Listings

Listing 1: **CODE HASHES**

Listing 2: hash.sh

```
# Hashes a file, ignoring all whitespace, comments, and includes. Use for
# verifying that code was correctly typed.

# usage:
#   chmod u+x hash.sh
#   cat <file> | ./hash.sh
# or just copy this command:
#   cat <file> | sed -r '/^#(include|pragma)/d' | cpp -dD -P -fpreprocessed -MM | tr -d
    ↪ '[:space:]' | md5sum | cut -c-6
sed -r '/^#(include|pragma)/d' | cpp -dD -P -fpreprocessed | tr -d '[:space:]' | md5sum
    ↪ | cut -c-6
```

Listing 3: **GRAPHS**

```
% cat graphs/bridges_and_cuts.h | ./misc/hash.sh
```

```
6b0c62
```

Listing 4: Bridges and Cuts

```
#pragma once

//tested on https://judge.yosupo.jp/problem/two_edge_connected_components and
    ↪ https://judge.yosupo.jp/problem/biconnected_components
//with asserts checking correctness of isBridge and isCut
//2 edge cc and bcc stuff doesn't depend on each other, so delete whatever is not needed

//To initialize `adj`:
//eid = 0
//for each edge (a,b):
//  adj[a].emplace_back(b, eid);
//  adj[b].emplace_back(a, eid++);

struct info {
    //2 edge connected component stuff (e.g. components split by bridge edges)
        ↪ https://cp-algorithms.com/graph/bridge-searching.html
    int num2EdgeCCs;
    vector<bool> isBridge;//edge id -> true iff bridge edge
    vector<int> TwoEdgeCCID;//node -> ID of 2-edge component (which are labeled 0, 1,
        ↪ ..., `num2EdgeCCs`-1)

    //bi-connected component stuff (e.g. components split by cut/articulation nodes)
        ↪ https://cp-algorithms.com/graph/cutpoints.html
    int numBCCs;
    vector<bool> isCut;//node -> true iff cut node
    vector<int> bccID;//edge id -> ID of BCC (which are labeled 0, 1, ..., `numBCCs`-1)
};

info bridge_and_cut(const vector<vector<pair<int/*neighbor*/, int/*edge id*/>>>&
    ↪ adj/*undirected graph*/, int m/*number of edges*/) {
    //stuff for both (always keep)
    int n = adj.size(), timer = 1;
```

```
    vector<int> tin(n, 0), low(n, 0);
    //2 edge CC stuff (delete if not needed)
    int num2EdgeCCs = 0;
    vector<bool> isBridge(m, false);
    vector<int> TwoEdgeCCID(n), nodeStack;
    //BCC stuff (delete if not needed)
    int numBCCs = 0;
    vector<bool> isCut(n, false);
    vector<int> bccID(m), edgeStack;
    auto dfs = [&](auto&& dfsPtr, int v, int pId) -> void {
        tin[v] = low[v] = timer++;
        int deg = 0;
        nodeStack.push_back(v);
        for (auto [to, eId] : adj[v]) {
            if (eId == pId) continue;
            if (!tin[to]) {
                edgeStack.push_back(eId);
                dfsPtr(dfsPtr, to, eId);
                if (low[to] >= tin[v]) {
                    isCut[v] = true;
                    while (true) {
                        int edge = edgeStack.back();
                        edgeStack.pop_back();
                        bccID[edge] = numBCCs;
                        if (edge == eId) break;
                    }
                    numBCCs++;
                }
                low[v] = min(low[v], low[to]);
                deg++;
            } else if (tin[to] < tin[v]) {
                edgeStack.push_back(eId);
                low[v] = min(low[v], tin[to]);
            }
        }
        if (pId == -1) isCut[v] = (deg > 1);
        if (tin[v] == low[v]) {
            if (pId != -1) isBridge[pId] = true;
            while (true) {
                int node = nodeStack.back();
                nodeStack.pop_back();
                TwoEdgeCCID[node] = num2EdgeCCs;
                if (node == v) break;
            }
            num2EdgeCCs++;
        }
    };
    for (int i = 0; i < n; i++) {
        if (!tin[i])
            dfs(dfs, i, -1);
    }
    return {num2EdgeCCs, isBridge, TwoEdgeCCID, numBCCs, isCut, bccID};
}
```

```
% cat graphs/centroid.h | ./misc/hash.sh
```

```
11398c
```

Listing 5: Centroid

```
#pragma once

//status: not tested

const int Max = 2e5 + 2;
vector<int> adj[Max];
int sizes[Max], parent[Max];
bool removed[Max];

void dfs2(int node, int par) {
    sizes[node] = 1;
    for (int to : adj[node]) {
        if (to != par && !removed[to]) {
            dfs2(to, node);
            sizes[node] += sizes[to];
        }
    }
}

int findCentroid(int node) {
    dfs2(node, node);
    bool found = true;
    int sizeCap = sizes[node] / 2;
    int par = node;
    while (found) {
        found = false;
        for (int to : adj[node]) {
            if (to != par && !removed[to] && sizes[to] > sizeCap) {
                found = true;
                par = node;
                node = to;
                break;
            }
        }
    }
    return node;
}

void dfs1(int node, int par) {
    removed[node] = true;
    parent[node] = par;
    for (int to : adj[node]) {
        if (!removed[to])
            dfs1(findCentroid(to), node);
    }
}
//dfs1(findCentroid(1), 0);
```

```
% cat graphs/countPathLengths.h | ./misc/hash.sh
```

```
470b6f
```

Listing 6: Count Path Lengths

```
#pragma once

//status: doesn't compile, but should be correct. Need to import FFT code (like
    ↪ https://github.com/kth-competitive-programming/kactl/blob/main/content/numerical/Fas
```

```
const int Max = 1e6 + 10;
int n, sizes[Max];
vector<int> adj[Max], cntPathLength[Max];
ll cntTotalPathLengths[Max];
bool removed[Max];

void dfs2(int node, int par, int root, int currDist) {
    while ((int)cntPathLength[root].size() <= currDist)
        cntPathLength[root].push_back(0);
    cntPathLength[root][currDist]++;
    sizes[node] = 1;
    for (int to : adj[node]) {
        if (to != par && !removed[to]) {
            dfs2(to, node, root, currDist + 1);
            sizes[node] += sizes[to];
        }
    }
}

int findCentroid(int node) {
    dfs2(node, node, node, 1);
    bool found = true;
    int sizeCap = sizes[node] / 2;
    int par = node;
    while (found) {
        found = false;
        for (int to : adj[node]) {
            if (to != par && !removed[to] && sizes[to] > sizeCap) {
                found = true;
                par = node;
                node = to;
                break;
            }
        }
    }
    return node;
}

void dfs1(int node, int par) {
    removed[node] = true;
    int maxLength = 1;
    for (int to : adj[node]) {
        if (to != par && !removed[to]) {
            cntPathLength[to].clear();
            cntPathLength[to].push_back(0);
            dfs2(to, to, to, 1);
            maxLength = max(maxLength, (int)cntPathLength[to].size());
        }
    }
    vector<int> temp(maxLength, 0);
    temp[0]++;
    for (int to : adj[node]) {
        if (to != par && !removed[to]) {
            vector<ll> prod = multiply(temp, cntPathLength[to]);
            for (int i = 0; i < (int)prod.size(); ++i)
                cntTotalPathLengths[i] += prod[i];
            for (int i = 0; i < (int)cntPathLength[to].size(); ++i)
                temp[i] += cntPathLength[to][i];
        }
    }
    for (int to : adj[node]) {
        if (to != par && !removed[to])
            dfs1(findCentroid(to), node);
    }
}
```

```
% cat graphs/disjointSet.h | ./misc/hash.sh
```

```
8369d6
```

Listing 7: Disjoint Set

```
#pragma once

//status: tested on random inputs, and on https://judge.yosupo.jp/problem/unionfind

struct disjointSet {
    int numSets;
    vector<int> par;
    disjointSet(int n) : numSets(n), par(n, -1) {}
    disjointSet(const disjointSet& rhs) : numSets(rhs.numSets), par(rhs.par) {}
    int find(int x) {
        return par[x] < 0 ? x : par[x] = find(par[x]);
    }
    int sizeOfSet(int x) {
        return -par[find(x)];
    }
    bool merge(int x, int y) {
        if ((x = find(x)) == (y = find(y))) return false;
        if (par[y] < par[x]) swap(x, y);
        par[x] += par[y];
        par[y] = x;
        numSets--;
        return true;
    }
};
```

```
% cat graphs/dijkstra.h | ./misc/hash.sh
```

```
27560a
```

Listing 8: Dijkstra

```
#pragma once

//returns array `len` where `len[i]` = shortest path from node `startNode` to node i
//For example len[startNode] will always = 0
//
//status: tested on https://judge.yosupo.jp/problem/shortest_path

const long long INF = 1e18;

vector<long long> dijkstra(const vector<vector<pair<int, long long>>>& adj /*directed or
    ↪ undirected, weighted graph*/, int startNode) {
    vector<long long> len(adj.size(), INF);
    len[startNode] = 0;
    set<pair<long long/*weight*/, int/*node*/>> q;
    q.insert({0LL, startNode});
    while (!q.empty()) {
```

```
        auto it = q.begin();
        int node = it->second;
        q.erase(it);
        for (auto [to, weight] : adj[node])
            if (len[to] > weight + len[node]) {
                q.erase({len[to], to});
                len[to] = weight + len[node];
                q.insert({len[to], to});
            }
    }
    return len;
}
```

% cat graphs/dsuTree.h | ./misc/hash.sh

da7e3b

Listing 9: DSU Tree

```
#pragma once

//status: not tested

const int Max = 1e5 + 3;
int color[Max], Time = 1, timeIn[Max], timeOut[Max], ver[Max], Size[Max], cnt[Max],
    ↪ heavyChild[Max], Depth[Max] = {0}, answer[Max];
vector<int> adj[Max];

void dfs(int node, int prev) {
    timeIn[node] = Time;
    ver[Time] = node;
    Time++;
    Size[node] = 1;
    int largest = heavyChild[node] = -1;
    Depth[node] = 1 + Depth[prev];
    for (int to : adj[node]) {
        if (to == prev) continue;
        dfs(to, node);
        Size[node] += Size[to];
        if (Size[to] > largest) {
            largest = Size[to];
            heavyChild[node] = to;
        }
    }
    timeOut[node] = Time;
}

void dfs1(int node, int prev, bool keep = true) {
    for (int to : adj[node]) {
        if (to == prev || to == heavyChild[node]) continue;
        dfs1(to, node, false);
    }
    if (heavyChild[node] != -1)
        dfs1(heavyChild[node], node, true);
    cnt[color[node]]++;
    for (int to : adj[node]) {
        if (to == prev || to == heavyChild[node]) continue;
        for (int i = timeIn[to]; i < timeOut[to]; ++i)
            cnt[color[ver[i]]]++;
    }
    if (!keep) {
        for (int i = timeIn[node]; i < timeOut[node]; ++i)
            cnt[color[ver[i]]]--;
    }
}
/*
int n;
cin >> n;
dfs(1, 1);
dfs1(1, 1);
for(int i = 1; i <= n; ++i) {
    cout << answer[i] << ' ';
}
cout << '\n';
*/
```

% cat graphs/floydWarshall.h | ./misc/hash.sh

84799a

Listing 10: Floyd Warshall

```
#pragma once

//status: not tested
//
//**for directed graphs only** if you initialize len[i][i] to infinity, then
//afterward floyds, len[i][i] = length of shortest cycle including node `i`
//
//another trick: change `len` to 2d array of *bools* where len[i][j] = true if
//there exists an edge from i -> j in initial graph. Also do:
//`len[i][j] |= len[i][k] & len[k][j]`
//Then after floyds, len[i][j] = true iff there's exists some path from node
//`i` to node `j`
//
//Changing the order of for-loops to i-j-k (instead of the current k-i-j)
//results in min-plus matrix multiplication. If adjacency matrix is M, then
//after computing M^k (with binary exponentiation), M[i][j] = min length path
//from i to j with at most k edges.

for (int k = 0; k < n; k++)
    for (int i = 0; i < n; i++)
        for (int j = 0; j < n; j++)
            len[i][j] = min(len[i][j], len[i][k] + len[k][j]);
```

% cat graphs/hld.h | ./misc/hash.sh

322dee

Listing 11: HLD

```
#pragma once

//status: all functions tested on random trees; also `lca` tested on
    ↪ https://judge.yosupo.jp/problem/lca

struct hld {
```

```
vector<int> Size, par, Depth, timeIn, Next, timeInToNode;
hld(vector<vector<int>>& adj /*forest of trees*/, int root = -1/*pass in to specify
    ↪ root, usually for a single component*/) :
    Size(adj.size(), 1), par(adj.size(), -1), Depth(adj.size(), 1),
        ↪ timeIn(adj.size()), Next(adj.size(), -1), timeInToNode(adj.size()) {
    int Time = 0;
    auto callDfss = [&](int node) -> void {
        Next[node] = par[node] = node;
        dfs1(node, adj);
        dfs2(node, adj, Time);
    };
    if (root != -1)
        callDfss(root);
    for (int i = 0; i < (int) adj.size(); i++) {
        if (par[i] == -1)   //roots each tree by node with min label
            callDfss(i);
    }
}
void dfs1(int node, vector<vector<int>>& adj) {
    for (auto& to : adj[node]) {
        if (to == par[node]) continue;
        Depth[to] = 1 + Depth[node];
        par[to] = node;
        dfs1(to, adj);
        Size[node] += Size[to];
        if (Size[to] > Size[adj[node][0]] || adj[node][0] == par[node])
            swap(to, adj[node][0]);
    }
}
void dfs2(int node, const vector<vector<int>>& adj, int& Time) {
    timeIn[node] = Time;
    timeInToNode[Time] = node;
    Time++;
    for (auto to : adj[node]) {
        if (to == par[node]) continue;
        Next[to] = (Time == timeIn[node] + 1 ? Next[node] : to);
        dfs2(to, adj, Time);
    }
}
// Returns intervals (of timeIn's) corresponding to the path between u and v, not
    ↪ necessarily in order
// This can answer queries for "is some node `x` on some path" by checking if the
    ↪ timeIn[x] is in any of these intervals
vector<pair<int, int>> path(int u, int v) const {
    vector<pair<int, int>> res;
    for (;; v = par[Next[v]]) {
        if (timeIn[v] < timeIn[u]) swap(u, v);
        if (timeIn[Next[v]] <= timeIn[u]) {
            res.push_back({timeIn[u], timeIn[v]});
            return res;
        }
        res.push_back({timeIn[Next[v]], timeIn[v]});
    }
}
// Returns interval (of timeIn's) corresponding to the subtree of node i
// This can answer queries for "is some node `x` in some other node's subtree" by
    ↪ checking if timeIn[x] is in this interval
pair<int, int> subtree(int i) const {
    return {timeIn[i], timeIn[i] + Size[i] - 1};
}
// Returns lca of nodes u and v
    int lca(int u, int v) const {
        for (;; v = par[Next[v]]) {
            if (timeIn[v] < timeIn[u]) swap(u, v);
            if (timeIn[Next[v]] <= timeIn[u]) return u;
        }
    }
};
```

```
% cat graphs/hopcroftKarp.h | ./misc/hash.sh
a3ff4a
```

Listing 12: Hopcroft Karp

```
#pragma once

//Modified from
    ↪ https://github.com/foreverbell/acm-icpc-cheat-sheet/blob/master/src/graph-algorithm/
//Worst case O(E*sqrt(V)) but faster in practice
//status: tested on https://judge.yosupo.jp/problem/bipartitematching with asserts
    ↪ checking correctness of min vertex cover

struct match {
    //# of edges in matching (which = size of min vertex cover by öKnig's theorem)
    int sizeOfMatching;
    //an arbitrary max matching is found. For this matching:
    //if ml[nodeLeft] == -1:
    //    `nodeLeft` is not in matching
    //else:
    //    the edge `nodeLeft` <=> ml[nodeLeft] is in the matching
    //
    //similarly for mr with edge mr[nodeRight] <=> nodeRight in matching if
        ↪ mr[nodeRight] != -1
    //matchings stored in ml and mr are the same matching
    //provides way to check if any node is in matching
    vector<int> ml, mr;
    //an arbitrary min vertex cover is found. For this MVC: leftMVC[`left node`] is true
        ↪ iff `left node` is in the min vertex cover (same for rightMVC)
    //if leftMVC[`left node`] is false, then `left node` is in the corresponding maximal
        ↪ independent set
    vector<bool> leftMVC, rightMVC;
};

//Think of the bipartite graph as having a left side (with size lSz) and a right side
    ↪ (with size rSz).
//Nodes on left side are indexed 0,1,...,lSz-1
//Nodes on right side are indexed 0,1,...,rSz-1
//
//`adj` is like a directed adjacency list containing edges from left side -> right side:
//To initialize `adj`: For every edge nodeLeft <=> nodeRight, do:
    ↪ adj[nodeLeft].push_back(nodeRight)
match hopcroftKarp(const vector<vector<int>>& adj /*bipartite graph*/, int rSz/*number
    ↪ of nodes on right side*/) {
    int sizeOfMatching = 0, lSz = adj.size();
    vector<int> level(lSz), ml(lSz, -1), mr(rSz, -1);
    vector<bool> visL(lSz, false);
    while (true) {
        queue<int> q;
        for (int i = 0; i < lSz; i++) {
```

```
            if (ml[i] == -1) level[i] = 0, q.push(i);
            else level[i] = -1;
        }
        while (!q.empty()) {
            int u = q.front();
            q.pop();
            for (int x : adj[u]) {
                int v = mr[x];
                if (v != -1 && level[v] < 0) {
                    level[v] = level[u] + 1;
                    q.push(v);
                }
            }
        }
        auto dfs = [&](auto&& dfsPtr, int u) -> bool {
            visL[u] = true;
            for (int x : adj[u]) {
                int v = mr[x];
                if (v == -1 || (!visL[v] && level[u] < level[v] && dfsPtr(dfsPtr, v))) {
                    ml[u] = x;
                    mr[x] = u;
                    return true;
                }
            }
            return false;
        };
        visL.assign(lSz, false);
        bool found = false;
        for (int i = 0; i < lSz; i++)
            if (ml[i] == -1 && dfs(dfs, i)) {
                found = true;
                sizeOfMatching++;
            }
        if (!found) break;
    }
    //find min vertex cover
    vector<bool> visR(rSz, false);
    auto dfs = [&](auto&& dfsPtr, int node) -> void {
        for (int to : adj[node]) {
            if (!visR[to] && mr[to] != -1) {
                visR[to] = true;
                dfsPtr(dfsPtr, mr[to]);
            }
        }
    };
    for (int i = 0; i < lSz; i++) {
        visL[i] = !visL[i];
        if (ml[i] == -1)
            dfs(dfs, i);
    }
    return {sizeOfMatching, ml, mr, visL, visR};
}
```

```
% cat graphs/lca.h | ./misc/hash.sh
```

```
eaac25
```

Listing 13: LCA

```
#pragma once

//https://codeforces.com/blog/entry/74847
//assumes a single tree, 1-based nodes is possible by passing in 'root' in range [1, n]
//status: all functions tested on random trees. 'getLca' also tested on
    ↪ https://judge.yosupo.jp/problem/lca

struct lca {
    vector<int> jmp, jmpEdges, par, subSize, depth;
    vector<long long> dist;

    lca(const vector<vector<pair<int, long long>>>& adj, int root) :
        jmp(adj.size(), root),
        jmpEdges(adj.size(), 0),
        par(adj.size(), root),
        subSize(adj.size(), 1),
        depth(adj.size(), 0),
        dist(adj.size(), 0LL) {
        dfs(root, -1, adj);
    }

    void dfs(int node, int parent, const vector<vector<pair<int, long long>>>& adj) {
        for (auto [ch, w] : adj[node]) {
            if (ch == parent) continue;
            depth[ch] = 1 + depth[node];
            par[ch] = node;
            dist[ch] = w + dist[node];
            if (parent != -1 && jmpEdges[node] == jmpEdges[jmp[node]])
                jmp[ch] = jmp[jmp[node]], jmpEdges[ch] = 2 * jmpEdges[node] + 1;
            else
                jmp[ch] = node, jmpEdges[ch] = 1;
            dfs(ch, node, adj);
            subSize[node] += subSize[ch];
        }
    }

    //traverse up k edges in O(log(k)). So with k=1 this returns 'node''s parent
    int kthPar(int node, int k) const {
        k = min(k, depth[node]);
        while (k > 0) {
            if (jmpEdges[node] <= k) {
                k -= jmpEdges[node];
                node = jmp[node];
            } else {
                k--;
                node = par[node];
            }
        }
        return node;
    }

    int getLca(int x, int y) const {
        if (depth[x] < depth[y]) swap(x, y);
        x = kthPar(x, depth[x] - depth[y]);
        while (x != y) {
            if (jmp[x] == jmp[y])
                x = par[x], y = par[y];
            else
                x = jmp[x], y = jmp[y];
        }
        return x;
```

```
    }

    int distEdges(int x, int y) const {
        return depth[x] + depth[y] - 2 * depth[getLca(x, y)];
    }

    long long distWeight(int x, int y) const {
        return dist[x] + dist[y] - 2 * dist[getLca(x, y)];
    }
};
```

```
% cat graphs/scc.h | ./misc/hash.sh
```

```
632fc2
```

Listing 14: SCC

```
#pragma once

//status: tested on https://judge.yosupo.jp/problem/scc
//building of condensation graph tested on https://cses.fi/problemset/task/1686/

struct sccInfo {
    int numSCCs;
    //scc's are labeled 0,1,...,`numSCCs-1`
    //sccId[i] is the id of the scc containing node `i`
    vector<int> sccId;
};

sccInfo getSCCs(const vector<vector<int>>& adj /*directed, unweighted graph*/) {
    int n = adj.size(), numSCCs = 0;
    vector<int> sccId(n);
    stack<int> seen;
    {
        vector<bool> vis(n, false);
        auto dfs = [&](auto&& dfsPtr, int curr) -> void {
            vis[curr] = true;
            for (int x : adj[curr]) {
                if (!vis[x])
                    dfsPtr(dfsPtr, x);
            }
            seen.push(curr);
        };
        for (int i = 0; i < n; ++i) {
            if (!vis[i])
                dfs(dfs, i);
        }
    }
    vector<vector<int>> adjInv(n);
    for (int i = 0; i < n; ++i) {
        for (int to : adj[i])
            adjInv[to].push_back(i);
    }
    vector<bool> vis(n, false);
    auto dfs = [&](auto&& dfsPtr, int curr) -> void {
        vis[curr] = true;
        sccId[curr] = numSCCs;
        for (int x : adjInv[curr]) {
            if (!vis[x])
                dfsPtr(dfsPtr, x);
        }
    };
    while (!seen.empty()) {
        int node = seen.top();
        seen.pop();
        if (vis[node])
            continue;
        dfs(dfs, node);
        numSCCs++;
    }
    return {numSCCs, sccId};
}
```

Listing 15: **RANGE DATA STRUCTURES**

```
% cat range_data_structures/segTreeBeats.h | ./misc/hash.sh
```

```
9c7ed3
```

Listing 16: Segment Tree Beats

```
#pragma once

//status: not tested, used in various problems

struct SegTreeBeats {
    typedef long long ll;
    struct Node {
        ll sum;
        ll mx;
        ll secondMx;
        ll cntMx;
    };
    vector<Node> tree;
    vector<int> lazy;
    int n, size;
    const ll inf = 1e18;

    /*implement these*/
    const Node zero = {0, -inf, -inf, 0};
    Node combine(const Node& L, const Node& R) {
        Node par;
        par.sum = L.sum + R.sum;
        if (L.mx == R.mx)
            par.cntMx = L.cntMx + R.cntMx;
        else if (L.mx > R.mx)
            par.cntMx = L.cntMx;
        else
            par.cntMx = R.cntMx;
        par.mx = max(L.mx, R.mx);
        par.secondMx = -inf;
        for (ll val : {
                    L.mx, R.mx, L.secondMx, R.secondMx
                }) {
            if (par.mx != val) {
                assert(par.mx > val);
                par.secondMx = max(par.secondMx, val);
```

```
        }
    }
    return par;
}
void push(int node, int start, int end) {
    if (start == end) return;
    assert(start < end);
    for (int child : {
                2 * node, 2 * node + 1
            }) {
        if (tree[child].mx <= tree[node].mx) continue;
        tree[child].sum -= (tree[child].mx - tree[node].mx) * tree[child].cntMx;
        tree[child].mx = tree[node].mx;
    }
}

SegTreeBeats(const vector<int>& arr) : n((int) arr.size()) {
    size = 1;
    while (size < n) size <<= 1;
    size <<= 1;
    tree.resize(size);
    lazy.resize(size, 0);
    build(arr, 1, 0, n - 1);
}
void build(const vector<int>& arr, int node, int start, int end) {
    if (start == end) {
        tree[node].sum = arr[start];
        tree[node].mx = arr[start];
        tree[node].secondMx = -inf;
        tree[node].cntMx = 1;
    } else {
        const int mid = (start + end) / 2;
        build(arr, 2 * node, start, mid);
        build(arr, 2 * node + 1, mid + 1, end);
        tree[node] = combine(tree[2 * node], tree[2 * node + 1]);
    }
}
//set a[i] = min(a[i], newMn), for i in range: [l,r]
void update(int l, int r, int newMn) {
    update(1, 0, n - 1, l, r, newMn);
}
void update(int node, int start, int end, int l, int r, int newMn) {
    assert(start <= end);
    push(node, start, end);
    if (start > r || end < l || tree[node].mx <= newMn) return;
    if (start >= l && end <= r && tree[node].secondMx < newMn) {
        tree[node].sum -= (tree[node].mx - newMn) * tree[node].cntMx;
        tree[node].mx = newMn;
        return;
    }
    assert(start < end);
    const int mid = (start + end) / 2;
    update(2 * node, start, mid, l, r, newMn);
    update(2 * node + 1, mid + 1, end, l, r, newMn);
    tree[node] = combine(tree[2 * node], tree[2 * node + 1]);
}
//query for sum/max in range [l,r]
Node query(int l, int r) {
    return query(1, 0, n - 1, l, r);
}
Node query(int node, int start, int end, int l, int r) {
    if (r < start || end < l) return zero;
    push(node, start, end);
    if (l <= start && end <= r) return tree[node];
    const int mid = (start + end) / 2;
    return combine(query(2 * node, start, mid, l, r), query(2 * node + 1, mid + 1,
        ↪ end, l, r));
}
};
```

```
% cat range_data_structures/implicitSegTree.h | ./misc/hash.sh
```

```
920519
```

Listing 17: Implicit Segment Tree

```
#pragma once

//status: tested on https://cses.fi/problemset/task/1144/ and
    ↪ https://judge.yosupo.jp/problem/point_add_range_sum

struct implicitSegTree {
    struct Node {
        int lCh, rCh;//children ptrs, indexes into 'tree'; 0 for null
        long long sum;
    };

    int sz;
    deque<Node> tree;

    implicitSegTree(int _sz): sz(_sz) {
        tree.push_back({0, 0, 0LL});//acts as null
        tree.push_back({0, 0, 0LL});//root node
    }

    void update(int idx, long long diff) {
        update(1, 0, sz - 1, idx, diff);
    }
    int update(int v, int tl, int tr, int idx, long long diff) {
        if (tl == tr) {
            if (v == 0) {
                tree.push_back(tree[0]);
                v = tree.size() - 1;
            }
            tree[v].sum += diff;
            return v;
        }
        int tm = (tl + tr) / 2;
        int lCh = tree[v].lCh;
        int rCh = tree[v].rCh;
        if (idx <= tm)
            lCh = update(lCh, tl, tm, idx, diff);
        else
            rCh = update(rCh, tm + 1, tr, idx, diff);
        if (v == 0) {
            tree.push_back(tree[0]);
            v = tree.size() - 1;
        }
        tree[v] = {lCh, rCh, tree[lCh].sum + tree[rCh].sum};
        return v;
```

```
    }

    //inclusive range: [l,r]
    long long query(int l, int r) const {
        return query(1, 0, sz - 1, l, r);
    }
    long long query(int v, int tl, int tr, int l, int r) const {
        if (tree[v].sum == 0LL || tr < l || r < tl) return 0LL;
        if (l <= tl && tr <= r) return tree[v].sum;
        int tm = (tl + tr) / 2;
        return query(tree[v].lCh, tl, tm, l, r) +
               query(tree[v].rCh, tm + 1, tr, l, r);
    }
};
```

% cat range_data_structures/kth_smallest.h | ./misc/hash.sh

8f3b27

Listing 18: Kth Smallest

```
#pragma once

//modified from
//  https://cp-algorithms.com/data_structures/segment_tree.html#preserving-the-history-of-its-values-persistent-segment-tree
//tested on https://judge.yosupo.jp/problem/range_kth_smallest
//works for -1e9 <= arr[i] <= 1e9

struct kth_smallest {

    const int mx = 1e9;

    struct Node {
        int lCh, rCh;//children, indexes into `tree`
        int sum;
    };

    deque<Node> tree;
    vector<int> roots;

    kth_smallest(const vector<int>& arr) {
        tree.push_back({0, 0, 0}); //acts as null
        roots.push_back(0);
        for (int i = 0; i < (int)arr.size(); i++) {
            assert(-mx <= arr[i] && arr[i] <= mx);
            roots.push_back(update(roots.back(), -mx, mx, arr[i], 1));
        }
    }
    int update(int v, int tl, int tr, int idx, int diff) {
        if (tl == tr) {
            tree.push_back({0, 0, tree[v].sum + diff});
            return tree.size() - 1;
        }
        int tm = tl + (tr - tl) / 2;
        int lCh = tree[v].lCh;
        int rCh = tree[v].rCh;
        if (idx <= tm)
            lCh = update(lCh, tl, tm, idx, diff);
        else
            rCh = update(rCh, tm + 1, tr, idx, diff);
        tree.push_back({lCh, rCh, tree[lCh].sum + tree[rCh].sum});
        return tree.size() - 1;
    }


    /* find kth smallest number among arr[l], arr[l+1], ..., arr[r]
     * k is 1-based, so find_kth(l,r,1) returns the min
     */
    int query(int l, int r, int k) const {
        assert(1 <= k && k <= r - l + 1); //note this condition implies L <= R
        assert(0 <= l && r + 1 < (int)roots.size());
        return query(roots[l], roots[r + 1], -mx, mx, k);
    }
    int query(int vl, int vr, int tl, int tr, int k) const {
        if (tl == tr)
            return tl;
        int tm = tl + (tr - tl) / 2;
        int left_count = tree[tree[vr].lCh].sum - tree[tree[vl].lCh].sum;
        if (left_count >= k) return query(tree[vl].lCh, tree[vr].lCh, tl, tm, k);
        return query(tree[vl].rCh, tree[vr].rCh, tm + 1, tr, k - left_count);
    }
};
```

% cat range_data_structures/sparseTable.h | ./misc/hash.sh

fcbc6d

Listing 19: Sparse Table

```
#pragma once

//usage:
//  vector<long long> arr;
//  ...
//  sparseTable<long long> st(arr, [](auto x, auto y) { return min(x,y); });
//
//to also get index of min element, do:
//  sparseTable<pair<long long,int>> st(arr, [](auto x, auto y) { return min(x,y); });
//and initialize second to index. If there are multiple indexes of min element,
//it'll return the smallest (left-most) one
//
//status: tested on random inputs, also on https://judge.yosupo.jp/problem/staticrmq
template <class T>
struct sparseTable {
    vector<vector<T>> dp;
    function<T(const T&, const T&)> func;
    sparseTable(const vector<T>& arr, const function<T(const T&, const T&)>& _func) :
        dp(1, arr), func(_func) {
        int n = arr.size();
        for (int pw = 1, k = 1; pw * 2 <= n; pw *= 2, k++) {
            dp.emplace_back(n - pw * 2 + 1);
            for (int j = 0; j < (int)dp[k].size(); j++)
                dp[k][j] = func(dp[k - 1][j], dp[k - 1][j + pw]);
        }
    }
    //inclusive range [l, r]
    T query(int l, int r) const {
        assert(0 <= l && l <= r && r < (int)dp[0].size());
```

```
        int lg = 31 - __builtin_clz(r - l + 1);
        return func(dp[lg][l], dp[lg][r - (1 << lg) + 1]);
    }
};
```

```
% cat range_data_structures/distinct_query.h | ./misc/hash.sh
```

```
116f83
```

Listing 20: Number Distinct Elements

```
#pragma once

//modified from
    ↪ https://cp-algorithms.com/data_structures/segment_tree.html#preserving-the-history-of-its-values-persistent-segment-tree
//tested on https://www.spoj.com/problems/DQUERY/ and stress tested

//works with negatives

struct persistentSegTree {

    struct Node {
        int lCh, rCh;//children, indexes into `tree`
        int sum;
    };

    int sz;
    deque<Node> tree;
    vector<int> roots;

    persistentSegTree(const vector<int>& arr) : sz(arr.size() + 1) {
        tree.push_back({0, 0, 0}); //acts as null
        roots.push_back(0);
        map<int, int> lastIdx;
        for (int i = 0; i < (int)arr.size(); i++) {
            roots.push_back(update(roots.back(), 0, sz - 1, lastIdx[arr[i]], 1));
            lastIdx[arr[i]] = i + 1;
        }
    }
    int update(int v, int tl, int tr, int idx, int diff) {
        if (tl == tr) {
            tree.push_back({0, 0, tree[v].sum + diff});
            return tree.size() - 1;
        }
        int tm = (tl + tr) / 2;
        int lCh = tree[v].lCh;
        int rCh = tree[v].rCh;
        if (idx <= tm)
            lCh = update(lCh, tl, tm, idx, diff);
        else
            rCh = update(rCh, tm + 1, tr, idx, diff);
        tree.push_back({lCh, rCh, tree[lCh].sum + tree[rCh].sum});
        return tree.size() - 1;
    }

    //returns number of distinct elements in range [l,r]
    int query(int l, int r) const {
        return query(roots[l], roots[r + 1], 0, sz - 1, l + 1);
    }
    int query(int vl, int vr, int tl, int tr, int idx) const {
        if (tree[vr].sum == 0 || idx <= tl)
            return 0;
        if (tr < idx)
            return tree[vr].sum - tree[vl].sum;
        int tm = (tl + tr) / 2;
        return query(tree[vl].lCh, tree[vr].lCh, tl, tm, idx) +
               query(tree[vl].rCh, tree[vr].rCh, tm + 1, tr, idx);
    }
};
```

```
% cat range_data_structures/buckets.h | ./misc/hash.sh
```

```
db53a3
```

Listing 21: Buckets

```
#pragma once

//this code isn't the best. It's meant as a rough start for sqrt-decomposition, and to
    ↪ be (heavily) modified
//doesn't handle overflow

//status: tested on random inputs, also used in various problems
struct buckets {
    const int BUCKET_SIZE = 50;//TODO: change - small value for testing

    struct bucket {
        int sumLazy = 0;
        int sumBucket = 0;
        int l, r;//inclusive range of bucket
        int len() const {
            return r - l + 1;
        }
    };

    vector<int> values;
    vector<bucket> _buckets;

    buckets(const vector<int>& initial) : values(initial) {
        int numBuckets = ((int) values.size() + BUCKET_SIZE - 1) / BUCKET_SIZE;
        _buckets.resize(numBuckets);
        for (int i = 0; i < numBuckets; i++) {
            _buckets[i].sumLazy = 0;
            _buckets[i].sumBucket = 0;
            _buckets[i].l = i * BUCKET_SIZE;
            _buckets[i].r = min((i + 1) * BUCKET_SIZE, (int) values.size()) - 1;
            for (int j = _buckets[i].l; j <= _buckets[i].r; j++)
                _buckets[i].sumBucket += values[j];
        }
    }

    void pushLazy(int bIdx) {
        bucket& b = _buckets[bIdx];
        if (!b.sumLazy) return;
        for (int i = b.l; i <= b.r; i++)
            values[i] += b.sumLazy;
        b.sumLazy = 0;
    }
```

```
    //update range [L,R]
    void update(int L, int R, int diff) {
        int startBucket = L / BUCKET_SIZE;
        int endBucket = R / BUCKET_SIZE;
        if (startBucket == endBucket) {   //range contained in same bucket case
            for (int i = L; i <= R; i++) {
                values[i] += diff;
                _buckets[startBucket].sumBucket += diff;
            }
            return;
        }
        for (int bIdx : {
                    startBucket, endBucket
                }) {  //handle "endpoint" buckets
            bucket& b = _buckets[bIdx];
            for (int i = max(b.l, L); i <= min(b.r, R); i++) {
                values[i] += diff;
                b.sumBucket += diff;
            }
        }
        for (int i = startBucket + 1; i < endBucket; i++) {   //handle all n/B buckets
            ↪ in middle
            bucket& b = _buckets[i];
            b.sumLazy += diff;
            b.sumBucket += b.len() * diff;
        }
    }

    //sum of range [L,R]
    int query(int L, int R) {
        int startBucket = L / BUCKET_SIZE;
        int endBucket = R / BUCKET_SIZE;
        if (startBucket == endBucket) {   //range contained in same bucket case
            pushLazy(startBucket);
            int sum = 0;
            for (int i = L; i <= R; i++)
                sum += values[i];
            return sum;
        }
        int sum = 0;
        for (int bIdx : {
                    startBucket, endBucket
                }) {  //handle "endpoint" buckets
            bucket& b = _buckets[bIdx];
            pushLazy(bIdx);
            for (int i = max(b.l, L); i <= min(b.r, R); i++)
                sum += values[i];
        }
        for (int i = startBucket + 1; i < endBucket; i++)   //handle all n/B buckets in
            ↪ middle
            sum += _buckets[i].sumBucket;
        return sum;
    }
};
```

```
% cat range_data_structures/implicitLazySegTree.h | ./misc/hash.sh
```

```
a13f2e
```

Listing 22: Implicit Lazy Segment Tree

```
#pragma once

//status: stress tested && AC's on https://cses.fi/problemset/task/1144
//see TODO for lines of code which usually need to change (not a complete list)

const int N = 1.5e7; //TODO

struct Node {
    long long val;//could represent max, sum, etc
    long long lazy;
    int lCh, rCh; // children, indexes into `tree`, -1 for null
} tree[N];

struct implicitLazySegTree {

    int NEW_NODE, rootL, rootR;//[rootL, rootR] defines range of root node; handles
        ↪ negatives

    implicitLazySegTree(int l, int r) : NEW_NODE(0), rootL(l), rootR(r) {
        assert(l <= r);
        tree[NEW_NODE++] = {0, 0, -1, -1}; //TODO
    }

    static long long combine(long long val_l, long long val_r) {
        return val_l + val_r; //TODO
    }

    void apply(int v, int tl, int tr, long long add) {
        tree[v].val += (tr - tl + 1) * add; //TODO
        if (tl != tr) {
            tree[tree[v].lCh].lazy += add; //TODO
            tree[tree[v].rCh].lazy += add;
        }
    }

    void push(int v, int tl, int tr) {
        if (tl != tr && tree[v].lCh == -1) {
            assert(NEW_NODE + 1 < N);
            tree[v].lCh = NEW_NODE;
            tree[NEW_NODE++] = {0, 0, -1, -1}; //TODO
            tree[v].rCh = NEW_NODE;
            tree[NEW_NODE++] = {0, 0, -1, -1};
        }
        if (tree[v].lazy) {
            apply(v, tl, tr, tree[v].lazy);
            tree[v].lazy = 0;
        }
    }

    //update range [l,r] with `add`
    void update(int l, int r, long long add) {
        update(0, rootL, rootR, l, r, add);
    }
    void update(int v, int tl, int tr, int l, int r, long long add) {
        push(v, tl, tr);
        if (tr < l || r < tl)
            return;
        if (l <= tl && tr <= r)
            return apply(v, tl, tr, add);
```

```
        int tm = tl + (tr - tl) / 2;
        update(tree[v].lCh, tl, tm, l, r, add);
        update(tree[v].rCh, tm + 1, tr, l, r, add);
        tree[v].val = combine(tree[tree[v].lCh].val, tree[tree[v].rCh].val);
    }

    //query range [l,r]
    long long query(int l, int r) {
        return query(0, rootL, rootR, l, r);
    }
    long long query(int v, int tl, int tr, int l, int r) {
        if (tr < l || r < tl)
            return 0; //TODO
        push(v, tl, tr);
        if (l <= tl && tr <= r)
            return tree[v].val;
        int tm = tl + (tr - tl) / 2;
        return combine(query(tree[v].lCh, tl, tm, l, r),
                       query(tree[v].rCh, tm + 1, tr, l, r));
    }
};
```

```
% cat range_data_structures/persistentLazySegTree.h | ./misc/hash.sh
```

```
87eace
```

Listing 23: Persistent Lazy Segment Tree

```
#pragma once

//tested on https://codeforces.com/contest/707/problem/D
struct persistentLazySegTree {

    struct Node {
        int lCh, rCh;//children, indexes into `tree`
        int sum;
        bool lazyTog;
    };

    int sz;
    deque<Node> tree;
    vector<int> roots;

    //implicit
    persistentLazySegTree(int _sz) : sz(_sz) {
        tree.push_back({0, 0, 0, 0}); //acts as null
        roots.push_back(0);
    }

    void push(int v, int tl, int tr) {
        if (tl != tr) {
            tree.push_back(tree[tree[v].lCh]);
            tree[v].lCh = tree.size() - 1;
            tree.push_back(tree[tree[v].rCh]);
            tree[v].rCh = tree.size() - 1;
        }
        if (tree[v].lazyTog) {
            tree[v].sum = (tr - tl + 1) - tree[v].sum;
            tree[v].lazyTog = false;
            if (tl != tr) {
                tree[tree[v].lCh].lazyTog ^= 1;
                tree[tree[v].rCh].lazyTog ^= 1;
            }
        }
    }

    void set(int idx, int new_val) {
        tree.push_back(tree[roots.back()]);//allocate top down
        roots.push_back(tree.size() - 1);
        set(roots.back(), 0, sz - 1, idx, new_val);
    }
    void set(int v, int tl, int tr, int idx, int new_val) {
        push(v, tl, tr);
        if (tr < idx || idx < tl)
            return;
        if (idx <= tl && tr <= idx) {
            tree[v].sum = new_val;
            return;
        }
        int tm = (tl + tr) / 2;
        int lCh = tree[v].lCh;
        int rCh = tree[v].rCh;
        set(lCh, tl, tm, idx, new_val);
        set(rCh, tm + 1, tr, idx, new_val);
        tree[v].sum = tree[lCh].sum + tree[rCh].sum;
    }

    void toggleRange(int l, int r) {
        tree.push_back(tree[roots.back()]);//allocate top down
        roots.push_back(tree.size() - 1);
        toggleRange(roots.back(), 0, sz - 1, l, r);
    }
    void toggleRange(int v, int tl, int tr, int l, int r) {
        push(v, tl, tr);
        if (tr < l || r < tl)
            return;
        int lCh = tree[v].lCh;
        int rCh = tree[v].rCh;
        if (l <= tl && tr <= r) {
            tree[v].sum = (tr - tl + 1) - tree[v].sum;
            if (tl != tr) {
                tree[lCh].lazyTog ^= 1;
                tree[rCh].lazyTog ^= 1;
            }
            return;
        }
        int tm = (tl + tr) / 2;
        toggleRange(lCh, tl, tm, l, r);
        toggleRange(rCh, tm + 1, tr, l, r);
        tree[v].sum = tree[lCh].sum + tree[rCh].sum;
    }

    //let's use implementation trick described here
        ↪ https://codeforces.com/blog/entry/72626
    //so that we don't have to propogate lazy vals and thus we don't have to allocate
        ↪ new nodes
    int query(int l, int r) const {
        int version = roots.size() - 1;
        int root = roots[version];
        return query(root, 0, sz - 1, l, r, tree[root].lazyTog);
```

```
    }
    int query(int v, int tl, int tr, int l, int r, bool tog) const {
        if (v == 0 || tr < l || r < tl)
            return 0;
        if (l <= tl && tr <= r) {
            int sum = tree[v].sum;
            if (tree[v].lazyTog) sum = (tr - tl + 1) - sum;
            return sum;
        }
        int tm = (tl + tr) / 2;
        tog ^= tree[v].lazyTog;
        return query(tree[v].lCh, tl, tm, l, r, tog) +
               query(tree[v].rCh, tm + 1, tr, l, r, tog);
    }
};
```

```
% cat range_data_structures/MosAlg.h | ./misc/hash.sh
```

```
e7b0af
```

Listing 24: Mos Algorithm

```
//status: not tested, but used in various problems

#include <bits/stdc++.h>
using namespace std;

const int Max = 1e6 + 2;
int block, answer[Max], answerToQuery;

struct query {
    int l, r, index;
};

bool cmp(query x, query y) {
    if (x.l / block == y.l / block) return x.r < y.r;
    return x.l < y.l;
}
void add(int pos) {
}
void remove(int pos) {
}

int main() {
    int q;
    cin >> q;
    vector<query> queries(q);
    for (int i = 0; i < q; ++i) {
        cin >> queries[i].l >> queries[i].r;
        queries[i].index = i;
        answer[i] = 0;
    }
    sort(queries.begin(), queries.end(), cmp);
    int left = 0, right = 0;//store inclusive ranges, start at [0,0]
    add(0);
    answerToQuery = 0;
    for (auto& q : queries) {
        while (left > q.l) {
            left--;
            add(left);
        }
        while (right < q.r) {
            right++;
            add(right);
        }
        while (left < q.l) {
            remove(left);
            left++;
        }
        while (right > q.r) {
            remove(right);
            right--;
        }
        answer[q.index] = answerToQuery;
    }
    for (int i = 0; i < q; ++i) cout << answer[i] << '\n';
    return 0;
}
```

```
% cat range_data_structures/persistentSegTree.h | ./misc/hash.sh
```

```
47efec
```

Listing 25: Persistent Segment Tree

```
#pragma once

//modified from
    ↪ https://cp-algorithms.com/data_structures/segment_tree.html#preserving-the-history-o
//tested on https://www.spoj.com/problems/PSEGTREE/ and
    ↪ https://cses.fi/problemset/task/1737/
struct persistentSegTree {

    struct Node {
        int lCh, rCh;//children, indexes into 'tree'
        long long sum;
    };

    int sz;
    deque<Node> tree;
    vector<int> roots;

    //implicit
    persistentSegTree(int _sz) : sz(_sz) {
        tree.push_back({0, 0, 0LL}); //acts as null
        roots.push_back(0);
    }

    persistentSegTree(const vector<long long>& arr) : sz(arr.size()) {
        tree.push_back({0, 0, 0LL}); //acts as null
        roots.push_back(build(arr, 0, sz - 1));
    }
    int build(const vector<long long>& arr, int tl, int tr) {
        if (tl == tr) {
            tree.push_back({0, 0, arr[tl]});
            return tree.size() - 1;
        }
        int tm = (tl + tr) / 2;
```

```
        int lCh = build(arr, tl, tm);
        int rCh = build(arr, tm + 1, tr);
        tree.push_back({lCh, rCh, tree[lCh].sum + tree[rCh].sum});
        return tree.size() - 1;
    }

    void update(int version, int idx, long long diff) {
        roots.push_back(update(roots[version], 0, sz - 1, idx, diff));
    }
    int update(int v, int tl, int tr, int idx, long long diff) {
        if (tl == tr) {
            tree.push_back({0, 0, tree[v].sum + diff});
            return tree.size() - 1;
        }
        int tm = (tl + tr) / 2;
        int lCh = tree[v].lCh;
        int rCh = tree[v].rCh;
        if (idx <= tm)
            lCh = update(lCh, tl, tm, idx, diff);
        else
            rCh = update(rCh, tm + 1, tr, idx, diff);
        tree.push_back({lCh, rCh, tree[lCh].sum + tree[rCh].sum});
        return tree.size() - 1;
    }

    long long query(int version, int l, int r) const {
        return query(roots[version], 0, sz - 1, l, r);
    }
    long long query(int v, int tl, int tr, int l, int r) const {
        if (tree[v].sum == 0LL || tr < l || r < tl)
            return 0LL;
        if (l <= tl && tr <= r)
            return tree[v].sum;
        int tm = (tl + tr) / 2;
        return query(tree[v].lCh, tl, tm, l, r) +
               query(tree[v].rCh, tm + 1, tr, l, r);
    }
};
```

```
% cat range_data_structures/mergeSortTree.h | ./misc/hash.sh
```

```
9cea9e
```

Listing 26: Merge Sort Tree

```
#pragma once

//status: stress-tested against persistent seg tree; used in various problems

struct MergeSortTree {
    struct Node {
        vector<int> vals;

        int l, r;
    };

    vector<Node> tree;

    Node combineChildren(const Node& L, const Node& R) {
        vector<int> par(L.vals.size() + R.vals.size());
        merge(L.vals.begin(), L.vals.end(), R.vals.begin(), R.vals.end(), par.begin());
        return Node{par, L.l, R.r};
    }

    //There's no constructor 'SegmentTree(int size)' because how to initialize l,r in
        ↪ nodes without calling build?
    //the whole point of this constructor was to be simpler by not calling build
    MergeSortTree(const vector<int>& arr)  {
        int n = arr.size(), size = 1;
        while (size < n) size <<= 1;
        size <<= 1;
        tree.resize(size);
        build(arr, 1, 0, n - 1);
    }
    void build(const vector<int>& arr, int node, int start, int end) {
        if (start == end) {
            tree[node] = Node {
                vector<int>{arr[start]},
                start,
                end
            };
        } else {
            int mid = (start + end) / 2;
            build(arr, 2 * node, start, mid);
            build(arr, 2 * node + 1, mid + 1, end);
            tree[node] = combineChildren(tree[2 * node], tree[2 * node + 1]);
        }
    }

    //inclusive range: [l,r]
    int query(int l, int r, int x) {
        return query(1, l, r, x);
    }
    int query(int node, int l, int r, int x) {
        int start = tree[node].l, end = tree[node].r;
        if (r < start || end < l) return 0;
        if (l <= start && end <= r) {
            vector<int>& v = tree[node].vals;
            return lower_bound(v.begin(), v.end(), x) - v.begin();
        }
        return query(2 * node, l, r, x) + query(2 * node + 1, l, r, x);
    }
};
```

```
% cat range_data_structures/segTree.h | ./misc/hash.sh
```

```
c81814
```

Listing 27: Segment Tree

```
#pragma once

//status: tested on random inputs

const long long inf = 1e18;

struct segTree {
    struct Node {
```

```
    long long sum, mx, mn;
    long long lazy;
    int l, r;

    int len() const {
        return r - l + 1;
    }
    //returns 1 + (# of nodes in left child's subtree)
    //https://cp-algorithms.com/data_structures/segment_tree.html#memory-efficient-implementation
    int rCh() const {
        return ((r - l) & ~1) + 2;
    }
};

vector<Node> tree;

//There's no constructor 'segTree(int size)' because how to initialize l,r in nodes
    ↪ without calling build?
//the whole point of 'segTree(int size)' was to be simpler by not calling build
segTree(const vector<long long>& arr) : tree(2 * (int) arr.size() - 1) {
    build(arr, 0, 0, (int) arr.size() - 1);
}
void build(const vector<long long>& arr, int v, int tl, int tr) {
    if (tl == tr) {
        tree[v] = {
            arr[tl],
            arr[tl],
            arr[tl],
            0,
            tl,
            tr
        };
    } else {
        int tm = tl + (tr - tl) / 2;
        build(arr, v + 1, tl, tm);
        build(arr, v + 2 * (tm - tl + 1), tm + 1, tr);
        tree[v] = combine(tree[v + 1], tree[v + 2 * (tm - tl + 1)]);
    }
}

Node combine(const Node& L, const Node& R) {
    return {
        L.sum + R.sum,
        max(L.mx, R.mx),
        min(L.mn, R.mn),
        0,
        L.l,
        R.r
    };
}

//what happens when 'add' is applied to every index in range [tree[v].l, tree[v].r]?
void apply(int v, long long add) {
    tree[v].sum += tree[v].len() * add;
    tree[v].mx += add;
    tree[v].mn += add;
    if (tree[v].len() > 1) {
        tree[v + 1].lazy += add;
        tree[v + tree[v].rCh()].lazy += add;
    }
}

    void push(int v) {
        if (tree[v].lazy) {
            apply(v, tree[v].lazy);
            tree[v].lazy = 0;
        }
    }

    //update range [l,r] with 'add'
    void update(int l, int r, long long add) {
        update(0, l, r, add);
    }
    void update(int v, int l, int r, long long add) {
        push(v);
        if (tree[v].r < l || r < tree[v].l)
            return;
        if (l <= tree[v].l && tree[v].r <= r)
            return apply(v, add);
        update(v + 1, l, r, add);
        update(v + tree[v].rCh(), l, r, add);
        tree[v] = combine(tree[v + 1], tree[v + tree[v].rCh()]);
    }

    //range [l,r]
    Node query(int l, int r) {
        return query(0, l, r);
    }
    Node query(int v, int l, int r) {
        if (tree[v].r < l || r < tree[v].l)
            return {0, -inf, inf, 0, 0, 0};
        push(v);
        if (l <= tree[v].l && tree[v].r <= r)
            return tree[v];
        return combine(query(v + 1, l, r),
                       query(v + tree[v].rCh(), l, r));
    }
};
```

```
% cat range_data_structures/fenwickTree.h | ./misc/hash.sh
```

```
836d135
```

Listing 28: Fenwick Tree

```
#pragma once

//status: tested on random inputs; also tested on
//https://judge.yosupo.jp/problem/point_add_range_sum, lower_bound tested on
//https://judge.yosupo.jp/problem/predecessor_problem

template<class T>
struct fenwickTree {
    vector<T> bit;
    fenwickTree(int n) : bit(n, 0) {}
    fenwickTree(const vector<T>& a) : bit(a.size()) {
        if (a.empty()) return;
        bit[0] = a[0];
        for (int i = 1; i < (int) a.size(); i++)
            bit[i] = bit[i - 1] + a[i];
```

```
        for (int i = (int) a.size() - 1; i > 0; i--) {
            int lower_i = (i & (i + 1)) - 1;
            if (lower_i >= 0)
                bit[i] -= bit[lower_i];
        }
    }
    void update(int idx, const T& d) {
        for (; idx < (int) bit.size(); idx = idx | (idx + 1))
            bit[idx] += d;
    }
    T sum(int r) const {
        T ret = 0;
        for (; r >= 0; r = (r & (r + 1)) - 1)
            ret += bit[r];
        return ret;
    }
    T sum(int l, int r) const {
        return sum(r) - sum(l - 1);
    }
    //Returns min pos such that sum of [0, pos] >= sum
    //Returns bit.size() if no sum is >= sum, or -1 if empty sum is.
    //Doesn't work with negatives (since it's greedy), counterexample: array: {1, -1},
        ↪ sum: 1, this returns 2, but should return 0
    int lower_bound(T sum) const {
        if (sum <= 0) return -1;
        int pos = 0;
        for (int pw = 1 << (31 - __builtin_clz(bit.size() | 1)); pw; pw >>= 1) {
            if (pos + pw <= (int)bit.size() && bit[pos + pw - 1] < sum)
                pos += pw, sum -= bit[pos - 1];
        }
        return pos;
    }
};

//status: tested on random inputs

template<class T>
struct rangeUpdatesAndPointQueries {
    fenwickTree<T> ft;
    rangeUpdatesAndPointQueries(int n) : ft(n) {}
    rangeUpdatesAndPointQueries(const vector<T>& arr) : ft(init(arr)) {}
    fenwickTree<T> init(vector<T> arr/*intentional pass by value*/) {
        for (int i = (int) arr.size() - 1; i >= 1; i--)
            arr[i] -= arr[i - 1];
        return fenwickTree<T> (arr);
    }
    //add `add` to inclusive range [l, r]
    void updateRange(int l, int r, const T& add) {
        ft.update(l, add);
        if (r + 1 < (int) ft.bit.size())
            ft.update(r + 1, -add);
    }
    //get value at index `idx`
    T queryIdx(int idx) const {
        return ft.sum(idx);
    }
};
```

Listing 29: **STRINGS**

```
% cat strings/suffix_array.h | ./misc/hash.sh
```

```
46840a
```

Listing 30: Suffix Array

```
#pragma once

//modified from here: https://judge.yosupo.jp/submission/37410
//
//status: tested on https://judge.yosupo.jp/problem/suffixarray
//
// SA-IS, linear-time suffix array construction
// Reference:
// G. Nong, S. Zhang, and W. H. Chan,
// Two Efficient Algorithms for Linear Time Suffix Array Construction
template<class T>
vector<int> sa_is(const T& s, int upper/*max element of `s`; for std::string, pass in
    ↪ 255*/) {
    int n = (int) s.size();
    if (n == 0) return {};
    if (n == 1) return {0};
    if (n == 2) {
        if (s[0] < s[1]) {
            return {0, 1};
        } else {
            return {1, 0};
        }
    }
    vector<int> sa(n);
    vector<bool> ls(n);
    for (int i = n - 2; i >= 0; i--)
        ls[i] = (s[i] == s[i + 1]) ? ls[i + 1] : (s[i] < s[i + 1]);
    vector<int> sum_l(upper + 1), sum_s(upper + 1);
    for (int i = 0; i < n; i++) {
        if (!ls[i])
            sum_s[s[i]]++;
        else
            sum_l[s[i] + 1]++;
    }
    for (int i = 0; i <= upper; i++) {
        sum_s[i] += sum_l[i];
        if (i < upper) sum_l[i + 1] += sum_s[i];
    }
    vector<int> buf(upper + 1);
    auto induce = [&](const vector<int>& lms) {
        fill(sa.begin(), sa.end(), -1);
        fill(buf.begin(), buf.end(), 0);
        copy(sum_s.begin(), sum_s.end(), buf.begin());
        for (auto d : lms) {
            if (d == n) continue;
            sa[buf[s[d]]++] = d;
        }
        copy(sum_l.begin(), sum_l.end(), buf.begin());
        sa[buf[s[n - 1]]++] = n - 1;
        for (int i = 0; i < n; i++) {
            int v = sa[i];
            if (v >= 1 && !ls[v - 1])
                sa[buf[s[v - 1]]++] = v - 1;
```

```
        }
        copy(sum_l.begin(), sum_l.end(), buf.begin());
        for (int i = n - 1; i >= 0; i--) {
            int v = sa[i];
            if (v >= 1 && ls[v - 1])
                sa[--buf[s[v - 1] + 1]] = v - 1;
        }
    };
    vector<int> lms_map(n + 1, -1);
    int m = 0;
    for (int i = 1; i < n; i++) {
        if (!ls[i - 1] && ls[i])
            lms_map[i] = m++;
    }
    vector<int> lms;
    lms.reserve(m);
    for (int i = 1; i < n; i++) {
        if (!ls[i - 1] && ls[i])
            lms.push_back(i);
    }
    induce(lms);
    if (m) {
        vector<int> sorted_lms;
        sorted_lms.reserve(m);
        for (int v : sa) {
            if (lms_map[v] != -1) sorted_lms.push_back(v);
        }
        vector<int> rec_s(m);
        int rec_upper = 0;
        rec_s[lms_map[sorted_lms[0]]] = 0;
        for (int i = 1; i < m; i++) {
            int l = sorted_lms[i - 1], r = sorted_lms[i];
            int end_l = (lms_map[l] + 1 < m) ? lms[lms_map[l] + 1] : n;
            int end_r = (lms_map[r] + 1 < m) ? lms[lms_map[r] + 1] : n;
            bool same = true;
            if (end_l - l != end_r - r)
                same = false;
            else {
                while (l < end_l) {
                    if (s[l] != s[r])
                        break;
                    l++;
                    r++;
                }
                if (l == n || s[l] != s[r]) same = false;
            }
            if (!same) rec_upper++;
            rec_s[lms_map[sorted_lms[i]]] = rec_upper;
        }
        auto rec_sa =
            sa_is(rec_s, rec_upper);
        for (int i = 0; i < m; i++)
            sorted_lms[i] = lms[rec_sa[i]];
        induce(sorted_lms);
    }
    return sa;
}
```

% cat strings/longest_common_prefix.h | ./misc/hash.sh

396173

Listing 31: Longest Common Prefix Array

```
#pragma once

//modified from here: https://judge.yosupo.jp/submission/37410
//
//status: tested on https://judge.yosupo.jp/problem/number_of_substrings (answer = (n *
    ↪ (n+1) / 2) - (sum of LCP array))
//
// Reference:
// T. Kasai, G. Lee, H. Arimura, S. Arikawa, and K. Park,
// Linear-Time Longest-Common-Prefix Computation in Suffix Arrays and Its
// Applications
template<class T>
vector<int> lcp_array(const T& s, const vector<int>& sa) {
    int n = s.size(), k = 0;
    vector<int> lcp(n, 0);
    vector<int> rank(n, 0);
    for (int i = 0; i < n; i++) rank[sa[i]] = i;
    for (int i = 0; i < n; i++, k ? k-- : 0) {
        if (rank[i] == n - 1) {
            k = 0;
            continue;
        }
        int j = sa[rank[i] + 1];
        while (i + k < n && j + k < n && s[i + k] == s[j + k]) k++;
        lcp[rank[i]] = k;
    }
    return lcp;
}
```

% cat strings/rotationally_equivalent.h | ./misc/hash.sh

7a608b

Listing 32: Rotational Equivalence

```
#pragma once

// Checks if two arrays are rotationally equvalent
// uses KMP with doubling trick
// usage:
//  string s1, s2;
//  ...
//  rot_eq(s1, s2)
// or
//  vector<int> arr1, arr2;
//  ...
//  rot_eq(arr1, arr2)
//
//status: tested on random inputs, also on https://open.kattis.com/problems/maze

template <class T>
bool rot_eq(const T& a, const T& b) {
    if (a.size() != b.size()) return false;
    if (a.empty()) return true;
```

```
    int n = a.size();
    vector<int> fail(n + 1, 0);
    auto update = [&](int val, int& p) -> void {
        while (p && val != a[p]) p = fail[p];
        if (val == a[p]) p++;
    };
    for (int i = 1, p = 0; i < n; i++) {
        update(a[i], p);
        fail[i + 1] = p;
    }
    for (int i = 0, p = 0; i < 2 * n; i++) {
        update(b[i % n], p);
        if (p == n) return true;
    }
    return false;
}
```

```
% cat strings/rollingHash.h | ./misc/hash.sh
8e807d
```

Listing 33: Rolling Hash

```
//usage:
//  string s;
//  Hash h(s);
//or
//  vector<int> arr;
//  Hash h(arr);
//
//  works for -2e9 - 1000 <= arr[i] <= 2e9 + 1000
//
//status: tested on random inputs, on https://judge.yosupo.jp/problem/zalgorithm, and on
    ↪ https://judge.yosupo.jp/problem/enumerate_palindromes

#pragma once

#include "../misc/random.h"

const unsigned mod = 4294967087; // largest prime p < UINT_MAX such that (p-1)/2 is also
    ↪ prime
const int mx = 2e9 + 1000;
const vector<unsigned> bases {
    getRand<unsigned>(2u * mx + 2, mod - 1),
    getRand<unsigned>(2u * mx + 2, mod - 1),
    getRand<unsigned>(2u * mx + 2, mod - 1)
};

template <class T>
struct Hash {
    vector<vector<unsigned>> prefix, powB;

    Hash(const T& s) :
        prefix(bases.size(), vector<unsigned> (s.size() + 1, 0)),
        powB(bases.size(), vector<unsigned> (s.size() + 1, 1)) {
        for (auto val : s) assert(-mx <= val && val <= mx);
        for (int i = 0; i < (int) bases.size(); i++) {
            for (int j = 0; j < (int) s.size(); j++) {
                powB[i][j + 1] = 1ULL * powB[i][j] * bases[i] % mod;
                prefix[i][j + 1] = (1ULL * prefix[i][j] * bases[i] + s[j] + mx + 1) %
                    ↪ mod;
            }
        }
    }

    void debugCollisionProbability() const {
        // (1 - (str_len / mod) ^ #bases) ^ #comparisons
        auto getProb = [&](long long num_comparisons) -> double {
            return pow(1 - pow(powB[0].size() / double(mod), bases.size()),
                ↪ num_comparisons);
        };
        long long num_comparisons = 1e5;
        cerr << fixed << setprecision(10) << "Probability of **no** collisions when
            ↪ doing "
            << num_comparisons << " comparisons is ~ " << getProb(num_comparisons) <<
                ↪ endl;
        long long k = 1e5;
        cerr << fixed << setprecision(10) << "Probability that " << k << " unique
            ↪ strings have "
            << k << " unique hashes (if storing hashes in a std::set) is "
            << getProb(k * (k - 1) / 2) << endl;
    }

    //returns hashes of substring/subarray [L,R] inclusive, one hash per base
    vector<unsigned> operator()(int L, int R) const {
        assert(0 <= L && L <= R && R + 1 < (int) prefix[0].size());
        vector<unsigned> res(bases.size());
        for (int i = 0; i < (int) bases.size(); i++) {
            long long x = 1LL * prefix[i][R + 1] + mod - 1ULL * prefix[i][L] * powB[i][R
                ↪ - L + 1] % mod;
            res[i] = x >= mod ? x - mod : x;
        }
        return res;
    }
};
```

```
% cat strings/kmp.h | ./misc/hash.sh
509da3
```

Listing 34: KMP

```
#pragma once

//usage:
//  string needle;
//  ...
//  KMP_Match kmp(needle);
//or
//  vector<int> needle;
//  ...
//  KMP_Match kmp(needle);
//
//status: tested on random inputs
template <class T>
struct KMP_Match {
public:
    KMP_Match(const T& needle_) : prefixFunction(needle_.size() + 1, 0), needle(needle_)
        ↪ {
```

```
        for (int i = 1, p = 0; i < (int) needle.size(); i++) {
            update(needle[i], p);
            prefixFunction[i + 1] = p;
        }
    };

    // if haystack = "bananas"
    // needle = "ana"
    //
    // then we find 2 matches:
    // bananas
    // _ana___
    // ___ana_
    // 0123456 (indexes)
    // and KMP_Match::find returns {1,3} - the indexes in kaystack where
    // each match starts.
    //
    // You can also pass in false for "all" and KMP_Match::find will only
    // return the first match: {1}. Useful for checking if there exists
    // some match:
    //
    // KMP_Match::find(<haystack>,false).size() > 0
    vector<int> find(const T& haystack, bool all = true) const {
        vector<int> matches;
        for (int i = 0, p = 0; i < (int) haystack.size(); i++) {
            update(haystack[i], p);
            if (p == (int) needle.size()) {
                matches.push_back(i - (int) needle.size() + 1);
                if (!all) return matches;
                p = prefixFunction[p];
            }
        }
        return matches;
    }
private:
    void update(char val, int& p) const {
        while (p && val != needle[p]) p = prefixFunction[p];
        if (val == needle[p]) p++;
    }
    vector<int> prefixFunction;
    T needle;
};
```

```
% cat strings/trie.h | ./misc/hash.sh
45e7a8
```

Listing 35: Trie

```
#pragma once

//status: not tested, but used on various problems
//intended to be a base template and to be modified

const int K = 26;//character size

struct trie {

    struct node {
        bool leaf = 0;
        int next[K], id, p = -1;
        char pch;
        node(int _p = -1, char ch = '#') : p(_p), pch(ch) {
            fill(next, next + K, -1);
        }
    };

    vector<node> t;

    trie() : t(1) {}

    void add_string(const string& s, int id) {
        int c = 0;
        for (char ch : s) {
            int v = ch - 'a';
            if (t[c].next[v] == -1) {
                t[c].next[v] = t.size();
                t.emplace_back(c, ch);
            }
            c = t[c].next[v];
        }
        t[c].leaf = 1;
        t[c].id = id;
    }

    void remove_string(const string& s) {
        int c = 0;
        for (char ch : s) {
            int v = ch - 'a';
            if (t[c].next[v] == -1)
                return;
            c = t[c].next[v];
        }
        t[c].leaf = 0;
    }

    int find_string(const string& s) {
        int c = 0;
        for (char ch : s) {
            int v = ch - 'a';
            if (t[c].next[v] == -1)
                return -1;
            c = t[c].next[v];
        }
        if (!t[c].leaf) return -1;
        return t[c].id;
    }
};
```

```
% cat strings/string_queries.h | ./misc/hash.sh
85ec3a
```

Listing 36: Longest Common Prefix Query

```
#pragma once

#include "suffix_array.h"
```

```cpp
#include "longest_common_prefix.h"
#include "../range_data_structures/sparseTable.h"

//status: tested on random inputs, and on
    ↪ https://open.kattis.com/problems/automatictrading
//computes suffix array, lcp array, and then sparse table over lcp array
//O(n log n)

struct str_queries {
    str_queries(const string& s) : sa(sa_is(s, 255)), inv_sa(s.size()), lcp(lcp_array(s,
        ↪ sa)), st(lcp, [](int x, int y) {
        return min(x, y);
    }) {
        for (int i = 0; i < (int) s.size(); i++)
            inv_sa[sa[i]] = i;
    }

    //length of longest common prefix of suffixes s[idx1 ... n-1], s[idx2 ... n-1],
        ↪ 0-based indexing
    //You can check if two substrings s[L1..R1], s[L2..R2] are equal in O(1) by:
    //
    //R2-L2 == R1-L1 && longest_common_prefix(L1, L2) >= R2-L2+1
    int longest_common_prefix(int idx1, int idx2) const {
        if (idx1 == idx2) return (int) inv_sa.size() - idx1;
        idx1 = inv_sa[idx1];
        idx2 = inv_sa[idx2];
        if (idx1 > idx2) swap(idx1, idx2);
        return st.query(idx1, idx2 - 1);
    }

    //returns true if suffix s[idx1 ... n-1] < s[idx2 ... n-1]
    //(so false if idx1 == idx2)
    bool less(int idx1, int idx2) const {
        return inv_sa[idx1] < inv_sa[idx2];
    }

    vector<int> sa, inv_sa, lcp;
    sparseTable<int> st;
};
```

Listing 37: **MATH**

```
% cat math/exp_mod.h | ./misc/hash.sh
530333
```

Listing 38: BIN EXP MOD

```cpp
#pragma once

//status: tested on random inputs, and used in misc. problems

//returns a^pw % mod in O(log(pw))
long long fastPow(long long a, long long pw, int mod) {
    long long res = 1;
    a %= mod;
    while (pw > 0) {
        if (pw & 1) res = (res * a) % mod;
        a = (a * a) % mod;
        pw >>= 1;
    }
    return res;
}
```

```
% cat math/fib.h | ./misc/hash.sh
1c28e8
```

Listing 39: Fibonacci

```cpp
#pragma once

//status: not tested
//https://codeforces.com/blog/entry/14516

const int mod = 1e9 + 7;
unordered_map<int, int> table;
int fib(int n) {    //**O(log(n))**
    if (n < 2) return 1;
    if (table.find(n) != table.end()) return table[n];
    table[n] = (1LL * fib((n + 1) / 2) * fib(n / 2) + 1LL * fib((n - 1) / 2) * fib((n -
        ↪ 2) / 2)) % mod;
    return table[n];
}
```

```
% cat math/matrixMultPow.h | ./misc/hash.sh
a37276
```

Listing 40: Matrix Mult and Pow

```cpp
#pragma once

//status: not tested, but used on misc. problems

const int mod = 1e9 + 7;

vector<vector<int>> mult(const vector<vector<int>>& a, const vector<vector<int>>& b) {
    if (a.size() == 0) return {};
    if (a[0].size() == 0) return {};
    if (b.size() == 0) return {};
    if (b[0].size() == 0) return {};
    if (a[0].size() != b.size()) return {};
    int resultRow = a.size(), resultCol = b[0].size(), n = a[0].size();
    vector<vector<int>> product(resultRow, vector<int> (resultCol, 0));
    for (int i = 0; i < resultRow; ++i) {
        for (int k = 0; k < n; ++k) {
            for (int j = 0; j < resultCol; ++j)
                product[i][j] = (product[i][j] + 1LL * a[i][k] * b[k][j]) % mod;
        }
    }
    return product;
}

vector<vector<int>> power(vector<vector<int>> matrix, int b) {
    vector<vector<int>> res(matrix.size(), vector<int> (matrix.size(), 0));
```

```
    for (int i = 0; i < (int) matrix.size(); i++)
        res[i][i] = 1;
    while (b > 0) {
        if (b % 2 == 1)
            res = mult(res, matrix);
        matrix = mult(matrix, matrix);
        b /= 2;
    }
    return res;
}
```

```
% cat math/n_choose_k_mod.h | ./misc/hash.sh
fa1e4f
```

Listing 41: N Choose K MOD

```
#pragma once

//status: tested on random inputs

//for mod inverse
#include "exp_mod.h"

// usage:
//     NchooseK nk(n+1, 1e9+7) to use `choose`, `inv` with inputs <= n
// or:
//     NchooseK nk(mod, mod) to use `chooseWithLucasTheorem`
struct NchooseK {
    // `factSz` is the size of the factorial array, so only call `choose`, `inv` with n
        ↪ < factSz
    NchooseK(int factSz, int currMod) : mod(currMod), fact(factSz, 1), invFact(factSz) {
        //this implimentation of doesn't work if factSz > mod because n! % mod = 0 when
            ↪ n >= mod. So `invFact` array will be all 0's
        assert(factSz <= mod);
        //assert mod is prime. mod is intended to fit inside an int so that
        //multiplications fit in a longlong before being modded down. So this
        //will take sqrt(2^31) time
        assert(mod >= 2);
        for (int i = 2; i * i <= mod; i++)
            assert(mod % i);
        for (int i = 1; i < factSz; i++)
            fact[i] = 1LL * fact[i - 1] * i % mod;
        invFact.back() = fastPow(fact.back(), mod - 2, mod);
        for (int i = factSz - 2; i >= 0; i--)
            invFact[i] = 1LL * invFact[i + 1] * (i + 1) % mod;
    }

    //classic n choose k
    //fails when n >= mod
    int choose(int n, int k) const {
        if (k < 0 || k > n) return 0;
        //now we know 0 <= k <= n so 0 <= n
        return 1LL * fact[n] * invFact[k] % mod * invFact[n - k] % mod;
    }

    //lucas theorem to calculate n choose k in O(log(k))
    //need to calculate all factorials in range [0,mod), so O(mod) time&space, so need
        ↪ smallish prime mod (< 1e6 maybe)
    //handles n >= mod correctly
    int chooseWithLucasTheorem(long long n, long long k) const {
        if (k < 0 || k > n) return 0;
        if (k == 0 || k == n) return 1;
        return 1LL * chooseWithLucasTheorem(n / mod, k / mod) * choose(n % mod, k % mod)
            ↪ % mod;
    }

    //returns inverse of n in O(1)
    int inv(int n) const {
        assert(1 <= n);    //don't divide by 0 :)
        return 1LL * fact[n - 1] * invFact[n] % mod;
    }

    int mod;
    vector<int> fact, invFact;
};
```

```
% cat math/partition.h | ./misc/hash.sh
456850
```

Listing 42: Partition

```
#pragma once

//status: not tested

struct partitionFunction {
    vector<long long> remember;
    //The number of ways you can add to a number
    long long getPartitionsModM(int n, int m) {
        if (n < 0) return 0;
        if (n == 0) return 1;
        if ((int) remember.size() <= n) remember.resize(n + 1, -1);
        if (remember[n] != -1) return remember[n];
        long long sum = 0;
        long long val = 1;
        for (int i = 1; val <= n; i++) {
            long long multiply = 1;
            if (i % 2 == 0) multiply = -1;
            val = ((3LL * i * i) + i) / 2;
            sum += getPartitionsModM(n - val, m) * multiply % m;
            val = ((3LL * i * i) - i) / 2;
            sum += getPartitionsModM(n - val, m) * multiply % m;
            sum %= m;
            if (sum < 0) sum += m;
        }
        return remember[n] = sum % m;
    }
};
```

```
% cat math/primeSieveMobius.h | ./misc/hash.sh
e657cb
```

Listing 43: Prime Sieve Mobius

```cpp
#pragma once

//status: not tested, but used on various problems

//mobius[i] = 0 iff there exists a prime p s.t. i%(p^2)=0
//mobius[i] = -1 iff i has an odd number of distinct prime factors
//mobius[i] = 1 iff i has an even number of distinct prime factors
const int N = 2e6 + 10;
int mobius[N];
void calcMobius() {
    mobius[1] = 1;
    for (int i = 1; i < N; ++i) {
        for (int j = i + i; j < N; j += i)
            mobius[j] -= mobius[i];
    }
}

int minPrime[N];
void calcSeive() {
    fill(minPrime, minPrime + N, N);
    for (int i = N - 1; i >= 2; --i) {
        for (int j = i; j < N; j += i)
            minPrime[j] = i;
    }
}
```

```
% cat math/solve_linear_mod.h | ./misc/hash.sh
971790
```

Listing 44: Solve Linear Equations MOD

```cpp
#pragma once

//for mod inverse
#include "exp_mod.h"

struct matrixInfo {
    int rank, det;
    vector<int> x;
};

//Solves A * x = b under prime mod.
//A is a n (rows) by m (cols) matrix, b is a length n column vector, x is a length m
    ↪ column vector.
//assumes n,m >= 1, else RTE
//Returns rank of A, determinant of A, and x (solution vector to A * x = b). x is empty
    ↪ if no solution. If multiple solutions, an arbitrary one is returned.
//Leaves A in reduced row echelon form (unlike kactl).
//O(n * m * min(n,m))
//
//status: tested on https://judge.yosupo.jp/problem/system_of_linear_equations and
    ↪ https://judge.yosupo.jp/problem/matrix_det
matrixInfo solve_linear_mod(vector<vector<int>>& A, vector<int>& b, const int mod) {
    assert(A.size() == b.size());
    int n = A.size(), m = A[0].size(), rank = 0, det = 1;
    //start of row reduce
    for (int col = 0; col < m && rank < n; ++col) {
        //find arbitrary pivot and swap pivot to current row
        for (int i = rank; i < n; ++i)
            if (A[i][col] != 0) {
                if (rank != i) det = det == 0 ? det : mod - det;
                swap(A[i], A[rank]);
                swap(b[i], b[rank]);
                break;
            }
        if (A[rank][col] == 0) {
            det = 0;
            continue;
        }
        det = (1LL * det * A[rank][col]) % mod;
        //make pivot 1 by dividing row by inverse of pivot
        const int aInv = fastPow(A[rank][col], mod - 2, mod);
        for (int j = 0; j < m; ++j)
            A[rank][j] = (1LL * A[rank][j] * aInv) % mod;
        b[rank] = (1LL * b[rank] * aInv) % mod;
        //zero-out all numbers above & below pivot
        for (int i = 0; i < n; ++i)
            if (i != rank && A[i][col] != 0) {
                const int val = A[i][col];
                for (int j = 0; j < m; ++j) {
                    A[i][j] -= 1LL * A[rank][j] * val % mod;
                    if (A[i][j] < 0) A[i][j] += mod;
                }
                b[i] -= 1LL * b[rank] * val % mod;
                if (b[i] < 0) b[i] += mod;
            }
        ++rank;
    }
    //end of row reduce, start of extracting answer (`x`) from `A` and `b`
    assert(rank <= min(n, m));
    //check if solution exists
    for (int i = rank; i < n; i++) {
        if (b[i] != 0) return {rank, det, {} }; //no solution exists
    }
    //initialize solution vector (`x`)
    vector<int> x(m, 0);
    for (int i = 0, j = 0; i < rank; i++) {
        while (A[i][j] == 0) j++; //find pivot column
        assert(A[i][j] == 1);
        x[j] = b[i];
    }
    return {rank, det, x};
}
```

```
% cat math/sumFloorArithSeries.h | ./misc/hash.sh
b12ad8
```

Listing 45: Sum Floors of Arithmetic Series

```cpp
#pragma once

//status: used on https://open.kattis.com/problems/itsamodmodmodmodworld

//computes:
//[p/q] + [2p/q] + [3p/q] + ... + [np/q]
//(p, q, n are natural numbers)
```

```
//[x] = floor(x)

long long cnt(long long p, long long q, long long n) {
    long long t = __gcd(p, q);
    p = p / t;
    q = q / t;
    long long s = 0;
    long long z = 1;
    while ((q > 0) && (n > 0)) {
        //(point A)
        t = p / q;
        s += z * t * n * (n + 1) / 2;
        p -= q * t;
        //(point B)
        t = n / q;
        s += z * p * t * (n + 1) - z * t * (p * q * t + p + q - 1) / 2;
        n -= q * t;
        //(point C)
        t = n * p / q;
        s += z * t * n;
        n = t;
        swap(p, q);
        z = -z;
    }
    return s;
}
```

```
% cat math/sumOfKthPowers.h | ./misc/hash.sh
```

```
5cdedb
```

Listing 46: Sum of Kth Powers

```
#pragma once

//status: not tested, but used on misc. problems

#define MAX 1000010
#define MOD 1000000007

//Faulhaber'the sum of the k-th powers of the first n positive integers
//1^k + 2^k + 3^k + 4^k + ... + n^k
//O(k*log(k))

//Usage: lgr::lagrange(n, k)

namespace lgr {
short factor[MAX];
int P[MAX], S[MAX], ar[MAX], inv[MAX];

inline int expo(int a, int b) {
    int res = 1;
    while (b) {
        if (b & 1) res = (long long) res * a % MOD;
        a = (long long) a * a % MOD;
        b >>= 1;
    }
    return res;
}

int lagrange(long long n, int k) {
    if (!k) return (n % MOD);
    int i, j, x, res = 0;
    if (!inv[0]) {
        for (i = 2, x = 1; i < MAX; i++) x = (long long) x * i % MOD;
        inv[MAX - 1] = expo(x, MOD - 2);
        for (i = MAX - 2; i >= 0; i--) inv[i] = ((long long) inv[i + 1] * (i + 1)) % MOD;
    }
    k++;
    for (i = 0; i <= k; i++) factor[i] = 0;
    for (i = 4; i <= k; i += 2) factor[i] = 2;
    for (i = 3; (i * i) <= k; i += 2) {
        if (!factor[i]) {
            for (j = (i * i), x = i << 1; j <= k; j += x)
                factor[j] = i;
        }
    }
    for (ar[1] = 1, ar[0] = 0, i = 2; i <= k; i++) {
        if (!factor[i]) ar[i] = expo(i, k - 1);
        else ar[i] = ((long long) ar[factor[i]] * ar[i / factor[i]]) % MOD;
    }
    for (i = 1; i <= k; i++) {
        ar[i] += ar[i - 1];
        if (ar[i] >= MOD) ar[i] -= MOD;
    }
    if (n <= k) return ar[n];
    P[0] = 1, S[k] = 1;
    for (i = 1; i <= k; i++) P[i] = ((long long) P[i - 1] * ((n - i + 1) % MOD)) % MOD;
    for (i = k - 1; i >= 0; i--) S[i] = ((long long) S[i + 1] * ((n - i - 1) % MOD)) %
        ↪ MOD;
    for (i = 0; i <= k; i++) {
        x = (long long) ar[i] * P[i] % MOD * S[i] % MOD * inv[k - i] % MOD * inv[i] %
            ↪ MOD;
        if ((k - i) & 1) {
            res -= x;
            if (res < 0) res += MOD;
        } else {
            res += x;
            if (res >= MOD) res -= MOD;
        }
    }
    return (res % MOD);
}
}
```

```
% cat math/totient.h | ./misc/hash.sh
```

```
dfa354
```

Listing 47: Euler's Totient Phi Function

```
#pragma once

//status: tested on n in range [1, 800]

//  Euler's totient function counts the positive integers
//  up to a given integer n that are relatively prime to n.
```

```
//To improve, use Pollard-rho to find prime factors

int phi(int n) {
    int result = n;
    for (int i = 2, tempN = n; i * i <= tempN; i++) {
        if (n % i == 0) {
            while (n % i == 0)
                n /= i;
            result -= result / i;
        }
    }
    if (n > 1) result -= result / n;
    return result;
}
```

Listing 48: **MAX FLOW**

% cat maxflow/dinic.h | ./misc/hash.sh

d421fb

Listing 49: Dinic

```
#pragma once

//status: no tests, but used in various problems

struct maxflow {
public:
    typedef long long ll;
    ll n, s, t;
    maxflow(int _n, int _s, int _t) : n(_n), s(_s), t(_t), d(n), ptr(n), q(n), g(n) {}
    void addedge(ll a, ll b, ll cap) {
        edgeMap[a * n + b] = e.size();
        edge e1 = { a, b, cap, 0 };
        edge e2 = { b, a, 0, 0 };
        g[a].push_back((ll) e.size());
        e.push_back(e1);
        g[b].push_back((ll) e.size());
        e.push_back(e2);
    }
    ll getflow() {
        ll flow = 0;
        for (;;) {
            if (!bfs())  break;
            ptr.assign(ptr.size(), 0);
            while (ll pushed = dfs(s, inf))
                flow += pushed;
        }
        return flow;
    }
    ll getFlowForEdge(ll a, ll b) {
        return e[edgeMap[a * n + b]].flow;
    }

private:
    const ll inf = 1e18;
    struct edge {
        ll a, b, cap, flow;
    };
    unordered_map<int, ll> edgeMap;
    vector<ll> d, ptr, q;
    vector<edge> e;
    vector<vector<ll>> g;
    bool bfs() {
        ll qh = 0, qt = 0;
        q[qt++] = s;
        d.assign(d.size(), -1);
        d[s] = 0;
        while (qh < qt && d[t] == -1) {
            ll v = q[qh++];
            for (size_t i = 0; i < g[v].size(); ++i) {
                ll id = g[v][i],
                   to = e[id].b;
                if (d[to] == -1 && e[id].flow < e[id].cap) {
                    q[qt++] = to;
                    d[to] = d[v] + 1;
                }
            }
        }
        return d[t] != -1;
    }
    ll dfs(ll v, ll flow) {
        if (!flow) return 0;
        if (v == t) return flow;
        for (; ptr[v] < (ll) g[v].size(); ++ptr[v]) {
            ll id = g[v][ptr[v]];
            ll to = e[id].b;
            if (d[to] != d[v] + 1)  continue;
            ll pushed = dfs(to, min(flow, e[id].cap - e[id].flow));
            if (pushed) {
                e[id].flow += pushed;
                e[id ^ 1].flow -= pushed;
                return pushed;
            }
        }
        return 0;
    }
};
```

% cat maxflow/hungarian.h | ./misc/hash.sh

422f16

Listing 50: Hungarian

```
#pragma once

const long long inf = 1e18;

// this is one-indexed
// jobs X workers cost matrix
// cost[i][j] is cost of job i done by worker j
// #jobs must be <= #workers
// Default finds min cost; to find max cost set all costs[i][j] to -costs[i][j], set all
    ↪ unused to positive inf, ***set main diagonal (self edges) to 0***
```

```
//status: tested on https://judge.yosupo.jp/problem/assignment

struct match {
    long long cost;
    vector<int> matching;
};

match HungarianMatch(const vector<vector<long long>>& cost) {
    long long n = cost.size() - 1;
    long long m = cost[0].size() - 1;
    vector<int> p(m + 1), way(m + 1);
    vector<long long> u(n + 1), v(m + 1);
    for (int i = 1; i <= n; ++i) {
        p[0] = i;
        int j0 = 0;
        vector<long long> minv(m + 1, inf);
        vector<char> used(m + 1, false);
        do {
            used[j0] = true;
            int i0 = p[j0], j1 = 0;
            long long delta = inf;
            for (int j = 1; j <= m; ++j)
                if (!used[j]) {
                    long long cur = cost[i0][j] - u[i0] - v[j];
                    if (cur < minv[j])
                        minv[j] = cur, way[j] = j0;
                    if (minv[j] < delta)
                        delta = minv[j], j1 = j;
                }
            for (int j = 0; j <= m; ++j)
                if (used[j])
                    u[p[j]] += delta, v[j] -= delta;
                else
                    minv[j] -= delta;
            j0 = j1;
        } while (p[j0] != 0);
        do {
            int j1 = way[j0];
            p[j0] = p[j1];
            j0 = j1;
        } while (j0);
    }
    // For each N, it contains the M it selected
    vector<int> ans(n + 1);
    for (int j = 1; j <= m ; ++j)
        ans[p[j]] = j;
    return {-v[0], ans};
}
```

```
% cat maxflow/minCostMaxFlow.h | ./misc/hash.sh
```

```
b76528
```

Listing 51: Min Cost Max Flow

```
#pragma once

//status: not tested, but used in various problems

const long long inf = 1e18;

struct mincostmaxflow {
    typedef long long ll;

    struct edge {
        ll a, b, cap, cost, flow;
        size_t back;
    };

    vector<edge> e;
    vector<vector<ll>> g;
    ll n, s, t;
    ll k = inf; // The maximum amount of flow allowed

    mincostmaxflow(int _n, int _s, int _t) : n(_n), s(_s), t(_t) {
        g.resize(n);
    }

    void addedge(ll a, ll b, ll cap, ll cost) {
        edge e1 = {a, b, cap, cost, 0, g[b].size() };
        edge e2 = {b, a, 0, -cost, 0, g[a].size() };
        g[a].push_back((ll) e.size());
        e.push_back(e1);
        g[b].push_back((ll) e.size());
        e.push_back(e2);
    }

    // Returns {flow,cost}
    pair<ll, ll> getflow() {
        ll flow = 0, cost = 0;
        while (flow < k) {
            vector<ll> id(n, 0);
            vector<ll> d(n, inf);
            vector<ll> q(n);
            vector<ll> p(n);
            vector<size_t> p_edge(n);
            ll qh = 0, qt = 0;
            q[qt++] = s;
            d[s] = 0;
            while (qh != qt) {
                ll v = q[qh++];
                id[v] = 2;
                if (qh == n) qh = 0;
                for (size_t i = 0; i < g[v].size(); ++i) {
                    edge& r = e[g[v][i]];
                    if (r.flow < r.cap && d[v] + r.cost < d[r.b]) {
                        d[r.b] = d[v] + r.cost;
                        if (id[r.b] == 0) {
                            q[qt++] = r.b;
                            if (qt == n) qt = 0;
                        } else if (id[r.b] == 2) {
                            if (--qh == -1) qh = n - 1;
                            q[qh] = r.b;
                        }
                        id[r.b] = 1;
                        p[r.b] = v;
                        p_edge[r.b] = i;
                    }
                }
            }
```

```
            if (d[t] == inf) break;
            ll addflow = k - flow;
            for (ll v = t; v != s; v = p[v]) {
                ll pv = p[v];
                size_t pr = p_edge[v];
                addflow = min(addflow, e[g[pv][pr]].cap - e[g[pv][pr]].flow);
            }
            for (ll v = t; v != s; v = p[v]) {
                ll pv = p[v];
                size_t pr = p_edge[v], r = e[g[pv][pr]].back;
                e[g[pv][pr]].flow += addflow;
                e[g[v][r]].flow -= addflow;
                cost += e[g[pv][pr]].cost * addflow;
            }
            flow += addflow;
        }
        return {flow, cost};
    }
};
```

Listing 52: **MISC**

% cat misc/cntRectangles.h | ./misc/hash.sh

ca4f09

Listing 53: Count Rectangles

```
#pragma once

//given a 2D boolean matrix, calculate cnt[i][j]
//cnt[i][j] = the number of times an (i * j) rectangle appears in the matrix
//such that all cells in the rectangle are false
//Note cnt[0][j] and cnt[i][0] will contain garbage values
//O(R*C)
//
//status: tested on random inputs
vector<vector<int>> getNumRectangles(const vector<vector<bool>>& grid) {
    vector<vector<int>> cnt;
    const int rows = grid.size(), cols = grid[0].size();
    if (rows == 0 || cols == 0) return cnt;
    cnt.resize(rows + 1, vector<int> (cols + 1, 0));
    vector<vector<int>> arr(rows + 2, vector<int> (cols + 1, 0));
    for (int i = 1; i <= rows; ++i) {
        for (int j = 1; j <= cols; ++j) {
            arr[i][j] = 1 + arr[i][j - 1];
            if (grid[i - 1][j - 1]) arr[i][j] = 0;
        }
    }
    for (int j = 1; j <= cols; ++j) {
        arr[rows + 1][j] = 0;
        stack<pair<int, int>> st;
        st.push({0, 0});
        for (int i = 1; i <= rows + 1; ++i) {
            pair<int, int> curr = {i, arr[i][j]};
            while (arr[i][j] < st.top().second) {
                curr = st.top();
                st.pop();
                cnt[i - curr.first][curr.second]++;
                cnt[i - curr.first][max(arr[i][j], st.top().second)]--;
            }
            st.push({curr.first, arr[i][j]});
        }
    }
    for (int j = 1; j <= cols; ++j) {
        for (int i = rows - 1; i >= 1; --i)
            cnt[i][j] += cnt[i + 1][j];
        for (int i = rows - 1; i >= 1; --i)
            cnt[i][j] += cnt[i + 1][j];
    }
    for (int i = 1; i <= rows; ++i) {
        for (int j = cols - 1; j >= 1; --j)
            cnt[i][j] += cnt[i][j + 1];
    }
    return cnt;
}
```

% cat misc/longest_increasing_subsequence.h | ./misc/hash.sh

e4162a

Listing 54: Longest Increasing Subsequence

```
#pragma once

// status: tested on https://open.kattis.com/problems/longincsubseq

//returns array of indexes representing the longest *strictly* increasing subsequence
//for non-decreasing: pass in a vector<pair<T, int>> where second is 0, 1, ..., n-1
template<class T>
vector<int> lis(const vector<T>& arr) {
    int n = arr.size();
    vector<int> dp/*array of indexes into `arr`*/, prev(n);
    for (int i = 0; i < n; i++) {
        auto it = lower_bound(dp.begin(), dp.end(), i, [&](int x, int y) -> bool {
            return arr[x] < arr[y];
        });
        if (it == dp.end()) {
            prev[i] = dp.empty() ? -1 : dp.back();
            dp.push_back(i);
        } else {
            prev[i] = it == dp.begin() ? -1 : *(it - 1);
            *it = i;
        }
    }
    vector<int> res(dp.size());
    int j = dp.size();
    for (int i = dp.back(); i != -1; i = prev[i])
        res[--j] = i;
    return res;
}

//returns length of longest *strictly* increasing subsequence
//alternatively, there's this https://codeforces.com/blog/entry/13225
template<class T>
int lisSize(const vector<T>& arr) {
    vector<int> dp;
```

```
    for (int val : arr) {
        auto it = lower_bound(dp.begin(), dp.end(), val);
        if (it == dp.end())
            dp.push_back(val);
        else
            *it = val;
        //here, `dp.size()` = length of LIS of prefix of `arr` so far
    }
    return dp.size();
}
```

```
% cat misc/policy_based_data_structures.h | ./misc/hash.sh
```

```
807de9
```

Listing 55: PBDS

```
//status: not tested

//place this include *before* the `#define int long long` else compile error
#include <bits/extc++.h>
using namespace __gnu_pbds;

//BST with extra functions https://codeforces.com/blog/entry/11080
//order_of_key - # of elements *strictly* less than given element
//find_by_order - find kth largest element, k is 0 based so find_by_order(0) returns min
    ↪ element
template<class T>
using indexed_set = tree<T, null_type, less<T>, rb_tree_tag,
    ↪ tree_order_statistics_node_update>;
//example initialization:
indexed_set<pair<long long, int>> is;


//hash table (apparently faster than unordered_map):
    ↪ https://codeforces.com/blog/entry/60737
//example initialization:
gp_hash_table<string, long long> ht;
```

```
% cat misc/random.h | ./misc/hash.sh
```

```
f2bc66
```

Listing 56: Random Number Generator

```
#pragma once

//MUCH RANDOM!!!
seed_seq seed{
    (uint32_t)chrono::duration_cast<chrono::nanoseconds>(chrono::high_resolution_clock::now().time_since_epoch()).count(),
    (uint32_t)random_device()(),
    (uint32_t)(uintptr_t)make_unique<char>().get(),
    (uint32_t)__builtin_ia32_rdtsc()
};
mt19937 rng(seed);

//intended types: int, unsigned
template<class T>
inline T getRand(T l, T r) {
    assert(l <= r);
    return uniform_int_distribution<T>(l, r)(rng);
}

inline double getRandReal(double l, double r) {
    assert(l < r);
    return uniform_real_distribution(l, r)(rng);
}
```

```
% cat misc/safehash.h | ./misc/hash.sh
```

```
a296c3
```

Listing 57: Safe Hash

```
#pragma once

struct custom_hash {
    static uint64_t splitmix64(uint64_t x) {
        // http://xorshift.di.unimi.it/splitmix64.c
        x += 0x9e3779b97f4a7c15;
        x = (x ^ (x >> 30)) * 0xbf58476d1ce4e5b9;
        x = (x ^ (x >> 27)) * 0x94d049bb133111eb;
        return x ^ (x >> 31);
    }

    size_t operator()(uint64_t x) const {
        static const uint64_t FIXED_RANDOM =
            ↪ chrono::steady_clock::now().time_since_epoch().count();
        return splitmix64(x + FIXED_RANDOM);
    }
};
```