

相变量作状态变量(无微分项)

$$y^{(n)}(t) + a_{n-1}y^{(n-1)}(t) + \dots + a_0y(t) = Cu$$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \vdots \\ \dot{x}_n \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & \dots & 0 \\ 0 & 0 & 1 & \dots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \dots & 1 \\ -a_0 & -a_1 & -a_2 & \dots & -a_{n-1} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ c_0 \end{bmatrix} u$$

$$\dot{x} = Ax + bu \quad \text{友矩阵}$$

$$y = cx \quad \text{能控标准型}(A, b, c)$$

(有微分项)  $\frac{Y}{U} = \frac{Y}{Z} \cdot \frac{Z}{U}$  (分子) \*  $\frac{Z}{U}$  (分母)

$$1y^{(n)}(t) + a_{n-1}y^{(n-1)}(t) + \dots + a_1y'(t) + a_0y(t) = CwU^{(n)}(t) + Cw_1U^{(n-1)}(t) + \dots + C_0U(t)$$

$$A = \frac{U}{s^n + a_{n-1}s^{n-1} + \dots + a_1s + a_0} \rightarrow \text{状态方程}$$

$$\begin{bmatrix} \dot{z}_1 \\ \dot{z}_2 \\ \vdots \\ \dot{z}_n \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & \dots & 0 \\ 0 & 0 & 1 & \dots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \dots & 1 \\ -a_0 & -a_1 & -a_2 & \dots & -a_{n-1} \end{bmatrix} \begin{bmatrix} z_1 \\ z_2 \\ \vdots \\ z_n \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix} U \quad (*)$$

$$Y(s) = \frac{C(sI - A)^{-1}bU(s)}{s^n + a_{n-1}s^{n-1} + \dots + a_1s + a_0} \rightarrow \text{输出方程}$$

$$y = [C_0 \ C_1 \ \dots \ C_{n-1} \ C_n] \begin{bmatrix} z_1 \\ z_2 \\ \vdots \\ z_n \end{bmatrix} \quad (*)$$

$$\text{或 } y = \frac{C(sI - A)^{-1}bU(s)}{s^n + a_{n-1}s^{n-1} + \dots + a_1s + a_0}$$

$$y = [C_0 \ C_1 \ C_2 \ \dots \ C_{n-1} \ C_n] \begin{bmatrix} z_1 \\ z_2 \\ \vdots \\ z_n \end{bmatrix} + C_n U$$

对角标准型

$$G(s) = \beta_n + \sum_{i=1}^n \beta_i \frac{1}{s - \lambda_i} = \frac{Y}{U}$$

$$Y(s) = \beta_n U + \sum_{i=1}^n \beta_i \frac{U}{s - \lambda_i} = \beta_n U + \sum_{i=1}^n \beta_i z_i$$

$$z_i = \frac{U}{s - \lambda_i} \Rightarrow \dot{z}_i = U + \lambda_i z_i$$

$$\dot{z} = \begin{bmatrix} \lambda_1 & 0 & \dots & 0 \\ 0 & \lambda_2 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & \lambda_n \end{bmatrix} z + \begin{bmatrix} 1 \\ 1 \\ \vdots \\ 1 \end{bmatrix} U$$

$$y = [f_1 \ f_2 \ \dots \ f_n] z + \beta_n U$$

能控标准型 同 (\*)

能观标准型

$$y = x_n + C_n u \quad \dot{x}_n = x_{n-1} - a_{n-1}y + C_n u + \dots$$

$$\dot{x} = \begin{bmatrix} 0 & 0 & \dots & -a_0 \\ 0 & 0 & \dots & -a_1 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & -a_{n-1} \end{bmatrix} x + \begin{bmatrix} C_0 - a_0 C_n \\ C_1 - a_1 C_n \\ \vdots \\ C_{n-1} - a_{n-1} C_n \end{bmatrix} u \quad (A_0, C_0)$$

$$y = [0 \ \dots \ 0 \ 1] x + C_n u \quad \text{能观标准型}$$

方块图  $\Leftrightarrow$  状态空间模型

关注每环节出来的  $x$  一条一条微分FC写

状态空间  $\Leftrightarrow$  传递HS

$$X(s) = (sI - A)^{-1} B U(s)$$

$$Y(s) = [C(sI - A)^{-1} B + D] U(s)$$

$$MIMO \rightarrow \begin{bmatrix} G_{11} & G_{12} \\ G_{21} & G_{22} \end{bmatrix} G = G_0 G_c$$

$$\text{机械系统 } f_m = MD\ddot{x} \quad f_k = Kx \quad f_b = B\dot{x} \quad \ddot{x} = \frac{1}{M} (F - B\dot{x} - Kx)$$

$$\text{液压系统 } \Delta p_{in} - \Delta p_{out} = A_1 \frac{dV_1}{dt} \quad \text{高升}$$

$$q_1 = \frac{h_1}{R_1} \quad q_{out} = \frac{h_2}{R_2} + q_f \quad \text{一阶+纯滞后}$$

$$\text{热力学 } q = CD(\theta_m - \theta_1) = \frac{1}{R} (\theta_m - \theta_1) \quad \text{Ts+1}$$

$$\text{直接蒸汽 } q_{in} = q_{out} + \frac{dq}{dt} \quad \theta_a \propto \theta_c$$

$$q_c C\theta_c + WH = q_a C\theta_a + \frac{dq}{dt} C$$

$$\text{纯滞后 } G = G_0(s) e^{-s} \quad \text{积分 } \text{输入无增量输出方波}$$

控制器的数学模型  $(K_c + \frac{K_i}{s} + K_d s)$

$$F(s) = \frac{K_c}{s} \frac{1}{(s - s_1)} + \frac{K_i}{s^2} \frac{1}{(s - s_1)} + \frac{K_d}{s} \frac{1}{(s - s_1)}$$

$$G_i = \frac{1}{(r-j)!} \lim_{s \rightarrow s_j} \frac{d^{(r-j)}}{ds^{(r-j)}} [ (s - s_j)^r F(s) ]$$

$$\cos \omega t \rightarrow \frac{s}{s^2 + \omega^2} \quad u(t) \rightarrow \frac{1}{s} \quad R \rightarrow \frac{R}{s^2} \quad R^2 \rightarrow \frac{2R}{s^3}$$

$$y(t) = y_{ss}(t) + y_t(t)$$

稳态特解 稳态分量

与输入同形式 非齐次通解

稳态响应 待定系数

暂态响应 单根, 重根, 共轭复根

$$\lambda_{k, k+1} = \pm j\omega_d \quad A_{k, k+1} \text{ 共振}$$

$$e^{st} (A_k e^{j\omega_d t} + A_{k+1} e^{-j\omega_d t}) \quad \zeta < 0 \text{ 稳}$$

$$= 2|A| e^{\zeta \omega_n t} \sin(\omega_d t + \phi)$$

$$= 2|A| e^{-\zeta \omega_n t} \sin(\omega_n \sqrt{1 - \zeta^2} t + \phi)$$

$$A_i = [(s - s_i) \frac{P(s)}{Q(s)}]_{s=s_i} \quad s_i: \text{+ 虚数的根}$$

$$\phi = A_i \text{ 角度} + 90^\circ$$

$$\text{阻尼比 } \zeta = \frac{a_1}{2\sqrt{a_0 a_2}}$$

$$\text{无阻尼振荡频率 (自然f)} \quad a_2 \lambda^2 + a_1 \lambda + a_0 = 0$$

$$\omega_n = \sqrt{\frac{a_0}{a_2}} \quad \lambda_{1,2} = \pm j\omega_n$$

$$\lambda^2 + 2\zeta \omega_n \lambda + \omega_n^2 = 0 \quad \lambda_{1,2} = -\zeta \omega_n \pm j\omega_n \sqrt{1 - \zeta^2}$$

$\zeta > 0$  稳定  $\zeta > 1$  过阻尼 (不共振)  $\zeta = 0$  临界稳定

$0 < \zeta < 1$  欠阻尼  $\zeta < 0$  不稳定

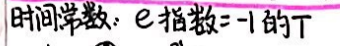
$$\text{重根: } Y(s) = \frac{A_{p1}}{(s - s_1)^p} + \frac{A_{p2}}{(s - s_2)^q} + \dots + \frac{A_{p1}}{s - s_1}$$

$$y(t) = \frac{1}{(p-1)!} A_{p1} t^{p-1} e^{s_1 t} + \frac{1}{(q-1)!} A_{p2} t^{q-1} e^{s_2 t} + \dots + A_{p1} e^{s_1 t}$$

$$A_{p1} = [(s - s_1)^p Y(s)]_{s=s_1} \quad A_{p1(p-k)} = \left[ \frac{1}{k!} \frac{d^k}{ds^k} \right]_{s=s_1}$$

$$A_{p1} = [(s - s_1)^p Y(s)]_{s=s_1} \quad [(s - s_1)^p Y(s)]_{s=s_1}$$

时间常数:  $e$  指数 = -1 的  $T$



$$\text{超调量 } \delta = \frac{y(T_p) - y(\infty)}{y(\infty)} \quad \text{衰减比 } n = \delta / \delta'$$

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高阶系统

- 直接解n阶微分FC - 转换为状态方程

- 二阶近似

- 一个不能忽略的零的影响: 向右速度 (微分)

- 极点: 调节时间 (衰减/阻尼作用)

$$\dot{x} = Ax + Bu$$

$$\Phi(t) = e^{At} (STM) \quad y = Cx + Du$$

$$= \sum_{k=0}^{\infty} \frac{(At)^k}{k!} \rightarrow \sum \frac{A^k}{s^{k+1}} \quad \text{带 } (-1)^{ij}$$

$$= L^{-1} [sI - A]^{-1}$$

$$\text{① 直接展开 } e^{At} = I + At + \dots \quad \frac{1}{s} \begin{bmatrix} \Delta_{11} & \Delta_{12} \\ \Delta_{21} & \Delta_{22} \end{bmatrix}$$

$$\text{② } L \Rightarrow \Phi(s) = [sI - A]^{-1}$$

$$\text{③ 对角化 } A = T \Lambda T^{-1} \quad \Phi(s) = T \exp(\Lambda t) T^{-1}$$

$$\text{若 } A_c \begin{bmatrix} 0 & 1 \\ -a_0 & -a_1 \end{bmatrix} \quad T = \begin{bmatrix} 1 & \dots & \lambda_n \\ \lambda_1 & \dots & \lambda_n \end{bmatrix}$$

$$\text{若非 } A_c \quad [\lambda_i I - A] v_i = 0 \quad T = [v_1 \ v_2]$$

$$\text{④ } e^{At} = \sum_{k=0}^{\infty} \alpha_k(t) A^k \quad (\text{seldom})$$

全解

$$\text{① } X(t) = \Phi(t-t_0) X(t_0) + \int_{t_0}^t \Phi(t-\tau) B U(\tau) d\tau, t \geq t_0$$

$$\text{② } L: X(s) = [sI - A]^{-1} X(0) + [sI - A]^{-1} B U(s)$$

$$X(t) = \Phi(t) X(0) + L^{-1} [\Phi(s) B U(s)]$$

STM性质

$$1. \Phi(t, t) = A \Phi(t, t) \text{ 且 } \Phi(t_0, t_0) = I$$

$$2. \Phi(t_1, t_2) \Phi(t_2, t_3) = \Phi(t_1, t_3) \quad \forall t_1, t_2, t_3$$

$$3. \Phi(t, t_0) = \Phi(t_0, t) \Rightarrow \Phi^{-1}(t) = \Phi(t, t_0)$$

$$4. \Phi(t_1, t_2) = \Phi(t_2, t_1) \quad \Phi(t_1, t_2) = \Phi(t_2, t_1)$$

$$5. \Phi(t) \text{ 为非奇异阵 (有限)}$$

$$\text{线性时不变 } \Phi(t) = e^{At} = \exp[At]$$

劳斯判据 必要非充分: ① 所有系数同号 ② 非0系数

充要: D1列符号无变化 [D1列变化次数: +R根个数]

$$s^n \mid a_n \ a_{n-2} \ \dots \ C_1 = -\frac{1}{a_{n-1}} \begin{vmatrix} a_n & a_{n-2} \\ a_{n-1} & a_{n-3} \end{vmatrix} \text{直到 } C \text{ 全0}$$

$$s^{n-1} \mid a_{n-1} \ a_{n-3} \ \dots \ C_2 = -\frac{1}{a_{n-1}} \begin{vmatrix} a_n & a_{n-3} \\ a_{n-1} & a_{n-4} \end{vmatrix}$$

$$s^{n-2} \mid a_{n-2} \ a_{n-4} \ \dots \ C_3 = -\frac{1}{a_{n-1}} \begin{vmatrix} a_n & a_{n-4} \\ a_{n-1} & a_{n-5} \end{vmatrix}$$

$$s^{n-3} \mid a_{n-3} \ a_{n-5} \ \dots \ C_4 = -\frac{1}{a_{n-1}} \begin{vmatrix} a_n & a_{n-5} \\ a_{n-1} & a_{n-6} \end{vmatrix}$$

$$s^{n-4} \mid a_{n-4} \ a_{n-6} \ \dots \ C_5 = -\frac{1}{a_{n-1}} \begin{vmatrix} a_n & a_{n-6} \\ a_{n-1} & a_{n-7} \end{vmatrix}$$

$$s^{n-5} \mid a_{n-5} \ a_{n-7} \ \dots \ C_6 = -\frac{1}{a_{n-1}} \begin{vmatrix} a_n & a_{n-7} \\ a_{n-1} & a_{n-8} \end{vmatrix}$$

$$s^{n-6} \mid a_{n-6} \ a_{n-8} \ \dots \ C_7 = -\frac{1}{a_{n-1}} \begin{vmatrix} a_n & a_{n-8} \\ a_{n-1} & a_{n-9} \end{vmatrix}$$

$$s^{n-7} \mid a_{n-7} \ a_{n-9} \ \dots \ C_8 = -\frac{1}{a_{n-1}} \begin{vmatrix} a_n & a_{n-9} \\ a_{n-1} & a_{n-10} \end{vmatrix}$$

$$s^{n-8} \mid a_{n-8} \ a_{n-10} \ \dots \ C_9 = -\frac{1}{a_{n-1}} \begin{vmatrix} a_n & a_{n-10} \\ a_{n-1} & a_{n-11} \end{vmatrix}$$

$$s^{n-9} \mid a_{n-9} \ a_{n-11} \ \dots \ C_{10} = -\frac{1}{a_{n-1}} \begin{vmatrix} a_n & a_{n-11} \\ a_{n-1} & a_{n-12} \end{vmatrix}$$

$$s^{n-10} \mid a_{n-10} \ a_{n-12} \ \dots \ C_{11} = -\frac{1}{a_{n-1}} \begin{vmatrix} a_n & a_{n-12} \\ a_{n-1} & a_{n-13} \end{vmatrix}$$

$$s^{n-11} \mid a_{n-11} \ a_{n-13} \ \dots \ C_{12} = -\frac{1}{a_{n-1}} \begin{vmatrix} a_n & a_{n-13} \\ a_{n-1} & a_{n-14} \end{vmatrix}$$

$$s^{n-12} \mid a_{n-12} \ a_{n-14} \ \dots \ C_{13} = -\frac{1}{a_{n-1}} \begin{vmatrix} a_n & a_{n-14} \\ a_{n-1} & a_{n-15} \end{vmatrix}$$

$$s^{n-13} \mid a_{n-13} \ a_{n-15} \ \dots \ C_{14} = -\frac{1}{a_{n-1}} \begin{vmatrix} a_n & a_{n-15} \\ a_{n-1} & a_{n-16} \end{vmatrix}$$

$$s^{n-14} \mid a_{n-14} \ a_{n-16} \ \dots \ C_{15} = -\frac{1}{a_{n-1}} \begin{vmatrix} a_n & a_{n-16} \\ a_{n-1} & a_{n-17} \end{vmatrix}$$

$$s^{n-15} \mid a_{n-15} \ a_{n-17} \ \dots \ C_{16} = -\frac{1}{a_{n-1}} \begin{vmatrix} a_n & a_{n-17} \\ a_{n-1} & a_{n-18} \end{vmatrix}$$

$$s^{n-16} \mid a_{n-16} \ a_{n-18} \ \dots \ C_{17} = -\frac{1}{a_{n-1}} \begin{vmatrix} a_n & a_{n-18} \\ a_{n-1} & a_{n-19} \end{vmatrix}$$

$$s^{n-17} \mid a_{n-17} \ a_{n-19} \ \dots \ C_{18} = -\frac{1}{a_{n-1}} \begin{vmatrix} a_n & a_{n-19} \\ a_{n-1} & a_{n-20} \end{vmatrix}$$

$$s^{n-18} \mid a_{n-18} \ a_{n-20} \ \dots \ C_{19} = -\frac{1}{a_{n-1}} \begin{vmatrix} a_n & a_{n-20} \\ a_{n-1} & a_{n-21} \end{vmatrix}$$

$$s^{n-19} \mid a_{n-19} \ a_{n-21} \ \dots \ C_{20} = -\frac{1}{a_{n-1}} \begin{vmatrix} a_n & a_{n-21} \\ a_{n-1} & a_{n-22} \end{vmatrix}$$

$$s^{n-20} \mid a_{n-20} \ a_{n-22} \ \dots \ C_{21} = -\frac{1}{a_{n-1}} \begin{vmatrix} a_n & a_{n-22} \\ a_{n-1} & a_{n-23} \end{vmatrix}$$

稳态误差  $e = r - y$

$$E(s) = Y(s) / G(s) \quad \text{稳态误差 } e_{ss}$$

$$e_{ss}(t) = \lim_{t \rightarrow \infty} (r(t) - y(t)) = \lim_{s \rightarrow 0} [s E(s)] = \lim_{s \rightarrow 0} [s Y(s) / G(s)]$$

$$s \rightarrow 0 \quad \frac{s Y(s)}{G(s)} = \frac{s Y(s)}{K_m} \quad \frac{s Y(s)}{K_m} = \frac{D^m Y(s)}{K_m}$$

$$\text{系统型别 } G(s) = \frac{K_m (1 + T_{s1}s + T_{s2}s^2 + \dots + T_{sn}s^n)}{s^m (1 + T_{d1}s + T_{d2}s^2 + \dots + T_{dn}s^n)}$$

$$E(s) = \frac{Y}{G} = \frac{R}{HG} \quad \text{稳态误差系数}$$

$$K_p = \lim_{s \rightarrow 0} G(s) \quad K_v = \lim_{s \rightarrow 0} sG(s) \quad K_a = \lim_{s \rightarrow 0} s^2 G(s)$$

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