G4J Manifesto

August 24, 2012

In the process of debugging my code, I've discovered that many (though possibly not all) of the differences between my code's results and Peter's stem from the way that the G4J values are calculated.

Equation (5.42) from Peter's cookbook [CITE] formulates the discretized version of the radiative transfer equation as:

$$T_{j+1} - T_j + \frac{(M_{j+1/2} - M_{j-1/2})GM_j}{4\pi r_j^4} 0.5(B_{j+1} + B_j) = 0 \tag{1}$$

where

$$B_j \equiv \frac{T_j}{P_j}\nabla \tag{2}$$

Here, ∇ represents the radiative/convective gradient value, and is defined as

$$\nabla = \frac{\partial \ln P}{\partial \ln T} = \frac{P}{T}\frac{\partial T}{\partial P} \tag{3}$$

Equation (1) in its pre-discretized form reads as

$$\frac{dT}{dM} = -\frac{GMT}{4\pi r^4 P}\nabla \tag{4}$$

This is, however, not the original form of the radiative transfer equation. Several assumptions and substitutions have transformed the original expression

$$\frac{\partial T}{\partial M} = -\frac{3}{64\pi^2 ac}\frac{\kappa L}{r^4 T^3} \tag{5}$$

to the form adopted by equation (4). I will go through those steps in detail, here. First,

$$\frac{\partial T}{\partial M} = \frac{\partial T}{\partial P}\frac{\partial P}{\partial M} = \frac{T}{P}\frac{\partial \ln P}{\partial \ln T}\frac{\partial P}{\partial M} \tag{6}$$

Substituting (5) into (6) produces

$$\frac{\partial T}{\partial M} = \frac{T}{P}\nabla\frac{\partial P}{\partial M} \tag{7}$$

If (and only if!) the system is in hydrostatic equilibrium, the following is true:

$$\frac{\partial P}{\partial M} = -\frac{GM}{4\pi r^4} \tag{8}$$

If the assumption of HSE holds, we could then combine (8) and (7) to get

$$\frac{\partial T}{\partial M} = -\frac{T}{P}\frac{GM}{4\pi r^4}\nabla \tag{9}$$

which Peter's cookbook uses as the basis for his equation (5.42).

The misleading thing about (5.42), however, is that it assumes your system is already in HSE. If you're cold-starting a Henyey code (e.g. from an n=3/2 polytropic model), that assumption is invalid. Your initial model will not, necessarily, be anywhere near HSE. The Henyey code cannot assume initial HSE; that is what it's trying to find, rather than what it's starting from.

This is reflected in Peter's Fortran code, if not in his equation (5.42). His code uses (7), rather than (9) to calculate the G4J values (i.e., a quantitative measure how far out of radiative transport equilibrium the model currently is). Because

$$\frac{\partial P}{\partial M} \neq -\frac{GM}{4\pi r^4} \tag{10}$$

when you're cold-starting a model, using (9) leads to wildly wrong G4J values. These incorrect G4J values, in turn, get carried into the CDE calculations, and end up wreaking havoc with the model's march towards convergence.