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#### CONTEST

# Tips and Tricks

## Tips and Tricks

```
- [C++ tips and tricks] (https://codeforces.com/blog/entry/74684)
- use '#pragma GCC optimize "trapy" to invoke RTE (Run Time Error) upon integer overflow
- use pramgas for C++ speed boost
#pragma GCC optimize("03,unroll-loops")
#pragma GCC target("avx2,bmi,bmi2,lzcnt,popcnt")
### Troubleshooting
/* stuff you should look for
   * int overflow, array bounds
   * special cases (n=1?)
   * do smth instead of nothing and stay organized
   * WRITE STUFF DOWN
   * DON'T GET STUCK ON ONE APPROACH
...
Author: Beng
- refer to [KACTL Troubleshoot](../../kactl/content/contest/troubleshoot.txt)
## Sources
- [[Tutorial] GCC Optimization Pragmas](https://codeforces.com/bloq/entry/96344)
- [Don't use rand(): a guide to random number generators in
```

### Hash codes

# Test on random inputs

```
#!/usr/bin/env bash
#runs 2 programs against each other on random inputs until they output different results
#source: https://github.com/Errichto/youtube/blob/master/testing/s.sh
#usage:
# chmod +x test.sh
# ./test.sh
for((i = 1; ; ++i)); do
    echo $i
    ./test.out > in
    diff --ignore-all-space <(./a.out < in) <(./brute.out < in) || break
done</pre>
```

#### MAX FLOW

## Hungarian

```
//cat hungarian.hpp / ./hash.sh
//625431
#pragma once
//source: https://e-maxx.ru/algo/assignment_hungary
//input: cost[1...n][1...m] with 1 <= n <= m
//n workers, indexed 1, 2, ..., n
//m jobs, indexed 1, 2, ..., m
//it costs 'cost[i][j]' to assign worker i to job j (1<=i<=n, 1<=j<=m)
//this returns *min* total cost to assign each worker to some distinct job
//0(n^2 * m)
//
//trick 1: set 'cost[i][j]' to INF to say: "worker 'i' cannot be assigned job 'j'"
//trick 2: 'cost[i][j]' can be negative, so to instead find max total cost over all
     \hookrightarrow matchings: set all 'cost[i][j]' to '-cost[i][j]'.
//Now max total cost = - hungarian(cost).min_cost
const long long INF = 1e18;
struct match {
    long long min_cost;
    vector<int> matching; //worker 'i' (1<=i<=n) is assigned to job 'matching[i]'
         \hookrightarrow (1 < = matching[i] < = m)
};
match hungarian(const vector<vector<long long>>& cost) {
    int n = cost.size() - 1, m = cost[0].size() - 1;
    assert(n <= m);</pre>
    vector\langle int \rangle p(m + 1), way(m + 1);
    vector<long long> u(n + 1), v(m + 1);
    for (int i = 1; i <= n; i++) {
        p[0] = i;
        int j0 = 0;
        vector<long long> minv(m + 1, INF);
        vector<bool> used(m + 1, 0);
        do {
             used[j0] = 1;
            int i0 = p[j0], j1 = 0;
             long long delta = INF;
             for (int j = 1; j <= m; j++)
                 if (!used[j]) {
                     long long cur = cost[i0][j] - u[i0] - v[j];
                     if (cur < minv[j])</pre>
                         minv[j] = cur, way[j] = j0;
```

```
if (minv[j] < delta)</pre>
                    delta = minv[j], j1 = j;
            }
        for (int j = 0; j \le m; j++)
            if (used[i])
                u[p[j]] += delta, v[j] -= delta;
                minv[j] -= delta;
        j0 = j1;
    } while (p[j0] != 0);
    do {
        int j1 = way[j0];
        p[j0] = p[j1];
        j0 = j1;
    } while (j0);
}
vector<int> ans(n + 1);
for (int j = 1; j \le m; j++)
    ans[p[i]] = i;
return {-v[0], ans};
```

# Min Cost Max Flow

```
//cat min_cost_max_flow.hpp | ./hash.sh
//f6b59d
#pragma once
//source: https://e-maxx.ru/alqo/min_cost_flow
const long long INF = 1e18;
struct mcmf {
   using 11 = long long;
   struct edge {
       int a. b:
       11 cap, cost, flow;
       int back;
   };
   const int N;
   vector<edge> e;
   vector<vector<int>> g;
   mcmf(int a_n) : N(a_n), g(N) {}
   void add_edge(int a, int b, ll cap, ll cost) {
       edge e1 = {a, b, cap, cost, 0, (int)g[b].size() };
       edge e2 = {b, a, 0, -cost, 0, (int)g[a].size() };
       g[a].push_back(e.size());
       e.push_back(e1);
       g[b].push_back(e.size());
       e.push back(e2):
   }
   pair<11, 11> get_flow(int s, int t, 11 total_flow) {
       11 flow = 0, cost = 0;
       while (flow < total_flow) {</pre>
           vector<11> d(N. INF):
            vector<int> p_edge(N), id(N, 0), q(N), p(N);
           int qh = 0, qt = 0;
           q[qt++] = s;
           d[s] = 0;
            while (qh != qt) {
                int v = q[qh++];
```

```
id[v] = 2:
                if (ah == N) ah = 0:
                for (int i = 0; i < (int)g[v].size(); i++) {</pre>
                    const edge& r = e[g[v][i]];
                    if (r.flow < r.cap && d[v] + r.cost < d[r.b]) {
                        d[r.b] = d[v] + r.cost;
                        if (id[r.b] == 0) {
                            q[qt++] = r.b;
                            if (qt == N) qt = 0;
                        } else if (id[r.b] == 2) {
                            if (--qh == -1) qh = N - 1;
                            q[qh] = r.b;
                        id[r.b] = 1;
                        p[r.b] = v;
                        p_{edge}[r.b] = i;
                    }
                }
            if (d[t] == INF) break:
            11 addflow = total_flow - flow;
            for (int v = t; v != s; v = p[v]) {
                int pv = p[v], pr = p_edge[v];
                addflow = min(addflow, e[g[pv][pr]].cap - e[g[pv][pr]].flow);
            for (int v = t; v != s; v = p[v]) {
                int pv = p[v], pr = p_edge[v], r = e[g[pv][pr]].back;
                e[g[pv][pr]].flow += addflow;
                e[g[v][r]].flow -= addflow;
                cost += e[g[pv][pr]].cost * addflow;
            flow += addflow;
        }
        return {flow, cost};
    }
};
```

# **GRAPHS**

### Block Vertex Tree

```
//cat block_vertex_tree.hpp / ./hash.sh
//ea8ef1
#pragma once
#include "bridges_and_cuts.hpp"
//returns adjacency list of block vertex tree
//usage:
// info cc = bridge_and_cut(adj, m);
// vector<vector<int>> but = block_vertex_tree(adj, cc);
//to loop over each *unique* bcc containing a node v:
// for (int bccid : but[v]) {
    bccid -= n;
    ...
// }
//to loop over each *unique* node inside a bcc:
// for (int v : but[bccid + n]) {
    ...
// ...
```

```
vector<vector<int>> block_vertex_tree(const vector<vector<pair<int, int>>>& adj, const
    \hookrightarrow info% cc) {
    int n = adi.size();
    vector<vector<int>> bvt(n + cc.num_bccs);
    vector<bool> vis(cc.num_bccs, 0);
   for (int v = 0: v < n: v++) {
        for (auto [_, e_id] : adj[v]) {
            int bccid = cc.bcc_id[e_id];
            if (!vis[bccid]) {
                vis[bccid] = 1;
                bvt[v].push_back(bccid + n); //add edge between original node, and bcc
                bvt[bccid + n].push_back(v);
           }
       }
       for (int bccid : bvt[v]) vis[bccid - n] = 0;
   }
    return bvt;
```

# Bridge Tree

```
//cat bridge_tree.hpp | ./hash.sh
//85f56b
#pragma once
#include "bridges_and_cuts.hpp"
//never adds multiple edges as bridges_and_cuts.hpp correctly marks them as non-bridges
//usage:
// info cc = bridge_and_cut(adj, m);
// vector<vector<int>> bt = bridge_tree(adj, cc);
vector<vector<int>> bridge_tree(const vector<vector<pair<int, int>>>& adj, const info&
    \hookrightarrow cc) {
    vector<vector<int>> tree(cc.num_2_edge_ccs);
    for (int i = 0; i < (int)adj.size(); i++)</pre>
        for (auto [to, e_id] : adj[i])
            if (cc.is_bridge[e_id])
                tree[cc.two_edge_ccid[i]].push_back(cc.two_edge_ccid[to]);
    return tree;
```

# Bridges and Cuts

```
//cat bridges_and_cuts.hpp | ./hash.sh
//5f0c60
#pragma once
//0(n+m) time & space
//2 edge cc and bcc stuff doesn't depend on each other, so delete whatever is not needed
//handles multiple edges
//
//example initialization of 'adj':
//for (int i = 0; i < m; i++) {
// int u, u;
// cin >> u >> u;
// u--, v--;
// adj[u].emplace_back(u, i);
// adj[v].emplace_back(u, i);
```

```
//}
struct info {
    //2 edge connected component stuff (e.g. components split by bridge edges)
         \hookrightarrow https://cp-algorithms.com/qraph/bridge-searching.html
    int num_2_edge_ccs;
    vector<bool> is_bridge;//edge id -> 1 iff bridge edge
    vector<int> two_edge_ccid;//node -> id of 2 edge component (which are labeled 0, 1,
         \hookrightarrow ..., 'num_2_edge_ccs'-1)
    //bi connected component stuff (e.g. components split by cut/articulation nodes)
         \hookrightarrow https://cp-algorithms.com/araph/cutpoints.html
    int num_bccs;
    vector<bool> is_cut;//node -> 1 iff cut node
    vector<int> bcc_id; //edge id -> id of bcc (which are labeled 0, 1, ..., 'num_bccs'-1)
};
info bridge_and_cut(const vector<vector<pair<int/*neiqhbor*/, int/*edqe id*/>>>&

→ adj/*undirected graph*/, int m/*number of edges*/) {
    //stuff for both (always keep)
    int n = adj.size(), timer = 1;
    vector<int> tin(n, 0);
    //2 edge cc stuff (delete if not needed)
    int num_2_edge_ccs = 0;
    vector<bool> is_bridge(m, 0);
    vector<int> two_edge_ccid(n), node_stack;
    node_stack.reserve(n);
    //bcc stuff (delete if not needed)
    int num_bccs = 0;
    vector<bool> is_cut(n, 0);
    vector<int> bcc_id(m), edge_stack;
    edge_stack.reserve(m);
    auto dfs = [&](auto self, int v, int p_id) -> int {
        int low = tin[v] = timer++, deg = 0;
        node_stack.push_back(v);
        for (auto [to, e_id] : adj[v]) {
             if (e_id == p_id) continue;
             if (!tin[to]) {
                 edge_stack.push_back(e_id);
                 int low_ch = self(self, to, e_id);
                 if (low ch >= tin[v]) {
                     is_cut[v] = 1;
                     while (1) {
                         int edge = edge_stack.back();
                         edge_stack.pop_back();
                         bcc_id[edge] = num_bccs;
                         if (edge == e_id) break;
                     }
                     num_bccs++;
                 }
                 low = min(low, low_ch);
                 deg++;
             } else if (tin[to] < tin[v]) {</pre>
                 edge_stack.push_back(e_id);
                 low = min(low, tin[to]);
        if (p_id == -1) is_cut[v] = (deg > 1);
        if (tin[v] == low) {
            if (p_id != -1) is_bridge[p_id] = 1;
            while (1) {
                 int node = node_stack.back();
```

```
node_stack.pop_back();
    two_edge_ccid[node] = num_2_edge_ccs;
    if (node == v) break;
}
    num_2_edge_ccs++;
}
    return low;
};
for (int i = 0; i < n; i++)
    if (!tin[i])
        dfs(dfs, i, -1);
return {num_2_edge_ccs, is_bridge, two_edge_ccid, num_bccs, is_cut, bcc_id};
}</pre>
```

# Centroid Decomp

```
//cat centroid_decomp.hpp / ./hash.sh
//31dd4e
#pragma once
// Time and Space complexity are given in terms of n where n is the number of nodes in
    \hookrightarrow the tree
// Time complexity O(n \log n)
// Space complexity O(n)
// Given an unweighted, undirected tree and a function,
// centroid_decomp runs the function on every decomposition
// see count_paths_per_node for example usage
struct centroid_decomp {
   vector<vector<int>> adi:
   function<void(const vector<vector<int>>&, int)> func;
    vector<int> sub sz:
    centroid_decomp(const vector<vector<int>>& a_adj,
                    const function<void(const vector<vector<int>>&. int)>& a func)
        : adj(a_adj), func(a_func), sub_sz(adj.size()) {
        decomp(find_centroid(0));
   }
   void calc_subtree_sizes(int u, int p = -1) {
       sub_sz[u] = 1;
       for (int v : adj[u]) {
            if (v == p) continue;
            calc_subtree_sizes(v, u);
            sub sz[u] += sub sz[v];
       }
   }
   int find_centroid(int root) {
        calc_subtree_sizes(root);
       int u = root, p = -1;
       while (1) {
            int big_ch = -1;
           for (int v : adj[u]) {
                if (v == p) continue;
                if (big_ch == -1 || sub_sz[big_ch] < sub_sz[v])</pre>
                    big_ch = v;
```

## Frequency Table of Tree Distance

```
//cat count_paths_per_length.hpp | ./hash.sh
//272170
#pragma once
#include "../../kactl/content/numerical/FastFourierTransform.h"
#include "centroid_decomp.hpp"
//returns array 'num_paths' where 'num_paths[i]' = # of paths in tree with 'i' edges
//0(n \log^2 n)
vector<long long> count_paths_per_length(const vector<vector<int>>& a_adj/*unrooted,
    \hookrightarrow connected tree*/) {
    vector<long long> num_paths(a_adj.size(), 0);
    auto func = [&](const vector<vector<int>>& adj, int root) -> void {
        vector<double> total_depth(1, 1.0);
        for (int to : adi[root]) {
            vector<double> cnt_depth(1, 0.0);
            for (queue<pair<int, int>> q({{to, root}}); !q.empty();) {
                cnt_depth.push_back(q.size());
                queue<pair<int, int>> new_q;
                while (!q.empty()) {
                    auto [curr, par] = q.front();
                    q.pop();
                    for (int ch : adj[curr]) {
                        if (ch == par) continue;
                        new_q.emplace(ch, curr);
                    }
                }
                swap(q, new_q);
                vector<double> prod = conv(total_depth, cnt_depth);
                for (int i = 1; i < (int)prod.size(); i++) num_paths[i] +=</pre>
                     ⇔ llround(prod[i]);
            if (total_depth.size() < cnt_depth.size())</pre>

    total_depth.resize(cnt_depth.size(), 0.0);

            for (int i = 1; i < (int)cnt_depth.size(); i++) total_depth[i] +=</pre>

    cnt_depth[i];

    };
```

```
centroid_decomp decomp(a_adj, func);
   return num_paths;
}
```

#### Count Paths Per Node

```
//cat count_paths_per_node.hpp / ./hash.sh
//8811b2
#pragma once
#include "centroid_decomp.hpp"
//0-based nodes
//returns array 'num_paths' where 'num_paths[i]' = number of paths with k edges where
    \hookrightarrow node 'i' is on the path
//0(n log n)
vector<long long> count_paths_per_node(const vector<vector<int>>& a_adj/*unrooted
    \hookrightarrow tree*/, int k) {
    vector<long long> num_paths(a_adj.size());
    auto func = [&](const vector<vector<int>>& adj, int root) -> void {
        vector<int> pre_d(1, 1), cur_d(1);
        auto dfs = [&](auto self, int u, int p, int d) -> long long {
            if (d > k)
                return 0;
            if (int(cur d.size()) <= d)</pre>
                cur_d.push_back(0);
            cur_d[d]++;
            long long cnt = 0;
            if (k - d < int(pre_d.size()))</pre>
                cnt += pre_d[k - d];
            for (int v : adj[u]) {
                if (v != p)
                    cnt += self(self, v, u, d + 1);
            num_paths[u] += cnt;
            return cnt;
        };
        auto dfs_child = [&](int child) -> long long {
            long long cnt = dfs(dfs, child, root, 1);
            pre_d.resize(cur_d.size());
            for (int i = 1; i < int(cur_d.size()) && cur_d[i]; i++) {</pre>
                pre_d[i] += cur_d[i];
                cur_d[i] = 0;
            }
            return cnt;
        };
        for (int child : adj[root])
            num_paths[root] += dfs_child(child);
        pre_d = vector<int>(1);
        cur d = vector<int>(1);
        for (auto it = adj[root].rbegin(); it != adj[root].rend(); it++)
            dfs_child(*it);
   };
    centroid_decomp decomp(a_adj, func);
    return num_paths;
```

# Dijkstra

```
//cat dijkstra.hpp | ./hash.sh
//8fe9d3
#pragma once
//returns array 'len' where 'len[i]' = shortest path from node 'start' to node 'i'
//For example 'len[start]' will always = 0
const long long INF = 1e18;
vector<long long> dijkstra(const vector<vector<pair<int, long long>>>& adj /*directed or
    using node = pair<long long, int>;
   vector<long long> len(adj.size(), INF);
   len[start] = 0;
   priority_queue<node, vector<node>, greater<node>> q;
   q.emplace(0, start);
   while (!q.empty()) {
       auto [curr_len, v] = q.top();
       if (len[v] < curr_len) continue;//important check: TLE without it</pre>
       for (auto [to, weight] : adj[v])
           if (len[to] > weight + len[v]) {
               len[to] = weight + len[v];
               q.emplace(len[to], to);
   }
   return len;
```

### HLD

```
//cat hld.hpp / ./hash.sh
//d08853
#pragma once
//source: https://codeforces.com/blog/entry/53170
//mnemonic: Heavy Light Decomposition
//NOLINTNEXTLINE(readability-identifier-naming)
struct HLD {
    struct node {
        int sub_sz = 1, par = -1, time_in = -1, next = -1;
    vector<node> tree;
    HLD(vector<vector<int>>& adj/*forest of unrooted trees*/) : tree(adj.size()) {
        for (int i = 0, timer = 0; i < (int)adj.size(); i++) {</pre>
            if (tree[i].next == -1) {//lowest indexed node in each tree becomes root
                tree[i].next = i:
                dfs1(i, adj);
                dfs2(i, adj, timer);
       }
    void dfs1(int v, vector<vector<int>>& adj) {
        auto par = find(adj[v].begin(), adj[v].end(), tree[v].par);
        if (par != adj[v].end()) adj[v].erase(par);
        for (int& to : adj[v]) {
            tree[to].par = v;
            dfs1(to, adj);
            tree[v].sub_sz += tree[to].sub_sz;
            if (tree[to].sub_sz > tree[adj[v][0]].sub_sz)
                swap(to, adj[v][0]);
```

```
}
    void dfs2(int v, const vector<vector<int>>& adj, int& timer) {
        tree[v].time in = timer++:
        for (int to : adj[v]) {
            tree[to].next = (timer == tree[v].time_in + 1 ? tree[v].next : to);
            dfs2(to, adi, timer):
        }
    // Returns inclusive-exclusive intervals (of time in's) corresponding to the path
         \hookrightarrow between u and v, not necessarily in order
    // This can answer queries for "is some node 'x' on some path" by checking if the
         \hookrightarrow tree[x].time_in is in any of these intervals
    // u, v must be in the same component
    vector<pair<int, int>> path(int u, int v) const {
        vector<pair<int, int>> res;
        for (;; v = tree[tree[v].next].par) {
            if (tree[v].time_in < tree[u].time_in) swap(u, v);</pre>
            if (tree[tree[v].next].time_in <= tree[u].time_in) {</pre>
                 res.emplace_back(tree[u].time_in, tree[v].time_in + 1);
                 return res;
            res.emplace_back(tree[tree[v].next].time_in, tree[v].time_in + 1);
    }
    // Returns interval (of time_in's) corresponding to the subtree of node i
    // This can answer queries for "is some node 'x' in some other node's subtree" by
         \hookrightarrow checking if tree[x].time_in is in this interval
    pair<int, int> subtree(int i) const {
        return {tree[i].time_in, tree[i].time_in + tree[i].sub_sz};
    // Returns lca of nodes u and v
    // u, v must be in the same component
    int lca(int u, int v) const {
        for (;; v = tree[tree[v].next].par) {
            if (tree[v].time_in < tree[u].time_in) swap(u, v);</pre>
            if (tree[tree[v].next].time_in <= tree[u].time_in) return u;</pre>
    }
};
```

# Hopcroft Karp

```
//cat hopcroft_karp.hpp | ./hash.sh
//de75d7
#pragma once
//source: https://github.com/foreverbell/acm-icpc-cheat-sheet/
// blob/master/src/graph-algorithm/hopcroft-karp.cpp
//Worst case O(E*sqrt(V)) but faster in practice
struct match {
    //# of edges in matching (which = size of min vertex cover by öKnig's theorem)
    int size_of_matching;
    //an arbitrary max matching is found. For this matching:
    //if l_to_r[node_left] == -1:
    // node_left is not in matching
    //else:
    // the edge 'node_left' <=> l_to_r[node_left] is in the matching
//else:
```

```
//similarly for r_to_l with edge r_to_l[node_right] <=> node_right in matching if
         \hookrightarrow r to l[node right] != -1
    //matchings stored in l_to_r and r_to_l are the same matching
    //provides way to check if any node/edge is in matching
    vector<int> l_to_r, r_to_l;
    //an arbitrary min vertex cover is found. For this mvc: mvc_l[node_left] is 1 iff
         \hookrightarrow node left is in the min vertex cover (same for muc r)
    //if muc_l[node_left] is 0, then node_left is in the corresponding maximal
         \hookrightarrow independent set
    vector<bool> mvc 1. mvc r:
};
//Think of the bipartite graph as having a left side (with size lsz) and a right side
     \hookrightarrow (with size rsz).
//Nodes on left side are indexed 0,1,...,lsz-1
//Nodes on right side are indexed 0.1....rsz-1
//'adj' is like a directed adjacency list containing edges from left side -> right side:
//To initialize 'adj': For every edge node_left <=> node_right, do:
     \hookrightarrow adj[node_left].push_back(node_right)
match hopcroft_karp(const vector<vector<int>>& adj/*bipartite graph*/, int rsz/*number
     \hookrightarrow of nodes on right side*/) {
    int size_of_matching = 0, lsz = adj.size();
    vector<int> l_to_r(lsz, -1), r_to_l(rsz, -1);
    while (1) {
        queue<int> q;
        vector<int> level(lsz, -1);
        for (int i = 0; i < lsz; i++)</pre>
            if (l_to_r[i] == -1)
                level[i] = 0, q.push(i);
        bool found = 0:
        vector<bool> mvc_l(lsz, 1), mvc_r(rsz, 0);
        while (!q.empty()) {
            int u = q.front();
            q.pop();
             mvc 1[u] = 0:
            for (int x : adj[u]) {
                mvc_r[x] = 1;
                int v = r to 1[x]:
                if (v == -1) found = 1;
                 else if (level[v] == -1) {
                     level[v] = level[u] + 1:
                     q.push(v);
                 }
            }
        if (!found) return {size_of_matching, l_to_r, r_to_l, mvc_l, mvc_r};
        auto dfs = [&] (auto self, int u) -> bool {
             for (int x : adj[u]) {
                int v = r to l[x]:
                 if (v == -1 || (level[u] + 1 == level[v] && self(self, v))) {
                     1 to r[u] = x:
                     r to 1[x] = u:
                     return 1;
                }
             level[u] = 1e9; //acts as visited array
             return 0:
        for (int i = 0; i < lsz; i++)</pre>
```

```
size_of_matching += (l_to_r[i] == -1 && dfs(dfs, i));
}
}
```

### Kth Node on Path

```
//cat kth_node_on_path.hpp / ./hash.sh
//c59307
#pragma once
#include "lca.hpp"
struct kth_node_on_path {
   LCA lca;
    kth node on path(const vector<vector<pair<int, long long>>>& adi/*forest of weighted
         \hookrightarrow trees*/) : lca(adj) {}
    //consider path \{u, u's par, \ldots, LCA(u,v), \ldots, v's par, v\}. This returns the node
         \hookrightarrow at index k
    //assumes 0 \le k \le number of edges on path from u to v
    // u. v must be in the same component
    int query(int u, int v, int k) const {
        int lca_uv = lca.get_lca(u, v);
        int u_lca = lca.tree[u].depth - lca.tree[lca_uv].depth;
        int v_lca = lca.tree[v].depth - lca.tree[lca_uv].depth;
        assert(0 <= k && k <= u_lca + v_lca);
        return k <= u_lca ? lca.kth_par(u, k) : lca.kth_par(v, u_lca + v_lca - k);</pre>
   }
};
```

# LCA

```
//cat lca.hpp / ./hash.sh
//f2cd84
#pragma once
//https://codeforces.com/blog/entru/74847
//mnemonic: Least/Lowest Common Ancestor
//NOLINTNEXTLINE(readability-identifier-naming)
struct LCA {
   struct node {
       int jmp = -1, jmp\_edges = 0, par = -1, depth = 0;
       long long dist = OLL;
   };
   vector<node> tree;
   LCA(const vector<vector<pair<int, long long>>>& adj/*forest of weighted trees*/):
        \hookrightarrow tree(adi.size()) {
       for (int i = 0; i < (int)adj.size(); i++) {</pre>
            if (tree[i].jmp == -1) {//lowest indexed node in each tree becomes root
                tree[i].jmp = i;
                dfs(i, adj);
       }
   void dfs(int v. const vector<vector<pair<int. long long>>>& adj) {
       int jmp, jmp_edges;
       if (tree[v].jmp != v && tree[v].jmp_edges == tree[tree[v].jmp].jmp_edges)
            jmp = tree[tree[v].jmp].jmp, jmp_edges = 2 * tree[v].jmp_edges + 1;
        else
            jmp = v, jmp_edges = 1;
        for (auto [ch, w] : adj[v]) {
```

```
if (ch == tree[v].par) continue;
            tree[ch] = {
                jmp,
                jmp_edges,
                1 + tree[v].depth,
                w + tree[v].dist
            };
            dfs(ch, adj);
        }
    }
    //traverse up k edges in O(log(k)). So with k=1 this returns 'v''s parent
    int kth_par(int v, int k) const {
        k = min(k, tree[v].depth);
        while (k > 0) {
            if (tree[v].jmp_edges <= k) {</pre>
                k -= tree[v].jmp_edges;
                v = tree[v].jmp;
            } else {
                k--;
                v = tree[v].par;
        }
        return v;
    // x, y must be in the same component
    int get_lca(int x, int y) const {
        if (tree[x].depth < tree[y].depth) swap(x, y);</pre>
        x = kth_par(x, tree[x].depth - tree[y].depth);
        while (x != y) {
            if (tree[x].jmp != tree[y].jmp)
                x = tree[x].jmp, y = tree[y].jmp;
                x = tree[x].par, y = tree[y].par;
        }
        return x;
    }
    int dist edges(int x. int v) const {
        return tree[x].depth + tree[y].depth - 2 * tree[get_lca(x, y)].depth;
    long long dist weight(int x, int v) const {
        return tree[x].dist + tree[y].dist - 2 * tree[get_lca(x, y)].dist;
};
```

# Rooted Tree Isomorphism

```
//cat subtree_isomorphism.hpp | ./hash.sh
//64d64a
#pragma once

// Complexity given in terms of n where n is the number of nodes in the tree
// Time complexity O(n log n)
// Space complexity O(n)

// Given an undirected or directed rooted tree
// rooted_subtree_isomorphism classifies each rooted subtree
struct info {
```

```
int num_distinct_subtrees; //0 <= id[i] < num_distinct_subtrees for all i</pre>
    vector<int> id; //id[u] == id[v] iff subtree u is isomorphic to subtree v
};
info subtree_iso(const vector<vector<int>>& adj, int root) {
    vector<int> id(adj.size());
    map<vector<int>, int> hashes:
    auto dfs = [&](auto self, int u, int p) -> int {
        vector<int> ch_ids;
        ch_ids.reserve(adj[u].size());
        for (int v : adj[u]) {
            if (v != p)
                ch_ids.push_back(self(self, v, u));
        }
        sort(ch_ids.begin(), ch_ids.end());
        auto it = hashes.find(ch_ids);
        if (it == hashes.end())
            return id[u] = hashes[ch_ids] = hashes.size();
        return id[u] = it->second;
   };
    dfs(dfs, root, root);
    return {(int)hashes.size(), id};
```

## MATH

## Derangements

```
//cat derangements.hpp | ./hash.sh
//c221bb
#pragma once
//https://oeis.org/A000166
//for a permutation of size i:
//there are (i-1) places to move 0 to not be at index 0. Let's say we moved 0 to index j
    \hookrightarrow (i>0).
//If we move value j to index 0 (forming a cycle of length 2), then there are dp[i-2]
    \hookrightarrow derangements of the remaining i-2 elements
//else there are dp[i-1] derangements of the remaining i-1 elements (including j)
vector<int> derangements(int n, int mod) {
   vector<int> dp(n, 0);
   dp[0] = 1:
   for (int i = 2; i < n; i++)
        dp[i] = 1LL * (i - 1) * (dp[i - 1] + dp[i - 2]) % mod;
   return dp;
```

# Binary Exponentiation MOD

```
//cat binary_exponentiation_mod.hpp | ./hash.sh

//c47ca5

#pragma once

//returns (base^pw)%mod in O(log(pw)), but returns 1 for 0^0

//

//What if base doesn't fit in long long?

//Since (base^pw)%mod == ((base%mod)^pw)%mod we can calculate base under mod of 'mod'
```

```
//What if pw doesn't fit in long long?
//case 1: mod is prime
//(base^pw), mod == (base^(pw; (mod-1))); mod (from Fermat's little theorem)
//so calculate pw under mod of 'mod-1'
//note 'mod-1' is not prime, so you need to be able to calculate 'pw%(mod-1)' without
    \hookrightarrow division
//case 2: non-prime mod
//let t = totient(mod)
//if pw >= log2(mod) then (base^pw)%mod == (base^(t+(pw%t)))%mod (proof)
    \hookrightarrow https://cp-algorithms.com/algebra/phi-function.html#generalization)
//so calculate pw under mod of 't'
//incidentally, totient(p) = p - 1 for every prime p, making this a more generalized
     \hookrightarrow version of case 1
int bin_exp(long long base, long long pw, int mod) {
    assert(0 <= pw && 0 <= base && 1 <= mod);</pre>
    int res = 1;
    base %= mod;
    while (pw > 0) {
        if (pw & 1) res = res * base % mod;
        base = base * base % mod:
        pw >>= 1:
    return res;
```

### Fibonacci

# Matrix Multiplication

# Mobius Inversion

```
//cat mobius_inversion.hpp | ./hash.sh
//811515
#pragma once
//mobius[i] = 0 iff there exists a prime p s.t. i%(p^2)=0
//mobius[i] = -1 iff i has an odd number of distinct prime factors
//mobius[i] = 1 iff i has an even number of distinct prime factors
const int N = 1e6 + 10;
int mobius[N];
void calc_mobius() {
    mobius[1] = 1;
    for (int i = 1; i < N; i++)
        for (int j = i + i; j < N; j += i)
            mobius[j] -= mobius[i];
}</pre>
```

## N Choose K MOD

```
//cat n choose k mod.hpp / ./hash.sh
//e7a700
#pragma once
//for mod inverse
#include "binary_exponentiation_mod.hpp"
// usage:
      n_{choose} k \ nk(n, 1e9+7) to use 'choose', 'inv' with inputs strictly < n
// or:
      n_choose_k nk(mod, mod) to use 'choose_with_lucas_theorem' with arbitrarily large
    \hookrightarrow inputs
struct n_choose_k {
    n_choose_k(int n, int a_mod) : mod(a_mod), fact(n, 1), inv_fact(n, 1) {
        //this implementation doesn't work if n > mod because n! " mod = 0 when n >=
             \hookrightarrow mod. So 'inv_fact' array will be all 0's
        assert(max(n, 2) <= mod);</pre>
        //assert mod is prime. mod is intended to fit inside an int so that
        //multiplications fit in a longlong before being modded down. So this
        //will take sqrt(2^31) time
        for (int i = 2; i * i <= mod; i++) assert(mod % i);</pre>
        for (int i = 2; i < n; i++)
            fact[i] = 1LL * fact[i - 1] * i % mod;
        inv_fact.back() = bin_exp(fact.back(), mod - 2, mod);
        for (int i = n - 2; i \ge 2; i--)
            inv_fact[i] = inv_fact[i + 1] * (i + 1LL) % mod;
    //classic n choose k
```

```
//fails when n \ge mod
    int choose(int n. int k) const {
        if (k < 0 \mid | k > n) return 0;
        //now we know 0 <= k <= n so 0 <= n
        return 1LL * fact[n] * inv_fact[k] % mod * inv_fact[n - k] % mod;
    //lucas theorem to calculate n choose k in O(log(k))
    //need to calculate all factorials in range [0,mod), so 0(mod) time&space, so need
         \hookrightarrow smallish prime mod (< 1e6 maybe)
    //handles n >= mod correctly
    int choose_lucas(long long n, long long k) const {
        if (k < 0 \mid | k > n) return 0;
        if (k == 0 | | k == n) return 1;
        return 1LL * choose_lucas(n / mod, k / mod) * choose(n % mod, k % mod) % mod;
    //returns x such that x * n % mod == 1
    int inv(int n) const {
        assert(1 <= n); //don't divide by 0 :)</pre>
        return 1LL * fact[n - 1] * inv_fact[n] % mod;
    }
    vector<int> fact, inv_fact;
};
```

#### Partitions

```
//cat partitions.hpp / ./hash.sh
//3356f6
#pragma once
//https://oeis.org/A000041
//0(n \ sqrt \ n) time, but small-ish constant factor (there does exist a O(n \ log \ n)
     \hookrightarrow solution as well)
vector<int> partitions(int n, int mod) {
    vector<int> dp(n, 1);
    for (int i = 1; i < n; i++) {
        long long sum = 0;
        for (int j = 1, pent = 1, sign = 1; pent <= i; j++, pent += 3 * j - 2, sign =
            if (pent + j \le i) sum += dp[i - pent - j] * sign + mod;
            sum += dp[i - pent] * sign + mod;
        dp[i] = sum % mod;
    }
    return dp;
```

# Prime Sieve

```
//cat prime_sieve.hpp | ./hash.sh
//1897a8
#pragma once
//a_prime[val] = some random prime factor of 'val'
//
//to check if 'val' is prime:
// if (a_prime[val] == val)
//
//to get all prime factors of a number 'val' in O(log(val)):
```

#### Row Reduce

```
//cat row_reduce.hpp / ./hash.sh
//5e4704
#pragma once
//for mod inverse
#include "binary_exponentiation_mod.hpp"
//First 'cols' columns of mat represents a matrix to be left in reduced row echelon form
//Row operations will be performed to all later columns
//example usage:
// row_reduce(mat, mat[0].size(), mod) //row reduce matrix with no extra columns
pair<int/*rank*/, int/*determinant*/> row_reduce(vector<vector<int>>& mat, int cols, int
    \hookrightarrow mod) {
    int n = mat.size(), m = mat[0].size(), rank = 0, det = 1;
    assert(cols <= m):</pre>
    for (int col = 0; col < cols && rank < n; col++) {</pre>
        //find arbitrary pivot and swap pivot to current row
        for (int i = rank: i < n: i++)</pre>
            if (mat[i][col] != 0) {
                if (rank != i) det = det == 0 ? 0 : mod - det;
                swap(mat[i], mat[rank]);
                break;
        if (mat[rank][col] == 0) {
            det = 0:
            continue;
        det = (1LL * det * mat[rank][col]) % mod;
        //make pivot 1 by dividing row by inverse of pivot
        int a_inv = bin_exp(mat[rank][col], mod - 2, mod);
        for (int i = 0: i < m: i++)
            mat[rank][j] = (1LL * mat[rank][j] * a_inv) % mod;
        //zero-out all numbers above & below pivot
        for (int i = 0; i < n; i++)
            if (i != rank && mat[i][col] != 0) {
                int val = mat[i][col];
                for (int j = 0; j < m; j++) {
                    mat[i][j] -= 1LL * mat[rank][j] * val % mod;
                    if (mat[i][j] < 0) mat[i][j] += mod;</pre>
            }
        rank++;
```

```
}
assert(rank <= min(n, cols));
return {rank, det};
}</pre>
```

## Solve Linear Equations MOD

```
//cat solve_linear_mod.hpp / ./hash.sh
//44cc6e
#pragma once
#include "row_reduce.hpp"
struct matrix_info {
   int rank. det:
    vector<int> x;
};
//Solves\ mat\ *\ x\ =\ b\ under\ prime\ mod.
//mat is a n (rows) by m (cols) matrix, b is a length n column vector, x is a length m
//assumes n,m >= 1, else RTE
//Returns rank of mat, determinant of mat, and x (solution vector to mat * x = b).
//x is empty if no solution. If rank < m, there are multiple solutions and an arbitrary
    \hookrightarrow one is returned.
//Leaves mat in reduced row echelon form (unlike kactl) with b appended.
//Trick: Number of unique solutions = mod ^ (# of free variables).
//# of free variables is generally equivalent to n - rank.
//0(n * m * min(n.m))
matrix_info solve_linear_mod(vector<vector<int>>& mat, const vector<int>& b, int mod) {
    assert(mat.size() == b.size());
    int n = mat.size(). m = mat[0].size();
    for (int i = 0; i < n; i++)
        mat[i].push_back(b[i]);
    auto [rank, det] = row_reduce(mat, m, mod); //row reduce not including the last column
    //check if solution exists
    for (int i = rank; i < n; i++) {</pre>
        if (mat[i].back() != 0) return {rank, det, {} }; //no solution exists
    //initialize solution vector ('x') from row-reduced matrix
    vector<int> x(m, 0);
    for (int i = 0, j = 0; i < rank; i++) {
        while (mat[i][j] == 0) j++; //find pivot column
        x[j] = mat[i].back();
    return {rank, det, x};
```

## Euler's Totient Phi Function

```
//cat totient.hpp | ./hash.sh
//36bd41
#pragma once
//Euler's totient function counts the positive integers
//up to a given integer n that are relatively prime to n.
//
//To improve, pre-calc prime factors or use Pollard-rho to find prime factors.
int totient(int n) {
   int res = n;
   for (int i = 2; i * i <= n; i++) {</pre>
```

```
if (n % i == 0) {
      while (n % i == 0) n /= i;
      res -= res / i;
    }
}
if (n > 1) res -= res / n;
return res;
```

#### **MISC**

#### Cartesian Tree

```
//cat cartesian_tree.hpp / ./hash.sh
//0b95bc
#pragma once
#include "monotonic_stack.hpp"
//min cartesian tree
vector<int> cartesian_tree(const vector<int>& arr) {
   int n = arr.size();
   auto rv /*reverse*/ = [&](int i) -> int {
       return n - 1 - i;
   };
   vector<int> left = monotonic_stack<int>(arr, greater());
    vector<int> right = monotonic_stack<int>(vector<int>(arr.rbegin(), arr.rend()),
        \hookrightarrow greater());
   vector<int> par(n);
   for (int i = 0; i < n; i++) {</pre>
        int l = left[i], r = rv(right[rv(i)]);
        if (1 >= 0 \&\& r < n) par[i] = arr[1] > arr[r] ? 1 : r;
        else if (1 >= 0) par[i] = 1;
        else if (r < n) par[i] = r;</pre>
        else par[i] = i;
   }
   return par;
```

# Count Rectangles

```
//cat count_rectangles.hpp | ./hash.sh
//b2cced
#pragma once
#include "monotonic_stack.hpp"
//qiven a 2D boolean matrix, calculate cnt[i][j]
//cnt[i][j] = the number of times an i-by-j rectangle appears in the matrix such that
    \hookrightarrow all i*j cells in the rectangle are 1
//Note cnt[0][i] and cnt[i][0] will contain garbage values
//0(n*m)
vector<vector<int>> count_rectangles(const vector<vector<bool>>& grid) {
   int n = grid.size(), m = grid[0].size();
   vector<vector<int>> cnt(n + 1, vector<int>(m + 1, 0));
   vector<int> arr(m, 0);
   auto rv /*reverse*/ = [&](int j) -> int {
       return m - 1 - j;
   }:
   for (int i = 0; i < n; i++) {
```

```
for (int j = 0; j < m; j++)
        arr[j] = grid[i][j] * (arr[j] + 1);
    vector<int> left = monotonic_stack<int>(arr, greater());
    vector<int> right = monotonic_stack<int>(vector<int>(arr.rbegin(), arr.rend()),
        for (int j = 0; j < m; j++) {
        int l = j - left[j] - 1, r = rv(right[rv(j)]) - j - 1;
        cnt[arr[j]][1 + r + 1]++;
        cnt[arr[j]][1]--;
        cnt[arr[i]][r]--:
}
for (int i = 1; i <= n; i++)
    for (int k = 0; k < 2; k++)
        for (int j = m; j > 1; j--)
           cnt[i][j - 1] += cnt[i][j];
for (int j = 1; j <= m; j++)</pre>
    for (int i = n; i > 1; i--)
        cnt[i - 1][j] += cnt[i][j];
return cnt:
```

# Max Rectangle in Histogram

```
//cat max_rect_histogram.hpp | ./hash.sh
//4e6291
#pragma once
#include "monotonic_stack.hpp"
long long max_rect_histogram(const vector<int>& arr) {
    int n = arr.size();
    auto rv /*reverse*/ = [&](int i) -> int {
        return n - 1 - i;
    vector<int> left = monotonic_stack<int>(arr, greater_equal());
    vector<int> right = monotonic_stack<int>(vector<int>(arr.rbegin(), arr.rend()),
        long long max_area = 0;
    for (int i = 0; i < n; i++) {
        int 1 = left[i], r = rv(right[rv(i)]);//arr[i] is the max of range (l, r)
        \max_{\text{area}} = \max(\max_{\text{area}}, 1LL * arr[i] * (r - 1 - 1));
   return max_area;
```

## Monotonic Stack

#### Iterate Chooses

```
//cat iterate_chooses.hpp | ./hash.sh
//c79083
#pragma once

// source: https://github.com/kth-competitive-programming/
// kactl/blob/main/content/various/chapter.tex
// iterates all bitmasks of size n with k bits set
// Time Complexity: O(n choose k)
// Space Complexity: O(1)

int next_subset(int mask) {
   int c = mask & -mask, r = mask + c;
   return r | (((r ^ mask) >> 2) / c);
}

void iterate_chooses(int n, int k, const function<void(int)>& func) {
   for (int mask = (1 << k) - 1; mask < (1 << n); mask = next_subset(mask))
      func(mask);
}</pre>
```

## Iterate Submasks

```
//cat iterate_submasks.hpp | ./hash.sh
//084c05
#pragma once

// iterates all submasks of mask
// Time Complexity: 0(3^n) to iterate every submask of every mask of size n
// Space Complexity: 0(1)

void iterate_submasks(int mask, const function<void(int)>& func) {
    for (int submask = mask; submask; submask = (submask - 1) & mask)
        func(submask);
}
```

# Iterate Supermasks

```
//cat iterate_supermasks.hpp | ./hash.sh
//76b38f
#pragma once
// iterates all supermasks of mask
```

```
// Time Complexity: 0(3^n) to iterate every supermask of every mask of size n
// Space Complexity: 0(1)

void iterate_supermasks(int mask, int n, const function<void(int)>& func) {
   for (int supermask = mask; supermask < (1 << n); supermask = (supermask + 1) | mask)
      func(supermask);
}</pre>
```

# Number of Distinct Subsequences DP

```
//cat num_distinct_subsequences.hpp / ./hash.sh
//9542f5
#pragma once
//returns number of distinct subsequences
//the empty subsequence is counted
int num_subsequences(const vector<int>& arr, int mod) {
    int n = arr.size();
    vector < int > dp(n + 1, 1);
    map<int, int> last;
    for (int i = 0; i < n; i++) {</pre>
        int& curr = dp[i + 1] = 2 * dp[i];
        if (curr >= mod) curr -= mod;
        auto it = last.find(arr[i]);
        if (it != last.end()) {
            curr -= dp[it->second];
            if (curr < 0) curr += mod;</pre>
            it->second = i;
       } else last[arr[i]] = i;
    return dp[n];
```

### **PBDS**

```
//cat policy_based_data_structures.hpp / ./hash.sh
//807de9
#pragma once
//place these includes *before* the '#define int long long' else compile error
//not using <bits/extc++.h> as it compile errors on codeforces c++20 compiler
#include <ext/pb_ds/assoc_container.hpp>
#include <ext/pb_ds/tree_policy.hpp>
using namespace __gnu_pbds;
//BST with extra functions https://codeforces.com/blog/entry/11080
//order_of_key - # of elements *strictly* less than given element
//find_by_order - find kth largest element, k is 0 based so find_by_order(0) returns min
template<class T> using indexed_set = tree<T, null_type, less<T>, rb_tree_tag,

    tree_order_statistics_node_update>;

//example initialization:
indexed_set<pair<long long, int>> is;
//hash table (apparently faster than unordered_map):
    \hookrightarrow https://codeforces.com/blog/entry/60737
//example initialization:
gp_hash_table<string, long long> ht;
```

# Random

```
//cat random.hpp | ./hash.sh
//46c199
#pragma once

//source: https://codeforces.com/blog/entry/61675

mt19937 rng(chrono::steady_clock::now().time_since_epoch().count());

//intended types: int, unsigned, long long
//returns a random number in range [l, r)
template<class T> inline T get_rand(T l, T r) {
    assert(l < r);
    return uniform_int_distribution<T>(l, r - 1)(rng);
}

//vector<int> a;
//shuffle(a.begin(), a.end(), rng);
```

#### Safe Hash

```
//cat safe_hash.hpp / ./hash.sh
//d9ea53
#pragma once
//source: https://codeforces.com/blog/entry/62393
struct custom_hash {
    static uint64_t splitmix64(uint64_t x) {
        // http://xorshift.di.unimi.it/splitmix64.c
        x += 0x9e3779b97f4a7c15;
        x = (x ^ (x >> 30)) * 0xbf58476d1ce4e5b9;
        x = (x ^ (x >> 27)) * 0x94d049bb133111eb;
        return x ^ (x >> 31);
   }
    size_t operator()(uint64_t x) const {
        static const uint64_t FIXED_RANDOM =

    chrono::steady_clock::now().time_since_epoch().count();

        return splitmix64(x + FIXED_RANDOM);
   }
};
//usage:
unordered_map<long long, int, custom_hash> safe_map;
#include "policy_based_data_structures.hpp"
gp_hash_table<long long, int, custom_hash> safe_hash_table;
```

# RANGE DATA STRUCTURES

# Number Distinct Elements

```
//cat distinct_query.hpp | ./hash.sh

//d0d922

#pragma once

//source: https://cp-algorithms.com/data_structures/segment_tree.html

// #preserving-the-history-of-its-values-persistent-segment-tree

//works with negatives

//O(n log n) time and space

struct distinct_query {
```

```
struct node {
        int lch, rch;//children, indexes into 'tree'
        node(int a_sum, int a_lch, int a_rch) : sum(a_sum), lch(a_lch), rch(a_rch) {}
   };
    const int N;
    vector<int> roots:
    deque<node> tree;
    distinct_query(const vector<int>& arr) : N(arr.size()), roots(N + 1, 0) {
        tree.emplace_back(0, 0, 0); //acts as null
        map<int, int> last_idx;
        for (int i = 0; i < N; i++) {</pre>
            roots[i + 1] = update(roots[i], 0, N, last_idx[arr[i]]);
            last_idx[arr[i]] = i + 1;
       }
   }
    int update(int v, int tl, int tr, int idx) {
        if (tr - tl == 1) {
            tree.emplace_back(tree[v].sum + 1, 0, 0);
            return tree.size() - 1;
        int tm = tl + (tr - tl) / 2;
        int lch = tree[v].lch:
        int rch = tree[v].rch;
        if (idx < tm)
            lch = update(lch, tl, tm, idx);
            rch = update(rch, tm, tr, idx);
        tree.emplace_back(tree[lch].sum + tree[rch].sum, lch, rch);
        return tree.size() - 1;
    //returns number of distinct elements in range [l,r)
    int query(int 1, int r) const {
        assert(0 <= 1 && 1 <= r && r <= N);
        return query(roots[1], roots[r], 0, N, 1 + 1);
    int query(int vl, int vr, int tl, int tr, int idx) const {
        if (tree[vr].sum == 0 || idx <= tl)</pre>
            return 0;
        if (tr <= idx)
            return tree[vr].sum - tree[vl].sum:
        int tm = tl + (tr - tl) / 2;
        return query(tree[v1].lch, tree[vr].lch, tl, tm, idx) +
               query(tree[v1].rch, tree[vr].rch, tm, tr, idx);
};
```

# Implicit Lazy Segment Tree

```
//cat implicit_seg_tree.hpp | ./hash.sh
//cbc0c0
#pragma once
//example initialization:
// implicit_seg_tree<10'000'000> ist(l, r);
template <int N> struct implicit_seg_tree {
   using dt = array<long long, 2>;//min, number of mins
   using ch = long long;
   static dt combine(const dt& 1, const dt& r) {
```

```
if (1[0] == r[0]) return \{1[0], 1[1] + r[1]\};
    return min(1, r);
}
static constexpr dt UNIT{(long long)1e18, OLL);
struct node {
    dt val;
    ch lazv:
    int lch, rch; // children, indexes into 'tree', -1 for null
    node() {}
    node(const dt& a val) : val(a val), lazv(0), lch(-1), rch(-1) {}
} tree[N];
int ptr = 0, root_l, root_r; //[root_l, root_r) defines range of root node; handles
     \hookrightarrow negatives
implicit_seg_tree(int 1, int r) : root_1(1), root_r(r) {
    tree[ptr++] = node(dt\{0, r - 1\});
}
void apply(int v, ch add) {
    tree[v].val[0] += add;
    tree[v].lazy += add;
}
void push(int v, int tl, int tr) {
    if (tr - tl > 1 && tree[v].lch == -1) {
        int tm = t1 + (tr - t1) / 2:
        assert(ptr + 1 < N);</pre>
        tree[v].lch = ptr;
        tree[ptr++] = node(dt{0, tm - tl});
        tree[v].rch = ptr;
        tree[ptr++] = node(dt{0, tr - tm});
    if (tree[v].lazy) {
        apply(tree[v].lch, tree[v].lazy);
        apply(tree[v].rch, tree[v].lazy);
        tree[v].lazy = 0;
    }
}
//update range [l,r)
void update(int 1, int r, ch add) {
    update(0, root 1, root r, 1, r, add):
}
void update(int v, int tl, int tr, int l, int r, ch add) {
    if (r <= tl || tr <= 1)
        return;
    if (1 <= t1 && tr <= r)
        return apply(v, add);
    push(v, tl, tr);
    int tm = tl + (tr - tl) / 2;
    update(tree[v].lch, tl, tm, l, r, add);
    update(tree[v].rch, tm, tr, 1, r, add);
    tree[v].val = combine(tree[tree[v].lch].val,
                          tree[tree[v].rch].val);
}
//queru range [l.r]
dt query(int 1, int r) {
    return query(0, root_1, root_r, 1, r);
dt query(int v, int tl, int tr, int l, int r) {
    if (r <= tl || tr <= 1)
        return UNIT;
    if (1 <= t1 && tr <= r)
```

### Kth Smallest

```
//cat kth_smallest.hpp / ./hash.sh
//023ee8
#include <bits/stdc++.h>
using namespace std;
#pragma once
//source: https://cp-algorithms.com/data_structures/segment_tree.html
// #preserving-the-history-of-its-values-persistent-segment-tree
struct kth smallest {
    struct node {
        int sum:
        int lch, rch;//children, indexes into 'tree'
        node(int a_sum, int a_lch, int a_rch) : sum(a_sum), lch(a_lch), rch(a_rch) {}
    int mn = INT_MAX, mx = INT_MIN;
    vector<int> roots;
    deque<node> tree;
    kth_smallest(const vector<int>& arr) : roots(arr.size() + 1, 0) {
        tree.emplace_back(0, 0, 0); //acts as null
        for (int val : arr) mn = min(mn, val), mx = max(mx, val + 1);
        for (int i = 0; i < (int)arr.size(); i++)</pre>
            roots[i + 1] = update(roots[i], mn, mx, arr[i]);
    int update(int v, int tl, int tr, int idx) {
        if (tr - tl == 1) {
            tree.emplace_back(tree[v].sum + 1, 0, 0);
            return tree.size() - 1:
        int tm = tl + (tr - tl) / 2;
        int lch = tree[v].lch;
        int rch = tree[v].rch;
        if (idx < tm)
            lch = update(lch, tl, tm, idx);
            rch = update(rch, tm, tr, idx);
        tree.emplace_back(tree[lch].sum + tree[rch].sum, lch, rch);
        return tree.size() - 1:
    /* find (k+1)th smallest number in range [l, r)
     * k is 0-based, so query(l.r.0) returns the min
    int query(int 1, int r, int k) const {
        assert(0 \le k \&\& k \le r - 1): //note this condition implies l \le r
        assert(0 <= 1 && r < (int)roots.size());</pre>
        return query(roots[1], roots[r], mn, mx, k);
    int query(int vl, int vr, int tl, int tr, int k) const {
        assert(tree[vr].sum > tree[vl].sum);
        if (tr - tl == 1)
```

```
return tl;
int tm = tl + (tr - tl) / 2;
int left_count = tree[tree[vr].lch].sum - tree[tree[vl].lch].sum;
if (left_count > k) return query(tree[vl].lch, tree[vr].lch, tl, tm, k);
return query(tree[vl].rch, tree[vr].rch, tm, tr, k - left_count);
}
};
```

# Merge Sort Tree

```
//cat merge_sort_tree.hpp | ./hash.sh
//c885bd
#pragma once
//For point updates: either switch to policy based BST, or use sgrt decomposition
struct merge_sort_tree {
    const int N, S/*smallest power of 2 >= N*/;
    vector<vector<int>> tree;
    merge_sort_tree(const vector<int>& arr) : N(arr.size()), S(N ? 1 << __lg(2 * N - 1)</pre>
         \hookrightarrow : 0), tree(2 * N) {
        for (int i = 0; i < N; i++)</pre>
            tree[i + N] = {arr[i]};
        rotate(tree.rbegin(), tree.rbegin() + S - N, tree.rbegin() + N);
        for (int i = N - 1; i >= 1; i--) {
            const auto& 1 = tree[2 * i]:
            const auto& r = tree[2 * i + 1];
            tree[i].reserve(l.size() + r.size()):
            merge(1.begin(), 1.end(), r.begin(), r.end(), back_inserter(tree[i]));
        }
    }
    int value(int v, int x) const {
        return lower_bound(tree[v].begin(), tree[v].end(), x) - tree[v].begin();
    }
    int to_leaf(int i) const {
        i += S:
        return i < 2 * N ? i : 2 * (i - N);
    }
    //How many values in range [l, r) are \langle x \rangle
    //0(log^2(n))
    int query(int 1, int r, int x) const {
        int res = 0:
        for (1 = to_leaf(1), r = to_leaf(r); 1 < r; 1 >>= 1, r >>= 1) {
            if (1 & 1) res += value(1++, x);
            if (r & 1) res += value(--r, x);
        }
        return res;
   }
};
```

#### BIT

```
//cat bit.hpp / ./hash.sh
//bc6b34
#pragma once
//mnemonic: Binary Indexed Tree
//NOLINTNEXTLINE(readability-identifier-naming)
template<class T> struct BIT {
    const int N;
```

```
vector<T> bit:
    BIT(int a_n) : N(a_n), bit(N, 0) {}
    BIT(const vector<T>& a) : N(a.size()), bit(a) {
        for (int i = 0; i < N; i++) {</pre>
            int j = i | (i + 1);
            if (j < N) bit[j] += bit[i];</pre>
    }
    void update(int i, const T& d) {
        assert(0 <= i && i < N):
        for (; i < N; i |= i + 1) bit[i] += d;
    T sum(int r) const {//sum of range [0, r)
        assert(0 <= r && r <= N);
        T ret = 0:
        for (; r > 0; r \&= r - 1) ret += bit[r - 1];
        return ret:
    T sum(int 1, int r) const {//sum of range [l, r)
        assert(0 <= 1 && 1 <= r && r <= N);
        return sum(r) - sum(1);
    }
    //Returns min pos (0 \le pos \le N+1) such that sum of [0, pos) \ge sum
    //Returns N + 1 if no sum is >= sum, or 0 if empty sum is.
    //Doesn't work with negatives
    int lower_bound(T sum) const {
        if (sum <= 0) return 0;</pre>
        int pos = 0;
        if (pos + pw <= N && bit[pos + pw - 1] < sum)
               pos += pw, sum -= bit[pos - 1];
        return pos + 1;
    }
};
```

# RMQ

```
//cat rmq.hpp | ./hash.sh
//082180
#pragma once
//source: https://github.com/kth-competitive-programming/
// kactl/blob/main/content/data-structures/RMQ.h
//usage:
// vector<long long> arr;
// RMQ<long long> rmq(arr, [@](auto x, auto y) \{ return min(x,y); \});
//to also get index of min element, do:
// RMQ < pair < T, int >> rmq(arr, [0](auto x, auto y) { return <math>min(x, y); });
//and\ initialize\ arr[i].second = i\ (0 <= i < n)
//If there are multiple indexes of min element, it'll return the smallest
//(left-most) one
//mnemonic: Range Min/Max Query
//NOLINTNEXTLINE(readability-identifier-naming)
template <class T> struct RMQ {
    vector<vector<T>> dp;
    function<T(const T&, const T&)> op;
    RMQ(const vector<T>& arr, const function<T(const T&, const T&)>& a_op) : dp(1, arr),
```

# Lazy Segment Tree

```
//cat lazy_segment_tree.hpp | ./hash.sh
//f51640
#pragma once
//source: https://codeforces.com/blog/entry/18051,
    → https://qithub.com/ecnerwala/cp-book/blob/master/src/seq_tree.hpp,
    \hookrightarrow \ https://qithub.com/yosupo06/Algorithm/blob/master/src/datastructure/segtree.hpp
//rotating leaves makes it a single complete binary tree (instead of a set of perfect
    \hookrightarrow binary trees)
//so standard implementations of
// - recursive seg tree
// - tree walks AKA binary search
//still work
struct seg_tree {
   using dt = long long;
    using ch = long long;
    static dt combine(const dt& 1, const dt& r) {
        return min(1, r):
   }
    static const dt UNIT = 1e18;
    struct node {
        dt val:
        ch lazy;
        int 1, r;//[l, r)
   };
    const int N, S/*smallest power of 2 >= N*/;
    vector<node> tree;
    seg\_tree(const\ vector<dt>\&\ arr): N(arr.size()), S(N ? 1 << __lg(2 * N - 1) : 0),
        \hookrightarrow tree(2 * N) {
        for (int i = 0; i < N; i++)</pre>
            tree[i + N] = \{arr[i], 0, i, i + 1\};
        rotate(tree.rbegin(), tree.rbegin() + S - N, tree.rbegin() + N);
        for (int i = N - 1; i >= 1; i--) {
            tree[i] = {
                combine(tree[2 * i].val, tree[2 * i + 1].val),
                tree[2 * i].1,
                tree[2 * i + 1].r
            };
        }
   }
    void apply(int v, ch change) {
```

```
tree[v].val += change:
    tree[v].lazy += change;
}
void push(int v) {
    if (tree[v].lazy) {
        apply(2 * v, tree[v].lazy);
        apply(2 * v + 1, tree[v].lazy);
        tree[v].lazy = 0;
}
void build(int v) {
    tree[v].val = combine(tree[2 * v].val, tree[2 * v + 1].val);
int to_leaf(int i) const {
    i += S:
    return i < 2 * N ? i : 2 * (i - N);
//update range [l, r)
void update(int 1, int r, ch change) {
    assert(0 <= 1 && 1 <= r && r <= N);</pre>
    l = to_leaf(l), r = to_leaf(r);
    int lca_l_r = __lg((l - 1) ^ r);
    for (int lg = __lg(1); lg > __builtin_ctz(1); lg--) push(1 >> lg);
    for (int lg = lca_l_r; lg > __builtin_ctz(r); lg--) push(r >> lg);
    for (int x = 1, y = r; x < y; x >>= 1, y >>= 1) {
        if (x & 1) apply(x++, change);
        if (y & 1) apply(--y, change);
    for (int lg = __builtin_ctz(r) + 1; lg <= lca_l_r; lg++) build(r >> lg);
    for (int lg = \_builtin\_ctz(l) + 1; lg <= \_lg(l); lg++) build(l >> lg);
void update(int v/* = 1*/, int 1, int r, ch change) {
    if (r <= tree[v].1 || tree[v].r <= 1)</pre>
        return:
    if (1 <= tree[v].1 && tree[v].r <= r)</pre>
        return apply(v, change);
    push(v);
    update(2 * v, 1, r, change);
    update(2 * v + 1, 1, r, change);
    build(v);
}
//query range [l, r)
dt query(int 1, int r) {
    assert(0 <= 1 && 1 <= r && r <= N);
    l = to_leaf(l), r = to_leaf(r);
    int lca_l_r = __lg((l - 1) ^ r);
    for (int lg = __lg(l); lg > __builtin_ctz(l); lg--) push(l >> lg);
    for (int lg = lca_l_r; lg > __builtin_ctz(r); lg--) push(r >> lg);
    dt resl = UNIT, resr = UNIT;
    for (; 1 < r; 1 >>= 1, r >>= 1) {
        if (1 & 1) resl = combine(resl, tree[l++].val);
        if (r & 1) resr = combine(tree[--r].val, resr);
    return combine(resl. resr):
dt query(int v/* = 1*/, int 1, int r) {
    if (r <= tree[v].1 || tree[v].r <= 1)</pre>
        return UNIT;
    if (1 <= tree[v].1 && tree[v].r <= r)</pre>
```

```
return tree[v].val:
        return combine(query(2 * v, 1, r), query(2 * v + 1, 1, r));
   }
};
```

#### **STRINGS**

# Binary Trie

```
//cat binary_trie.hpp / ./hash.sh
//330030
#pragma once
struct binary_trie {
   const int MX_BIT = 62;
   struct node {
       long long val = -1;
       int sub_sz = 0;//number of inserted values in subtree
       array<int, 2> next = {-1, -1};
   };
   vector<node> t;
   binary_trie() : t(1) {}
   //delta = 1 to insert val, -1 to remove val, 0 to get the # of val's in this data
        \hookrightarrow structure
   int update(long long val, int delta) {
       int c = 0;
       t[0].sub sz += delta:
       for (int bit = MX_BIT; bit >= 0; bit--) {
            bool v = (val >> bit) & 1;
           if (t[c].next[v] == -1) {
                t[c].next[v] = t.size();
                t.emplace_back();
            c = t[c].next[v];
            t[c].sub sz += delta:
       t[c].val = val;
       return t[c].sub_sz;
   }
   int size() const {
       return t[0].sub_sz;
   }
   //returns x such that:
   // x is in this data structure
   // value of (x ^val) is minimum
   long long min_xor(long long val) const {
       assert(size() > 0);
       int c = 0:
       for (int bit = MX_BIT; bit >= 0; bit--) {
            bool v = (val >> bit) & 1;
           int ch = t[c].next[v]:
           if (ch != -1 && t[ch].sub_sz > 0)
                c = ch:
            else
                c = t[c].next[!v];
       }
       return t[c].val;
```

```
};
```

#### **KMP**

```
//cat kmp.hpp | ./hash.sh
//66aeae
#pragma once
//mnemonic: Knuth Morris Pratt
#include "prefix_function.hpp"
//usage:
// string needle;
// ...
// KMP kmp(needle);
//or
// vector<int> needle;
// ...
// KMP kmp(needle):
//kmp doubling trick: to check if 2 arrays are rotationally equivalent: run kmp
//with one array as the needle and the other array doubled (excluding the first
//8 last characters) as the haystack or just use kactl's min rotation code
//NOLINTNEXTLINE(readability-identifier-naming)
template <class T> struct KMP {
    KMP(const T& a_needle) : needle(a_needle), pi(prefix_function(needle)) {}
    // if haystack = "bananas"
    // needle = "ana"
    // then we find 2 matches:
    // bananas
    // _ana___
    // ___ana_
    // 0123456 (indexes)
    // and KMP::find returns {1,3} - the indexes in haystack where
    // each match starts.
    // You can also pass in 0 for "all" and KMP::find will only
    // return the first match: {1}. Useful for checking if there exists
    // some match:
    // KMP::find(<haystack>,0).size() > 0
    vector<int> find(const T& haystack, bool all = 1) const {
        vector<int> matches:
        for (int i = 0, j = 0; i < (int)haystack.size(); i++) {</pre>
            while (j > 0 && needle[j] != haystack[i]) j = pi[j - 1];
            if (needle[j] == haystack[i]) j++;
            if (j == (int)needle.size()) {
                matches.push_back(i - (int)needle.size() + 1);
                if (!all) return matches;
                j = pi[j - 1];
            }
        return matches;
    T needle;
    vector<int> pi;
};
```

## Longest Common Prefix Query

```
//cat longest_common_prefix_query.hpp | ./hash.sh
//178fb7
#pragma once
#include "../../ac-library/atcoder/string.hpp"
#include "../range_data_structures/rmq.hpp"
//computes suffix array, lcp array, and then sparse table over lcp array
//0(n \log n)
template<typename T> struct lcp_query {
   const int N;
    vector<int> sa, lcp, inv_sa;
   RMQ<int> rmq;
   lcp_query(const T& s) : N(s.size()), sa(atcoder::suffix_array(s)),
        return min(x, y);
   }) {
       for (int i = 0; i < N; i++) inv_sa[sa[i]] = i;</pre>
   }
    //length of longest common prefix of suffixes s[idx1 ... n), s[idx2 ... n), 0-based
        \hookrightarrow indexina
   //You can check if two substrings s[11..r1), s[12..r2) are equal in O(1) by:
   //r1-l1 == r2-l2 && longest_common_prefix(l1, l2) >= r1-l1
    int get_lcp(int idx1, int idx2) const {
       if (idx1 == idx2) return N - idx1;
       idx1 = inv_sa[idx1];
       idx2 = inv_sa[idx2];
       if (idx1 > idx2) swap(idx1, idx2);
       return rmq.query(idx1, idx2);
   }
   //returns \ 1 \ if \ suffix \ s[idx1 ... n) < s[idx2 ... n)
   //(so 0 if idx1 == idx2)
   bool less(int idx1, int idx2) const {
       return inv_sa[idx1] < inv_sa[idx2];</pre>
   }
};
```

# Palindrome Query

```
//cat palindrome_query.hpp / ./hash.sh
//7326d0
#pragma once
#include "../../kactl/content/strings/Manacher.h"
struct pal_query {
    const int N;
    array<vi, 2> pal_len;
    pal_query(const string& s) : N(s.size()), pal_len(manacher(s)) {}
    //returns 1 if substring s[l...r) is a palindrome
    //(returns\ 1\ when\ l == r)
   bool is_pal(int 1, int r) const {
        assert(0 <= 1 && 1 <= r && r <= N);
        int len = r - 1:
        return pal_len[len & 1][l + len / 2] >= len / 2;
   }
};
```

```
Trie
```

```
//cat trie.hpp / ./hash.sh
//91612d
#pragma once
//source: https://cp-algorithms.com/string/aho_corasick.html#construction-of-the-trie
const int K = 26;//alphabet size
struct trie {
    const char MIN_CH = 'A';//'a' for lowercase, '0' for digits
    struct node {
        int next[K], cnt_words = 0, par = -1;
        node(int a_par = -1, char a_ch = '#') : par(a_par), ch(a_ch) {
            fill(next, next + K, -1):
       }
   };
    vector<node> t;
    trie(): t(1) {}
    void insert(const string& s) {
        int v = 0:
        for (char ch : s) {
            int let = ch - MIN CH:
            if (t[v].next[let] == -1) {
                t[v].next[let] = t.size():
                t.emplace_back(v, ch);
            v = t[v].next[let]:
        t[v].cnt_words++;
    bool find(const string& s) const {
        int v = 0:
        for (char ch : s) {
            int let = ch - MIN CH:
            if (t[v].next[let] == -1) return 0;
            v = t[v].next[let];
        return t[v].cnt_words;
   }
};
```

# Suffix Array and LCP Array

```
//cat string.hpp | ./hash.sh
//67378f
#ifndef ATCODER_STRING_HPP
#define ATCODER_STRING_HPP 1

#include <algorithm>
#include <numeric>
#include <string>
#include <vector>

namespace atcoder {

namespace internal {

std::vector<int> sa_naive(const std::vector<int>& s) {
    int n = int(s.size());
}
```

South Dakota Mines

```
std::vector<int> sa(n):
    std::iota(sa.begin(), sa.end(), 0);
    std::sort(sa.begin(), sa.end(), [&](int 1, int r) {
        if (1 == r) return false:
        while (1 < n \&\& r < n) {
            if (s[1] != s[r]) return s[1] < s[r];</pre>
            r++;
        }
        return 1 == n:
    });
    return sa;
std::vector<int> sa_doubling(const std::vector<int>& s) {
    int n = int(s.size());
    std::vector<int> sa(n), rnk = s, tmp(n);
    std::iota(sa.begin(), sa.end(), 0);
    for (int k = 1; k < n; k *= 2) {
        auto cmp = [&](int x, int y) {
            if (rnk[x] != rnk[y]) return rnk[x] < rnk[y];</pre>
            int rx = x + k < n ? rnk[x + k] : -1;
            int ry = y + k < n ? rnk[y + k] : -1;
            return rx < ry;</pre>
        };
        std::sort(sa.begin(), sa.end(), cmp);
        tmp[sa[0]] = 0;
        for (int i = 1; i < n; i++) {</pre>
            tmp[sa[i]] = tmp[sa[i - 1]] + (cmp(sa[i - 1], sa[i]) ? 1 : 0);
        std::swap(tmp, rnk);
    }
    return sa;
// SA-IS, linear-time suffix array construction
// Reference:
// G. Nong, S. Zhang, and W. H. Chan,
// Two Efficient Algorithms for Linear Time Suffix Array Construction
template <int THRESHOLD_NAIVE = 10, int THRESHOLD_DOUBLING = 40>
std::vector<int> sa_is(const std::vector<int>& s, int upper) {
    int n = int(s.size());
    if (n == 0) return {};
    if (n == 1) return {0};
    if (n == 2) {
        if (s[0] < s[1]) {
            return {0, 1};
        } else {
            return {1, 0};
    }
    if (n < THRESHOLD NAIVE) {</pre>
        return sa_naive(s);
    }
    if (n < THRESHOLD_DOUBLING) {</pre>
        return sa_doubling(s);
    }
    std::vector<int> sa(n);
```

```
std::vector<bool> ls(n):
for (int i = n - 2; i \ge 0; i--) {
   ls[i] = (s[i] == s[i + 1]) ? ls[i + 1] : (s[i] < s[i + 1]);
std::vector<int> sum_l(upper + 1), sum_s(upper + 1);
for (int i = 0; i < n; i++) {
    if (!ls[i]) {
        sum_s[s[i]]++;
   } else {
        sum l[s[i] + 1]++:
   }
}
for (int i = 0; i <= upper; i++) {</pre>
    sum_s[i] += sum_l[i];
    if (i < upper) sum_l[i + 1] += sum_s[i];</pre>
}
auto induce = [&](const std::vector<int>& lms) {
    std::fill(sa.begin(), sa.end(), -1);
    std::vector<int> buf(upper + 1);
    std::copy(sum_s.begin(), sum_s.end(), buf.begin());
    for (auto d : lms) {
        if (d == n) continue:
        sa[buf[s[d]]++] = d;
   }
    std::copy(sum_l.begin(), sum_l.end(), buf.begin());
    sa[buf[s[n-1]]++] = n-1;
    for (int i = 0; i < n; i++) {</pre>
        int v = sa[i];
        if (v >= 1 && !ls[v - 1]) {
            sa[buf[s[v - 1]] ++] = v - 1;
    std::copy(sum_l.begin(), sum_l.end(), buf.begin());
    for (int i = n - 1; i \ge 0; i--) {
        int v = sa[i]:
        if (v >= 1 && ls[v - 1]) {
            sa[--buf[s[v-1]+1]] = v-1;
   }
}:
std::vector<int> lms_map(n + 1, -1);
int m = 0:
for (int i = 1; i < n; i++) {
    if (!ls[i - 1] && ls[i]) {
        lms_map[i] = m++;
std::vector<int> lms;
lms.reserve(m):
for (int i = 1; i < n; i++) {
    if (!ls[i - 1] && ls[i]) {
        lms.push_back(i);
}
induce(lms);
```

```
if (m) {
        std::vector<int> sorted lms:
        sorted_lms.reserve(m);
        for (int v : sa) {
            if (lms_map[v] != -1) sorted_lms.push_back(v);
        }
        std::vector<int> rec s(m):
        int rec_upper = 0;
        rec_s[lms_map[sorted_lms[0]]] = 0;
        for (int i = 1: i < m: i++) {
            int l = sorted_lms[i - 1], r = sorted_lms[i];
            int end_1 = (lms_map[1] + 1 < m) ? lms[lms_map[1] + 1] : n;</pre>
            int end_r = (lms_map[r] + 1 < m) ? lms[lms_map[r] + 1] : n;
            bool same = true:
            if (end 1 - 1 != end r - r) {
                same = false;
            } else {
                while (1 < end 1) {
                    if (s[1] != s[r]) {
                        break;
                    1++:
                    r++:
                if (1 == n || s[1] != s[r]) same = false;
            if (!same) rec_upper++;
            rec_s[lms_map[sorted_lms[i]]] = rec_upper;
        auto rec_sa =
            sa_is<THRESHOLD_NAIVE, THRESHOLD_DOUBLING>(rec_s, rec_upper);
        for (int i = 0; i < m; i++) {</pre>
            sorted lms[i] = lms[rec sa[i]]:
        induce(sorted_lms);
   }
    return sa;
} // namespace internal
std::vector<int> suffix_array(const std::vector<int>& s, int upper) {
    assert(0 <= upper);</pre>
   for (int d : s) {
        assert(0 <= d && d <= upper);</pre>
   auto sa = internal::sa_is(s, upper);
    return sa;
template <class T> std::vector<int> suffix_array(const std::vector<T>& s) {
    int n = int(s.size());
    std::vector<int> idx(n);
    iota(idx.begin(), idx.end(), 0);
    sort(idx.begin(), idx.end(), [&](int 1, int r) { return s[1] < s[r]; });</pre>
    std::vector<int> s2(n);
    int now = 0;
```

```
for (int i = 0: i < n: i++) {</pre>
        if (i && s[idx[i - 1]] != s[idx[i]]) now++;
        s2[idx[i]] = now;
    return internal::sa_is(s2, now);
std::vector<int> suffix_array(const std::string& s) {
    int n = int(s.size());
    std::vector<int> s2(n):
    for (int i = 0; i < n; i++) {
        s2[i] = s[i];
    return internal::sa_is(s2, 255);
// Reference:
// T. Kasai, G. Lee, H. Arimura, S. Arikawa, and K. Park,
// Linear-Time Longest-Common-Prefix Computation in Suffix Arrays and Its
// Applications
template <class T>
std::vector<int> lcp_array(const std::vector<T>& s,
                            const std::vector<int>& sa) {
    int n = int(s.size());
    assert(n >= 1);
    std::vector<int> rnk(n);
    for (int i = 0; i < n; i++) {
        rnk[sa[i]] = i;
    std::vector<int> lcp(n - 1);
    int h = 0:
    for (int i = 0; i < n; i++) {</pre>
        if (h > 0) h--;
        if (rnk[i] == 0) continue;
        int j = sa[rnk[i] - 1];
        for (; j + h < n && i + h < n; h++) {
            if (s[j + h] != s[i + h]) break;
        lcp[rnk[i] - 1] = h;
    return lcp:
std::vector<int> lcp_array(const std::string& s, const std::vector<int>& sa) {
    int n = int(s.size());
    std::vector<int> s2(n):
    for (int i = 0; i < n; i++) {</pre>
        s2[i] = s[i];
    return lcp_array(s2, sa);
// Reference:
// D. Gusfield.
// Algorithms on Strings, Trees, and Sequences: Computer Science and
// Computational Biology
template <class T> std::vector<int> z_algorithm(const std::vector<T>& s) {
    int n = int(s.size());
    if (n == 0) return {};
```

```
std::vector<int> z(n);
   for (int i = 1, j = 0; i < n; i++) {
        int & k = z[i];
       k = (j + z[j] \le i) ? 0 : std::min(j + z[j] - i, z[i - j]);
        while (i + k < n \&\& s[k] == s[i + k]) k++;
        if (j + z[j] < i + z[i]) j = i;
   }
   z[0] = n;
    return z;
std::vector<int> z_algorithm(const std::string& s) {
    int n = int(s.size());
    std::vector<int> s2(n);
   for (int i = 0; i < n; i++) {</pre>
        s2[i] = s[i];
    return z_algorithm(s2);
} // namespace atcoder
#endif // ATCODER_STRING_HPP
```