

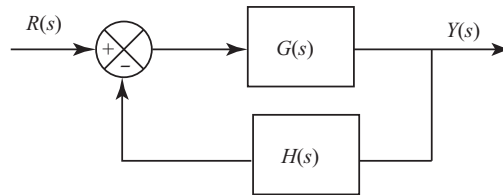
ME 6202 Control Systems I

Formula Sheet for Final Exam

Fall 2022

- Closed-loop transfer function of a negative feedback system:

$$\frac{Y(s)}{R(s)} = \frac{G(s)}{1 + G(s)H(s)}$$



- Transfer function of a general second-order transfer function:

$$G(s) = \frac{\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2}$$

- Transient-response specifications of the second-order system:

1. Rise time t_r :

$$t_r = \frac{\pi - \beta}{\omega_d}, \quad \text{where } \omega_d = \omega_n \sqrt{1 - \xi^2}, \quad \cos \beta = \xi$$

2. Peak time t_p :

$$t_p = \frac{\pi}{\omega_d}$$

3. Maximum overshoot M_p :

$$M_p = e^{-(\xi/\sqrt{1-\xi^2})\pi}$$

4. Settling time t_s :

$$t_s = \frac{4}{\xi\omega_n} \quad (2\% \text{ criterion})$$

$$t_s = \frac{3}{\xi\omega_n} \quad (5\% \text{ criterion})$$

- Final value theorem:

$$\lim_{t \rightarrow \infty} f(t) = \lim_{s \rightarrow 0} sF(s)$$

where $F(s)$ is the Laplace transform of $f(t)$.

- **The Routh-Hurwitz criterion:**

The Routh-Hurwitz criterion is based on ordering the coefficients of the characteristic equation

$$a_n s^n + a_{n-1} s^{n-1} + a_{n-2} s^{n-2} + \dots + a_1 s + a_0 = 0$$

into an array or schedule as follows

$$\begin{array}{c|cccc} s^n & a_n & a_{n-2} & a_{n-4} & \dots \\ s^{n-1} & a_{n-1} & a_{n-3} & a_{n-5} & \dots \end{array}$$

Further rows of the schedule are then completed as

$$\begin{array}{c|cccc} s^n & a_n & a_{n-2} & a_{n-4} & \dots \\ s^{n-1} & a_{n-1} & a_{n-3} & a_{n-5} & \dots \\ s^{n-2} & b_{n-1} & b_{n-3} & b_{n-5} & \dots \\ s^{n-3} & c_{n-1} & c_{n-3} & c_{n-5} & \dots \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ s^0 & h_{n-1} & & & \end{array}$$

where

$$\begin{aligned} b_{n-1} &= \frac{a_{n-1}a_{n-2} - a_n a_{n-3}}{a_{n-1}} = \frac{-1}{a_{n-1}} \begin{vmatrix} a_n & a_{n-2} \\ a_{n-1} & a_{n-3} \end{vmatrix}, \\ b_{n-3} &= -\frac{1}{a_{n-1}} \begin{vmatrix} a_n & a_{n-4} \\ a_{n-1} & a_{n-5} \end{vmatrix}, \dots \\ c_{n-1} &= \frac{-1}{b_{n-1}} \begin{vmatrix} a_{n-1} & a_{n-3} \\ b_{n-1} & b_{n-3} \end{vmatrix}, \dots \end{aligned}$$

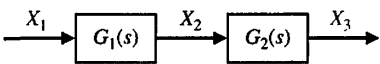
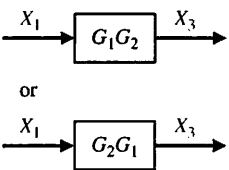
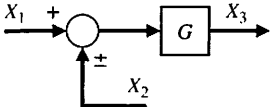
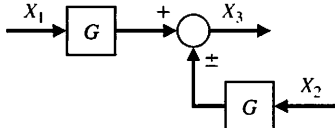
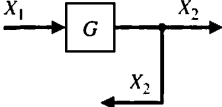
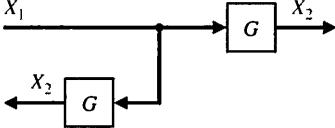
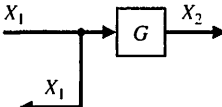
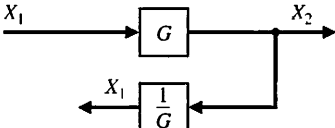
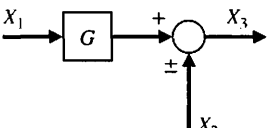
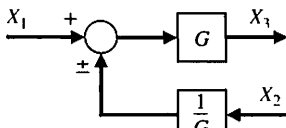
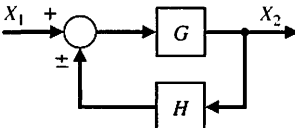
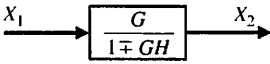
and so on. The algorithm for calculating the entries in the array can be followed on a determinant basis or by using the form of the equation for b_{n-1} .

- **System sensitivity:**

System sensitivity is the ratio of the change in the system transfer function $T(s)$ to the change of a process transfer function $G(s)$ (or parameter) for a small incremental change:

$$S = \frac{\partial T/T}{\partial G/G} = \frac{\partial \ln T}{\partial \ln G}$$

- Block diagram transformation:

Transformation	Original Diagram	Equivalent Diagram
1. Combining blocks in cascade		
2. Moving a summing point behind a block		
3. Moving a pickoff point ahead of a block		
4. Moving a pickoff point behind a block		
5. Moving a summing point ahead of a block		
6. Eliminating a feedback loop		

• **Root-locus procedure:**

1. *Locate the poles and zeros of $G(s)H(s)$ on the s plane. The root-locus branches start from open-loop poles and terminate at zeros (finite zeros or zeros at infinity).*
2. *Determine the root loci on the real axis.* If the total number of real poles and real zeros to the right of this test point is odd, then this point lies on a root locus.
3. *Determine the asymptotes of root loci.*

$$\text{Angles of asymptotes} = \frac{\pm 180^\circ(2k+1)}{n-m} \quad (k = 0, 1, 2, \dots)$$

where n = number of finite poles of $G(s)H(s)$, m = number of finite zeros of $G(s)H(s)$.
The asymptotes intersection point on the real axis is given by

$$s = \frac{\sum \text{real parts of poles of } G(s)H(s) - \sum \text{real parts of zeros of } G(s)H(s)}{n-m}$$

4. *Find the breakaway and break-in points.* Suppose that the characteristic equation is given by

$$B(s) + KA(s) = 0$$

The breakaway or break-in points can be determined from the roots of

$$\frac{dK}{ds} = -\frac{B'(s)A(s) - B(s)A'(s)}{A^2(s)} = 0$$

where the prime indicates differentiation with respect to s .

5. *Determine the angle of departure (angle of arrival) of the root locus from a complex pole (at a complex zero).*

Angle of departure from a complex pole = 180°

– (sum of the angles of vectors to a complex pole in question from other poles)

+ (sum of the angles of vectors to a complex pole in question from zeros)

Angle of arrival at a complex zero = 180°

– (sum of the angles of vectors to a complex zero in question from other zeros)

+ (sum of the angles of vectors to a complex zero in question from poles)

6. *Find the points where the root loci may cross the imaginary axis.* The points where the root loci intersect the $j\omega$ axis can be found easily by (a) use of Routh's stability criterion or (b) letting $s = j\omega$ in the characteristic equation, equating both the real part and the imaginary part to zero, and solving for ω and K .

- **Frequency response:**

1. **Constant gain K :**

The magnitude $M = 20 \log K$ and the phase angle $\phi = 0^\circ$

2. **Integral and derivative factors:**

(a) For $G(s) = 1/s$,

Magnitude: $M = 20 \log |G(j\omega)| = -20 \log \omega$, slope = -20 dB/decade;

Phase angle: $\phi = -90^\circ$

(b) For $G(s) = s$,

Magnitude: $M = 20 \log |G(j\omega)| = 20 \log \omega$, slope = 20 dB/decade;

Phase angle: $\phi = 90^\circ$

(c) For $G(s) = s^n$,

Magnitude: $M = 20 \log |G(j\omega)| = 20n \log |j\omega|$, slope = $n \times (20$ dB/decade);

Phase angle: $\phi = n \times 90^\circ$

3. **First-order system:**

(a) For $G(s) = \frac{1}{1 + Ts}$,

For $\omega T \ll 1$, the asymptotic curve is 0 dB, $\phi = 0^\circ$;

For $\omega T \gg 1$, the asymptotic curve is -20 dB/decade, $\phi = -90^\circ$.

(b) For $G(s) = 1 + Ts$,

For $\omega T \ll 1$, the asymptotic curve is 0 dB, $\phi = 0^\circ$;

For $\omega T \gg 1$, the asymptotic curve is 20 dB/decade, $\phi = 90^\circ$.

4. **Second-order system:**

corner frequency: $\omega = \omega_n$; The asymptotic curve for $\omega \ll \omega_n$ is 0 dB, $\phi = 0^\circ$;

The asymptotic curve for $\omega \gg \omega_n$ is -40 dB/decade, $\phi = -180^\circ$;

The resonant frequency $\omega_r = \omega_n \sqrt{1 - 2\xi^2}$.