1.

Exercise

Let $a_n = \frac{\sin(\frac{1}{n})}{n}$, starting from $n = 1$ Is a_n monotone? Is it convergent / divergent?

Direct solution

$$\begin{aligned}\lim_{n\to\infty} a_n &= \lim\left(\sin\left(\frac{1}{n}\right)\right)\frac{1}{\lim n} \\ &= \lim\left(\sin\left(\frac{1}{n}\right)\right)\lim\left(\frac{1}{n}\right) \\ &= \lim\left(\sin\left(\frac{1}{n}\right)\right)0 \\ &= 0\end{aligned}$$

Alternative solution using sqeeze theorem

Since

$$-\frac{1}{n} \leq \frac{\sin(\frac{1}{n})}{n} \leq \frac{1}{n}$$

trivially holds, and $\lim \frac{1}{n} = \lim -\frac{1}{n} = 0$, we have $\lim a_n = 0$

2a

Exercise

Consider the sequence $a_n = \frac{n}{4n-1}$. Is it monotone? Is it convergent / divergent?

Through intuition:

- monotonely increasing
- diverges to infinity

Squeeze theorem

$$a_n = n\left(\frac{1}{4n-1}\right)$$

We can factor out the highest degree (n^1)

$$a_n = \frac{1}{4 - \frac{1}{n}}$$

as $\frac{1}{n} \overset{n\to\infty}{\longrightarrow} 0$, we have

$$a_n = \frac{1}{4-0} = \frac{1}{4}$$