Consider the optimization problem:

$$\begin{aligned} \min_{\boldsymbol{x},\boldsymbol{y},\boldsymbol{z}} & \|\boldsymbol{y}\|_1 + \lambda\|\boldsymbol{z}\|_1 \\ \text{subject to } & \boldsymbol{z} - \boldsymbol{x} = 0 \\ & \boldsymbol{y} - (\boldsymbol{s} - \boldsymbol{D}\boldsymbol{x}) = 0 \end{aligned} \tag{1}$$

This optimization problem has the Lagrangian function:

$$\mathrm{L}(\boldsymbol{x},\boldsymbol{y},\boldsymbol{z},\boldsymbol{\gamma},\boldsymbol{\eta})\|\boldsymbol{y}\|_1 + \lambda\|\boldsymbol{z}\|_1 + \boldsymbol{\gamma}^H(\boldsymbol{z}-\boldsymbol{x}) + \boldsymbol{\eta}^H(\boldsymbol{y}-\boldsymbol{s}+\boldsymbol{D}\boldsymbol{x}) \tag{2}$$

Now, we can augment the Lagrangian. I will use the modification from the normalization document in my notes for one of the augmentation terms.

$$\mathrm{L}_{\rho,\boldsymbol{\Lambda}}(\boldsymbol{x},\boldsymbol{y},\boldsymbol{z},\boldsymbol{\eta},\boldsymbol{\gamma}) = \|\boldsymbol{y}\|_1+\lambda\|\boldsymbol{z}\|_1+\boldsymbol{\gamma}^H(\boldsymbol{z}-\boldsymbol{x})+\boldsymbol{\eta}^H(\boldsymbol{y}-\boldsymbol{s}+\boldsymbol{D}\boldsymbol{x})+\frac{\rho}{2}\|\boldsymbol{\Lambda}^{\frac{1}{2}}(\boldsymbol{z}-\boldsymbol{x})\|_2^2+\frac{\rho}{2}\|\boldsymbol{y}-\boldsymbol{s}+\boldsymbol{D}\boldsymbol{x}\|_2^2 \tag{3}$$

$$\nabla_{\boldsymbol{x}}\mathrm{L}_{\rho,\boldsymbol{\Lambda}}(\boldsymbol{x},\boldsymbol{y},\boldsymbol{z},\boldsymbol{\eta},\boldsymbol{\gamma}) = -\boldsymbol{\gamma}-\boldsymbol{D}^H\boldsymbol{\eta}+\rho\boldsymbol{\Lambda}\boldsymbol{x}-\rho\boldsymbol{\Lambda}\boldsymbol{z}+\rho\boldsymbol{D}^H\boldsymbol{D}\boldsymbol{x}-\rho\boldsymbol{D}^H(\boldsymbol{y}-\boldsymbol{s}) \tag{4}$$

$$\min_{\boldsymbol{x}}\mathrm{L}_{\rho,\boldsymbol{\Lambda}}(\boldsymbol{x},\boldsymbol{y},\boldsymbol{z},\boldsymbol{\eta},\boldsymbol{\gamma}) = (\rho\boldsymbol{\Lambda}+\rho\boldsymbol{D}^H\boldsymbol{D})^{-1}(\rho\boldsymbol{D}^H(\boldsymbol{y}-\boldsymbol{s}+\frac{\boldsymbol{\eta}}{\rho})+\rho\boldsymbol{\Lambda}\boldsymbol{z}+\boldsymbol{\gamma}) \tag{5}$$

$$\min_{\boldsymbol{x}}\mathrm{L}_{\rho,\boldsymbol{\Lambda}}(\boldsymbol{x},\boldsymbol{y},\boldsymbol{z},\boldsymbol{\eta},\boldsymbol{\gamma}) = (\boldsymbol{\Lambda}+\boldsymbol{D}^H\boldsymbol{D})^{-1}(\boldsymbol{D}^H(\boldsymbol{y}-\boldsymbol{s}+\frac{\boldsymbol{\eta}}{\rho})+\boldsymbol{\Lambda}\boldsymbol{z}+\frac{\boldsymbol{\gamma}}{\rho}) \tag{6}$$

$$\min_{\boldsymbol{y}}\mathrm{L}_{\rho,\boldsymbol{\Lambda}}(\boldsymbol{x},\boldsymbol{y},\boldsymbol{z},\boldsymbol{\eta},\boldsymbol{\gamma}) = \mathrm{S}_{\frac{1}{\rho}}(\boldsymbol{s}-\boldsymbol{D}\boldsymbol{x}-\frac{\boldsymbol{\eta}}{\rho}) \tag{7}$$

$$\min_{\boldsymbol{z}}\mathrm{L}_{\rho,\boldsymbol{\Lambda}}(\boldsymbol{x},\boldsymbol{y},\boldsymbol{z},\boldsymbol{\eta},\boldsymbol{\gamma}) = \mathrm{S}_{\frac{\lambda\boldsymbol{\Lambda}^{-1}}{\rho}}(\boldsymbol{x}-\frac{\boldsymbol{\Lambda}^{-1}\boldsymbol{\gamma}}{\rho}) \tag{8}$$

From this, we can get the update equations:

$$\boldsymbol{x}^{(k+1)} = (\boldsymbol{\Lambda}+\boldsymbol{D}^H\boldsymbol{D})^{-1}(\boldsymbol{D}^H(\boldsymbol{y}^{(k)}-\boldsymbol{s}+\frac{\boldsymbol{\eta}^{(k)}}{\rho})+\boldsymbol{\Lambda}\boldsymbol{z}^{(k)}+\frac{\boldsymbol{\gamma}^{(k)}}{\rho}) \tag{9}$$

$$\boldsymbol{y}^{(k+1)} = \mathrm{S}_{\frac{1}{\rho}}(\boldsymbol{s}-\boldsymbol{D}\boldsymbol{x}^{(k+1)}-\frac{\boldsymbol{\eta}^{(k)}}{\rho}) \tag{10}$$

$$\boldsymbol{z}^{(k+1)} = \mathrm{S}_{\frac{\lambda\boldsymbol{\Lambda}^{-1}}{\rho}}(\boldsymbol{x}^{(k+1)}-\frac{\boldsymbol{\Lambda}^{-1}\boldsymbol{\gamma}^{(k)}}{\rho}) \tag{11}$$

$$\boldsymbol{\gamma}^{(k+1)} = \boldsymbol{\gamma}^{(k)}+\rho\boldsymbol{\Lambda}(\boldsymbol{z}^{(k+1)}-\boldsymbol{x}^{(k+1)}) \tag{12}$$

$$\boldsymbol{\eta}^{(k+1)} = \boldsymbol{\eta}^{(k)}+\rho(\boldsymbol{y}^{(k+1)}-\boldsymbol{s}+\boldsymbol{D}\boldsymbol{x}^{(k+1)}) \tag{13}$$

For mask decoupling, only one change is needed:

$$\boldsymbol{y}^{(k+1)} = \begin{cases} \mathrm{S}_{\frac{1}{\rho}}(\boldsymbol{s} - \boldsymbol{D}\boldsymbol{x}^{(k+1)} - \frac{\boldsymbol{\eta}^{(k)}}{\rho}) & \text{inside signal domain} \\ \boldsymbol{s} - \boldsymbol{D}\boldsymbol{x}^{(k+1)} - \frac{\boldsymbol{\eta}^{(k)}}{\rho} & \text{outside signal domain} \end{cases} \tag{14}$$