation with respect to s .

5. Determine the angle of departure (angle of arrival) of the root locus from a complex pole (at a complex zero).

Angle of departure from a complex pole = 180°

- (sum of the angles of vectors to a complex pole in question from other poles)
- + (sum of the angles of vectors to a complex pole in question from zeros)

Angle of arrival at a complex zero = 180°

- (sum of the angles of vectors to a complex zero in question from other zeros)
- + (sum of the angles of vectors to a complex zero in question from poles)

6. Find the points where the root loci may cross the imaginary axis. The points where the root loci intersect the $j\omega$ axis can be found easily by (a) use of Routh's stability criterion or (b) letting $s = j\omega$ in the characteristic equation, equating both the real part and the imaginary part to zero, and solving for ω and K .

- Frequency response:

1. Constant gain K :

The magnitude $M = 20 \log K$ and the phase angle $\phi = 0^\circ$

2. Integral and derivative factors:

(a) For $G(s) = 1/s$,

Magnitude: $M = 20 \log |G(j\omega)| = -20 \log \omega$, slope = -20 dB/decade;

Phase angle: $\phi = -90^\circ$

(b) For $G(s) = s$,

Magnitude: $M = 20 \log |G(j\omega)| = 20 \log \omega$, slope = 20 dB/decade;

Phase angle: $\phi = 90^\circ$

(c) For $G(s) = s^n$,

Magnitude: $M = 20 \log |G(j\omega)| = 20n \log |j\omega|$, slope = $n \times (20$ dB/decade);

Phase angle: $\phi = n \times 90^\circ$

3. First-order system:

(a) For $G(s) = \frac{1}{1 + Ts}$,

For $\omega T \ll 1$, the asymptotic curve is 0 dB, $\phi = 0^\circ$;

For $\omega T \gg 1$, the asymptotic curve is -20 dB/decade, $\phi = -90^\circ$.

(b) For $G(s) = 1 + Ts$,

For $\omega T \ll 1$, the asymptotic curve is 0 dB, $\phi = 0^\circ$;

For $\omega T \gg 1$, the asymptotic curve is 20 dB/decade, $\phi = 90^\circ$.

4. Second-order system:

corner frequency: $\omega = \omega_n$; The asymptotic curve for $\omega \ll \omega_n$ is 0 dB, $\phi = 0^\circ$;

The asymptotic curve for $\omega \gg \omega_n$ is -40 dB/decade, $\phi = -180^\circ$;

The resonant frequency $\omega_r = \omega_n \sqrt{1 - 2\xi^2}$.