6/10/2012

0

$$\frac{dG}{dP} = \frac{G_1 - G_2}{P_1 - P_2}$$

$$dP = \frac{-G_1(P_1 - P_2)}{G_1 - G_2}$$

$$dM = -\frac{4\pi r^2}{\kappa} d\tau$$

$$dP = \frac{GM}{\kappa r^2} d\tau$$

$$dr = -\frac{1}{\rho\kappa} d\tau$$

$$dT = \frac{GMT\nabla}{r^2 \kappa P} d\tau$$

$$\boxed{\nabla_{ad} = \frac{\delta P}{c_P T \rho}}$$

$$\nabla_{rad} = \frac{3\kappa L P}{16\pi G a c M T^4}$$

$$\boxed{\nabla_{rad} = \frac{3}{16\pi G a c} \frac{\kappa L P}{M T^4}}$$

$$(P_1 - P_{out})\kappa - GD = 0$$

$$P_1 = P_{out} + \frac{GD}{\kappa}$$

$$dP_1 = P_1 - P_{out} = \frac{GD}{\kappa}$$

$$dP_1 = \frac{GD}{\kappa}$$

$$g\,d\tau = \kappa\,dP = GD$$

$$GD = d\tau \frac{G M}{R^2} = g\,d\tau$$

(1) 6/10/2012

$$(1)\quad dP = \frac{d\tau}{\kappa}\frac{GM_{atm}}{R_{atm}^2} \Rightarrow \frac{dP}{d\tau} = \frac{GM_{atm}}{\kappa R_{atm}^2}$$

$$(2)\quad dR = \frac{d\tau}{\kappa}\cdot\frac{1}{\rho} \rightarrow \frac{dR}{d\tau} = \frac{1}{\kappa\rho}$$

$$(3)\quad dM = \frac{d\tau}{\kappa}\left(4\pi R_{atm}^2\right) \longrightarrow \frac{dM}{d\tau} = \frac{4\pi R_{atm}^2}{\kappa}$$

$$(4)\quad dT = \frac{GM_{atm}}{R_{atm}^2}\nabla\frac{d\tau}{\kappa}\frac{T}{P} \longrightarrow \frac{dT}{d\tau} = \frac{GM_{atm}\nabla T}{R_{atm}^2\kappa P}$$

$$(5)\quad d\tau = -\kappa\rho\, dR \rightarrow dR = \frac{d\tau}{-\kappa\rho}$$

$$(6)\quad \frac{dr}{dM} = \frac{1}{4\pi r^2\rho} \Rightarrow \cancel{dr = \frac{dM}{4\pi r^2\rho}}$$

$$\cancel{(7)\quad [(5)+(6)]\quad dr = \frac{dM}{4\pi r^2\rho} = \frac{d\tau}{-\kappa\rho}}$$

$$(7)\quad dM = 4\pi r^2\rho\, dR$$

$$= 4\pi r^2\cancel{\rho}\,\frac{d\tau}{-\kappa\cancel{\rho}}$$

$$\boxed{dM = -\frac{4\pi r^2 d\tau}{\kappa}}$$ ← matches (3), except for the sign.

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(8) $$\frac{1}{\rho}\frac{dP}{dr} = -\frac{GM}{r^2}$$

$$\frac{dP}{dr} = -\frac{GM\rho}{r^2} \cancel{\frac{GM\rho}{\kappa r^2 \rho}}$$

$$dP = -\frac{GM\rho}{r^2}\,dr$$

$$= \cancel{+}\frac{GM\cancel{\rho}}{r^2}\left(\frac{d\tau}{\cancel{+}\kappa\cancel{\rho}}\right)$$

$$\boxed{dP = \frac{GM}{\kappa R^2}\,d\tau}$$ ← matches (1).

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(9) $d\tau \equiv^{-} dr \cdot \rho \cdot \kappa$ ← fr. the definition of ~~opit~~ optical depth.

$$\boxed{dR = \frac{-d\tau}{\rho\kappa}}$$ ← matches (2), except for the sign.

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(10) $$\frac{dT}{dM} = \frac{-GMT\nabla}{4\pi r^4 P}$$

$$dT = \frac{-GMT\nabla}{4\pi r^4 P}\,dM$$

$$= \frac{+GMT\nabla}{\cancel{4\pi r^4}\,r^2 P}\left(\frac{-\cancel{4\pi r^2}\,d\tau}{\kappa}\right)$$

$$\boxed{dT = \frac{GMT\,d\tau\,\nabla}{r^2 P\kappa}}$$ ← matches (4)