Estimating the polarization error of the IAGPOL ETC

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December 15, 2018

This document describes the expressions and the values used in Exposure Time Calculator (ETC) of the instrument IAGPOL (Magalhães et al., 1996).

The relation between the polarization error, σ_P, and signal to noise ratio, SNR, is given by:

$$\sigma_P = K\,100\,\frac{1}{\sqrt{N_{WP}}}\frac{1}{SNR}, \tag{1}$$

where the polarization is expressed in percentage, N_{WP} is the number of waveplate positions used in the measurement and K is a constant that depends on the type of waveplate: K = 1, for half-wave plate and $K = \sqrt{2}$, for quarter-wave plate.

We adopted the following expression for SNR (Howell et al., 2006):

$$SNR = \frac{N t_{exp}}{\sqrt{N t_{exp} + 2 n_{pix}\left[N_s t_{exp} + B\,N_R^2 + (0.289G)^2\right]}}, \tag{2}$$

where

- N is the number of source photons per second;
- t_{exp} is the exposure time;
- N_s is the number of sky photons per second per pixel;
- n_{pix} is the number of pixels (after binning) used in the aperture photometry;
- N_R is the readout noise;
- B is related to the binning. Specifically, it represents a $B \times B$ binning. See this link;
- G is the gain.

To calculate n_{pix}, we need to consider the aperture radius to be used in the aperture photometry extraction:

$$n_{pix} = \pi \left(\frac{R_{ap}}{B\, P_{scale}} \right)^2 , \tag{3}$$

where

- R_{ap} is the aperture radius in arcsec;
- P_{scale} is the plate scale in arcsec/pixel without binning. The binning effect is taken into account by the B factor.

The expression to calculate the photon number from the source is presented below as well as the variables.

$$N = f_{calib}\, f_{area}\, t_{tel}\, t_{instr}\, C\, Q\, t_{sky} f_0 \frac{\Delta\lambda}{\lambda_{eff}} d_{tel}^2 10^{-0.4m}. \tag{4}$$

- f_{calib} is a factor to correct possible difference between this ETC results and the real measurements. In the present version, it is set as 1.0.
- f_{area} is the fraction of the telescope area that effectively collects photons. The Perkin & Elmer 1.6-m telescope has f_{area} = 0.804. We adopt the same value for the IAG 60cm telescope, but we are not sure about it.
- t_{tel} is the transmission of the telescope surface. It depends on the wavelength. We assume t_{tel} = 0.85 for the two telescopes and for all bands.
- t_{filter} is the transmission of the filter. See curve "old" in Figure 1. The values are presented in Table 1.
- t_{instr} is the transmission in the instrument. We adopt t_{instr} = 0.95 without focal reducer and t_{instr} = 0.90 with the focal reducer.
- C is a constant that takes into account all constants and units conversion. $C = 1.1853\, 10^{10}$.
- Q is the quantum efficiency of the detector.
- t_{sky} is the sky transparence.
- f_0 is the flux of a zero magnitude source (see below).
- $\Delta\lambda$ is the filter width.
- λ_{eff} is the effective wavelength of the filter.
- d_{tel} is the telescope diameter (in meters).

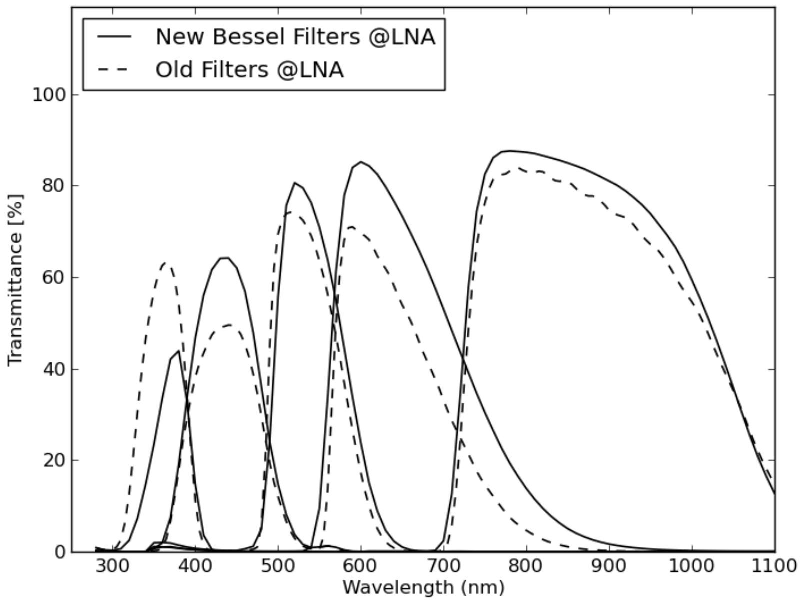

Figure 1: Transmission curves of the filters used at LNA. The IAGPOL instrument uses the "old" filters.

- m is the source magnitude.

The expression to calculate the photon number from the sky (per second and per pixel) is:

$$N_{sky} = f_{calib}\, f_{area}\, t_{tel}\, t_{instr}\, C\, Q\, t_{sky}\, f_0\, d_{tel}^2 \frac{\Delta\lambda}{\lambda_{eff}} (B\, P_{scale})^2\, 10^{-0.4 m_{sky}}, \quad (5)$$

where

- m_{sky} is the sky magnitude (in mag arcsec^2);
- and the remaining variables are the same from the previous equations.

To characterize the UBVRI bands, we adopt λ_{eff}, $\Delta\lambda$, and f_0 presented in Table 1 from Bessell (1979, 2005).

The user can choose the CCD and its operation mode from the many options available at OPD with one click. From this choice, the ETC picks up the correct value of G, N_R, and P_{scale}. The last also takes into account whether the focal reducer is used or not. The value of Q is selected considering the chosen CCD and filter.

Table 2 shows the sky magnitude for OPD from Dias et al. (2010)[1]. This paper presents an average value of the sky magnitudes in each band, but it does not present them as a

[1] https://sab-astro.org.br/wp-content/uploads/2018/10/Paper5.pdf

Table 1: Filters properties.

Filter	λ_{eff} (nm)	$\Delta\lambda$ (nm)	f_0 (10^{-23} Wm^{-2}Hz^{-1})	t_{filter}
U	366.3	65	1.81	0.65
B	436.1	89	4.26	0.50
V	544.8	84	3.64	0.75
R	640.7	158	3.08	0.70
I	798.0	154	2.55	0.82

function of the Moon phase. In order to take into account the Moon phase, we consider three values for m_{sky}: new Moon, quarter and full Moon. We consider that Dias et al. (2010)'s values correspond to quarter Moon. The difference for other Moon-s phase are from Walker (1987): these values can also be found in `https://www.noao.edu/noao/noaonews/mar94/art20.html`. We assume quarter Moon as 7 days from New Moon and Full, as 14. We consider that m_{sky} is not a function of the object and the Moon.

Table 2: Adopted sky magnitude used in the IAGPOL ETC from Dias et al. (2010) and Walker (1987). See text for details.

Filter	m_{sky}		
	New Moon	Quarter Moon	Full Moon
U	20.2	18.1	15.2
B	20.9	19.8	17.7
V	20.5	20.1	18.7
R	20.7	20.4	19.7
I	20.3	20.1	19.6

The sky transparence can be calculate by the following expression:

$$t_{sky} = 10^{-0.4\,k\,\chi}, \tag{6}$$

where k is the extinction coefficient and χ is the air mass.

Dias et al. (2010) also provide the OPD extinction coefficients for some nights. We select the minimum and maximum values (k$_{min}$ and k$_{max}$, respectively) for each filter and calculate the average value, k$_{ave}$ (see Table 4). These three values are used to calculate the sky transparence for photometric, good and regular conditions, respectively. This sky quality is an user choice. The air mass is also an input from the user.

References

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Table 3: Variation of m_{sky} with Moon's phases from Walker (1987).

Days from new Moon	U	B	V	R	I
3	-0.5	-0.3	-0.1	-0.1	0
7	-2.1	-1.1	-0.4	-0.3	-0.2
10	-3.5	-2	-1.1	-0.6	-0.4
14	-5	-3.2	-1.8	-1	-0.7

Table 4: Sky transparence from Dias et al. (2010).

Filter	k_{min}	k_{ave}	k_{max}
U	0.344	0.5135	0.683
B	0.195	0.277	0.359
V	0.113	0.1655	0.218
R	0.03	0.0975	0.165
I	0.01	0.0665	0.123

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