$$-G_i^j = C_{ik}^j \delta x_k^{j-1} + D_{il}^j \delta x_l^j + E_{im}^j \delta x_m^{j+1}$$

$\hookrightarrow$ Assume (why?!)

$$\delta x_k^{j-1} = A_k^{j-1} + B_{kl}^{j-1} \delta x_l^j$$

$$-G_i^j = C_{ik}^j \left(A_k^{j-1} + B_{kl}^{j-1} \delta x_l^j\right) + D_{il} \delta x_l^j + E_{im}^j \delta x_m^{j+1}$$

$$= C_{ik}^j A_k^{j-1} + C_{ik}^j B_{kl}^{j-1} \delta x_l^j + D_{il} \delta x_l^j + E_{im}^j \delta x_m^{j+1}$$

$$\boxed{-G_i^j} = \boxed{C_{ik}^j}\boxed{A_k^{j-1}} + \left(\boxed{C_{ik}^j}\boxed{B_{kl}^{j-1}} + \boxed{D_{il}^j}\right)\boxed{\delta x_l^j} + \boxed{E_{im}^j}\boxed{\delta x_m^{j+1}}$$

= value is known

= ... profit? (Intermediate vars. Values = currently unknown?)

= values unknown, but want to solve for them eventually

$$S_{il}^j \equiv C_{ik}^j B_{kl}^{j-1} + D_{il}^j$$ ← Basically, introduce a short-hand notation.

$$-G_i^j = C_{ik}^j A_k^{j-1} + S_{il}^j \delta x_l^j + E_{im}^j \delta x_m^{j+1}$$

$$-S_{il}^j \delta x_l^j = G_i^j + C_{ik}^j A_k^{j-1} + E_{im}^j \delta x_m^{j+1}$$

$$\delta x_l^j = -\left(S_{il}^j\right)^{-1} \left(G_i^j + C_{ik}^j A_k^{j-1} + E_{im}^j \delta x_m^{j+1}\right)$$ (← eqn. *)

Now, before, we'd postulated that the corrections at cell # j could be expressed in terms of their δx^{j+1} neighbors thusly:

$$\delta x_k^{j-1} = \underbrace{A_k^{j-1}}_{\substack{\text{constants} \\ \text{"unique" to} \\ \text{that } j-1 \text{ cell}}} + \underbrace{B_{kl}^{j-1}\, \delta x_k^j}_{\substack{\text{Some factor} \\ \text{times the} \\ \text{variables next} \\ \text{door at } j.}}$$

If you look at Eqn. "*", you can see (after a second...) that it has a similar form:

$$\delta x_l^j = -\left[S_{il}^j\right]^{-1} \left\{ \underbrace{G_i^j + C_{ik}^j A_k^{j-1}}_{\text{"constants"}} + \underbrace{E_{im}^j\, \delta x_m^{j+1}}_{\substack{\text{a factor times} \\ \text{the variable next} \\ \text{door at } j+1}} \right\}$$

So, we can call the "constants" A_k^j, and the extra factor-thing "B_{lm}^j", and re-format the eqn. above like this:

$$\cancel{\delta x_l^j = -[}$$

$$\delta x_l^j = A_l^j + B_{lm}^j\, \delta_m^{j+1}$$

where

$$A_l^j \equiv -\left[S_{il}^j\right]^{-1} \left\{G_i^j + C_{ik}^j A_k^{j-1}\right\}$$

← oh God, it's a recursive relation. :-(. "Simplifying" the eqns my <u>foot</u>...

$$B_{lm}^j \equiv -\left[S_{il}^j\right]^{-1} \left\{E_{im}^j\right\}$$

Although we've stuck w/ a recursive (ugh!) relation for A_l^j, we're not totally screwed b/c you <u>do</u> know all of the $C^{j=0}$ values: zero. (Right? You can't calculated $\frac{\partial f}{\partial x}$ where the $\frac{\partial}{\partial x}$ element is outside the range of your function. Or in other words, there <u>is</u> no $\delta x^{j=-1}$!)
So, this means:

$$A_l^{j=0} = -\left[S_{il}^0\right]^{-1}\left\{G_i^0 + \cancelto{0}{\emptyset A^{-1}}\right\}$$

$$A_l^{j=0} = -\left[S_{il}^0\right]^{-1} G_i^0$$

$$= -\left[\cancel{C_{ik}^{j=0}}^{\,0}\,\cancel{B_{kl}^{j=-1}} + D_{il}^{j=0}\right]^{-1} G_i^0$$

$$A_l^{j=0} = -\left[D_{il}^{j=0}\right]^{-1} G_i^0$$

and we <u>do</u> already know what all of these values are.

To calculate all of the rest of the A & B matrix values at the remaining mass cells, you need to know A^{j-1} & B^{j-1} at each cell # j.

So, starting fr. $j=0$, where A & B are purely functs. of <u>known</u> C, D, E, & G values, we can forward-substitute to calculate all of the A & B "coefficients" we need in order to actually calculate all of the δx^j values.

Now, remember that we've postulated that:

$$\delta x_l^{j} = A_l^j + B_{lm}^j \, \delta x_l^{j+1}$$

At the outer boundary, $\delta x_l^{j+1} = 0$ (or more accurately, doesn't really... exist.) That means

$$\delta x_l^J \equiv A_l^J$$ ← one eqn & one unknown → we can solve this.

In fact, it's kind of already solved itself...

$$\cancel{\delta x_l^{J-1} = A_l^{J-1} + B_{lm}^{J-1}(A_l^J)}$$

Error propagation in this process

↳ or, how ~~can~~ can/do things go wrong?

→ You need to have the correct A matrix values @ the outer boundary in order for all the δx values to be right as you do that inward ~~calc~~ moving calculation.

A values depend on the G values.

B values depend on (C, B, D, E) values.

→ If the G values are off, they'll effect the A values (and thus the δx correction values), but not the B matrix values, nor the S (& S^{-1}) matrix values.

→ Errors in the G-values will propagate forward thru the A matrix value calculations. So, if $G^{j=0}$ is wrong, all of your A values will end up being wrong, too.