APPENDIX A: DEFINING THE SPECTRAL INDEX FOR PHOTOMETRY AND SPECTROSCOPY

The number of photons arriving in a CCD per time and per area, after passing through a filter whose passband is $R_X(\lambda)$, is

$$S_X \equiv \int_0^\infty \frac{\lambda}{hc} f_\lambda R_X(\lambda) \mathrm{d}\lambda\,. \tag{A1}$$

In a Vega photometric system, the magnitude in the X band is defined by

$$X - X_{\mathrm{Vega}} \equiv -\frac{5}{2}\log\left(\frac{S_X}{S_{X,\mathrm{Vega}}}\right), \tag{A2}$$

where $S_{X,\mathrm{Vega}}$ is the photon flux of Vega in the band X and X_{Vega} is the arbitrarily defined magnitude of Vega in the same band.

Thus, the difference between two magnitudes taken at different filters, X and Y, of the same object (the color index) is given by

$$X - Y = C_{XY} - \frac{5}{2}\log\left(\frac{S_X}{S_Y}\right), \tag{A3}$$

where the constant C_{XY} is

$$C_{XY} = X_{\mathrm{Vega}} - Y_{\mathrm{Vega}} + \frac{5}{2}\log\left(\frac{S_{X,\mathrm{Vega}}}{S_{Y,\mathrm{Vega}}}\right). \tag{A4}$$

In this work, we also defined the integrated flux as

$$F_X \equiv \int_0^\infty f_\lambda R_X(\lambda)\,\mathrm{d}\lambda \tag{A5}$$

We have the approximate relations between S_X and F_X:

$$\lambda_X^* F_X \approx hcS_X \approx \lambda_X^* f_{\lambda_X^*} \Delta\lambda_X\,, \tag{A6}$$

where we used the commonly defined bandwidth, $\Delta\lambda_X$, and mean wavelength, λ_X^*, of the filter X, respectively defined by

$$\Delta\lambda_X \equiv \int_0^\infty R_X(\lambda)\mathrm{d}\lambda\,, \tag{A7}$$

and

$$\lambda_X^*\Delta\lambda_X \equiv \int_0^\infty \lambda R_X(\lambda)\mathrm{d}\lambda\,. \tag{A8}$$

In order to demonstrate the validity of Eq. (A6), we begin by Taylor-expanding λf_λ in Eq. (A1) around a certain λ_0. We obtain

$$\int_0^\infty \lambda f_\lambda R_X(\lambda)\mathrm{d}\lambda = \int_0^\infty \lambda_0 f_{\lambda_0} R_X(\lambda)\mathrm{d}\lambda + \int_0^\infty \left.\frac{\mathrm{d}\lambda f_\lambda}{\mathrm{d}\lambda}\right|_{\lambda_0} (\lambda - \lambda_0)\, R_X(\lambda)\mathrm{d}\lambda + \dots \tag{A9}$$

or, introducing the parameters λ_X^* and $\Delta\lambda_X$, we have

$$\int_0^\infty \lambda f_\lambda R_X(\lambda)\mathrm{d}\lambda = \lambda_0 f_{\lambda_0}\Delta\lambda_X + \left.\frac{\mathrm{d}\lambda f_\lambda}{\mathrm{d}\lambda}\right|_{\lambda_0} (\lambda_X^* - \lambda_0)\,\Delta\lambda_X + \dots\,, \tag{A10}$$

Now we see that, by choosing $\lambda_0 = \lambda_X^*$ in Eq. (A10) and ignoring second order terms, we have that $hcS_X \approx \lambda_X^* f_{\lambda_X^*}\Delta\lambda_X$ (one of the approximations of Eq. A6). We may, then, apply the same reasoning to Eq. (A5). We Taylor-expand λf_λ around λ_X^* and we find that $F_X \approx f_{\lambda_X^*}\Delta\lambda_X$ (the other approximation of Eq. A6). Thus, Eq. (A6) follows, in which the approximations are better the narrower the function $R_X(\lambda)$ is.

Table A1. Correspondency between WISE spectral indexes and WISE colors

α_{X-Y}	$W1-W2$ [mag]	$W2-W3$ [mag]	$W3-W4$ [mag]
−5.0	−0.764	−2.185	−1.536
−4.5	−0.588	−1.747	−1.123
−4.0	−0.413	−1.298	−0.720
−3.5	−0.239	−0.837	−0.328
−3.0	−0.066	−0.365	0.052
−2.5	0.107	0.118	0.421
−2.0	0.280	0.613	0.778
−1.5	0.452	1.119	1.124
−1.0	0.623	1.636	1.459
−0.5	0.794	2.163	1.784
0.0	0.965	2.700	2.100
0.5	1.135	3.245	2.407
1.0	1.305	3.800	2.706
1.5	1.475	4.361	2.997

A1 The spectral index

With no loss of generality, we can write λf_λ as

$$\lambda f_\lambda = A\lambda^{a_\lambda}\,, \tag{A11}$$

where A is a non-negative constant and a_λ is a function of λ.

Thus, the color index $X-Y$ relates with a_λ by

$$X - Y = C_{XY} - \frac{5}{2}\log\left(\frac{\int_0^\infty \lambda^{a_\lambda} R_X(\lambda)\mathrm{d}\lambda}{\int_0^\infty \lambda^{a_\lambda} R_Y(\lambda)\mathrm{d}\lambda}\right), \tag{A12}$$

from which we see that the constant A is unimportant.

Now, using the fact that $\lambda f_\lambda = A\lambda^{a_\lambda} = Ae^{a_\lambda \ln\lambda}$, it is straightforward to show that the spectral index α_λ is given by

$$\alpha_\lambda \equiv \frac{\partial \ln \lambda f_\lambda}{\partial \ln\lambda} = a_\lambda + \frac{\mathrm{d}a_\lambda}{\mathrm{d}\ln\lambda}\ln\lambda \tag{A13}$$

Hence, the approximation that λf_λ nearly follows a power-law around the domains of R_X and R_Y corresponds to assuming that

$$\left|\frac{\mathrm{d}a_\lambda}{\mathrm{d}\ln\lambda}\right|\ln\lambda \ll |a_\lambda|\,, \tag{A14}$$

for all λ's around that domain.

Thus, we define the spectral index α by the following equation

$$\frac{S_X}{S_Y} \equiv \frac{\int_0^\infty \lambda^\alpha R_X(\lambda)\mathrm{d}\lambda}{\int_0^\infty \lambda^\alpha R_Y(\lambda)\mathrm{d}\lambda}, \tag{A15}$$

from which we obtain the following implicit relation between the spectral index α and the color index $X-Y$:

$$X - Y = C_{XY} - \frac{5}{2}\log\left(\frac{\int_0^\infty \lambda^\alpha R_X(\lambda)\mathrm{d}\lambda}{\int_0^\infty \lambda^\alpha R_Y(\lambda)\mathrm{d}\lambda}\right), \tag{A16}$$

which is a very good approximation of Eq. (A12), in the situation that the power-law approximation (Eq. A14) is valid, like in the continuum portions of the visible and IR spectra of stars.

Table A1 gives the WISE color indexes that correspond to certain values of the spectral index α, defined by Eq. (A16)

By applying the approximation of Eq. (**??**), typically valid for narrow passbands, into the definition of α given by Eq. (A15), we may solve for α in that approximation, giving

$$\alpha \approx \frac{\frac{2}{5}(X-Y) - \left[\frac{2}{5}C_{XY} - \log\left(\frac{\Delta\lambda_X}{\Delta\lambda_Y}\right)\right]}{\log\left(\frac{\lambda_Y^*}{\lambda_X^*}\right)}. \tag{A17}$$

Thus, α is a linear transformation of $X - Y$. It gives a more intuitive way of comparing models and observations.

$$\frac{\lambda_X^* F_X}{\lambda_Y^* F_Y} \approx \frac{\left(\lambda_X^*\right)^\alpha \Delta\lambda_X}{\left(\lambda_Y^*\right)^\alpha \Delta\lambda_Y}, \tag{A18}$$

from which we solve for α as

$$\alpha \approx 1 - \frac{\log\left(\frac{F_X}{\Delta\lambda_X}\frac{\Delta\lambda_Y}{F_Y}\right)}{\log\left(\frac{\lambda_Y^*}{\lambda_X^*}\right)} \tag{A19}$$

Assuming a passband R_X to be equal to 1 in the domain $\lambda_1 \leq \lambda \leq \lambda_2$ and zero otherwise, we will have

$$\Delta\lambda_X = \lambda_{2X} - \lambda_{1X} \tag{A20}$$

and

$$\lambda_X^* = \frac{1}{2}(\lambda_{1X} + \lambda_{2X}) \tag{A21}$$

APPENDIX B: A SIMPLE LTE MODEL OF BE STARS

APPENDIX C: LIGHT CURVES OF THE SAMPLE STARS