

Listings

1	CONTEST	1	38	Max Rectangle in Histogram	13
2	Tips and Tricks	1	39	Monotonic Stack	13
3	Hash codes	1	40	GCD Convolution	13
4	Test on random inputs	2	41	Iterate Chooses	13
5	MAX FLOW	2	42	Iterate Submasks	13
6	Hungarian	2	43	Iterate Supermasks	14
7	Min Cost Max Flow	2	44	Number of Distinct Subsequences DP	14
8	GRAPHS	3	45	PBDS	14
9	Block Vertex Tree	3	46	Random	14
10	Bridge Tree	3	47	Safe Hash	14
11	Bridges and Cuts	3	48	RANGE DATA STRUCTURES	15
12	Strongly Connected Components	4	49	Number Distinct Elements	15
13	Centroid Decomposition	5	50	Implicit Lazy Segment Tree	15
14	Frequency Table of Tree Distance	5	51	Kth Smallest	16
15	Count Paths Per Node	6	52	Merge Sort Tree	16
16	Dijkstra	6	53	BIT	17
17	HLD	6	54	RMQ	17
18	Hopcroft Karp	7	55	Disjoint RMQ	17
19	Kth Node on Path	8	56	Lazy Segment Tree	18
20	LCA	8	57	STRINGS	19
21	Rooted Tree Isomorphism	8	58	Binary Trie	19
22	MATH	9	59	Prefix Function	19
23	Derangements	9	60	KMP String Matching	19
24	Binary Exponentiation MOD	9	61	Suffix Array Related Queries	20
25	Fibonacci	9	62	Palindrome Query	21
26	Matrix Multiplication	9	63	Trie	21
27	Mobius Inversion	10			
28	N Choose K MOD	10			
29	Partitions	10			
30	Prime Sieve	10			
31	Row Reduce	11			
32	Solve Linear Equations MOD	11			
33	Euler’s Totient Phi Function	12			
34	Tetration MOD	12			
35	MISC	12			
36	Cartesian Tree	12			
37	Count Rectangles	12			

CONTEST

Tips and Tricks

```
## Tips and Tricks
- [C++ tips and tricks](https://codeforces.com/blog/entry/74684)
- invokes RTE (Run Time Error) upon integer overflow
...

#pragma GCC optimize "trapv"
...

- invoke RTE for input error (e.g. reading a long long into an int)
...
cin.exceptions(cin.failbit);
...

- use pramgas for C++ speed boost
...

#pragma GCC optimize("O3,unroll-loops")
#pragma GCC target("avx2,bmi,bmi2,lzcnt,popcnt")
...

### Troubleshooting
...

/* stuff you should look for
 * int overflow, array bounds
 * special cases (n=1?)
 * do smth instead of nothing and stay organized
 * WRITE STUFF DOWN
 * DON'T GET STUCK ON ONE APPROACH
 */
...

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- refer to [KACTL
  ↳ Troubleshoot](https://github.com/kth-competitive-programming/kactl/blob/main/content/contest/troubleshoot.cpp)

## Sources

- [[Tutorial] GCC Optimization Pragmas](https://codeforces.com/blog/entry/96344)
- [Don't use rand(): a guide to random number generators in
  ↳ C++](https://codeforces.com/blog/entry/61587)
```

Hash codes

```
#!/bin/bash
#Hashes a file, ignoring all:
# - whitespace
# - comments
# - asserts
# - includes
# - pragmas
#Use to verify that code was correctly typed.

#usage:
#  chmod +x hash.sh
#  cat a.cpp | ./hash.sh
#or just copy this command:
```

```
#  cat a.cpp | sed -r '/(assert|include|pragma)/d' | cpp -fpreprocessed -P | tr -d
↳ '[:space:]' | md5sum | cut -c-6
sed -r '/(assert|include|pragma)/d' | cpp -fpreprocessed -P | tr -d '[:space:]' | md5sum | cut
↳ -c-6
```

Test on random inputs

```
#!/bin/bash
#runs 2 programs against each other on random inputs until they output different results
#source: https://github.com/Errichto/youtube/blob/master/testing/s.sh
#usage:
#  chmod +x test.sh
#  ./test.sh
for((i = 1; ; ++i)); do
    echo $i
    ./test.out > in
    diff --ignore-all-space <./a.out < in <./brute.out < in || break
done
```

MAX FLOW

Hungarian

```
//cat hungarian.hpp | ./hash.sh
//935a16
#pragma once
//source: https://e-maxx.ru/algo/assignment_hungary
//
//input: cost[1...n][1...m] with 1 <= n <= m
//n workers, indexed 1, 2, ..., n
//m jobs, indexed 1, 2, ..., m
//it costs `cost[i][j]` to assign worker i to job j (1<=i<=n, 1<=j<=m)
//find minimum total cost to assign each worker to some distinct job
//0(n^2 * m)
//
//trick 1: set `cost[i][j]` to INF to say: "worker `i` cannot be assigned job `j`"
//trick 2: `cost[i][j]` can be negative, so to instead find max total cost over all matchings:
↳ set all `cost[i][j]` to `-cost[i][j]`.
//Now max total cost = - hungarian(cost).min_cost
const long long INF = 1e18;
struct weighted_match {
    long long min_cost;
    vector<int> matching;//worker `i` (1<=i<=n) is assigned to job `matching[i]`
↳ (1<=matching[i]<=m)
};
weighted_match hungarian(const vector<vector<long long>>& cost) {
    int n = ssize(cost) - 1, m = ssize(cost[0]) - 1;
    assert(n <= m);
    vector<int> p(m + 1), way(m + 1);
    vector<long long> u(n + 1), v(m + 1);
    for (int i = 1; i <= n; i++) {
        p[0] = i;
        int j0 = 0;
        vector<long long> minv(m + 1, INF);
        vector<bool> used(m + 1, 0);
        do {
```

```
used[j0] = 1;
int i0 = p[j0], j1 = 0;
long long delta = INF;
for (int j = 1; j <= m; j++)
    if (!used[j]) {
        long long cur = cost[i0][j] - u[i0] - v[j];
        if (cur < minv[j])
            minv[j] = cur, way[j] = j0;
        if (minv[j] < delta)
            delta = minv[j], j1 = j;
    }
for (int j = 0; j <= m; j++)
    if (used[j])
        u[p[j]] += delta, v[j] -= delta;
    else
        minv[j] -= delta;
j0 = j1;
} while (p[j0] != 0);
do {
    int j1 = way[j0];
    p[j0] = p[j1];
    j0 = j1;
} while (j0);
}
vector<int> ans(n + 1);
for (int j = 1; j <= m; j++)
    ans[p[j]] = j;
return {-v[0], ans};
}
```

Min Cost Max Flow

```
//cat min_cost_max_flow.hpp | ./hash.sh
//9dd6b6
#pragma once
//source: https://e-maxx.ru/algo/min_cost_flow
const long long INF = 1e18;
struct mcmf {
    using ll = long long;
    struct edge {
        int a, b;
        ll cap, cost, flow;
        int back;
    };
    const int N;
    vector<edge> e;
    vector<vector<int>> g;
    mcmf(int a_n : N(a_n), g(N) {}
    void add_edge(int a, int b, ll cap, ll cost) {
        edge e1 = {a, b, cap, cost, 0, ssize(g[b])};
        edge e2 = {b, a, 0, -cost, 0, ssize(g[a])};
        g[a].push_back(ssize(e));
        e.push_back(e1);
        g[b].push_back(ssize(e));
        e.push_back(e2);
    }
    pair<ll, ll> get_flow(int s, int t, ll total_flow) {
        ll flow = 0, cost = 0;
```

```
while (flow < total_flow) {
    vector<ll> d(N, INF);
    vector<int> p_edge(N), id(N, 0), q(N), p(N);
    int qh = 0, qt = 0;
    q[qt++] = s;
    d[s] = 0;
    while (qh != qt) {
        int v = q[qh++];
        id[v] = 2;
        if (qh == N)
            qh = 0;
        for (int i = 0; i < ssize(g[v]); i++) {
            const edge& r = e[g[v][i]];
            if (r.flow < r.cap && d[v] + r.cost < d[r.b]) {
                d[r.b] = d[v] + r.cost;
                if (id[r.b] == 0) {
                    q[qt++] = r.b;
                    if (qt == N)
                        qt = 0;
                } else if (id[r.b] == 2) {
                    if (--qh == -1)
                        qh = N - 1;
                    q[qh] = r.b;
                }
                id[r.b] = 1;
                p[r.b] = v;
                p_edge[r.b] = i;
            }
        }
    }
    if (d[t] == INF)
        break;
    ll addflow = total_flow - flow;
    for (int v = t; v != s; v = p[v]) {
        int pv = p[v], pr = p_edge[v];
        addflow = min(addflow, e[g[pv][pr]].cap - e[g[pv][pr]].flow);
    }
    for (int v = t; v != s; v = p[v]) {
        int pv = p[v], pr = p_edge[v], r = e[g[pv][pr]].back;
        e[g[pv][pr]].flow += addflow;
        e[g[v][r]].flow -= addflow;
        cost += e[g[pv][pr]].cost * addflow;
    }
    flow += addflow;
}
return {flow, cost};
};
```

GRAPHS

Block Vertex Tree

```
//cat block_vertex_tree.hpp | ./hash.sh
//a5c2b9
#pragma once
#include "bridges_and_cuts.hpp"
```

```
//returns adjacency list of block vertex tree
//usage:
// graph_info cc = bridge_and_cut(adj, m);
// vector<vector<int>> bvt = block_vertex_tree(adj, cc);
//to loop over each *unique* bcc containing a node v:
// for (int bccid : bvt[v]) {
//     bccid -= n;
//     ...
// }
//to loop over each *unique* node inside a bcc:
// for (int v : bvt[bccid + n]) {
//     ...
// }
vector<vector<int>> block_vertex_tree(const vector<vector<pair<int, int>>& adj, const
    graph_info& cc) {
    int n = ssize(adj);
    vector<vector<int>> bvt(n + cc.num_bccs);
    vector<bool> vis(cc.num_bccs, 0);
    for (int v = 0; v < n; v++) {
        for (auto [_, e_id] : adj[v]) {
            int bccid = cc.bcc_id[e_id];
            if (!vis[bccid]) {
                vis[bccid] = 1;
                bvt[v].push_back(bccid + n); //add edge between original node, and bcc node
                bvt[bccid + n].push_back(v);
            }
        }
        for (int bccid : bvt[v])
            vis[bccid - n] = 0;
    }
    return bvt;
}
```

Bridge Tree

```
//cat bridge_tree.hpp | ./hash.sh
//8eb014
#pragma once
#include "bridges_and_cuts.hpp"
//never adds multiple edges as bridges_and_cuts.hpp correctly marks them as non-bridges
//usage:
// graph_info cc = bridge_and_cut(adj, m);
// vector<vector<int>> bt = bridge_tree(adj, cc);
vector<vector<int>> bridge_tree(const vector<vector<pair<int, int>>& adj, const graph_info&
    cc) {
    vector<vector<int>> tree(cc.num_2_edge_ccs);
    for (int i = 0; i < ssize(adj); i++)
        for (auto [to, e_id] : adj[i])
            if (cc.is_bridge[e_id])
                tree[cc.two_edge_ccid[i]].push_back(cc.two_edge_ccid[to]);
    return tree;
}
```

Bridges and Cuts

```
//cat bridges_and_cuts.hpp | ./hash.sh
//3f21b9
```

```
#pragma once
//O(n+m) time & space
//2 edge cc and bcc stuff doesn't depend on each other, so delete whatever is not needed
//handles multiple edges
//
//example initialization of `adj`:
//for (int i = 0; i < m; i++) {
//    int u, v;
//    cin >> u >> v;
//    u--, v--;
//    adj[u].emplace_back(v, i);
//    adj[v].emplace_back(u, i);
//}
struct graph_info {
    //2 edge connected component stuff (e.g. components split by bridge edges)
    //    https://cp-algorithms.com/graph/bridge-searching.html
    int num_2_edge_ccs;
    vector<bool> is_bridge; //edge id -> 1 iff bridge edge
    vector<int> two_edge_ccid; //node -> id of 2 edge component (which are labeled 0, 1, ...,
        //    num_2_edge_ccs-1)
    //bi connected component stuff (e.g. components split by cut/articulation nodes)
    //    https://cp-algorithms.com/graph/cutpoints.html
    int num_bccs;
    vector<bool> is_cut; //node -> 1 iff cut node
    vector<int> bcc_id; //edge id -> id of bcc (which are labeled 0, 1, ..., num_bccs-1)
};
graph_info bridge_and_cut(const vector<vector<pair<int/*neighbor*/, int/*edge id*/>>&
    adj/*undirected graph*/, int m/*number of edges*/) {
    //stuff for both (always keep)
    int n = ssize(adj), timer = 1;
    vector<int> tin(n, 0);
    //2 edge cc stuff (delete if not needed)
    int num_2_edge_ccs = 0;
    vector<bool> is_bridge(m, 0);
    vector<int> two_edge_ccid(n), node_stack;
    node_stack.reserve(n);
    //bcc stuff (delete if not needed)
    int num_bccs = 0;
    vector<bool> is_cut(n, 0);
    vector<int> bcc_id(m), edge_stack;
    edge_stack.reserve(m);
    auto dfs = [&](auto self, int v, int p_id) -> int {
        int low = tin[v] = timer++; deg = 0;
        node_stack.push_back(v);
        for (auto [to, e_id] : adj[v]) {
            if (e_id == p_id)
                continue;
            if (!tin[to]) {
                edge_stack.push_back(e_id);
                int low_ch = self(self, to, e_id);
                if (low_ch >= tin[v]) {
                    is_cut[v] = 1;
                    while (1) {
                        int edge = edge_stack.back();
                        edge_stack.pop_back();
                        bcc_id[edge] = num_bccs;
                        if (edge == e_id)
                            break;
                    }
                }
            }
        }
    };
}
```

```
        num_bccs++;
    }
    low = min(low, low_ch);
    deg++;
} else if (tin[to] < tin[v]) {
    edge_stack.push_back(e_id);
    low = min(low, tin[to]);
}
}
if (p_id == -1)
    is_cut[v] = (deg > 1);
if (tin[v] == low) {
    if (p_id != -1)
        is_bridge[p_id] = 1;
    while (1) {
        int node = node_stack.back();
        node_stack.pop_back();
        two_edge_ccid[node] = num_2_edge_ccs;
        if (node == v)
            break;
    }
    num_2_edge_ccs++;
}
return low;
};
for (int i = 0; i < n; i++)
    if (!tin[i])
        dfs(dfs, i, -1);
return {num_2_edge_ccs, is_bridge, two_edge_ccid, num_bccs, is_cut, bcc_id};
}
```

Strongly Connected Components

```
//cat strongly_connected_components.hpp | ./hash.sh
//69acd7
#pragma once
//source: https://github.com/kth-competitive-programming/
// kactl/blob/main/content/graph/SCC.h
struct scc_info {
    int num_sccs;
    //scc's are labeled 0,1,...,`num_sccs-1`
    //scc_id[i] is the id of the scc containing node `i`
    //for each edge i -> j: scc_id[i] >= scc_id[j] (reverse topo order of scc's)
    vector<int> scc_id;
};
//NOLINTNEXTLINE(readability-identifier-naming)
scc_info SCC(const vector<vector<int>>& adj/*directed, unweighted graph*/) {
    int n = ssize(adj), timer = 1, num_sccs = 0;
    vector<int> tin(n, 0), scc_id(n, -1), node_stack;
    node_stack.reserve(n);
    auto dfs = [&](auto self, int v) -> int {
        int low = tin[v] = timer++;
        node_stack.push_back(v);
        for (int to : adj[v]) {
            if (scc_id[to] < 0)
                low = min(low, tin[to] ? tin[to] : self(self, to));
        }
        if (tin[v] == low) {

```

```
            while (1) {
                int node = node_stack.back();
                node_stack.pop_back();
                scc_id[node] = num_sccs;
                if (node == v)
                    break;
            }
            num_sccs++;
        }
        return low;
    };
    for (int i = 0; i < n; i++) {
        if (!tin[i])
            dfs(dfs, i);
    }
    return {num_sccs, scc_id};
}
```

Centroid Decomposition

```
//cat centroid_decomposition.hpp | ./hash.sh
//71e2e5
#pragma once

// Time and Space complexity are given in terms of n where n is the number of nodes in the
//   ⇔ forest
// Time complexity O(n log n)
// Space complexity O(n)

// Given an unweighted, undirected forest and a function,
// centroid_decomp runs the function on the centroid of every decomposition

// Example usage:
// centroid_decomp decomp(adj, [&](const vector<vector<int>>& adj_removed_edges, int cent) ->
//   ⇔ void {
//     ...
// });
template <typename F> struct centroid_decomp {
    vector<vector<int>> adj;
    F func;
    vector<int> sub_sz;
    centroid_decomp(const vector<vector<int>>& a_adj/*undirected forest*/, const F& a_func)
        : adj(a_adj), func(a_func), sub_sz(ssize(adj), -1) {
        for (int i = 0; i < ssize(adj); i++)
            if (sub_sz[i] == -1)
                dfs(i);
    }
    void calc_subtree_sizes(int u, int p = -1) {
        sub_sz[u] = 1;
        for (int v : adj[u]) {
            if (v == p)
                continue;
            calc_subtree_sizes(v, u);
            sub_sz[u] += sub_sz[v];
        }
    }
    void dfs(int u) {
        calc_subtree_sizes(u);

```

```
for (int p = -1, sz_root = sub_sz[u];) {
    auto big_ch = find_if(adj[u].begin(), adj[u].end(), [&](int v) -> bool {
        return v != p && 2 * sub_sz[v] > sz_root;
    });
    if (big_ch == adj[u].end())
        break;
    p = u, u = *big_ch;
}
func(adj, u);
for (int v : adj[u]) {
    //each node is adjacent to O(logn) centroids
    adj[v].erase(find(adj[v].begin(), adj[v].end(), u));
    dfs(v);
}
};
```

Frequency Table of Tree Distance

```
//cat count_paths_per_length.hpp | ./hash.sh
//182b70
#pragma once
#include "../kactl/content/numerical/FastFourierTransform.h"
#include "centroid_decomposition.hpp"
//returns array `num_paths` where `num_paths[i]` = # of paths in tree with `i` edges
//O(n log^2 n)
vector<long long> count_paths_per_length(const vector<vector<int>>& adj/*unrooted, connected
    ⇨ tree*/) {
    vector<long long> num_paths(ssize(adj), 0);
    centroid_decomp decomp(adj, [&](const vector<vector<int>>& adj_removed_edges, int cent) ->
        ⇨ void {
        vector<vector<double>> child_depths;
        for (int to : adj_removed_edges[cent]) {
            child_depths.emplace_back(1, 0.0);
            for (queue<pair<int, int>> q({{to, cent}}); !q.empty();) {
                child_depths.back().push_back(ssize(q));
                queue<pair<int, int>> new_q;
                while (!q.empty()) {
                    auto [curr, par] = q.front();
                    q.pop();
                    for (int ch : adj_removed_edges[curr]) {
                        if (ch == par)
                            continue;
                        new_q.emplace(ch, curr);
                    }
                }
                swap(q, new_q);
            }
        }
    sort(child_depths.begin(), child_depths.end(), [&](const auto & x, const auto & y) {
        return x.size() < y.size();
    });
    vector<double> total_depth(1, 1.0);
    for (const auto& cnt_depth : child_depths) {
        auto prod = conv(total_depth, cnt_depth);
        for (int i = 1; i < ssize(prod); i++)
            num_paths[i] += llround(prod[i]);
        total_depth.resize(ssize(cnt_depth), 0.0);
    }
}
```

```
for (int i = 1; i < ssize(cnt_depth); i++)
    total_depth[i] += cnt_depth[i];
}
return num_paths;
}
```

Count Paths Per Node

```
//cat count_paths_per_node.hpp | ./hash.sh
//2077d2
#pragma once
#include "centroid_decomposition.hpp"
//0-based nodes
//returns array `num_paths` where `num_paths[i]` = number of paths with k edges where node `i`
    ⇨ is on the path
//O(n log n)
vector<long long> count_paths_per_node(const vector<vector<int>>& adj/*unrooted tree*/, int k)
    ⇨ {
    vector<long long> num_paths(ssize(adj));
    centroid_decomp decomp(adj, [&](const vector<vector<int>>& adj_removed_edges, int cent) ->
        ⇨ void {
        vector<int> pre_d(1, 1), cur_d(1);
        auto dfs = [&](auto self, int u, int p, int d) -> long long {
            if (d > k)
                return 0;
            if (ssize(cur_d) <= d)
                cur_d.push_back(0);
            cur_d[d]++;
            long long cnt = 0;
            if (k - d < ssize(pre_d))
                cnt += pre_d[k - d];
            for (int v : adj_removed_edges[u])
                if (v != p)
                    cnt += self(self, v, u, d + 1);
            num_paths[u] += cnt;
            return cnt;
        };
        auto dfs_child = [&](int child) -> long long {
            long long cnt = dfs(dfs, child, cent, 1);
            pre_d.resize(ssize(cur_d));
            for (int i = 1; i < ssize(cur_d) && cur_d[i]; i++)
                pre_d[i] += cur_d[i], cur_d[i] = 0;
            return cnt;
        };
        for (int child : adj_removed_edges[cent])
            num_paths[cent] += dfs_child(child);
        pre_d = vector<int>(1);
        cur_d = vector<int>(1);
        for (auto it = adj_removed_edges[cent].rbegin(); it != adj_removed_edges[cent].rend();
            ⇨ it++)
            dfs_child(*it);
    });
    return num_paths;
}
```

```
//cat dijkstra.hpp | ./hash.sh
//aa6eda
#pragma once
//returns array `len` where `len[i]` = shortest path from node `start` to node `i`
//For example `len[start]` will always = 0
const long long INF = 1e18;
vector<long long> dijkstra(const vector<vector<pair<int, long long>>>& adj /*directed or
    ↳ undirected, weighted graph*/, int start) {
    using node = pair<long long, int>;
    vector<long long> len(ssize(adj), INF);
    len[start] = 0;
    priority_queue<node, vector<node>, greater<node>> q;
    q.emplace(0, start);
    while (!q.empty()) {
        auto [curr_len, v] = q.top();
        q.pop();
        if (len[v] < curr_len)
            continue; //important check: O(n*m) without it
        for (auto [to, weight] : adj[v])
            if (len[to] > weight + len[v]) {
                len[to] = weight + len[v];
                q.emplace(len[to], to);
            }
    }
    return len;
}
```

HLD

```
//cat hld.hpp | ./hash.sh
//d30c4a
#pragma once
//source: https://codeforces.com/blog/entry/53170
//mnemonic: Heavy Light Decomposition
//NOLINTNEXTLINE(readability-identifier-naming)
struct HLD {
    struct node {
        int sub_sz = 1, par = -1, time_in = -1, next = -1;
    };
    vector<node> tree;
    HLD(vector<vector<int>>& adj /*forest of unrooted trees*/) : tree(ssize(adj)) {
        for (int i = 0, timer = 0; i < ssize(adj); i++) {
            if (tree[i].next == -1) { //lowest indexed node in each tree becomes root
                tree[i].next = i;
                dfs1(i, adj);
                dfs2(i, adj, timer);
            }
        }
    }
    void dfs1(int v, vector<vector<int>>& adj) {
        auto par = find(adj[v].begin(), adj[v].end(), tree[v].par);
        if (par != adj[v].end())
            adj[v].erase(par);
        for (int& to : adj[v]) {
            tree[to].par = v;
            dfs1(to, adj);
            tree[v].sub_sz += tree[to].sub_sz;
            if (tree[to].sub_sz > tree[adj[v][0]].sub_sz)
```

```
                swap(to, adj[v][0]);
            }
        }
    }
    void dfs2(int v, const vector<vector<int>>& adj, int& timer) {
        tree[v].time_in = timer++;
        for (int to : adj[v]) {
            tree[to].next = (timer == tree[v].time_in + 1 ? tree[v].next : to);
            dfs2(to, adj, timer);
        }
    }
    // Returns inclusive-exclusive intervals (of time_in's) corresponding to the path between
    ↳ u and v, not necessarily in order
    // This can answer queries for "is some node `x` on some path" by checking if the
    ↳ tree[x].time_in is in any of these intervals
    // u, v must be in the same component
    vector<pair<int, int>> path(int u, int v) const {
        vector<pair<int, int>> res;
        for (; v = tree[tree[v].next].par) {
            if (tree[v].time_in < tree[u].time_in)
                swap(u, v);
            if (tree[tree[v].next].time_in <= tree[u].time_in) {
                res.emplace_back(tree[u].time_in, tree[v].time_in + 1);
                return res;
            }
            res.emplace_back(tree[tree[v].next].time_in, tree[v].time_in + 1);
        }
    }
    // Returns interval (of time_in's) corresponding to the subtree of node i
    // This can answer queries for "is some node `x` in some other node's subtree" by checking
    ↳ if tree[x].time_in is in this interval
    pair<int, int> subtree(int i) const {
        return {tree[i].time_in, tree[i].time_in + tree[i].sub_sz};
    }
    // Returns lca of nodes u and v
    // u, v must be in the same component
    int lca(int u, int v) const {
        for (; v = tree[tree[v].next].par) {
            if (tree[v].time_in < tree[u].time_in)
                swap(u, v);
            if (tree[tree[v].next].time_in <= tree[u].time_in)
                return u;
        }
    }
};
```

Hopcroft Karp

```
//cat hopcroft_karp.hpp | ./hash.sh
//5d1682
#pragma once
//source: https://github.com/foreverbell/acm-icpc-cheat-sheet/
// blob/master/src/graph-algorithm/hopcroft-karp.cpp
//Worst case O(E*sqrt(V)) but faster in practice
struct match {
    // # of edges in matching (which = size of min vertex cover by König's theorem)
    int size_of_matching;
    //an arbitrary max matching is found. For this matching:
    //if l_to_r[node_left] == -1:
```

```
// node_left is not in matching
//else:
// the edge `node_left` <=> l_to_r[node_left] is in the matching
//
//similarly for r_to_l with edge r_to_l[node_right] <=> node_right in matching if
    ↪ r_to_l[node_right] != -1
//matchings stored in l_to_r and r_to_l are the same matching
//provides way to check if any node/edge is in matching
vector<int> l_to_r, r_to_l;
//an arbitrary min vertex cover is found. For this mvc: mvc_l[node_left] is 1 iff
    ↪ node_left is in the min vertex cover (same for mvc_r)
//if mvc_l[node_left] is 0, then node_left is in the corresponding maximal independent set
vector<bool> mvc_l, mvc_r;
};
//Think of the bipartite graph as having a left side (with size lsz) and a right side (with
    ↪ size rsz).
//Nodes on left side are indexed 0,1,...,lsz-1
//Nodes on right side are indexed 0,1,...,rsz-1
//
//`adj` is like a directed adjacency list containing edges from left side -> right side:
//To initialize `adj`: For every edge node_left <=> node_right, do:
    ↪ adj[node_left].push_back(node_right)
match hopcroft_karp(const vector<vector<int>>& adj/*bipartite graph*/, int rsz/*number of
    ↪ nodes on right side*/) {
    int size_of_matching = 0, lsz = ssize(adj);
    vector<int> l_to_r(lsz, -1), r_to_l(rsz, -1);
    while (1) {
        queue<int> q;
        vector<int> level(lsz, -1);
        for (int i = 0; i < lsz; i++)
            if (l_to_r[i] == -1)
                level[i] = 0, q.push(i);
        bool found = 0;
        vector<bool> mvc_l(lsz, 1), mvc_r(rsz, 0);
        while (!q.empty()) {
            int u = q.front();
            q.pop();
            mvc_l[u] = 0;
            for (int x : adj[u]) {
                mvc_r[x] = 1;
                int v = r_to_l[x];
                if (v == -1)
                    found = 1;
                else if (level[v] == -1) {
                    level[v] = level[u] + 1;
                    q.push(v);
                }
            }
        }
        if (!found)
            return {size_of_matching, l_to_r, r_to_l, mvc_l, mvc_r};
        auto dfs = [&](auto self, int u) -> bool {
            for (int x : adj[u]) {
                int v = r_to_l[x];
                if (v == -1 || (level[u] + 1 == level[v] && self(self, v))) {
                    l_to_r[u] = x;
                    r_to_l[x] = u;
                    return 1;
                }
            }
        };
    }
```

```
    }
    level[u] = 1e9; //acts as visited array
    return 0;
};
for (int i = 0; i < lsz; i++)
    size_of_matching += (l_to_r[i] == -1 && dfs(dfs, i));
}
}
```

Kth Node on Path

```
//cat kth_node_on_path.hpp | ./hash.sh
//c59307
#pragma once
#include "lca.hpp"
//consider path {u, u's par, ..., LCA(u,v), ..., v's par, v}. This returns the node at index k
//assumes 0 <= k <= number of edges on path from u to v
// u, v must be in the same component
//
//To loop over all nodes on a path from u to v:
//kth_node_on_path kpath(adj);
//for (int i = 0; i <= kpath.lca.dist_edges(u, v); i++) {
//    int node = kpath.query(u, v, i);
//    ...
//}
struct kth_node_on_path {
    LCA lca;
    kth_node_on_path(const vector<vector<pair<int, long long>>&& adj/*forest of weighted
        ↪ trees*/) : lca(adj) {}
    int query(int u, int v, int k) const {
        int lca_uv = lca.get_lca(u, v);
        int u_lca = lca.tree[u].depth - lca.tree[lca_uv].depth;
        int v_lca = lca.tree[v].depth - lca.tree[lca_uv].depth;
        assert(0 <= k && k <= u_lca + v_lca);
        return k <= u_lca ? lca.kth_par(u, k) : lca.kth_par(v, u_lca + v_lca - k);
    }
};
```

LCA

```
//cat lca.hpp | ./hash.sh
//b28532
#pragma once
//https://codeforces.com/blog/entry/74847
//mnemonic: Least/Lowest Common Ancestor
//NOLINTNEXTLINE(readability-identifier-naming)
struct LCA {
    struct node {
        int jmp = -1, jmp_edges = 0, par = -1, depth = 0;
        long long dist = 0LL;
    };
    vector<node> tree;
    LCA(const vector<vector<pair<int, long long>>&& adj/*forest of weighted trees*/) :
        ↪ tree(ssize(adj)) {
        for (int i = 0; i < ssize(adj); i++) {
            if (tree[i].jmp == -1) { //lowest indexed node in each tree becomes root
                tree[i].jmp = i;
                dfs(i, adj);
            }
        }
    }
```



```
    }
}
}
void dfs(int v, const vector<vector<pair<int, long long>>& adj) {
    int jmp, jmp_edges;
    if (tree[v].jmp != v && tree[v].jmp_edges == tree[tree[v].jmp].jmp_edges)
        jmp = tree[tree[v].jmp].jmp, jmp_edges = 2 * tree[v].jmp_edges + 1;
    else
        jmp = v, jmp_edges = 1;
    for (auto [ch, w] : adj[v]) {
        if (ch == tree[v].par)
            continue;
        tree[ch] = {
            jmp,
            jmp_edges,
            v,
            1 + tree[v].depth,
            w + tree[v].dist
        };
        dfs(ch, adj);
    }
}
//traverse up k edges in O(log(k)). So with k=1 this returns `v`'s parent
int kth_par(int v, int k) const {
    k = min(k, tree[v].depth);
    while (k > 0) {
        if (tree[v].jmp_edges <= k) {
            k -= tree[v].jmp_edges;
            v = tree[v].jmp;
        } else {
            k--;
            v = tree[v].par;
        }
    }
    return v;
}
// x, y must be in the same component
int get_lca(int x, int y) const {
    if (tree[x].depth < tree[y].depth)
        swap(x, y);
    x = kth_par(x, tree[x].depth - tree[y].depth);
    while (x != y) {
        if (tree[x].jmp != tree[y].jmp)
            x = tree[x].jmp, y = tree[y].jmp;
        else
            x = tree[x].par, y = tree[y].par;
    }
    return x;
}
int dist_edges(int x, int y) const {
    return tree[x].depth + tree[y].depth - 2 * tree[get_lca(x, y)].depth;
}
long long dist_weight(int x, int y) const {
    return tree[x].dist + tree[y].dist - 2 * tree[get_lca(x, y)].dist;
}
};
```

Rooted Tree Isomorphism

```
//cat subtree_isomorphism.hpp | ./hash.sh
//455aef
#pragma once

// Complexity given in terms of n where n is the number of nodes in the forest
// Time complexity O(n log n)
// Space complexity O(n)

// Given an undirected or directed rooted forest
// subtree_iso classifies each rooted subtree
// minimum label of each tree becomes root
struct iso_info {
    int num_distinct_subtrees; //0 <= id[i] < num_distinct_subtrees for all i
    vector<int> id; //id[u] == id[v] iff subtree u is isomorphic to subtree v
};

iso_info subtree_iso(const vector<vector<int>>& adj) {
    vector<int> id(ssize(adj), -1);
    map<vector<int>, int> hashes;
    auto dfs = [&](auto self, int u, int p) -> int {
        vector<int> ch_ids;
        ch_ids.reserve(ssize(adj[u]));
        for (int v : adj[u]) {
            if (v != p)
                ch_ids.push_back(self(self, v, u));
        }
        sort(ch_ids.begin(), ch_ids.end());
        auto it = hashes.find(ch_ids);
        if (it == hashes.end())
            return id[u] = hashes[ch_ids] = ssize(hashes);
        return id[u] = it->second;
    };
    for (int i = 0; i < ssize(adj); i++)
        if (id[i] == -1)
            dfs(dfs, i, i);
    return {ssize(hashes), id};
}
```

MATH

Derangements

```
//cat derangements.hpp | ./hash.sh
//64d325
#pragma once
//https://oeis.org/A000166
//
//for a permutation of size i:
//there are (i-1) places to move 0 to not be at index 0. Let's say we moved 0 to index j (j>0).
//If we move value j to index 0 (forming a cycle of length 2), then there are dp[i-2]
//    ↳ derangements of the remaining i-2 elements
//else there are dp[i-1] derangements of the remaining i-1 elements (including j)
vector<long long> derangements(int n, long long mod) {
    vector<long long> dp(n, 0);
    dp[0] = 1;
    for (int i = 2; i < n; i++)
```

```
    dp[i] = (i - 1) * (dp[i - 1] + dp[i - 2]) % mod;
    return dp;
}
```

Binary Exponentiation MOD

```
//cat binary_exponentiation_mod.hpp | ./hash.sh
//92a3ef
#pragma once
//returns (base^pw)%mod in O(log(pw)), but returns 1 for 0^0
//
//What if base doesn't fit in long long?
//Since (base^pw)%mod == ((base%mod)^pw)%mod we can calculate base under mod of `mod`
//
//What if pw doesn't fit in long long?
//assuming mod is prime:
//((base^pw)%mod == (base^(pw%(mod-1))))%mod (from Fermat's little theorem)
//so calculate pw under mod of `mod-1`
//note `mod-1` is not prime, so you need to be able to calculate `pw%(mod-1)` without division
long long bin_exp(long long base, long long pw, long long mod) {
    assert(0 <= pw && 0 <= base && 1 <= mod);
    long long res = 1;
    base %= mod;
    while (pw > 0) {
        if (pw & 1)
            res = res * base % mod;
        base = base * base % mod;
        pw >>= 1;
    }
    return res;
}
```

Fibonacci

```
//cat fibonacci.hpp | ./hash.sh
//78a41f
#pragma once
//https://codeforces.com/blog/entry/14516
//O(log(n))
unordered_map<long long, long long> table;
long long fib(long long n, long long mod) {
    if (n < 2)
        return 1;
    if (table.find(n) != table.end())
        return table[n];
    table[n] = (fib((n + 1) / 2, mod) * fib(n / 2, mod) + fib((n - 1) / 2, mod) * fib((n - 2) / 2, mod)) % mod;
    return table[n];
}
```

Matrix Multiplication

```
//cat matrix_mult.hpp | ./hash.sh
//910018
#pragma once
```

```
// source: https://codeforces.com/blog/entry/80195

// generic matrix multiplication (not overflow safe)
// will RTE if the given matrices are not compatible
// Time: O(n * m * inner)
// Space: O(n * m)

template <typename T> vector<vector<T>> operator * (const vector<vector<T>>& a, const
    ⇨ vector<vector<T>>& b) {
    assert(ssize(a[0]) == ssize(b));
    int n = ssize(a), m = ssize(b[0]), inner = ssize(b);
    vector<vector<T>> c(n, vector<T>(m));
    for (int i = 0; i < n; i++)
        for (int k = 0; k < inner; k++)
            for (int j = 0; j < m; j++)
                c[i][j] += a[i][k] * b[k][j];

    return c;
}
```

Mobius Inversion

```
//cat mobius_inversion.hpp | ./hash.sh
//811515
#pragma once
//mobius[i] = 0 iff there exists a prime p s.t. i%(p^2)=0
//mobius[i] = -1 iff i has an odd number of distinct prime factors
//mobius[i] = 1 iff i has an even number of distinct prime factors
const int N = 1e6 + 10;
int mobius[N];
void calc_mobius() {
    mobius[1] = 1;
    for (int i = 1; i < N; i++)
        for (int j = i + i; j < N; j += i)
            mobius[j] -= mobius[i];
}
```

N Choose K MOD

```
//cat n_choose_k_mod.hpp | ./hash.sh
//c7fb35
#pragma once
//for mod inverse
#include "binary_exponentiation_mod.hpp"
// usage:
//     n_choose_k nk(n, 1e9+7) to use `choose`, `inv` with inputs strictly < n
// or:
//     n_choose_k nk(mod, mod) to use `choose_lucas` with arbitrarily large inputs
struct n_choose_k {
    n_choose_k(int n, long long a_mod) : mod(a_mod), fact(n, 1), inv_fact(n, 1) {
        //this implementation doesn't work if n > mod because n! % mod = 0 when n >= mod. So
        ⇨ `inv_fact` array will be all 0's
        assert(max(n, 2) <= mod);
        //assert mod is prime. mod is intended to fit inside an int so that
        //multiplications fit in a longlong before being modded down. So this
        //will take sqrt(2^31) time
        for (int i = 2; i * i <= mod; i++)
            assert(mod % i);
    }
```

```
for (int i = 2; i < n; i++)
    fact[i] = fact[i - 1] * i % mod;
inv_fact.back() = bin_exp(fact.back(), mod - 2, mod);
for (int i = n - 2; i >= 2; i--)
    inv_fact[i] = inv_fact[i + 1] * (i + 1) % mod;
}
//classic n choose k
//fails when n >= mod
long long choose(int n, int k) const {
    if (k < 0 || k > n)
        return 0;
    //now we know 0 <= k <= n so 0 <= n
    return fact[n] * inv_fact[k] % mod * inv_fact[n - k] % mod;
}
//lucas theorem to calculate n choose k in O(log(k))
//need to calculate all factorials in range [0,mod), so O(mod) time&space, so need
    ↪ smallish prime mod (< 1e6 maybe)
//handles n >= mod correctly
long long choose_lucas(long long n, long long k) const {
    if (k < 0 || k > n)
        return 0;
    if (k == 0 || k == n)
        return 1;
    return choose_lucas(n / mod, k / mod) * choose(int(n % mod), int(k % mod)) % mod;
}
//returns x such that x * n % mod == 1
long long inv(int n) const {
    assert(1 <= n); //don't divide by 0 :)
    return fact[n - 1] * inv_fact[n] % mod;
}
long long mod;
vector<long long> fact, inv_fact;
};
```

Partitions

```
//cat partitions.hpp | ./hash.sh
//e7ae42
#pragma once
//https://oeis.org/A000041
//O(n sqrt n) time, but small-ish constant factor (there does exist a O(n log n) solution as
    ↪ well)
vector<long long> partitions(int n, long long mod) {
    vector<long long> dp(n, 1);
    for (int i = 1; i < n; i++) {
        long long sum = 0;
        for (int j = 1, pent = 1, sign = 1; pent <= i; j++, pent += 3 * j - 2, sign = -sign) {
            if (pent + j <= i)
                sum += dp[i - pent - j] * sign + mod;
            sum += dp[i - pent] * sign + mod;
        }
        dp[i] = sum % mod;
    }
    return dp;
}
```

Prime Sieve

```
//cat prime_sieve.hpp | ./hash.sh
//25a877
#pragma once
bool is_prime(int val, const vector<int>& sieve) {
    assert(val < ssize(sieve));
    return val >= 2 && sieve[val] == val;
}
vector<int> get_prime_factors(int val, const vector<int>& sieve) {
    assert(val < ssize(sieve));
    vector<int> factors;
    while (val > 1) {
        int p = sieve[val];
        factors.push_back(p);
        val /= p;
    }
    return factors;
}
//returns array `sieve` where `sieve[i]` = some prime factor of `i`
vector<int> get_sieve(int n) {
    vector<int> sieve(n);
    iota(sieve.begin(), sieve.end(), 0);
    for (int i = 2; i * i < n; i++)
        if (sieve[i] == i)
            for (int j = i * i; j < n; j += i)
                sieve[j] = i;
    return sieve;
}
```

Row Reduce

```
//cat row_reduce.hpp | ./hash.sh
//c812f1
#pragma once
//for mod inverse
#include "binary_exponentiation_mod.hpp"
//First `cols` columns of mat represents a matrix to be left in reduced row echelon form
//Row operations will be performed to all later columns
//
//example usage:
// auto [rank, det] = row_reduce(mat, ssize(mat[0]), mod) //row reduce matrix with no extra
    ↪ columns
pair<int/*rank*/, long long/*determinant*/> row_reduce(vector<vector<long long>>& mat, int
    ↪ cols, long long mod) {
    int n = ssize(mat), m = ssize(mat[0]), rank = 0;
    long long det = 1;
    assert(cols <= m);
    for (int col = 0; col < cols && rank < n; col++) {
        //find arbitrary pivot and swap pivot to current row
        for (int i = rank; i < n; i++)
            if (mat[i][col] != 0) {
                if (rank != i)
                    det = det == 0 ? 0 : mod - det;
                swap(mat[i], mat[rank]);
                break;
            }
        if (mat[rank][col] == 0) {
            det = 0;
            continue;
        }
    }
}
```

```
    }
    det = det * mat[rank][col] % mod;
    //make pivot 1 by dividing row by inverse of pivot
    long long a_inv = bin_exp(mat[rank][col], mod - 2, mod);
    for (int j = 0; j < m; j++)
        mat[rank][j] = mat[rank][j] * a_inv % mod;
    //zero-out all numbers above & below pivot
    for (int i = 0; i < n; i++)
        if (i != rank && mat[i][col] != 0) {
            long long val = mat[i][col];
            for (int j = 0; j < m; j++) {
                mat[i][j] -= mat[rank][j] * val % mod;
                if (mat[i][j] < 0)
                    mat[i][j] += mod;
            }
        }
    rank++;
}
assert(rank <= min(n, cols));
return {rank, det};
}
```

Solve Linear Equations MOD

```
//cat solve_linear_mod.hpp | ./hash.sh
//0a302e
#pragma once
#include "row_reduce.hpp"
struct matrix_info {
    int rank;
    long long det;
    vector<long long> x;
};
//Solves mat * x = b under prime mod.
//mat is a n (rows) by m (cols) matrix, b is a length n column vector, x is a length m vector.
//assumes n,m >= 1, else RTE
//Returns rank of mat, determinant of mat, and x (solution vector to mat * x = b).
//x is empty if no solution. If rank < m, there are multiple solutions and an arbitrary one is
    ↪ returned.
//Leaves mat in reduced row echelon form (unlike kactl) with b appended.
//Trick: Number of unique solutions = (size of domain) ^ (# of free variables).
//# of free variables is generally equivalent to n - rank.
//O(n * m * min(n,m))
matrix_info solve_linear_mod(vector<vector<long long>>& mat, const vector<long long>& b, long
    ↪ long mod) {
    assert(ssize(mat) == ssize(b));
    int n = ssize(mat), m = ssize(mat[0]);
    for (int i = 0; i < n; i++)
        mat[i].push_back(b[i]);
    auto [rank, det] = row_reduce(mat, m, mod); //row reduce not including the last column
    //check if solution exists
    for (int i = rank; i < n; i++) {
        if (mat[i].back() != 0)
            return {rank, det, {}}; //no solution exists
    }
    //initialize solution vector (`x`) from row-reduced matrix
    vector<long long> x(m, 0);
    for (int i = 0, j = 0; i < rank; i++) {
```

```
        while (mat[i][j] == 0)
            j++; //find pivot column
        x[j] = mat[i].back();
    }
    return {rank, det, x};
}
```

Euler’s Totient Phi Function

```
//cat totient.hpp | ./hash.sh
//36bd41
#pragma once
//Euler's totient function counts the positive integers
//up to a given integer n that are relatively prime to n.
//
//To improve, pre-calc prime factors or use Pollard-rho to find prime factors.
int totient(int n) {
    int res = n;
    for (int i = 2; i * i <= n; i++) {
        if (n % i == 0) {
            while (n % i == 0)
                n /= i;
            res -= res / i;
        }
    }
    if (n > 1)
        res -= res / n;
    return res;
}
```

Tetration MOD

```
//cat tetration_mod.hpp | ./hash.sh
//e2153e
#pragma once
#include "binary_exponentiation_mod.hpp"
#include "totient.hpp"
//to calculate (base^pw)%mod with huge pw and non-prime mod:
//let t = totient(mod)
//if log2(mod) <= pw then (base^pw)%mod == (base^(t+(pw/t)))%mod
//source: https://cp-algorithms.com/algebra/phi-function.html#generalization
//
//returns base ^ (base ^ (base ^ ... )) % mod, where the height of the tower is pw
long long tetration(long long base, long long pw, long long mod) {
    if (mod == 1)
        return 0;
    if (base == 0)
        return (pw + 1) % 2 % mod;
    if (base == 1 || pw == 0)
        return 1;
    if (pw == 1)
        return base % mod;
    if (base == 2 && pw == 2)
        return 4 % mod;
    if (base == 2 && pw == 3)
        return 16 % mod;
    if (base == 3 && pw == 2)
```

```
    return 27 % mod;
//need enough base cases such that the following is true
//log2(mod) <= tetration(base, pw - 1) (before modding)
int t = totient(int(mod));
long long exp = tetration(base, pw - 1, t);
return bin_exp(base, exp + t, mod);
}
```

MISC

Cartesian Tree

```
//cat cartesian_tree.hpp | ./hash.sh
//204c45
#pragma once
#include "monotonic_stack.hpp"
//min cartesian tree
vector<int> cartesian_tree(const vector<int>& arr) {
    int n = ssize(arr);
    auto rv /*reverse*/ = [&](int i) -> int {
        return n - 1 - i;
    };
    vector<int> left = monotonic_stack<int>(arr, greater());
    vector<int> right = monotonic_stack<int>(vector<int>(arr.rbegin(), arr.rend()), greater());
    vector<int> par(n);
    for (int i = 0; i < n; i++) {
        int le = left[i], ri = rv(right[rv(i)]);
        if (le >= 0 && ri < n)
            par[i] = arr[le] > arr[ri] ? le : ri;
        else if (le >= 0)
            par[i] = le;
        else if (ri < n)
            par[i] = ri;
        else
            par[i] = i;
    }
    return par;
}
```

Count Rectangles

```
//cat count_rectangles.hpp | ./hash.sh
//8990f7
#pragma once
#include "monotonic_stack.hpp"
//given a 2D boolean matrix, calculate cnt[i][j]
//cnt[i][j] = the number of times an i-by-j rectangle appears in the matrix such that all i*j
//    -> cells in the rectangle are 1
//Note cnt[0][j] and cnt[i][0] will contain garbage values
//O(n*m)
vector<vector<int>>> count_rectangles(const vector<vector<bool>>& grid) {
    int n = ssize(grid), m = ssize(grid[0]);
    vector<vector<int>>> cnt(n + 1, vector<int>(m + 1, 0));
    vector<int> arr(m, 0);
    auto rv /*reverse*/ = [&](int j) -> int {
        return m - 1 - j;
    };
    for (int i = 0; i < n; i++) {
        vector<int> left = monotonic_stack<int>(arr, greater());
        vector<int> right = monotonic_stack<int>(vector<int>(arr.rbegin(), arr.rend()), greater());
        vector<int> par(m, 0);
        for (int j = 0; j < m; j++) {
            int le = left[j], ri = rv(right[rv(j)]);
            if (le >= 0 && ri < m)
                par[j] = arr[le] > arr[ri] ? le : ri;
            else if (le >= 0)
                par[j] = le;
            else if (ri < m)
                par[j] = ri;
            else
                par[j] = j;
        }
        for (int k = 0; k <= par[j]; k++)
            cnt[i + 1][k] += cnt[i][k];
    }
    return cnt;
}
```

```
};
for (int i = 0; i < n; i++) {
    for (int j = 0; j < m; j++)
        arr[j] = grid[i][j] * (arr[j] + 1);
    vector<int> left = monotonic_stack<int>(arr, greater());
    vector<int> right = monotonic_stack<int>(vector<int>(arr.rbegin(), arr.rend()),
        -> greater_equal());
    for (int j = 0; j < m; j++) {
        int le = j - left[j] - 1, ri = rv(right[rv(j)]) - j - 1;
        cnt[arr[j]][le + ri + 1]++;
        cnt[arr[j]][le]--;
        cnt[arr[j]][ri]--;
    }
}
for (int i = 1; i <= n; i++)
    for (int k = 0; k < 2; k++)
        for (int j = m - 1; j >= 1; j--)
            cnt[i][j] += cnt[i][j + 1];
for (int j = 1; j <= m; j++)
    for (int i = n - 1; i >= 1; i--)
        cnt[i][j] += cnt[i + 1][j];
return cnt;
}
```

Max Rectangle in Histogram

```
//cat max_rect_histogram.hpp | ./hash.sh
//95288f
#pragma once
#include "monotonic_stack.hpp"
long long max_rect_histogram(const vector<int>& arr) {
    auto rv /*reverse*/ = [&](int i) -> int {
        return ssize(arr) - 1 - i;
    };
    vector<int> left = monotonic_stack<int>(arr, greater_equal());
    vector<int> right = monotonic_stack<int>(vector<int>(arr.rbegin(), arr.rend()),
        -> greater_equal());
    long long max_area = 0;
    for (int i = 0; i < ssize(arr); i++) {
        int le = left[i], ri = rv(right[rv(i)]); //arr[i] is the max of range (le, ri)
        max_area = max(max_area, 1LL * arr[i] * (ri - le - 1));
    }
    return max_area;
}
```

Monotonic Stack

```
//cat monotonic_stack.hpp | ./hash.sh
//35c95c
#pragma once
//usages:
// vector<int> le = monotonic_stack<int>(arr, less()); //or replace `less` with: less_equal,
//    -> greater, greater_equal
// vector<int> le = monotonic_stack<int>(arr, [&](int x, int y) {return x < y;});
//
//returns array `le` where `le[i]` = max index such that:
// `le[i]` < i && !op(arr[le[i]], arr[i])
```

```
//or -1 if no index exists
//O(n)
template <typename T> vector<int> monotonic_stack(const vector<T>& arr, const
    ↪ function<bool(const T&, const T&)>& op) {
    vector<int> le(ssize(arr));
    for (int i = 0; i < ssize(arr); i++) {
        le[i] = i - 1;
        while (le[i] >= 0 && op(arr[le[i]], arr[i]))
            le[i] = le[le[i]];
    }
    return le;
}
```

GCD Convolution

```
//cat gcd_convolution.hpp | ./hash.sh
//d92c44
#pragma once
// c[k] = the sum for all pairs where gcd(i,j) == k of a[i] * b[j]
template<int MOD> vector<int> gcd_convolution(const vector<int>& a, const vector<int>& b) {
    assert(ssize(a) == ssize(b));
    int n = ssize(a);
    vector<int> c(n);
    for (int gcd = n - 1; gcd >= 1; gcd--) {
        int sum_a = 0, sum_b = 0;
        for (int i = gcd; i < n; i += gcd) {
            sum_a = (sum_a + a[i]) % MOD, sum_b = (sum_b + b[i]) % MOD;
            c[gcd] = (c[gcd] - c[i] + MOD) % MOD;
        }
        c[gcd] = int((c[gcd] + 1LL * sum_a * sum_b) % MOD);
    }
    return c;
}
```

Iterate Chooses

```
//cat iterate_chooses.hpp | ./hash.sh
//c79083
#pragma once

// source: https://github.com/kth-competitive-programming/
// kactl/blob/main/content/various/chapter.tex
// iterates all bitmasks of size n with k bits set
// Time Complexity: O(n choose k)
// Space Complexity: O(1)

int next_subset(int mask) {
    int c = mask & -mask, r = mask + c;
    return r | (((r ^ mask) >> 2) / c);
}

void iterate_chooses(int n, int k, const function<void(int)>& func) {
    for (int mask = (1 << k) - 1; mask < (1 << n); mask = next_subset(mask))
        func(mask);
}
```

Iterate Submasks

```
//cat iterate_submasks.hpp | ./hash.sh
//084c05
#pragma once

// iterates all submasks of mask
// Time Complexity: O(3^n) to iterate every submask of every mask of size n
// Space Complexity: O(1)

void iterate_submasks(int mask, const function<void(int)>& func) {
    for (int submask = mask; submask; submask = (submask - 1) & mask)
        func(submask);
}
```

Iterate Supermasks

```
//cat iterate_supermasks.hpp | ./hash.sh
//76b38f
#pragma once

// iterates all supermasks of mask
// Time Complexity: O(3^n) to iterate every supermask of every mask of size n
// Space Complexity: O(1)

void iterate_supermasks(int mask, int n, const function<void(int)>& func) {
    for (int supermask = mask; supermask < (1 << n); supermask = (supermask + 1) | mask)
        func(supermask);
}
```

Number of Distinct Subsequences DP

```
//cat num_distinct_subsequences.hpp | ./hash.sh
//d94bdc
#pragma once
//returns number of distinct subsequences
//the empty subsequence is counted
int num_subsequences(const vector<int>& arr, int mod) {
    vector<int> dp(ssize(arr) + 1, 1);
    map<int, int> last;
    for (int i = 0; i < ssize(arr); i++) {
        int& curr = dp[i + 1] = 2 * dp[i];
        if (curr >= mod)
            curr -= mod;
        auto it = last.find(arr[i]);
        if (it != last.end()) {
            curr -= dp[it->second];
            if (curr < 0)
                curr += mod;
            it->second = i;
        } else
            last[arr[i]] = i;
    }
    return dp.back();
}
```

PBDS

```
//cat policy_based_data_structures.hpp | ./hash.sh
//a777d7
#pragma once
//place these includes *before* the `#define int long long` else compile error
//not using <bits/extc++.h> as it compile errors on codeforces c++20 compiler
#include <ext/pb_ds/assoc_container.hpp>
#include <ext/pb_ds/tree_policy.hpp>
using namespace __gnu_pbds;
//BST with extra functions https://codeforces.com/blog/entry/11080
//order_of_key - # of elements *strictly* less than given element
//find_by_order - find kth largest element, k is 0 based so find_by_order(0) returns min
    ↪ element
template <typename T> using indexed_set = tree<T, null_type, less<T>, rb_tree_tag,
    ↪ tree_order_statistics_node_update>;
//example initialization:
indexed_set<pair<long long, int>> is;
//hash table (apparently faster than unordered_map): https://codeforces.com/blog/entry/60737
//example initialization:
gp_hash_table<string, long long> ht;
```

Random

```
//cat random.hpp | ./hash.sh
//ab9111
#pragma once

//source: https://codeforces.com/blog/entry/61675

mt19937 rng(chrono::steady_clock::now().time_since_epoch().count());

//intended types: int, unsigned, long long
//returns a random number in range [le, ri)
template <typename T> inline T get_rand(T le, T ri) {
    assert(le < ri);
    return uniform_int_distribution<T>(le, ri - 1)(rng);
}

//vector<int> a;
//shuffle(a.begin(), a.end(), rng);
```

Safe Hash

```
//cat safe_hash.hpp | ./hash.sh
//d9ea53
#pragma once
//source: https://codeforces.com/blog/entry/62393
struct custom_hash {
    static uint64_t splitmix64(uint64_t x) {
        // http://xorshift.di.unimi.it/splitmix64.c
        x += 0x9e3779b97f4a7c15;
        x = (x ^ (x >> 30)) * 0xbf58476d1ce4e5b9;
        x = (x ^ (x >> 27)) * 0x94d049bb133111eb;
        return x ^ (x >> 31);
    }
    size_t operator()(uint64_t x) const {
        static const uint64_t FIXED_RANDOM =
            ↪ chrono::steady_clock::now().time_since_epoch().count();
```

```
        return splitmix64(x + FIXED_RANDOM);
    }
};
//usage:
unordered_map<long long, int, custom_hash> safe_map;
#include "policy_based_data_structures.hpp"
gp_hash_table<long long, int, custom_hash> safe_hash_table;
```

RANGE DATA STRUCTURES

Number Distinct Elements

```
//cat distinct_query.hpp | ./hash.sh
//79534f
#pragma once
//source: https://cp-algorithms.com/data_structures/segment_tree.html
// #preserving-the-history-of-its-values-persistent-segment-tree
//works with negatives
//O(n log n) time and space
struct distinct_query {
    struct node {
        int sum;
        int lch, rch; //children, indexes into `tree`
        node(int a_sum, int a_lch, int a_rch) : sum(a_sum), lch(a_lch), rch(a_rch) {}
    };
    const int N;
    vector<int> roots;
    deque<node> tree;
    distinct_query(const vector<int>& arr) : N(ssize(arr)), roots(N + 1, 0) {
        tree.emplace_back(0, 0, 0); //acts as null
        map<int, int> last_idx;
        for (int i = 0; i < N; i++) {
            roots[i + 1] = update(roots[i], 0, N, last_idx[arr[i]]);
            last_idx[arr[i]] = i + 1;
        }
    }
    int update(int v, int tl, int tr, int idx) {
        if (tr - tl == 1) {
            tree.emplace_back(tree[v].sum + 1, 0, 0);
            return ssize(tree) - 1;
        }
        int tm = tl + (tr - tl) / 2;
        int lch = tree[v].lch;
        int rch = tree[v].rch;
        if (idx < tm)
            lch = update(lch, tl, tm, idx);
        else
            rch = update(rch, tm, tr, idx);
        tree.emplace_back(tree[lch].sum + tree[rch].sum, lch, rch);
        return ssize(tree) - 1;
    }
    //returns number of distinct elements in range [le,ri)
    int query(int le, int ri) const {
        assert(0 <= le && le <= ri && ri <= N);
        return query(roots[le], roots[ri], 0, N, le + 1);
    }
    int query(int vl, int vr, int tl, int tr, int idx) const {
```

```

    if (tree[vr].sum == 0 || idx <= tl)
        return 0;
    if (tr <= idx)
        return tree[vr].sum - tree[vl].sum;
    int tm = tl + (tr - tl) / 2;
    return query(tree[vl].lch, tree[vr].lch, tl, tm, idx) +
           query(tree[vl].rch, tree[vr].rch, tm, tr, idx);
}
};
```

Implicit Lazy Segment Tree

```

//cat implicit_seg_tree.hpp | ./hash.sh
//ab5eb9
#pragma once
//example initialization:
// implicit_seg_tree<10'000'000> ist(le, ri);
template <int N> struct implicit_seg_tree {
    using dt = array<long long, 2>; //min, number of mins
    using ch = long long;
    static dt combine(const dt& le, const dt& ri) {
        if (le[0] == ri[0])
            return {le[0], le[1] + ri[1]};
        return min(le, ri);
    }
    static constexpr dt UNIT{(long long)1e18, 0LL};
    struct node {
        dt val;
        ch lazy = 0;
        int lch = -1, rch = -1; // children, indexes into `tree`, -1 for null
    } tree[N];
    int ptr = 0, root_l, root_r; // [root_l, root_r) defines range of root node; handles
    ⇨ negatives
    implicit_seg_tree(int le, int ri) : root_l(le), root_r(ri) {
        tree[ptr++].val = {0, ri - le};
    }
    void apply(int v, ch add) {
        tree[v].val[0] += add;
        tree[v].lazy += add;
    }
    void push(int v, int tl, int tr) {
        if (tr - tl > 1 && tree[v].lch == -1) {
            int tm = tl + (tr - tl) / 2;
            assert(ptr + 1 < N);
            tree[v].lch = ptr;
            tree[ptr++].val = {0, tm - tl};
            tree[v].rch = ptr;
            tree[ptr++].val = {0, tr - tm};
        }
        if (tree[v].lazy) {
            apply(tree[v].lch, tree[v].lazy);
            apply(tree[v].rch, tree[v].lazy);
            tree[v].lazy = 0;
        }
    }
    //update range [le,ri)
    void update(int le, int ri, ch add) {
        update(0, root_l, root_r, le, ri, add);
    }
};
```

```

    }
    void update(int v, int tl, int tr, int le, int ri, ch add) {
        if (ri <= tl || tr <= le)
            return;
        if (le <= tl && tr <= ri)
            return apply(v, add);
        push(v, tl, tr);
        int tm = tl + (tr - tl) / 2;
        update(tree[v].lch, tl, tm, le, ri, add);
        update(tree[v].rch, tm, tr, le, ri, add);
        tree[v].val = combine(tree[tree[v].lch].val,
                              tree[tree[v].rch].val);
    }
    //query range [le,ri)
    dt query(int le, int ri) {
        return query(0, root_l, root_r, le, ri);
    }
    dt query(int v, int tl, int tr, int le, int ri) {
        if (ri <= tl || tr <= le)
            return UNIT;
        if (le <= tl && tr <= ri)
            return tree[v].val;
        push(v, tl, tr);
        int tm = tl + (tr - tl) / 2;
        return combine(query(tree[v].lch, tl, tm, le, ri),
                       query(tree[v].rch, tm, tr, le, ri));
    }
};
```

Kth Smallest

```

//cat kth_smallest.hpp | ./hash.sh
//b86dd0
#pragma once
//source: https://cp-algorithms.com/data_structures/segment_tree.html
// #preserving-the-history-of-its-values-persistent-segment-tree
struct kth_smallest {
    struct node {
        int sum;
        int lch, rch; //children, indexes into `tree`
        node(int a_sum, int a_lch, int a_rch) : sum(a_sum), lch(a_lch), rch(a_rch) {}
    };
    int mn = INT_MAX, mx = INT_MIN;
    vector<int> roots;
    deque<node> tree;
    kth_smallest(const vector<int>& arr) : roots(ssize(arr) + 1, 0) {
        tree.emplace_back(0, 0, 0); //acts as null
        for (int val : arr)
            mn = min(mn, val), mx = max(mx, val + 1);
        for (int i = 0; i < ssize(arr); i++)
            roots[i + 1] = update(roots[i], mn, mx, arr[i]);
    }
    int update(int v, int tl, int tr, int idx) {
        if (tr - tl == 1) {
            tree.emplace_back(tree[v].sum + 1, 0, 0);
            return ssize(tree) - 1;
        }
        int tm = tl + (tr - tl) / 2;
```



```
int lch = tree[v].lch;
int rch = tree[v].rch;
if (idx < tm)
    lch = update(lch, tl, tm, idx);
else
    rch = update(rch, tm, tr, idx);
tree.emplace_back(tree[lch].sum + tree[rch].sum, lch, rch);
return ssize(tree) - 1;
}
/* find (k+1)th smallest number in range [le, ri)
 * k is 0-based, so query(le,ri,0) returns the min
 */
int query(int le, int ri, int k) const {
    assert(0 <= k && k < ri - le); //note this condition implies le < ri
    assert(0 <= le && ri < ssize(root));
    return query(root[l], root[r], mn, mx, k);
}
int query(int vl, int vr, int tl, int tr, int k) const {
    assert(tree[vr].sum > tree[vl].sum);
    if (tr - tl == 1)
        return tl;
    int tm = tl + (tr - tl) / 2;
    int left_count = tree[tree[vr].lch].sum - tree[tree[vl].lch].sum;
    if (left_count > k)
        return query(tree[vl].lch, tree[vr].lch, tl, tm, k);
    return query(tree[vl].rch, tree[vr].rch, tm, tr, k - left_count);
}
};
```

Merge Sort Tree

```
//cat merge_sort_tree.hpp | ./hash.sh
//e110ed
#pragma once
//For point updates: either switch to policy based BST, or use sqrt decomposition
struct merge_sort_tree {
    const int N, S/*smallest power of 2 >= N*/;
    vector<vector<int>> tree;
    merge_sort_tree(const vector<int>& arr) : N(ssize(arr)), S(N ? 1 << __lg(2 * N - 1) : 0),
        tree(2 * N) {
        for (int i = 0; i < N; i++)
            tree[i + N] = {arr[i]};
        rotate(tree.rbegin(), tree.rbegin() + S - N, tree.rbegin() + N);
        for (int i = N - 1; i >= 1; i--) {
            const auto& le = tree[2 * i];
            const auto& ri = tree[2 * i + 1];
            tree[i].reserve(ssize(le) + ssize(ri));
            merge(le.begin(), le.end(), ri.begin(), ri.end(), back_inserter(tree[i]));
        }
    }
    int value(int v, int x) const {
        return int(lower_bound(tree[v].begin(), tree[v].end(), x) - tree[v].begin());
    }
    int to_leaf(int i) const {
        i += S;
        return i < 2 * N ? i : 2 * (i - N);
    }
    //How many values in range [le, ri) are < x?
```

```
//O(log^2(n))
int query(int le, int ri, int x) const {
    int res = 0;
    for (le = to_leaf(le), ri = to_leaf(ri); le < ri; le >>= 1, ri >>= 1) {
        if (le & 1)
            res += value(le++, x);
        if (ri & 1)
            res += value(--ri, x);
    }
    return res;
}
};
```

BIT

```
//cat bit.hpp | ./hash.sh
//ab7995
#pragma once
//mnemonic: Binary Indexed Tree
//NOLINTNEXTLINE(readability-identifier-naming)
template <typename T> struct BIT {
    vector<T> bit;
    BIT(int n) : bit(n, 0) {}
    BIT(const vector<T>& a) : bit(a) {
        for (int i = 0; i < ssize(a); i++) {
            int j = i | (i + 1);
            if (j < ssize(a))
                bit[j] += bit[i];
        }
    }
    void update(int i, const T& d) {
        assert(0 <= i && i < ssize(bit));
        for (; i < ssize(bit); i |= i + 1)
            bit[i] += d;
    }
    T sum(int ri) const { //sum of range [0, ri)
        assert(0 <= ri && ri <= ssize(bit));
        T ret = 0;
        for (; ri > 0; ri &= ri - 1)
            ret += bit[ri - 1];
        return ret;
    }
    T sum(int le, int ri) const { //sum of range [le, ri)
        assert(0 <= le && le <= ri && ri <= ssize(bit));
        return sum(ri) - sum(le);
    }
    //Returns min pos (0<=pos<=ssize(bit)+1) such that sum of [0, pos) >= sum
    //Returns ssize(bit) + 1 if no sum is >= sum, or 0 if empty sum is.
    //Doesn't work with negatives
    int lower_bound(T sum) const {
        if (sum <= 0)
            return 0;
        int pos = 0;
        for (int pw = 1 << __lg(ssize(bit) | 1); pw; pw >>= 1)
            if (pos + pw <= ssize(bit) && bit[pos + pw - 1] < sum)
                pos += pw, sum -= bit[pos - 1];
        return pos + 1;
    }
};
```

```
};
```

RMQ

```
//cat rmq.hpp | ./hash.sh
//59ae14
#pragma once
//source: https://github.com/kth-competitive-programming/
// kactl/blob/main/content/data-structures/RMQ.h
//usage:
// vector<long long> arr;
// ...
// RMQ<long long> rmq(arr, [&](auto x, auto y) { return min(x, y); });
//
//to also get index of min element, do:
// RMQ<pair<long long, int>> rmq(arr, [&](auto x, auto y) { return min(x, y); });
//and initialize arr[i].second = i
//If there are multiple indexes of min element, it'll return the smallest
//(left-most) one
//mnemonic: Range Min/Max Query
//NOLINTNEXTLINE(readability-identifier-naming)
template <typename T> struct RMQ {
    vector<vector<T>> dp;
    function<T(const T&, const T&>> op;
    RMQ(const vector<T>& arr, const function<T(const T&, const T&>& a_op) : dp(1, arr),
        ↳ op(a_op) {
        for (int i = 0; (2 << i) <= ssize(arr); i++) {
            dp.emplace_back(ssize(arr) - (2 << i) + 1);
            for (int j = 0; j < ssize(dp.back()); j++)
                dp[i + 1][j] = op(dp[i][j], dp[i][j + (1 << i)]);
        }
    }
    //inclusive-exclusive range [le, ri)
    T query(int le, int ri) const {
        assert(0 <= le && le < ri && ri <= ssize(dp[0]));
        int lg = __lg(ri - le);
        return op(dp[lg][le], dp[lg][ri - (1 << lg)]);
    }
};
```

Disjoint RMQ

```
//cat disjoint_rmq.hpp | ./hash.sh
//848e2b
#pragma once
//sources: https://codeforces.com/blog/entry/87940,
↳ https://github.com/sgtlaugh/algovault/blob/
↳ master/code_library/disjoint_sparse_table.cpp
//Disjoint RMQ is like normal RMQ except these ranges never overlap. So
//now, you can do a rmq with +, although it's overkill as you can do prefix sums
//instead. Also note seg trees can do everything this can do. It is useful for:
// - min and # of mins.
// - product under composite mod
// - 2-by-2 matrix multiply
//usage for min and # of mins:
// vector<pair<long long, int>> arr; //initialize arr[i].second = 1
// ...
```

```
// disjoint_rmq<pair<long long, int>> rmq(arr, {LLONG_MAX, 0}, [&](auto x, auto y) {
//     if (x.first == y.first)
//         return make_pair(x.first, x.second + y.second);
//     return min(x, y);
// });
template <typename T> struct disjoint_rmq {
    const int N;
    vector<vector<T>> dp;
    function<T(const T&, const T&> op; // any associative operation
    disjoint_rmq(const vector<T>& arr, const T& identity, const function<T(const T&, const
        ↳ T&>& a_op) : N(ssize(arr)), op(a_op) {
        for (int i = 0, len = 1; len <= N; i++, len *= 2) {
            dp.emplace_back(N + 1, identity);
            for (int center = len; center < N + len; center += 2 * len) {
                for (int j = center + 1; j <= min(N, center + len); j++)
                    dp[i][j] = op(dp[i][j - 1], arr[j - 1]);
                for (int j = min(N, center) - 1; j >= center - len; j--)
                    dp[i][j] = op(arr[j], dp[i][j + 1]);
            }
        }
    }
    //range [le, ri)
    T query(int le, int ri) const {
        assert(0 <= le && le < ri && ri <= N);
        int lg = __lg(le ^ ri);
        return op(dp[lg][le], dp[lg][ri]);
    }
};
```

Lazy Segment Tree

```
//cat lazy_segment_tree.hpp | ./hash.sh
//96535f
#pragma once
//source: https://codeforces.com/blog/entry/18051,
↳ https://github.com/ecnerwala/cp-book/blob/master/src/seg_tree.hpp,
↳ https://github.com/yosupo06/Algorithm/blob/master/src/datastructure/segtree.hpp
//rotating leaves makes it a single complete binary tree (instead of a set of perfect binary
↳ trees)
//so standard implementations of
// - recursive seg tree
// - tree walks AKA binary search
//still work
struct seg_tree {
    using dt = long long;
    using ch = long long;
    static dt combine(const dt& le, const dt& ri) {
        return min(le, ri);
    }
    static const dt UNIT = 1e18;
    struct node {
        dt val;
        ch lazy;
        int le, ri; //[le, ri)
    };
    const int N, S/*smallest power of 2 >= N*/;
    vector<node> tree;
    seg_tree(const vector<dt>& arr) : N(ssize(arr)), S(N ? 1 << __lg(2 * N - 1) : 0), tree(2 *
```

```
    ↪ N) {
    for (int i = 0; i < N; i++)
        tree[i + N] = {arr[i], 0, i, i + 1};
    rotate(tree.rbegin(), tree.rbegin() + S - N, tree.rbegin() + N);
    for (int i = N - 1; i >= 1; i--) {
        tree[i] = {
            combine(tree[2 * i].val, tree[2 * i + 1].val),
            0,
            tree[2 * i].le,
            tree[2 * i + 1].ri
        };
    }
}

void apply(int v, ch change) {
    tree[v].val += change;
    tree[v].lazy += change;
}

void push(int v) {
    if (tree[v].lazy) {
        apply(2 * v, tree[v].lazy);
        apply(2 * v + 1, tree[v].lazy);
        tree[v].lazy = 0;
    }
}

void build(int v) {
    tree[v].val = combine(tree[2 * v].val, tree[2 * v + 1].val);
}

int to_leaf(int i) const {
    i += S;
    return i < 2 * N ? i : 2 * (i - N);
}

//update range [le, ri)
void update(int le, int ri, ch change) {
    assert(0 <= le && le <= ri && ri <= N);
    le = to_leaf(le), ri = to_leaf(ri);
    int lca_l_r = __lg((le - 1) ^ ri);
    for (int lg = __lg(le); lg > __builtin_ctz(le); lg--)
        push(le >> lg);
    for (int lg = lca_l_r; lg > __builtin_ctz(ri); lg--)
        push(ri >> lg);
    for (int x = le, y = ri; x < y; x >>= 1, y >>= 1) {
        if (x & 1)
            apply(x++, change);
        if (y & 1)
            apply(--y, change);
    }
    for (int lg = __builtin_ctz(ri) + 1; lg <= lca_l_r; lg++)
        build(ri >> lg);
    for (int lg = __builtin_ctz(le) + 1; lg <= __lg(le); lg++)
        build(le >> lg);
}

void update(int v/* = 1*/, int le, int ri, ch change) {
    if (ri <= tree[v].le || tree[v].ri <= le)
        return;
    if (le <= tree[v].le && tree[v].ri <= ri)
        return apply(v, change);
    push(v);
    update(2 * v, le, ri, change);
    update(2 * v + 1, le, ri, change);
}
```

```
    build(v);
}

//query range [le, ri)
dt query(int le, int ri) {
    assert(0 <= le && le <= ri && ri <= N);
    le = to_leaf(le), ri = to_leaf(ri);
    int lca_l_r = __lg((le - 1) ^ ri);
    for (int lg = __lg(le); lg > __builtin_ctz(le); lg--)
        push(le >> lg);
    for (int lg = lca_l_r; lg > __builtin_ctz(ri); lg--)
        push(ri >> lg);
    dt resl = UNIT, resr = UNIT;
    for (; le < ri; le >>= 1, ri >>= 1) {
        if (le & 1)
            resl = combine(resl, tree[le++].val);
        if (ri & 1)
            resr = combine(tree[--ri].val, resr);
    }
    return combine(resl, resr);
}

dt query(int v/* = 1*/, int le, int ri) {
    if (ri <= tree[v].le || tree[v].ri <= le)
        return UNIT;
    if (le <= tree[v].le && tree[v].ri <= ri)
        return tree[v].val;
    push(v);
    return combine(query(2 * v, le, ri), query(2 * v + 1, le, ri));
}

};
```

STRINGS

Binary Trie

```
//cat binary_trie.hpp | ./hash.sh
//88fa9c
#pragma once
struct binary_trie {
    const int MX_BIT = 62;
    struct node {
        long long val = -1;
        int sub_sz = 0; //number of inserted values in subtree
        array<int, 2> next = {-1, -1};
    };
    vector<node> t;
    binary_trie() : t(1) {}
    //delta = 1 to insert val, -1 to remove val, 0 to get the # of val's in this data structure
    int update(long long val, int delta) {
        int c = 0;
        t[0].sub_sz += delta;
        for (int bit = MX_BIT; bit >= 0; bit--) {
            bool v = (val >> bit) & 1;
            if (t[c].next[v] == -1) {
                t[c].next[v] = ssize(t);
                t.emplace_back();
            }
            c = t[c].next[v];
        }
    }
};
```

```
        t[c].sub_sz += delta;
    }
    t[c].val = val;
    return t[c].sub_sz;
}
int size() const {
    return t[0].sub_sz;
}
//returns x such that:
// x is in this data structure
// value of (x ^ val) is minimum
long long min_xor(long long val) const {
    assert(size() > 0);
    int c = 0;
    for (int bit = MX_BIT; bit >= 0; bit--) {
        bool v = (val >> bit) & 1;
        int ch = t[c].next[v];
        if (ch != -1 && t[ch].sub_sz > 0)
            c = ch;
        else
            c = t[c].next[!v];
    }
    return t[c].val;
}
};
```

Prefix Function

```
//cat prefix_function.hpp | ./hash.sh
//65fea7
#pragma once
//source: https://cp-algorithms.com/string/prefix-function.html#implementation
template <typename T> vector<int> prefix_function(const T& s) {
    vector<int> pi(ssize(s), 0);
    for (int i = 1; i < ssize(s); i++) {
        int j = pi[i - 1];
        while (j > 0 && s[i] != s[j])
            j = pi[j - 1];
        pi[i] = j + (s[i] == s[j]);
    }
    return pi;
}
```

KMP String Matching

```
//cat kmp.hpp | ./hash.sh
//ab4dba
#pragma once
//mnemonic: Knuth Morris Pratt
#include "prefix_function.hpp"
//usage:
// string needle;
// ...
// KMP kmp(needle);
//or
// vector<int> needle;
// ...
```

```
// KMP kmp(needle);
//kmp doubling trick: to check if 2 arrays are rotationally equivalent: run kmp
//with one array as the needle and the other array doubled (excluding the first
//& last characters) as the haystack or just use kactl's min rotation code
//
// if haystack = "bananas"
// needle = "ana"
//
// then we find 2 matches:
// bananas
// _ana___
// ____ana_
// 0123456 (indexes)
// and KMP::find returns {1,3} - the indexes in haystack where
// each match starts.
//
// You can also pass in 0 for "all" and KMP::find will only
// return the first match: {1}. Useful for checking if there exists
// some match:
//
// ssize(KMP::find(<haystack>,0)) > 0
//NOLINTNEXTLINE(readability-identifier-naming)
template <typename T> struct KMP {
    T needle;
    vector<int> pi;
    KMP(const T& a_needle) : needle(a_needle), pi(prefix_function(needle)) {}
    vector<int> find(const T& haystack, bool all = 1) const {
        vector<int> matches;
        for (int i = 0, j = 0; i < ssize(haystack); i++) {
            while (j > 0 && needle[j] != haystack[i])
                j = pi[j - 1];
            if (needle[j] == haystack[i])
                j++;
            if (j == ssize(needle)) {
                matches.push_back(i - ssize(needle) + 1);
                if (!all)
                    return matches;
                j = pi[j - 1];
            }
        }
        return matches;
    }
};
```

Suffix Array Related Queries

```
//cat suffix_array_query.hpp | ./hash.sh
//7a48a6
#pragma once
#include "../hackpack-cpp/content/strings/SuffixArray.h"
#include "../range_data_structures/rmq.hpp"
//various queries you can do based on Suffix Array
/*
suffixes of "banana":

0 banana$
1 anana$
2 nana$
```

```
3 ana$
4 na$
5 a$
6 $

sorted:
      lcp
      0

6 $
      0

5 a$
|      1
3 ana$
|||    3
1 anana$
      0

0 banana$
      0

4 na$
||      2
2 nana$

suffix array = [6, 5, 3, 1, 0, 4, 2]
lcp array = [0, 0, 1, 3, 0, 0, 2]
*/
struct sa_query {
    string s;
    SuffixArray info;
    RMQ<int> rmq_lcp, rmq_sa;
    sa_query(string& a_s) :
        s(a_s),
        info(SuffixArray(s)),
        rmq_lcp(vi(info.lcp.begin() + 1, info.lcp.end()), [](int i, int j) -> int {return
            ↪ min(i, j)}),
        rmq_sa(info.sa, [](int i, int j) -> int {return min(i, j)})) {}
    //length of longest common prefix of suffixes s[idx1 ... N), s[idx2 ... N), 0-based
    ↪ indexing
    //
    //You can check if two substrings s[l1..r1), s[l2..r2) are equal in O(1) by:
    //r1-l1 == r2-l2 && longest_common_prefix(l1, l2) >= r1-l1
    int get_lcp(int idx1, int idx2) const {
        if (idx1 == idx2)
            return ssize(s) - idx1;
        auto [le, ri] = minmax(info.rank[idx1], info.rank[idx2]);
        return rmq_lcp.query(le, ri);
    }
    //returns 1 if suffix s[idx1 ... N) < s[idx2 ... N)
    //(so 0 if idx1 == idx2)
    bool less(int idx1, int idx2) const {
        return info.rank[idx1] < info.rank[idx2];
    }
    //returns range [le, ri) such that:
    // - for all i in [le, ri): t == s.substr(info.sa[i], ssize(t))
    // - `ri - le` is the # of matches of t in s
    //O(|t| * log(|s|))
    pair<int, int> find(const string& t) const {
        auto cmp = [&](int i, int cmp_val) -> bool {
            return s.compare(i, ssize(t), t) < cmp_val;
        };
};
```

```
        auto le = lower_bound(info.sa.begin(), info.sa.end(), 0, cmp);
        auto ri = lower_bound(info.sa.begin(), info.sa.end(), 1, cmp);
        return {le - info.sa.begin(), ri - info.sa.begin()};
    }
    //returns min i such that t == s.substr(i, ssize(t)) or -1
    //for example, replace RMQ with kth-smallest PST/Wavelet to solve
    ↪ https://open.kattis.com/problems/anothersubstringqueryproblem
    //O(|t| * log(|s|))
    int find_first(const string& t) const {
        auto [le, ri] = find(t);
        if (le == ri)
            return -1;
        return rmq_sa.query(le, ri);
    }
};
```

Palindrome Query

```
//cat palindrome_query.hpp | ./hash.sh
//68c8e1
#pragma once
#include "../kactl/content/strings/Manacher.h"
struct pal_query {
    const int N;
    array<vi, 2> pal_len;
    pal_query(const string& s) : N(ssize(s)), pal_len(manacher(s)) {}
    //returns 1 if substring s[le...ri) is a palindrome
    //(returns 1 when le == ri)
    bool is_pal(int le, int ri) const {
        assert(0 <= le && le <= ri && ri <= N);
        int len = ri - le;
        return pal_len[len & 1][le + len / 2] >= len / 2;
    }
};
```

Trie

```
//cat trie.hpp | ./hash.sh
//2aa8c6
#pragma once
//source: https://cp-algorithms.com/string/aho_corasick.html#construction-of-the-trie
const int K = 26; //alphabet size
struct trie {
    const char MIN_CH = 'A'; // 'a' for lowercase, '0' for digits
    struct node {
        int next[K], cnt_words = 0, par = -1;
        char ch;
        node(int a_par = -1, char a_ch = '#') : par(a_par), ch(a_ch) {
            fill(next, next + K, -1);
        }
    };
    vector<node> t;
    trie() : t(1) {}
    void insert(const string& s) {
        int v = 0;
        for (char ch : s) {
            int let = ch - MIN_CH;
```

```
        if (t[v].next[let] == -1) {
            t[v].next[let] = ssize(t);
            t.emplace_back(v, ch);
        }
        v = t[v].next[let];
    }
    t[v].cnt_words++;
}

int find(const string& s) const {
    int v = 0;
    for (char ch : s) {
        int let = ch - MIN_CH;
        if (t[v].next[let] == -1)
            return 0;
        v = t[v].next[let];
    }
    return t[v].cnt_words;
}
};
```