Problem:
Let $\boldsymbol{Z} \in \mathcal{R}^{m,n}, m < n$.
Known: $\boldsymbol{L}_k$, $\boldsymbol{\Lambda}_k$ satisfying

$$\mathbf{I} + \boldsymbol{Z}\boldsymbol{Z}^T = \boldsymbol{L}_k \boldsymbol{\Lambda}_k \boldsymbol{L}_k^T \tag{1}$$

Let $\boldsymbol{u} \in \mathcal{R}^m$, and $\boldsymbol{v} \in \mathcal{R}^n$.
Desired: $\boldsymbol{L}_{k+1}$, $\boldsymbol{\Lambda}_{k+1}$ satisfying

$$\mathbf{I} + (\boldsymbol{Z} + \boldsymbol{u}\boldsymbol{v}^T)(\boldsymbol{Z} + \boldsymbol{u}\boldsymbol{v}^T)^T = \boldsymbol{L}_{k+1} \boldsymbol{\Lambda}_{k+1} \boldsymbol{L}_{k+1}^T \tag{2}$$

So, how do we get there?
First, expand the LHS.

$$\mathbf{I} + (\boldsymbol{Z} + \boldsymbol{u}\boldsymbol{v}^T)(\boldsymbol{Z} + \boldsymbol{u}\boldsymbol{v}^T)^T = \mathbf{I} + \boldsymbol{Z}\boldsymbol{Z}^T + \boldsymbol{Z}\boldsymbol{v}\boldsymbol{u}^T + \boldsymbol{u}\boldsymbol{v}^T\boldsymbol{Z} + \boldsymbol{u}\boldsymbol{v}^T\boldsymbol{v}\boldsymbol{u}^T \tag{3}$$

$$\mathbf{I} + \boldsymbol{Z}\boldsymbol{Z}^T + \boldsymbol{Z}\boldsymbol{v}\boldsymbol{u}^T + \boldsymbol{u}\boldsymbol{v}^T\boldsymbol{Z} + \boldsymbol{u}\boldsymbol{v}^T\boldsymbol{v}\boldsymbol{u}^T = \boldsymbol{L}_{k+1} \boldsymbol{\Lambda}_{k+1} \boldsymbol{L}_{k+1}^T \tag{4}$$

Use $\boldsymbol{L}_k^{-1}$ and $(\boldsymbol{L}_k^T)^{-1}$ to get a $\boldsymbol{\Lambda}_k$ term on the LHS.

$$\boldsymbol{L}_k^{-1}(\mathbf{I}+\boldsymbol{Z}\boldsymbol{Z}^T)(\boldsymbol{L}_k^T)^{-1}+\boldsymbol{L}_k^{-1}(\boldsymbol{Z}\boldsymbol{v}\boldsymbol{u}^T+\boldsymbol{u}\boldsymbol{v}^T\boldsymbol{Z}+\boldsymbol{u}\boldsymbol{v}^T\boldsymbol{v}\boldsymbol{u}^T)(\boldsymbol{L}_k^T)^{-1} = \boldsymbol{L}_k^{-1}\boldsymbol{L}_{k+1}\boldsymbol{\Lambda}_{k+1}\boldsymbol{L}_{k+1}^T(\boldsymbol{L}_k^T)^{-1} \tag{5}$$

$$\boldsymbol{\Lambda}_k+\boldsymbol{L}_k^{-1}\boldsymbol{Z}\boldsymbol{v}(\boldsymbol{L}_k^{-1}\boldsymbol{u})^T+\boldsymbol{L}_k^{-1}\boldsymbol{u}(\boldsymbol{L}_k^{-1}\boldsymbol{Z}\boldsymbol{v})^T+\boldsymbol{v}^T\boldsymbol{v}\boldsymbol{L}_k^{-1}\boldsymbol{u}(\boldsymbol{L}_k^{-1}\boldsymbol{u})^T = \boldsymbol{L}_k^{-1}\boldsymbol{L}_{k+1}\boldsymbol{\Lambda}_{k+1}\boldsymbol{L}_{k+1}^T(\boldsymbol{L}_k^T)^{-1} \tag{6}$$

Compute LDLT factorization on LHS. Then, solve for the new factorization.