Listings

CONTEST

Tips and Tricks

```
## Tips and Tricks
- [C++ tips and tricks](https://codeforces.com/blog/entry/74684)
- invokes RTE (Run Time Error) upon integer overflow
```
#pragma GCC optimize "trapv"
```
- invoke RTE for input error (e.g. reading a long long into an int)
```
cin.exceptions(cin.failbit);
```

- use pramgas for C++ speed boost
```
#pragma GCC optimize("O3,unroll-loops")
#pragma GCC target("avx2,bmi,bmi2,lzcnt,popcnt")
```

### Troubleshooting
```
/* stuff you should look for
    * int overflow, array bounds
    * special cases (n=1?)
    * do smth instead of nothing and stay organized
    * WRITE STUFF DOWN
    * DON'T GET STUCK ON ONE APPROACH
*/
```
Author: Benq

- refer to [KACTL
    ↪ Troubleshoot](https://github.com/kth-competitive-programming/kactl/blob/main/content/contest/troubleshoot.txt)

## Sources

- [[Tutorial] GCC Optimization Pragmas](https://codeforces.com/blog/entry/96344)
- [Don't use rand(): a guide to random number generators in
    ↪ C++](https://codeforces.com/blog/entry/61587)
```

Hash codes

```
#! /bin/bash
#Hashes a file, ignoring all:
#   - whitespace
#   - comments
#   - asserts
#   - includes
#   - pragmas
#Use to verify that code was correctly typed.

#usage:
#   chmod +x hash.sh
#   cat a.cpp | ./hash.sh
#or just copy this command:
#   cat a.cpp | sed -r '/(assert|include|pragma)/d' | cpp -fpreprocessed -P | tr -d
    ↪ '[:space:]' | md5sum | cut -c-6
sed -r '/(assert|include|pragma)/d' | cpp -fpreprocessed -P | tr -d '[:space:]' | md5sum | cut
    ↪ -c-6
```

Test on random inputs

```
#! /bin/bash
#runs 2 programs against each other on random inputs until they output different results
#source: https://github.com/Errichto/youtube/blob/master/testing/s.sh
#usage:
#   chmod +x test.sh
#   ./test.sh
for((i = 1; ; ++i)); do
    echo $i
    ./test.out > in
    diff --ignore-all-space <(./a.out < in) <(./brute.out < in) || break
done
```

MAX FLOW

Hungarian

```
//cat hungarian.hpp | ./hash.sh
//935a16
#pragma once
const long long INF = 1e18;
struct weighted_match {
    long long min_cost;
    vector<int> matching;//worker `i` (1<=i<=n) is assigned to job `matching[i]`
        ↪ (1<=matching[i]<=m)
};
/**
 * Given cost[1...n][1...m] with 1 <= n <= m.
 * n workers, indexed 1, 2, ..., n
 * m jobs, indexed 1, 2, ..., m
 * It costs `cost[i][j]` to assign worker i to job j (1<=i<=n, 1<=j<=m). This
 * returns *min* sum of costs to assign each worker to some distinct job.
 *
 * Set `cost[i][j]` to INF to say: "worker i cannot be assigned job j".
 * This works for negatives, so negating cost array gives max matching.
 *
 * @see https://e-maxx.ru/algo/assignment_hungary
 * @time O(n^2 * m)
 * @memory O(n * m)
 */
weighted_match hungarian(const vector<vector<long long>>& cost) {
    int n = ssize(cost) - 1, m = ssize(cost[0]) - 1;
    assert(n <= m);
    vector<int> p(m + 1), way(m + 1);
    vector<long long> u(n + 1), v(m + 1);
    for (int i = 1; i <= n; i++) {
        p[0] = i;
        int j0 = 0;
        vector<long long> minv(m + 1, INF);
        vector<bool> used(m + 1, 0);
```

```
        do {
            used[j0] = 1;
            int i0 = p[j0], j1 = 0;
            long long delta = INF;
            for (int j = 1; j <= m; j++)
                if (!used[j]) {
                    long long cur = cost[i0][j] - u[i0] - v[j];
                    if (cur < minv[j])
                        minv[j] = cur, way[j] = j0;
                    if (minv[j] < delta)
                        delta = minv[j], j1 = j;
                }
            for (int j = 0; j <= m; j++)
                if (used[j])
                    u[p[j]] += delta, v[j] -= delta;
                else
                    minv[j] -= delta;
            j0 = j1;
        } while (p[j0] != 0);
        do {
            int j1 = way[j0];
            p[j0] = p[j1];
            j0 = j1;
        } while (j0);
    }
    vector<int> ans(n + 1);
    for (int j = 1; j <= m; j++)
        ans[p[j]] = j;
    return {-v[0], ans};
}
```

Min Cost Max Flow

```
//cat min_cost_max_flow.hpp | ./hash.sh
//9dd6b6
#pragma once
const long long INF = 1e18;
/**
 * @see https://e-maxx.ru/algo/min_cost_flow
 */
struct mcmf {
    using ll = long long;
    struct edge {
        int a, b;
        ll cap, cost, flow;
        int back;
    };
    const int N;
    vector<edge> e;
    vector<vector<int>> g;
    mcmf(int a_n) : N(a_n), g(N) {}
    void add_edge(int a, int b, ll cap, ll cost) {
        edge e1 = {a, b, cap, cost, 0, ssize(g[b])};
        edge e2 = {b, a, 0, -cost, 0, ssize(g[a])};
        g[a].push_back(ssize(e));
        e.push_back(e1);
        g[b].push_back(ssize(e));
        e.push_back(e2);
    }
    pair<ll, ll> get_flow(int s, int t, ll total_flow) {
        ll flow = 0, cost = 0;
        while (flow < total_flow) {
            vector<ll> d(N, INF);
            vector<int> p_edge(N), id(N, 0), q(N), p(N);
            int qh = 0, qt = 0;
            q[qt++] = s;
            d[s] = 0;
            while (qh != qt) {
                int v = q[qh++];
                id[v] = 2;
                if (qh == N) qh = 0;
                for (int i = 0; i < ssize(g[v]); i++) {
                    const edge& r = e[g[v][i]];
                    if (r.flow < r.cap && d[v] + r.cost < d[r.b]) {
                        d[r.b] = d[v] + r.cost;
                        if (id[r.b] == 0) {
                            q[qt++] = r.b;
                            if (qt == N) qt = 0;
                        } else if (id[r.b] == 2) {
                            if (--qh == -1) qh = N - 1;
                            q[qh] = r.b;
                        }
                        id[r.b] = 1;
                        p[r.b] = v;
                        p_edge[r.b] = i;
                    }
                }
            }
            if (d[t] == INF) break;
            ll addflow = total_flow - flow;
            for (int v = t; v != s; v = p[v]) {
                int pv = p[v], pr = p_edge[v];
                addflow = min(addflow, e[g[pv][pr]].cap - e[g[pv][pr]].flow);
            }
            for (int v = t; v != s; v = p[v]) {
                int pv = p[v], pr = p_edge[v], r = e[g[pv][pr]].back;
                e[g[pv][pr]].flow += addflow;
                e[g[v][r]].flow -= addflow;
                cost += e[g[pv][pr]].cost * addflow;
            }
            flow += addflow;
        }
        return {flow, cost};
    }
};
```

GRAPHS

Block Vertex Tree

```
//cat block_vertex_tree.hpp | ./hash.sh
//a5c2b9
#pragma once
#include "bridges_and_cuts.hpp"
/**
```

```
 * Returns adjacency list of block vertex tree.
 * @code{.cpp}
 *     graph_info cc = bridge_and_cut(adj, m);
 *     vector<vector<int>> bvt = block_vertex_tree(adj, cc);
 *     //to loop over each *unique* bcc containing a node v:
 *     for (int bccid : bvt[v]) {
 *         bccid -= n;
 *     }
 *     //to loop over each *unique* node inside a bcc:
 *     for (int v : bvt[bccid + n]) {}
 * @endcode
 * @time O(n + m)
 * @memory O(n + m)
 */
vector<vector<int>> block_vertex_tree(const vector<vector<pair<int, int>>>& adj, const
     ↪ graph_info& cc) {
    int n = ssize(adj);
    vector<vector<int>> bvt(n + cc.num_bccs);
    vector<bool> vis(cc.num_bccs, 0);
    for (int v = 0; v < n; v++) {
        for (auto [_, e_id] : adj[v]) {
            int bccid = cc.bcc_id[e_id];
            if (!vis[bccid]) {
                vis[bccid] = 1;
                bvt[v].push_back(bccid + n);//add edge between original node, and bcc node
                bvt[bccid + n].push_back(v);
            }
        }
        for (int bccid : bvt[v]) vis[bccid - n] = 0;
    }
    return bvt;
}
```

Bridge Tree

```
//cat bridge_tree.hpp | ./hash.sh
//8eb014
#pragma once
#include "bridges_and_cuts.hpp"
/**
 * Never adds multiple edges as bridges_and_cuts.hpp correctly marks them as
 * non-bridges.
 * @code{.cpp}
 *     graph_info cc = bridge_and_cut(adj, m);
 *     vector<vector<int>> bt = bridge_tree(adj, cc);
 * @endcode
 * @time O(n + m)
 * @memory O(n + m)
 */
vector<vector<int>> bridge_tree(const vector<vector<pair<int, int>>>& adj, const graph_info&
     ↪ cc) {
    vector<vector<int>> tree(cc.num_2_edge_ccs);
    for (int i = 0; i < ssize(adj); i++)
        for (auto [to, e_id] : adj[i])
            if (cc.is_bridge[e_id])
                tree[cc.two_edge_ccid[i]].push_back(cc.two_edge_ccid[to]);
    return tree;
}
```

Bridges and Cuts

```
//cat bridges_and_cuts.hpp | ./hash.sh
//3f21b9
#pragma once
struct graph_info {
    //2 edge connected component stuff (e.g. components split by bridge edges)
    int num_2_edge_ccs;
    vector<bool> is_bridge;//edge id -> 1 iff bridge edge
    vector<int> two_edge_ccid;//node -> id of 2 edge component (which are labeled 0, 1, ...,
         ↪ `num_2_edge_ccs`-1)
    //bi connected component stuff (e.g. components split by cut/articulation nodes)
    int num_bccs;
    vector<bool> is_cut;//node -> 1 iff cut node
    vector<int> bcc_id;//edge id -> id of bcc (which are labeled 0, 1, ..., `num_bccs`-1)
};
/**
 * @code{.cpp}
 *     //example initialization of `adj`:
 *     for (int i = 0; i < m; i++) {
 *         int u, v;
 *         cin >> u >> v;
 *         u--, v--;
 *         adj[u].emplace_back(v, i);
 *         adj[v].emplace_back(u, i);
 *     }
 * @endcode
 * @see https://cp-algorithms.com/graph/bridge-searching.html
 *     https://cp-algorithms.com/graph/cutpoints.html
 * @time O(n + m)
 * @memory O(n + m)
 */
graph_info bridge_and_cut(const vector<vector<pair<int/*neighbor*/, int/*edge id*/>>>&
     ↪ adj/*undirected graph*/, int m/*number of edges*/) {
    //stuff for both (always keep)
    int n = ssize(adj), timer = 1;
    vector<int> tin(n, 0);
    //2 edge cc stuff (delete if not needed)
    int num_2_edge_ccs = 0;
    vector<bool> is_bridge(m, 0);
    vector<int> two_edge_ccid(n), node_stack;
    node_stack.reserve(n);
    //bcc stuff (delete if not needed)
    int num_bccs = 0;
    vector<bool> is_cut(n, 0);
    vector<int> bcc_id(m), edge_stack;
    edge_stack.reserve(m);
    auto dfs = [&](auto self, int v, int p_id) -> int {
        int low = tin[v] = timer++, deg = 0;
        node_stack.push_back(v);
        for (auto [to, e_id] : adj[v]) {
            if (e_id == p_id) continue;
            if (!tin[to]) {
                edge_stack.push_back(e_id);
                int low_ch = self(self, to, e_id);
                if (low_ch >= tin[v]) {
                    is_cut[v] = 1;
                    while (1) {
                        int edge = edge_stack.back();
```

```
                    edge_stack.pop_back();
                    bcc_id[edge] = num_bccs;
                    if (edge == e_id) break;
                }
                num_bccs++;
            }
            low = min(low, low_ch);
            deg++;
        } else if (tin[to] < tin[v]) {
            edge_stack.push_back(e_id);
            low = min(low, tin[to]);
        }
    }
    if (p_id == -1) is_cut[v] = (deg > 1);
    if (tin[v] == low) {
        if (p_id != -1) is_bridge[p_id] = 1;
        while (1) {
            int node = node_stack.back();
            node_stack.pop_back();
            two_edge_ccid[node] = num_2_edge_ccs;
            if (node == v) break;
        }
        num_2_edge_ccs++;
    }
    return low;
};
for (int i = 0; i < n; i++)
    if (!tin[i])
        dfs(dfs, i, -1);
return {num_2_edge_ccs, is_bridge, two_edge_ccid, num_bccs, is_cut, bcc_id};
}
```

Strongly Connected Components

```
//cat strongly_connected_components.hpp | ./hash.sh
//f6f849
#pragma once
//source: https://github.com/kth-competitive-programming/
//  kactl/blob/main/content/graph/SCC.h
struct scc_info {
    int num_sccs;
    //scc's are labeled 0,1,...,`num_sccs-1`
    //scc_id[i] is the id of the scc containing node `i`
    //for each edge i -> j: scc_id[i] >= scc_id[j] (reverse topo order of scc's)
    vector<int> scc_id;
};
//NOLINTNEXTLINE(readability-identifier-naming)
scc_info SCC(const vector<vector<int>>& adj/*directed, unweighted graph*/) {
    int n = ssize(adj), timer = 1, num_sccs = 0;
    vector<int> tin(n, 0), scc_id(n, -1), node_stack;
    node_stack.reserve(n);
    auto dfs = [&](auto self, int v) -> int {
        int low = tin[v] = timer++;
        node_stack.push_back(v);
        for (int to : adj[v])
            if (scc_id[to] < 0)
                low = min(low, tin[to] ? tin[to] : self(self, to));
        if (tin[v] == low) {
            while (1) {
                int node = node_stack.back();
                node_stack.pop_back();
                scc_id[node] = num_sccs;
                if (node == v) break;
            }
            num_sccs++;
        }
        return low;
    };
    for (int i = 0; i < n; i++)
        if (!tin[i])
            dfs(dfs, i);
    return {num_sccs, scc_id};
}
```

Centroid Decomposition

```
//cat centroid_decomposition.hpp | ./hash.sh
//71e2e5
#pragma once
/**
 * Given an unweighted, undirected forest and a function, centroid_decomp runs
 * the function on the centroid of every decomposition.
 * @code{.cpp}
 *     //example usage
 *     centroid_decomp decomp(adj, [&](const vector<vector<int>>& adj_removed_edges, int cent)
       ↪ -> void {
 *     });
 * @endcode
 * @time O(n log n)
 * @memory O(n)
 */
template <typename F> struct centroid_decomp {
    vector<vector<int>> adj;
    F func;
    vector<int> sub_sz;
    centroid_decomp(const vector<vector<int>>& a_adj/*undirected forest*/, const F& a_func)
        : adj(a_adj), func(a_func), sub_sz(ssize(adj), -1) {
        for (int i = 0; i < ssize(adj); i++)
            if (sub_sz[i] == -1)
                dfs(i);
    }
    void calc_subtree_sizes(int u, int p = -1) {
        sub_sz[u] = 1;
        for (int v : adj[u]) {
            if (v == p) continue;
            calc_subtree_sizes(v, u);
            sub_sz[u] += sub_sz[v];
        }
    }
    void dfs(int u) {
        calc_subtree_sizes(u);
        for (int p = -1, sz_root = sub_sz[u];;) {
            auto big_ch = find_if(adj[u].begin(), adj[u].end(), [&](int v) -> bool {
                return v != p && 2 * sub_sz[v] > sz_root;
            });
            if (big_ch == adj[u].end()) break;
```

```
            p = u, u = *big_ch;
        }
        func(adj, u);
        for (int v : adj[u]) {
            //each node is adjacent to O(logn) centroids
            adj[v].erase(find(adj[v].begin(), adj[v].end(), u));
            dfs(v);
        }
    }
};
```

Frequency Table of Tree Distance

```
//cat count_paths_per_length.hpp | ./hash.sh
//182b70
#pragma once
#include "../../kactl/content/numerical/FastFourierTransform.h"
#include "centroid_decomposition.hpp"
/**
 * Returns array `num_paths` where `num_paths[i]` = # of paths in tree with `i`
 * edges.
 * @time O(n log^2 n)
 */
vector<long long> count_paths_per_length(const vector<vector<int>>& adj/*unrooted, connected
    ↪ tree*/) {
    vector<long long> num_paths(ssize(adj), 0);
    centroid_decomp decomp(adj, [&](const vector<vector<int>>& adj_removed_edges, int cent) ->
        ↪ void {
        vector<vector<double>> child_depths;
        for (int to : adj_removed_edges[cent]) {
            child_depths.emplace_back(1, 0.0);
            for (queue<pair<int, int>> q({{to, cent}}); !q.empty();) {
                child_depths.back().push_back(ssize(q));
                queue<pair<int, int>> new_q;
                while (!q.empty()) {
                    auto [curr, par] = q.front();
                    q.pop();
                    for (int ch : adj_removed_edges[curr]) {
                        if (ch == par) continue;
                        new_q.emplace(ch, curr);
                    }
                }
                swap(q, new_q);
            }
        }
        sort(child_depths.begin(), child_depths.end(), [&](const auto & x, const auto & y) {
            return x.size() < y.size();
        });
        vector<double> total_depth(1, 1.0);
        for (const auto& cnt_depth : child_depths) {
            auto prod = conv(total_depth, cnt_depth);
            for (int i = 1; i < ssize(prod); i++)
                num_paths[i] += llround(prod[i]);
            total_depth.resize(ssize(cnt_depth), 0.0);
            for (int i = 1; i < ssize(cnt_depth); i++)
                total_depth[i] += cnt_depth[i];
        }
    });
```

```
    return num_paths;
}
```

Count Paths Per Node

```
//cat count_paths_per_node.hpp | ./hash.sh
//2077d2
#pragma once
#include "centroid_decomposition.hpp"
/**
 * Returns array `num_paths` where `num_paths[i]` = number of paths with k
 * edges where node `i` is on the path. 0-based nodes.
 * @time O(n log n)
 */
vector<long long> count_paths_per_node(const vector<vector<int>>& adj/*unrooted tree*/, int k)
    ↪ {
    vector<long long> num_paths(ssize(adj));
    centroid_decomp decomp(adj, [&](const vector<vector<int>>& adj_removed_edges, int cent) ->
        ↪ void {
        vector<int> pre_d(1, 1), cur_d(1);
        auto dfs = [&](auto self, int u, int p, int d) -> long long {
            if (d > k) return 0;
            if (ssize(cur_d) <= d) cur_d.push_back(0);
            cur_d[d]++;
            long long cnt = 0;
            if (k - d < ssize(pre_d)) cnt += pre_d[k - d];
            for (int v : adj_removed_edges[u])
                if (v != p)
                    cnt += self(self, v, u, d + 1);
            num_paths[u] += cnt;
            return cnt;
        };
        auto dfs_child = [&](int child) -> long long {
            long long cnt = dfs(dfs, child, cent, 1);
            pre_d.resize(ssize(cur_d));
            for (int i = 1; i < ssize(cur_d) && cur_d[i]; i++)
                pre_d[i] += cur_d[i], cur_d[i] = 0;
            return cnt;
        };
        for (int child : adj_removed_edges[cent])
            num_paths[cent] += dfs_child(child);
        pre_d = vector<int>(1);
        cur_d = vector<int>(1);
        for (auto it = adj_removed_edges[cent].rbegin(); it != adj_removed_edges[cent].rend();
            ↪ it++)
            dfs_child(*it);
    });
    return num_paths;
}
```

Dijkstra

```
//cat dijkstra.hpp | ./hash.sh
//aa6eda
#pragma once
const long long INF = 1e18;
/**
```

```
 * Returns array `len` where `len[i]` = shortest path from node `start` to node
 * `i`. For example `len[start]` = 0.
 *
 * @time O((n + m) log n) Note log(m) < log(n^2) = 2*log(n), so O(log n) ==
 * O(log m)
 * @memory O(n + m)
 */
vector<long long> dijkstra(const vector<vector<pair<int, long long>>>& adj /*directed or
    ↪ undirected, weighted graph*/, int start) {
    using node = pair<long long, int>;
    vector<long long> len(ssize(adj), INF);
    len[start] = 0;
    priority_queue<node, vector<node>, greater<node>> q;
    q.emplace(0, start);
    while (!q.empty()) {
        auto [curr_len, v] = q.top();
        q.pop();
        if (len[v] < curr_len) continue;//important check: O(n*m) without it
        for (auto [to, weight] : adj[v])
            if (len[to] > weight + len[v]) {
                len[to] = weight + len[v];
                q.emplace(len[to], to);
            }
    }
    return len;
}
```

HLD

```
//cat hld.hpp | ./hash.sh
//d30c4a
#pragma once
/**
 * @see https://codeforces.com/blog/entry/53170
 */
//NOLINTNEXTLINE(readability-identifier-naming)
struct HLD {
    struct node {
        int sub_sz = 1, par = -1, time_in = -1, next = -1;
    };
    vector<node> tree;
    /**
     * @time O(n)
     * @memory O(n)
     */
    HLD(vector<vector<int>>& adj/*forest of unrooted trees*/) : tree(ssize(adj)) {
        for (int i = 0, timer = 0; i < ssize(adj); i++) {
            if (tree[i].next == -1) {//lowest indexed node in each tree becomes root
                tree[i].next = i;
                dfs1(i, adj);
                dfs2(i, adj, timer);
            }
        }
    }
    void dfs1(int v, vector<vector<int>>& adj) {
        auto par = find(adj[v].begin(), adj[v].end(), tree[v].par);
        if (par != adj[v].end()) adj[v].erase(par);
        for (int& to : adj[v]) {
            tree[to].par = v;
            dfs1(to, adj);
            tree[v].sub_sz += tree[to].sub_sz;
            if (tree[to].sub_sz > tree[adj[v][0]].sub_sz)
                swap(to, adj[v][0]);
        }
    }
    void dfs2(int v, const vector<vector<int>>& adj, int& timer) {
        tree[v].time_in = timer++;
        for (int to : adj[v]) {
            tree[to].next = (timer == tree[v].time_in + 1 ? tree[v].next : to);
            dfs2(to, adj, timer);
        }
    }
    /**
     * Returns inclusive-exclusive intervals (of time_in's) corresponding to
     * the path between u and v, not necessarily in order. u, v must be in the
     * same component.
     * @time O(log n)
     * @memory O(log n)
     */
    vector<pair<int, int>> path(int u, int v) const {
        vector<pair<int, int>> res;
        for (;; v = tree[tree[v].next].par) {
            if (tree[v].time_in < tree[u].time_in) swap(u, v);
            if (tree[tree[v].next].time_in <= tree[u].time_in) {
                res.emplace_back(tree[u].time_in, tree[v].time_in + 1);
                return res;
            }
            res.emplace_back(tree[tree[v].next].time_in, tree[v].time_in + 1);
        }
    }
    pair<int, int> subtree(int i) const {
        return {tree[i].time_in, tree[i].time_in + tree[i].sub_sz};
    }
    /**
     * u, v must be in the same component.
     * @time O(log n)
     * @memory O(1)
     */
    int lca(int u, int v) const {
        for (;; v = tree[tree[v].next].par) {
            if (tree[v].time_in < tree[u].time_in) swap(u, v);
            if (tree[tree[v].next].time_in <= tree[u].time_in) return u;
        }
    }
};
```

Hopcroft Karp

```
//cat hopcroft_karp.hpp | ./hash.sh
//5d1682
#pragma once
struct match {
    //# of edges in matching (which = size of min vertex cover by öKnig's theorem)
    int size_of_matching;
    //an arbitrary max matching is found. For this matching:
    //if l_to_r[node_left] == -1:
```

```cpp
    //  node_left is not in matching
    //else:
    //  the edge `node_left` <=> l_to_r[node_left] is in the matching
    //
    //similarly for r_to_l with edge r_to_l[node_right] <=> node_right in matching if
        ↪ r_to_l[node_right] != -1
    //matchings stored in l_to_r and r_to_l are the same matching
    //provides way to check if any node/edge is in matching
    vector<int> l_to_r, r_to_l;
    //an arbitrary min vertex cover is found. For this mvc: mvc_l[node_left] is 1 iff
        ↪ node_left is in the min vertex cover (same for mvc_r)
    //if mvc_l[node_left] is 0, then node_left is in the corresponding maximal independent set
    vector<bool> mvc_l, mvc_r;
};
/**
 * Think of the bipartite graph as having a left side (with size lsz) and a
 * right side (with size rsz).
 * Nodes on left side are indexed 0,1,...,lsz-1.
 * Nodes on right side are indexed 0,1,...,rsz-1.
 * @code{.cpp}
 *     //for every edge node_left <=> node_right
 *     adj[node_left].push_back(node_right);
 * @endcode
 * @see https://github.com/foreverbell/acm-icpc-cheat-sheet/
 *     blob/master/src/graph-algorithm/hopcroft-karp.cpp
 * @time O(m * sqrt(n)) n = lsz + rsz
 * @memory O(n + m)
 */
match hopcroft_karp(const vector<vector<int>>& adj/*bipartite graph*/, int rsz/*number of
    ↪ nodes on right side*/) {
    int size_of_matching = 0, lsz = ssize(adj);
    vector<int> l_to_r(lsz, -1), r_to_l(rsz, -1);
    while (1) {
        queue<int> q;
        vector<int> level(lsz, -1);
        for (int i = 0; i < lsz; i++)
            if (l_to_r[i] == -1)
                level[i] = 0, q.push(i);
        bool found = 0;
        vector<bool> mvc_l(lsz, 1), mvc_r(rsz, 0);
        while (!q.empty()) {
            int u = q.front();
            q.pop();
            mvc_l[u] = 0;
            for (int x : adj[u]) {
                mvc_r[x] = 1;
                int v = r_to_l[x];
                if (v == -1) found = 1;
                else if (level[v] == -1) {
                    level[v] = level[u] + 1;
                    q.push(v);
                }
            }
        }
        if (!found) return {size_of_matching, l_to_r, r_to_l, mvc_l, mvc_r};
        auto dfs = [&](auto self, int u) -> bool {
            for (int x : adj[u]) {
                int v = r_to_l[x];
                if (v == -1 || (level[u] + 1 == level[v] && self(self, v))) {
                    l_to_r[u] = x;
                    r_to_l[x] = u;
                    return 1;
                }
            }
            level[u] = 1e9; //acts as visited array
            return 0;
        };
        for (int i = 0; i < lsz; i++)
            size_of_matching += (l_to_r[i] == -1 && dfs(dfs, i));
    }
}
```

Kth Node on Path

```cpp
//cat kth_node_on_path.hpp | ./hash.sh
//c59307
#pragma once
#include "lca.hpp"
struct kth_node_on_path {
    LCA lca;
    kth_node_on_path(const vector<vector<pair<int, long long>>>& adj/*forest of weighted
        ↪ trees*/) : lca(adj) {}
    /**
     * Consider path {u, u's par, ..., LCA(u,v), ..., v's par, v}. This returns
     * the node at index k. So k=0 returns u, k=#path_edges returns v.
     * u, v must be in the same component.
     * @time O(log n)
     */
    int query(int u, int v, int k) const {
        int lca_uv = lca.get_lca(u, v);
        int u_lca = lca.tree[u].depth - lca.tree[lca_uv].depth;
        int v_lca = lca.tree[v].depth - lca.tree[lca_uv].depth;
        assert(0 <= k && k <= u_lca + v_lca);
        return k <= u_lca ? lca.kth_par(u, k) : lca.kth_par(v, u_lca + v_lca - k);
    }
};
```

LCA

```cpp
//cat lca.hpp | ./hash.sh
//b28532
#pragma once
/**
 * @see https://codeforces.com/blog/entry/74847
 */
//NOLINTNEXTLINE(readability-identifier-naming)
struct LCA {
    struct node {
        int jmp = -1, jmp_edges = 0, par = -1, depth = 0;
        long long dist = 0LL;
    };
    vector<node> tree;
    /**
     * @time O(n)
     * @memory O(n)
     */
```

```
LCA(const vector<vector<pair<int, long long>>>& adj/*forest of weighted trees*/) :
    ↪ tree(ssize(adj)) {
    for (int i = 0; i < ssize(adj); i++) {
        if (tree[i].jmp == -1) {//lowest indexed node in each tree becomes root
            tree[i].jmp = i;
            dfs(i, adj);
        }
    }
}
void dfs(int v, const vector<vector<pair<int, long long>>>& adj) {
    int jmp, jmp_edges;
    if (tree[v].jmp != v && tree[v].jmp_edges == tree[tree[v].jmp].jmp_edges)
        jmp = tree[tree[v].jmp].jmp, jmp_edges = 2 * tree[v].jmp_edges + 1;
    else
        jmp = v, jmp_edges = 1;
    for (auto [ch, w] : adj[v]) {
        if (ch == tree[v].par) continue;
        tree[ch] = {
            jmp,
            jmp_edges,
            v,
            1 + tree[v].depth,
            w + tree[v].dist
        };
        dfs(ch, adj);
    }
}
/**
 * Traverse up k edges. So with k=1 this returns v's parent.
 * @time O(log k)
 */
int kth_par(int v, int k) const {
    k = min(k, tree[v].depth);
    while (k > 0) {
        if (tree[v].jmp_edges <= k) {
            k -= tree[v].jmp_edges;
            v = tree[v].jmp;
        } else {
            k--;
            v = tree[v].par;
        }
    }
    return v;
}
/**
 * x, y must be in the same component.
 * @time O(log n)
 */
int get_lca(int x, int y) const {
    if (tree[x].depth < tree[y].depth) swap(x, y);
    x = kth_par(x, tree[x].depth - tree[y].depth);
    while (x != y) {
        if (tree[x].jmp != tree[y].jmp)
            x = tree[x].jmp, y = tree[y].jmp;
        else
            x = tree[x].par, y = tree[y].par;
    }
    return x;
}
int dist_edges(int x, int y) const {
    return tree[x].depth + tree[y].depth - 2 * tree[get_lca(x, y)].depth;
}
long long dist_weight(int x, int y) const {
    return tree[x].dist + tree[y].dist - 2 * tree[get_lca(x, y)].dist;
}
};
```

Rooted Tree Isomorphism

```
//cat subtree_isomorphism.hpp | ./hash.sh
//455aef
#pragma once
struct iso_info {
    int num_distinct_subtrees; //0 <= id[i] < num_distinct_subtrees
    vector<int> id; //id[u] == id[v] iff rooted subtree u is isomorphic to rooted subtree v
};
/**
 * Classifies each rooted subtree by isomorphism.
 * @time O(n log n)
 * @memory O(n)
 */
iso_info subtree_iso(const vector<vector<int>>& adj) {
    vector<int> id(ssize(adj), -1);
    map<vector<int>, int> hashes;
    auto dfs = [&](auto self, int u, int p) -> int {
        vector<int> ch_ids;
        ch_ids.reserve(ssize(adj[u]));
        for (int v : adj[u]) {
            if (v != p)
                ch_ids.push_back(self(self, v, u));
        }
        sort(ch_ids.begin(), ch_ids.end());
        auto it = hashes.find(ch_ids);
        if (it == hashes.end())
            return id[u] = hashes[ch_ids] = ssize(hashes);
        return id[u] = it->second;
    };
    for (int i = 0; i < ssize(adj); i++)
        if (id[i] == -1)
            dfs(dfs, i, i);
    return {ssize(hashes), id};
}
```

MATH

Derangements

```
//cat derangements.hpp | ./hash.sh
//64d325
#pragma once
/**
 * @see https://oeis.org/A000166
 * @time O(n)
 */
vector<long long> derangements(int n, long long mod) {
```

```
    vector<long long> dp(n, 0);
    dp[0] = 1;
    for (int i = 2; i < n; i++)
        dp[i] = (i - 1) * (dp[i - 1] + dp[i - 2]) % mod;
    return dp;
}
```

Binary Exponentiation MOD

```
//cat binary_exponentiation_mod.hpp | ./hash.sh
//92a3ef
#pragma once
/**
 * Returns (base^pw)%mod; returns 1 for 0^0.
 * @time O(log pw)
 */
long long bin_exp(long long base, long long pw, long long mod) {
    assert(0 <= pw && 0 <= base && 1 <= mod);
    long long res = 1;
    base %= mod;
    while (pw > 0) {
        if (pw & 1) res = res * base % mod;
        base = base * base % mod;
        pw >>= 1;
    }
    return res;
}
```

Fibonacci

```
//cat fibonacci.hpp | ./hash.sh
//78a41f
#pragma once
//https://codeforces.com/blog/entry/14516
//O(log(n))
unordered_map<long long, long long> table;
long long fib(long long n, long long mod) {
    if (n < 2) return 1;
    if (table.find(n) != table.end()) return table[n];
    table[n] = (fib((n + 1) / 2, mod) * fib(n / 2, mod) + fib((n - 1) / 2, mod) * fib((n - 2)
        ↪ / 2, mod)) % mod;
    return table[n];
}
```

Matrix Multiplication

```
//cat matrix_mult.hpp | ./hash.sh
//910018
#pragma once
/**
 * Generic matrix multiplication (not overflow safe). This will RTE if the
 * given matricies are not compatible.
 * @see https://codeforces.com/blog/entry/80195
 * @time O(n * m * inner)
 * @memory O(n * m)
 */
template <typename T> vector<vector<T>> operator * (const vector<vector<T>>& a, const
    ↪ vector<vector<T>>& b) {
    assert(ssize(a[0]) == ssize(b));
    int n = ssize(a), m = ssize(b[0]), inner = ssize(b);
    vector<vector<T>> c(n, vector<T>(m));
    for (int i = 0; i < n; i++)
        for (int k = 0; k < inner; k++)
            for (int j = 0; j < m; j++)
                c[i][j] += a[i][k] * b[k][j];
    return c;
}
```

Mobius Inversion

```
//cat mobius_inversion.hpp | ./hash.sh
//811515
#pragma once
const int N = 1e6 + 10;
int mobius[N];
/**
 * mobius[i] = 0 iff there exists a prime p s.t. i%(p^2)=0
 * mobius[i] = -1 iff i has an odd number of distinct prime factors
 * mobius[i] = 1 iff i has an even number of distinct prime factors
 * @time O(n log n)
 */
void calc_mobius() {
    mobius[1] = 1;
    for (int i = 1; i < N; i++)
        for (int j = i + i; j < N; j += i)
            mobius[j] -= mobius[i];
}
```

N Choose K MOD

```
//cat n_choose_k_mod.hpp | ./hash.sh
//daaaa8
#pragma once
#include "binary_exponentiation_mod.hpp"
/**
 * @code{.cpp}
 *     n_choose_k nk(n, 1e9+7); // to use `choose`, `inv` with inputs strictly < n
 *     n_choose_k nk(mod, mod); // to use `choose_lucas` with arbitrarily large inputs
 * @endcode
 */
struct n_choose_k {
    long long mod;
    vector<long long> fact, inv_fact;
    /**
     * Only works for `n <= mod` and prime mod.
     * @time O(n + sqrt(mod)) The sqrt is only to assert mod is prime.
     * @memory O(n)
     */
    n_choose_k(int n, long long a_mod) : mod(a_mod), fact(n, 1), inv_fact(n, 1) {
        assert(max(n, 2) <= mod);
        for (int i = 2; i * i <= mod; i++) assert(mod % i);
        for (int i = 2; i < n; i++)
            fact[i] = fact[i - 1] * i % mod;
```

```
        inv_fact.back() = bin_exp(fact.back(), mod - 2, mod);
        for (int i = n - 2; i >= 2; i--)
            inv_fact[i] = inv_fact[i + 1] * (i + 1) % mod;
    }
    /**
     * n choose k for n,k < ssize(fact). Fails when n >= mod.
     * @time O(1)
     * @memory O(n) precomp
     */
    long long choose(int n, int k) const {
        if (k < 0 || k > n) return 0;
        //now we know 0 <= k <= n so 0 <= n
        return fact[n] * inv_fact[k] % mod * inv_fact[n - k] % mod;
    }
    /**
     * Lucas theorem - n choose k for n, k up to LLONG_MAX. Handles n>=mod
     * correctly.
     * @time O(log(k))
     * @memory O(mod) precomp, so can't use 1e9 + 7.
     */
    long long choose_lucas(long long n, long long k) const {
        if (k < 0 || k > n) return 0;
        if (k == 0 || k == n) return 1;
        return choose_lucas(n / mod, k / mod) * choose(int(n % mod), int(k % mod)) % mod;
    }
    /**
     * Returns x such that x * n % mod == 1.
     * @time O(1)
     */
    long long inv(int n) const {
        assert(1 <= n); //don't divide by 0 :)
        return fact[n - 1] * inv_fact[n] % mod;
    }
};
```

Partitions

```
//cat partitions.hpp | ./hash.sh
//e7ae42
#pragma once
//https://oeis.org/A000041
//O(n sqrt n) time, but small-ish constant factor (there does exist a O(n log n) solution as
    ↪ well)
vector<long long> partitions(int n, long long mod) {
    vector<long long> dp(n, 1);
    for (int i = 1; i < n; i++) {
        long long sum = 0;
        for (int j = 1, pent = 1, sign = 1; pent <= i; j++, pent += 3 * j - 2, sign = -sign) {
            if (pent + j <= i) sum += dp[i - pent - j] * sign + mod;
            sum += dp[i - pent] * sign + mod;
        }
        dp[i] = sum % mod;
    }
    return dp;
}
```

Prime Sieve

```
//cat prime_sieve.hpp | ./hash.sh
//25a877
#pragma once
bool is_prime(int val, const vector<int>& sieve) {
    assert(val < ssize(sieve));
    return val >= 2 && sieve[val] == val;
}
vector<int> get_prime_factors(int val, const vector<int>& sieve) {
    assert(val < ssize(sieve));
    vector<int> factors;
    while (val > 1) {
        int p = sieve[val];
        factors.push_back(p);
        val /= p;
    }
    return factors;
}
/**
 * Returns array `sieve` where `sieve[i]` = some prime factor of `i`.
 * @time O(n * log(logn))
 */
vector<int> get_sieve(int n) {
    vector<int> sieve(n);
    iota(sieve.begin(), sieve.end(), 0);
    for (int i = 2; i * i < n; i++)
        if (sieve[i] == i)
            for (int j = i * i; j < n; j += i)
                sieve[j] = i;
    return sieve;
}
```

Row Reduce

```
//cat row_reduce.hpp | ./hash.sh
//c812f1
#pragma once
#include "binary_exponentiation_mod.hpp"
/**
 * First `cols` columns of mat represents a matrix to be left in reduced row
 * echelon form. Row operations will be performed to all later columns.
 * @code{.cpp}
 *     auto [rank, det] = row_reduce(mat, ssize(mat[0]), mod);
 * @endcode
 * @time O(n * m * min(cols, n))
 * @memory O(n * m)
 */
pair<int/*rank*/, long long/*determinant*/> row_reduce(vector<vector<long long>>& mat, int
    ↪ cols, long long mod) {
    int n = ssize(mat), m = ssize(mat[0]), rank = 0;
    long long det = 1;
    assert(cols <= m);
    for (int col = 0; col < cols && rank < n; col++) {
        //find arbitrary pivot and swap pivot to current row
        for (int i = rank; i < n; i++)
            if (mat[i][col] != 0) {
                if (rank != i) det = det == 0 ? 0 : mod - det;
                swap(mat[i], mat[rank]);
                break;
```

```
        }
        if (mat[rank][col] == 0) {
            det = 0;
            continue;
        }
        det = det * mat[rank][col] % mod;
        //make pivot 1 by dividing row by inverse of pivot
        long long a_inv = bin_exp(mat[rank][col], mod - 2, mod);
        for (int j = 0; j < m; j++)
            mat[rank][j] = mat[rank][j] * a_inv % mod;
        //zero-out all numbers above & below pivot
        for (int i = 0; i < n; i++)
            if (i != rank && mat[i][col] != 0) {
                long long val = mat[i][col];
                for (int j = 0; j < m; j++) {
                    mat[i][j] -= mat[rank][j] * val % mod;
                    if (mat[i][j] < 0) mat[i][j] += mod;
                }
            }
        rank++;
    }
    assert(rank <= min(n, cols));
    return {rank, det};
}
```

Solve Linear Equations MOD

```
//cat solve_linear_mod.hpp | ./hash.sh
//eba3e8
#pragma once
#include "row_reduce.hpp"
struct matrix_info {
    int rank;
    long long det;
    //solution vector
    //empty if no solution exists
    //if there are multiple solutions, this is an arbitrary one
    vector<long long> x;
};
/**
 * Solves mat * x = b under prime mod.
 * `mat` is a n (rows) by m (cols) matrix, b is a length n column vector, x is
 * a length m vector.
 * Assumes n,m >= 1, else RTE. Leaves mat in reduced row echelon form with b
 * appended.
 *
 * Number of unique solutions = (size of domain) ^ (# of free variables).
 * (# of free variables) is generally equivalent to n - rank.
 * @time O(n * m * min(n, m))
 * @memory O(n * m)
 */
matrix_info solve_linear_mod(vector<vector<long long>>& mat, const vector<long long>& b, long
    ↪ long mod) {
    assert(ssize(mat) == ssize(b));
    int n = ssize(mat), m = ssize(mat[0]);
    for (int i = 0; i < n; i++)
        mat[i].push_back(b[i]);
    auto [rank, det] = row_reduce(mat, m, mod);//row reduce not including the last column
    //check if solution exists
    for (int i = rank; i < n; i++)
        if (mat[i].back() != 0)
            return {rank, det, {} }; //no solution exists
    //initialize solution vector (`x`) from row-reduced matrix
    vector<long long> x(m, 0);
    for (int i = 0, j = 0; i < rank; i++) {
        while (mat[i][j] == 0) j++; //find pivot column
        x[j] = mat[i].back();
    }
    return {rank, det, x};
}
```

Euler's Totient Phi Function

```
//cat totient.hpp | ./hash.sh
//36bd41
#pragma once
/**
 * Euler's totient function counts the positive integers up to a given integer
 * n that are relatively prime to n. To improve, pre-calc prime factors or use
 * Pollard-rho to find prime factors.
 * @time O(sqrt n)
 */
int totient(int n) {
    int res = n;
    for (int i = 2; i * i <= n; i++) {
        if (n % i == 0) {
            while (n % i == 0) n /= i;
            res -= res / i;
        }
    }
    if (n > 1) res -= res / n;
    return res;
}
```

Tetration MOD

```
//cat tetration_mod.hpp | ./hash.sh
//e2153e
#pragma once
#include "binary_exponentiation_mod.hpp"
#include "totient.hpp"
/**
 * Returns base ^ (base ^ (base ^ ... )) % mod, where the height of the tower
 * is pw.
 * Let t = totient(mod).
 * If log2(mod) <= pw then (base^pw)%mod == (base^(t+(pw%t)))%mod
 * @see https://cp-algorithms.com/algebra/phi-function.html#generalization
 * @time O(sqrt(mod) * log(mod))
 */
long long tetration(long long base, long long pw, long long mod) {
    if (mod == 1)
        return 0;
    if (base == 0)
        return (pw + 1) % 2 % mod;
    if (base == 1 || pw == 0)
```

```
        return 1;
    if (pw == 1)
        return base % mod;
    if (base == 2 && pw == 2)
        return 4 % mod;
    if (base == 2 && pw == 3)
        return 16 % mod;
    if (base == 3 && pw == 2)
        return 27 % mod;
    //need enough base cases such that the following is true
    //log2(mod) <= tetration(base, pw - 1) (before modding)
    int t = totient(int(mod));
    long long exp = tetration(base, pw - 1, t);
    return bin_exp(base, exp + t, mod);
}
```

MISC

Cartesian Tree

```
//cat cartesian_tree.hpp | ./hash.sh
//204c45
#pragma once
#include "monotonic_stack.hpp"
/**
 * Min cartesian tree - root stores min.
 * @time O(n)
 * @memory O(n)
 */
vector<int> cartesian_tree(const vector<int>& arr) {
    int n = ssize(arr);
    auto rv /*reverse*/ = [&](int i) -> int {
        return n - 1 - i;
    };
    vector<int> left = monotonic_stack<int>(arr, greater());
    vector<int> right = monotonic_stack<int>(vector<int>(arr.rbegin(), arr.rend()), greater());
    vector<int> par(n);
    for (int i = 0; i < n; i++) {
        int le = left[i], ri = rv(right[rv(i)]);
        if (le >= 0 && ri < n) par[i] = arr[le] > arr[ri] ? le : ri;
        else if (le >= 0) par[i] = le;
        else if (ri < n) par[i] = ri;
        else par[i] = i; //true only for root
    }
    return par;
}
```

Count Rectangles

```
//cat count_rectangles.hpp | ./hash.sh
//8990f7
#pragma once
#include "monotonic_stack.hpp"
/**
 * Given a n-by-m boolean matrix, calculate cnt[i][j]. cnt[i][j] = the number
 * of times an i-by-j sub rectangle appears in the matrix such that all i*j
 * cells in the sub rectangle are 1.
 *
 * cnt[0][j] and cnt[i][0] will contain garbage values.
 * @time O(n * m)
 * @memory O(n * m)
 */
vector<vector<int>> count_rectangles(const vector<vector<bool>>& grid) {
    int n = ssize(grid), m = ssize(grid[0]);
    vector<vector<int>> cnt(n + 1, vector<int>(m + 1, 0));
    vector<int> arr(m, 0);
    auto rv /*reverse*/ = [&](int j) -> int {
        return m - 1 - j;
    };
    for (int i = 0; i < n; i++) {
        for (int j = 0; j < m; j++)
            arr[j] = grid[i][j] * (arr[j] + 1);
        vector<int> left = monotonic_stack<int>(arr, greater());
        vector<int> right = monotonic_stack<int>(vector<int>(arr.rbegin(), arr.rend()),
            ↪ greater_equal());
        for (int j = 0; j < m; j++) {
            int le = j - left[j] - 1, ri = rv(right[rv(j)]) - j - 1;
            cnt[arr[j]][le + ri + 1]++;
            cnt[arr[j]][le]--;
            cnt[arr[j]][ri]--;
        }
    }
    for (int i = 1; i <= n; i++)
        for (int k = 0; k < 2; k++)
            for (int j = m - 1; j >= 1; j--)
                cnt[i][j] += cnt[i][j + 1];
    for (int j = 1; j <= m; j++)
        for (int i = n - 1; i >= 1; i--)
            cnt[i][j] += cnt[i + 1][j];
    return cnt;
}
```

Max Rectangle in Histogram

```
//cat max_rect_histogram.hpp | ./hash.sh
//95288f
#pragma once
#include "monotonic_stack.hpp"
/**
 * @time O(n)
 * @memory O(n)
 */
long long max_rect_histogram(const vector<int>& arr) {
    auto rv /*reverse*/ = [&](int i) -> int {
        return ssize(arr) - 1 - i;
    };
    vector<int> left = monotonic_stack<int>(arr, greater_equal());
    vector<int> right = monotonic_stack<int>(vector<int>(arr.rbegin(), arr.rend()),
        ↪ greater_equal());
    long long max_area = 0;
    for (int i = 0; i < ssize(arr); i++) {
        int le = left[i], ri = rv(right[rv(i)]);//arr[i] is the max of range (le, ri)
        max_area = max(max_area, 1LL * arr[i] * (ri - le - 1));
    }
```

```
    return max_area;
}
```

Monotonic Stack

```
//cat monotonic_stack.hpp | ./hash.sh
//35c95c
#pragma once
/**
 * Returns array `le` where `le[i]` = max number such that: `le[i]` < i and
 * !op(arr[le[i]], arr[i]). Returns -1 if no number exists.
 * @code{.cpp}
 *     vector<int> le = monotonic_stack<int>(arr, less()); //(or replace `less` with:
    ↪ less_equal, greater, greater_equal
 *     vector<int> le = monotonic_stack<int>(arr, [&](int x, int y) {return x < y;});
 * @endcode
 * @time O(n)
 * @memory O(n)
 */
template <typename T> vector<int> monotonic_stack(const vector<T>& arr, const
    ↪ function<bool(const T&, const T&)>& op) {
    vector<int> le(ssize(arr));
    for (int i = 0; i < ssize(arr); i++) {
        le[i] = i - 1;
        while (le[i] >= 0 && op(arr[le[i]], arr[i])) le[i] = le[le[i]];
    }
    return le;
}
```

GCD Convolution

```
//cat gcd_convolution.hpp | ./hash.sh
//d92c44
#pragma once
/**
 * Returns array `c` where `c[k]` = the sum for all pairs where gcd(i,j) == k
 * of a[i] * b[j].
 * @time O(n log n)
 * @memory O(n)
 */
template<int MOD> vector<int> gcd_convolution(const vector<int>& a, const vector<int>& b) {
    assert(ssize(a) == ssize(b));
    int n = ssize(a);
    vector<int> c(n);
    for (int gcd = n - 1; gcd >= 1; gcd--) {
        int sum_a = 0, sum_b = 0;
        for (int i = gcd; i < n; i += gcd) {
            sum_a = (sum_a + a[i]) % MOD, sum_b = (sum_b + b[i]) % MOD;
            c[gcd] = (c[gcd] - c[i] + MOD) % MOD;
        }
        c[gcd] = int((c[gcd] + 1LL * sum_a * sum_b) % MOD);
    }
    return c;
}
```

Iterate Chooses

```
//cat iterate_chooses.hpp | ./hash.sh
//c79083
#pragma once

int next_subset(int mask) {
    int c = mask & -mask, r = mask + c;
    return r | (((r ^ mask) >> 2) / c);
}
/**
 * Iterates over all bitmasks of size n with k bits set.
 * @see https://github.com/kth-competitive-programming/
 *     kactl/blob/main/content/various/chapter.tex
 * @time O(n choose k)
 * @memory O(1)
 */
void iterate_chooses(int n, int k, const function<void(int)>& func) {
    for (int mask = (1 << k) - 1; mask < (1 << n); mask = next_subset(mask))
        func(mask);
}
```

Iterate Submasks

```
//cat iterate_submasks.hpp | ./hash.sh
//084c05
#pragma once
/**
 * Iterates over all submasks of mask.
 * @time O(3^n) to iterate every submask of every mask of size n
 * @memory O(1)
 */
void iterate_submasks(int mask, const function<void(int)>& func) {
    for (int submask = mask; submask; submask = (submask - 1) & mask)
        func(submask);
}
```

Iterate Supermasks

```
//cat iterate_supermasks.hpp | ./hash.sh
//76b38f
#pragma once
/**
 * Iterates over all supermasks of mask.
 * @time O(3^n) to iterate every supermask of every mask of size n
 * @memory O(1)
 */
void iterate_supermasks(int mask, int n, const function<void(int)>& func) {
    for (int supermask = mask; supermask < (1 << n); supermask = (supermask + 1) | mask)
        func(supermask);
}
```

Number of Distinct Subsequences DP

```
//cat num_distinct_subsequences.hpp | ./hash.sh
//d94bdc
#pragma once
/**
```

```
 * Returns the number of distinct subsequences of `arr`. The empty subsequence
 * is counted.
 * @time O(n log n)
 * @memory O(n)
 */
int num_subsequences(const vector<int>& arr, int mod) {
    vector<int> dp(ssize(arr) + 1, 1);
    map<int, int> last;
    for (int i = 0; i < ssize(arr); i++) {
        int& curr = dp[i + 1] = 2 * dp[i];
        if (curr >= mod) curr -= mod;
        auto it = last.find(arr[i]);
        if (it != last.end()) {
            curr -= dp[it->second];
            if (curr < 0) curr += mod;
            it->second = i;
        } else last[arr[i]] = i;
    }
    return dp.back();
}
```

PBDS

```
//cat policy_based_data_structures.hpp | ./hash.sh
//a777d7
#pragma once
//place these includes *before* the `#define int long long` else compile error
//not using <bits/extc++.h> as it compile errors on codeforces c++20 compiler
#include <ext/pb_ds/assoc_container.hpp>
#include <ext/pb_ds/tree_policy.hpp>
using namespace __gnu_pbds;
//BST with extra functions https://codeforces.com/blog/entry/11080
//order_of_key - # of elements *strictly* less than given element
//find_by_order - find kth largest element, k is 0 based so find_by_order(0) returns min
    ↪ element
template <typename T> using indexed_set = tree<T, null_type, less<T>, rb_tree_tag,
    ↪ tree_order_statistics_node_update>;
//example initialization:
indexed_set<pair<long long, int>> is;
//hash table (apparently faster than unordered_map): https://codeforces.com/blog/entry/60737
//example initialization:
gp_hash_table<string, long long> ht;
```

Random

```
//cat random.hpp | ./hash.sh
//ab9111
#pragma once

//source: https://codeforces.com/blog/entry/61675

mt19937 rng(chrono::steady_clock::now().time_since_epoch().count());

//intended types: int, unsigned, long long
//returns a random number in range [le, ri)
template <typename T> inline T get_rand(T le, T ri) {
    assert(le < ri);
    return uniform_int_distribution<T>(le, ri - 1)(rng);
}

//vector<int> a;
//shuffle(a.begin(), a.end(), rng);
```

Safe Hash

```
//cat safe_hash.hpp | ./hash.sh
//d9ea53
#pragma once
//source: https://codeforces.com/blog/entry/62393
struct custom_hash {
    static uint64_t splitmix64(uint64_t x) {
        // http://xorshift.di.unimi.it/splitmix64.c
        x += 0x9e3779b97f4a7c15;
        x = (x ^ (x >> 30)) * 0xbf58476d1ce4e5b9;
        x = (x ^ (x >> 27)) * 0x94d049bb133111eb;
        return x ^ (x >> 31);
    }
    size_t operator()(uint64_t x) const {
        static const uint64_t FIXED_RANDOM =
            ↪ chrono::steady_clock::now().time_since_epoch().count();
        return splitmix64(x + FIXED_RANDOM);
    }
};
//usage:
unordered_map<long long, int, custom_hash> safe_map;
#include "policy_based_data_structures.hpp"
gp_hash_table<long long, int, custom_hash> safe_hash_table;
```

RANGE DATA STRUCTURES

Number Distinct Elements

```
//cat distinct_query.hpp | ./hash.sh
//79534f
#pragma once
/**
 * Can't handle updates; works with negatives.
 * @see https://cp-algorithms.com/data_structures/segment_tree.html
 *     #preserving-the-history-of-its-values-persistent-segment-tree
 */
struct distinct_query {
    struct node {
        int sum;
        int lch, rch;//children, indexes into `tree`
        node(int a_sum, int a_lch, int a_rch) : sum(a_sum), lch(a_lch), rch(a_rch) {}
    };
    const int N;
    vector<int> roots;
    deque<node> tree;
    /**
     * @time O(n log n)
     * @memory O(n log n)
     */
```

```
    distinct_query(const vector<int>& arr) : N(ssize(arr)), roots(N + 1, 0) {
        tree.emplace_back(0, 0, 0); //acts as null
        map<int, int> last_idx;
        for (int i = 0; i < N; i++) {
            roots[i + 1] = update(roots[i], 0, N, last_idx[arr[i]]);
            last_idx[arr[i]] = i + 1;
        }
    }
    int update(int v, int tl, int tr, int idx) {
        if (tr - tl == 1) {
            tree.emplace_back(tree[v].sum + 1, 0, 0);
            return ssize(tree) - 1;
        }
        int tm = tl + (tr - tl) / 2;
        int lch = tree[v].lch;
        int rch = tree[v].rch;
        if (idx < tm) lch = update(lch, tl, tm, idx);
        else rch = update(rch, tm, tr, idx);
        tree.emplace_back(tree[lch].sum + tree[rch].sum, lch, rch);
        return ssize(tree) - 1;
    }
    /**
     * Returns number of distinct elements in range [le, ri).
     * @time O(log n)
     */
    int query(int le, int ri) const {
        assert(0 <= le && le <= ri && ri <= N);
        return query(roots[le], roots[ri], 0, N, le + 1);
    }
    int query(int vl, int vr, int tl, int tr, int idx) const {
        if (tree[vr].sum == 0 || idx <= tl) return 0;
        if (tr <= idx) return tree[vr].sum - tree[vl].sum;
        int tm = tl + (tr - tl) / 2;
        return query(tree[vl].lch, tree[vr].lch, tl, tm, idx) +
               query(tree[vl].rch, tree[vr].rch, tm, tr, idx);
    }
};
```

Implicit Lazy Segment Tree

```
//cat implicit_seg_tree.hpp | ./hash.sh
//ab5eb9
#pragma once
//example initialization:
//  implicit_seg_tree<10'000'000> ist(le, ri);
template <int N> struct implicit_seg_tree {
    using dt = array<long long, 2>;//min, number of mins
    using ch = long long;
    static dt combine(const dt& le, const dt& ri) {
        if (le[0] == ri[0]) return {le[0], le[1] + ri[1]};
        return min(le, ri);
    }
    static constexpr dt UNIT{(long long)1e18, 0LL};
    struct node {
        dt val;
        ch lazy = 0;
        int lch = -1, rch = -1; // children, indexes into `tree`, -1 for null
    } tree[N];
    int ptr = 0, root_l, root_r;//[root_l, root_r) defines range of root node; handles
        ↪ negatives
    implicit_seg_tree(int le, int ri) : root_l(le), root_r(ri) {
        tree[ptr++].val = {0, ri - le};
    }
    void apply(int v, ch add) {
        tree[v].val[0] += add;
        tree[v].lazy += add;
    }
    void push(int v, int tl, int tr) {
        if (tr - tl > 1 && tree[v].lch == -1) {
            int tm = tl + (tr - tl) / 2;
            assert(ptr + 1 < N);
            tree[v].lch = ptr;
            tree[ptr++].val = {0, tm - tl};
            tree[v].rch = ptr;
            tree[ptr++].val = {0, tr - tm};
        }
        if (tree[v].lazy) {
            apply(tree[v].lch, tree[v].lazy);
            apply(tree[v].rch, tree[v].lazy);
            tree[v].lazy = 0;
        }
    }
    //update range [le,ri)
    void update(int le, int ri, ch add) {
        update(0, root_l, root_r, le, ri, add);
    }
    void update(int v, int tl, int tr, int le, int ri, ch add) {
        if (ri <= tl || tr <= le)
            return;
        if (le <= tl && tr <= ri)
            return apply(v, add);
        push(v, tl, tr);
        int tm = tl + (tr - tl) / 2;
        update(tree[v].lch, tl, tm, le, ri, add);
        update(tree[v].rch, tm, tr, le, ri, add);
        tree[v].val = combine(tree[tree[v].lch].val,
                              tree[tree[v].rch].val);
    }
    //query range [le,ri)
    dt query(int le, int ri) {
        return query(0, root_l, root_r, le, ri);
    }
    dt query(int v, int tl, int tr, int le, int ri) {
        if (ri <= tl || tr <= le)
            return UNIT;
        if (le <= tl && tr <= ri)
            return tree[v].val;
        push(v, tl, tr);
        int tm = tl + (tr - tl) / 2;
        return combine(query(tree[v].lch, tl, tm, le, ri),
                       query(tree[v].rch, tm, tr, le, ri));
    }
};
```

Kth Smallest

```cpp
//cat kth_smallest.hpp | ./hash.sh
//b86dd0
#pragma once
/**
 * Can't handle updates.
 * @see https://cp-algorithms.com/data_structures/segment_tree.html
 *     preserving-the-history-of-its-values-persistent-segment-tree
 */
struct kth_smallest {
    struct node {
        int sum;
        int lch, rch;//children, indexes into `tree`
        node(int a_sum, int a_lch, int a_rch) : sum(a_sum), lch(a_lch), rch(a_rch) {}
    };
    int mn = INT_MAX, mx = INT_MIN;
    vector<int> roots;
    deque<node> tree;
    /**
     * @time O(n log max)
     * @memory O(n log max)
     */
    kth_smallest(const vector<int>& arr) : roots(ssize(arr) + 1, 0) {
        tree.emplace_back(0, 0, 0); //acts as null
        for (int val : arr) mn = min(mn, val), mx = max(mx, val + 1);
        for (int i = 0; i < ssize(arr); i++)
            roots[i + 1] = update(roots[i], mn, mx, arr[i]);
    }
    int update(int v, int tl, int tr, int idx) {
        if (tr - tl == 1) {
            tree.emplace_back(tree[v].sum + 1, 0, 0);
            return ssize(tree) - 1;
        }
        int tm = tl + (tr - tl) / 2;
        int lch = tree[v].lch;
        int rch = tree[v].rch;
        if (idx < tm) lch = update(lch, tl, tm, idx);
        else rch = update(rch, tm, tr, idx);
        tree.emplace_back(tree[lch].sum + tree[rch].sum, lch, rch);
        return ssize(tree) - 1;
    }
    /**
     * Returns (k+1)th smallest number in range [le, ri). k is 0-based, so
     * query(le,ri,0) returns the min.
     * @time O(log max)
     */
    int query(int le, int ri, int k) const {
        assert(0 <= k && k < ri - le); //note this condition implies le < ri
        assert(0 <= le && ri < ssize(roots));
        return query(roots[le], roots[ri], mn, mx, k);
    }
    int query(int vl, int vr, int tl, int tr, int k) const {
        assert(tree[vr].sum > tree[vl].sum);
        if (tr - tl == 1)
            return tl;
        int tm = tl + (tr - tl) / 2;
        int left_count = tree[tree[vr].lch].sum - tree[tree[vl].lch].sum;
        if (left_count > k) return query(tree[vl].lch, tree[vr].lch, tl, tm, k);
        return query(tree[vl].rch, tree[vr].rch, tm, tr, k - left_count);
    }
};
```

Merge Sort Tree

```cpp
//cat merge_sort_tree.hpp | ./hash.sh
//e110ed
#pragma once
//For point updates: either switch to policy based BST, or use sqrt decomposition
struct merge_sort_tree {
    const int N, S/*smallest power of 2 >= N*/;
    vector<vector<int>> tree;
    merge_sort_tree(const vector<int>& arr) : N(ssize(arr)),  S(N ? 1 << __lg(2 * N - 1) : 0),
        ↪ tree(2 * N) {
        for (int i = 0; i < N; i++)
            tree[i + N] = {arr[i]};
        rotate(tree.rbegin(), tree.rbegin() + S - N, tree.rbegin() + N);
        for (int i = N - 1; i >= 1; i--) {
            const auto& le = tree[2 * i];
            const auto& ri = tree[2 * i + 1];
            tree[i].reserve(ssize(le) + ssize(ri));
            merge(le.begin(), le.end(), ri.begin(), ri.end(), back_inserter(tree[i]));
        }
    }
    int value(int v, int x) const {
        return int(lower_bound(tree[v].begin(), tree[v].end(), x) - tree[v].begin());
    }
    int to_leaf(int i) const {
        i += S;
        return i < 2 * N ? i : 2 * (i - N);
    }
    /**
     * Returns the number of values in range [le, ri) which are < x.
     * @time O(log^2(n))
     */
    int query(int le, int ri, int x) const {
        int res = 0;
        for (le = to_leaf(le), ri = to_leaf(ri); le < ri; le >>= 1, ri >>= 1) {
            if (le & 1) res += value(le++, x);
            if (ri & 1) res += value(--ri, x);
        }
        return res;
    }
};
```

BIT

```cpp
//cat bit.hpp | ./hash.sh
//ab7995
#pragma once
//NOLINTNEXTLINE(readability-identifier-naming)
template <typename T> struct BIT {
    vector<T> bit;
    BIT(int n) : bit(n, 0) {}
    /**
     * @time O(n)
     */
    BIT(const vector<T>& a) : bit(a) {
```

```
        for (int i = 0; i < ssize(a); i++) {
            int j = i | (i + 1);
            if (j < ssize(a)) bit[j] += bit[i];
        }
    }
    /**
     * @time O(log n)
     */
    void update(int i, const T& d) {
        assert(0 <= i && i < ssize(bit));
        for (; i < ssize(bit); i |= i + 1) bit[i] += d;
    }
    /**
     * sum of range [0, ri)
     * @time O(log n)
     */
    T sum(int ri) const {
        assert(0 <= ri && ri <= ssize(bit));
        T ret = 0;
        for (; ri > 0; ri &= ri - 1) ret += bit[ri - 1];
        return ret;
    }
    /**
     * sum of range [le, ri)
     * @time O(log n)
     */
    T sum(int le, int ri) const {
        assert(0 <= le && le <= ri && ri <= ssize(bit));
        return sum(ri) - sum(le);
    }
    /**
     * Returns min pos such that sum of range [0, pos) >= sum. Returns
     * ssize(bit) + 1 if no sum is >= sum.
     * Doesn't work if BIT::sum(i, i + 1) < 0
     * @time O(log n)
     */
    int lower_bound(T sum) const {
        if (sum <= 0) return 0;
        int pos = 0;
        for (int pw = 1 << __lg(ssize(bit) | 1); pw; pw >>= 1)
            if (pos + pw <= ssize(bit) && bit[pos + pw - 1] < sum)
                pos += pw, sum -= bit[pos - 1];
        return pos + 1;
    }
};
```

RMQ

```
//cat rmq.hpp | ./hash.sh
//59ae14
#pragma once
/**
 * @code{.cpp}
 *     vector<long long> arr;
 *     RMQ<long long> rmq(arr, [&](auto x, auto y) { return min(x, y); });
 *
 *     //To get index of min element:
 *     vector<pair<long long, int>> arr; //initialize arr[i].second = i
 *     RMQ<pair<long long, int>> rmq(arr, [&](auto x, auto y) { return min(x, y); });
 * @endcode
 * @see https://github.com/kth-competitive-programming/
 *     kactl/blob/main/content/data-structures/RMQ.h
 */
//NOLINTNEXTLINE(readability-identifier-naming)
template <typename T> struct RMQ {
    vector<vector<T>> dp;
    function<T(const T&, const T&)> op;
    /**
     * @time O(n log n)
     * @memory O(n log n)
     */
    RMQ(const vector<T>& arr, const function<T(const T&, const T&)>& a_op) : dp(1, arr),
        ↪ op(a_op) {
        for (int i = 0; (2 << i) <= ssize(arr); i++) {
            dp.emplace_back(ssize(arr) - (2 << i) + 1);
            for (int j = 0; j < ssize(dp.back()); j++)
                dp[i + 1][j] = op(dp[i][j], dp[i][j + (1 << i)]);
        }
    }
    /**
     * range [le, ri)
     * @time O(1)
     */
    T query(int le, int ri) const {
        assert(0 <= le && le < ri && ri <= ssize(dp[0]));
        int lg = __lg(ri - le);
        return op(dp[lg][le], dp[lg][ri - (1 << lg)]);
    }
};
```

Disjoint RMQ

```
//cat disjoint_rmq.hpp | ./hash.sh
//848e2b
#pragma once
/**
 * Disjoint RMQ is like normal RMQ except these ranges never overlap. It is
 * useful for:
 * - min and # of mins.
 * - product under composite mod
 * - 2-by-2 matrix multiply
 * @code{.cpp}
 *     //usage for min and # of mins:
 *     vector<pair<long long, int>> arr; //initialize arr[i].second = 1
 *     disjoint_rmq<pair<long long, int>> rmq(arr, {LLONG_MAX, 0}, [&](auto x, auto y) {
 *         if (x.first == y.first) return make_pair(x.first, x.second + y.second);
 *         return min(x, y);
 *     });
 * @endcode
 * @see https://codeforces.com/blog/entry/87940,
 *     https://github.com/sgtlaugh/algovault/blob/
 *     master/code_library/disjoint_sparse_table.cpp
 */
template <typename T> struct disjoint_rmq {
    const int N;
    vector<vector<T>> dp;
```

```
    function<T(const T&, const T&)> op; // any associative operation
    /**
     * @time O(n log n)
     * @memory O(n log n)
     */
    disjoint_rmq(const vector<T>& arr, const T& identity, const function<T(const T&, const
        ↪ T&)>& a_op) : N(ssize(arr)), op(a_op) {
        for (int i = 0, len = 1; len <= N; i++, len *= 2) {
            dp.emplace_back(N + 1, identity);
            for (int center = len; center < N + len; center += 2 * len) {
                for (int j = center + 1; j <= min(N, center + len); j++)
                    dp[i][j] = op(dp[i][j - 1], arr[j - 1]);
                for (int j = min(N, center) - 1; j >= center - len; j--)
                    dp[i][j] = op(arr[j], dp[i][j + 1]);
            }
        }
    }
    /**
     * range [le, ri)
     * @time O(1)
     */
    T query(int le, int ri) const {
        assert(0 <= le && le < ri && ri <= N);
        int lg = __lg(le ^ ri);
        return op(dp[lg][le], dp[lg][ri]);
    }
};
```

Lazy Segment Tree

```
//cat lazy_segment_tree.hpp | ./hash.sh
//96535f
#pragma once
/**
 * Internal nodes are [1, n), leaf nodes are [n, 2 * n).
 * Rotating leaves makes it a single complete binary tree (instead of a set of
 * perfect binary trees). So now, even for non-power of 2 size:
 * - recursive seg tree works
 * - recursive tree walks AKA binary search works
 * - root is at tree[1]
 * @see https://codeforces.com/blog/entry/18051
 *     https://github.com/ecnerwala/cp-book/blob/master/src/seg_tree.hpp
 *     https://github.com/yosupo06/Algorithm/blob/master/src/datastructure/segtree.hpp
 */
struct seg_tree {
    using dt = long long;
    using ch = long long;
    static dt combine(const dt& le, const dt& ri) {
        return min(le, ri);
    }
    static const dt UNIT = 1e18;
    struct node {
        dt val;
        ch lazy;
        int le, ri;//[le, ri)
    };
    const int N, S/*smallest power of 2 >= N*/;
    vector<node> tree;
    seg_tree(const vector<dt>& arr) : N(ssize(arr)), S(N ? 1 << __lg(2 * N - 1) : 0), tree(2 *
        ↪ N) {
        for (int i = 0; i < N; i++)
            tree[i + N] = {arr[i], 0, i, i + 1};
        rotate(tree.rbegin(), tree.rbegin() + S - N, tree.rbegin() + N);
        for (int i = N - 1; i >= 1; i--) {
            tree[i] = {
                combine(tree[2 * i].val, tree[2 * i + 1].val),
                0,
                tree[2 * i].le,
                tree[2 * i + 1].ri
            };
        }
    }
    void apply(int v, ch change) {
        tree[v].val += change;
        tree[v].lazy += change;
    }
    void push(int v) {
        if (tree[v].lazy) {
            apply(2 * v, tree[v].lazy);
            apply(2 * v + 1, tree[v].lazy);
            tree[v].lazy = 0;
        }
    }
    void build(int v) {
        tree[v].val = combine(tree[2 * v].val, tree[2 * v + 1].val);
    }
    int to_leaf(int i) const {
        i += S;
        return i < 2 * N ? i : 2 * (i - N);
    }
    //update range [le, ri)
    void update(int le, int ri, ch change) {
        assert(0 <= le && le <= ri && ri <= N);
        le = to_leaf(le), ri = to_leaf(ri);
        int lca_l_r = __lg((le - 1) ^ ri);
        for (int lg = __lg(le); lg > __builtin_ctz(le); lg--) push(le >> lg);
        for (int lg = lca_l_r; lg > __builtin_ctz(ri); lg--) push(ri >> lg);
        for (int x = le, y = ri; x < y; x >>= 1, y >>= 1) {
            if (x & 1) apply(x++, change);
            if (y & 1) apply(--y, change);
        }
        for (int lg = __builtin_ctz(ri) + 1; lg <= lca_l_r; lg++) build(ri >> lg);
        for (int lg = __builtin_ctz(le) + 1; lg <= __lg(le); lg++) build(le >> lg);
    }
    void update(int v/* = 1*/, int le, int ri, ch change) {
        if (ri <= tree[v].le || tree[v].ri <= le) return;
        if (le <= tree[v].le && tree[v].ri <= ri) return apply(v, change);
        push(v);
        update(2 * v, le, ri, change);
        update(2 * v + 1, le, ri, change);
        build(v);
    }
    //query range [le, ri)
    dt query(int le, int ri) {
        assert(0 <= le && le <= ri && ri <= N);
        le = to_leaf(le), ri = to_leaf(ri);
        int lca_l_r = __lg((le - 1) ^ ri);
```

```
        for (int lg = __lg(le); lg > __builtin_ctz(le); lg--) push(le >> lg);
        for (int lg = lca_l_r; lg > __builtin_ctz(ri); lg--) push(ri >> lg);
        dt resl = UNIT, resr = UNIT;
        for (; le < ri; le >>= 1, ri >>= 1) {
            if (le & 1) resl = combine(resl, tree[le++].val);
            if (ri & 1) resr = combine(tree[--ri].val, resr);
        }
        return combine(resl, resr);
    }
    dt query(int v/* = 1*/, int le, int ri) {
        if (ri <= tree[v].le || tree[v].ri <= le) return UNIT;
        if (le <= tree[v].le && tree[v].ri <= ri) return tree[v].val;
        push(v);
        return combine(query(2 * v, le, ri), query(2 * v + 1, le, ri));
    }
};
```

STRINGS

Binary Trie

```
//cat binary_trie.hpp | ./hash.sh
//88fa9c
#pragma once
struct binary_trie {
    const int MX_BIT = 62;
    struct node {
        long long val = -1;
        int sub_sz = 0;//number of inserted values in subtree
        array<int, 2> next = {-1, -1};
    };
    vector<node> t;
    binary_trie() : t(1) {}
    /**
     * Pass delta = 1 to insert val, -1 to remove val, 0 to get the # of val's
     * in this data structure.
     * @time O(MX_BIT)
     */
    int update(long long val, int delta) {
        int c = 0;
        t[0].sub_sz += delta;
        for (int bit = MX_BIT; bit >= 0; bit--) {
            bool v = (val >> bit) & 1;
            if (t[c].next[v] == -1) {
                t[c].next[v] = ssize(t);
                t.emplace_back();
            }
            c = t[c].next[v];
            t[c].sub_sz += delta;
        }
        t[c].val = val;
        return t[c].sub_sz;
    }
    int size() const {
        return t[0].sub_sz;
    }
    /**
     * Returns x such that x is in this data structure, and the value of
     * (x^val) is minimum.
     * @time O(MX_BIT)
     */
    long long min_xor(long long val) const {
        assert(size() > 0);
        int c = 0;
        for (int bit = MX_BIT; bit >= 0; bit--) {
            bool v = (val >> bit) & 1;
            int ch = t[c].next[v];
            if (ch != -1 && t[ch].sub_sz > 0)
                c = ch;
            else
                c = t[c].next[!v];
        }
        return t[c].val;
    }
};
```

Prefix Function

```
//cat prefix_function.hpp | ./hash.sh
//65fea7
#pragma once
/**
 * @see https://cp-algorithms.com/string/prefix-function.html#implementation
 * @time O(n)
 */
template <typename T> vector<int> prefix_function(const T& s) {
    vector<int> pi(ssize(s), 0);
    for (int i = 1; i < ssize(s); i++) {
        int j = pi[i - 1];
        while (j > 0 && s[i] != s[j]) j = pi[j - 1];
        pi[i] = j + (s[i] == s[j]);
    }
    return pi;
}
```

KMP String Matching

```
//cat kmp.hpp | ./hash.sh
//1edfb2
#pragma once
#include "prefix_function.hpp"
/**
 * @code{.cpp}
 *     string s;
 *     KMP kmp(s);
 *     vector<int> a;
 *     KMP kmp(a);
 * @endcode
 * KMP doubling trick: to check if 2 arrays are rotationally equivalent: run
 * kmp with one array as the needle and the other array doubled (excluding the
 * first & last characters) as the haystack or just use kactl's min rotation
 * code.
 */
//NOLINTNEXTLINE(readability-identifier-naming)
```

```cpp
template <typename T> struct KMP {
    T needle;
    vector<int> pi;
    /**
     * @time O(|needle|)
     * @memory O(|needle|)
     */
    KMP(const T& a_needle) : needle(a_needle), pi(prefix_function(needle)) {}
    /**
     * Returns array `matches` where:
     * haystack.substr(matches[i], ssize(needle)) == needle
     * @time O(|needle| + |haystack|)
     */
    vector<int> find(const T& haystack) const {
        vector<int> matches;
        for (int i = 0, j = 0; i < ssize(haystack); i++) {
            while (j > 0 && needle[j] != haystack[i]) j = pi[j - 1];
            if (needle[j] == haystack[i]) j++;
            if (j == ssize(needle)) {
                matches.push_back(i - ssize(needle) + 1);
                j = pi[j - 1];
            }
        }
        return matches;
    }
};
```

Suffix and LCP Arrays

```cpp
//cat suffix_array.hpp | ./hash.sh
//755406
#pragma once
/**
 * suffixes of "banana":
 * 0 banana
 * 1 anana
 * 2 nana
 * 3 ana
 * 4 na
 * 5 a
 *
 * sorted,  lcp
 * 5 a
 *   |      1
 * 3 ana
 *   |||    3
 * 1 anana
 *          0
 * 0 banana
 *          0
 * 4 na
 *   ||     2
 * 2 nana
 *
 * suffix array = [5, 3, 1, 0, 4, 2]
 * rank array satisfies sa[rank[i]] == i and rank[sa[i]] == i
 * lcp array = [1, 3, 0, 0, 2, 0] - last 0 is a dummy value
 *
 * @code{.cpp}
 *     string s;
 *     suffix_array info(s, 128);
 *     vector<int> arr;
 *     suffix_array info(arr, 1e5 + 1);
 * @endcode
 * @see https://github.com/kth-competitive-programming/kactl
 * /blob/main/content/strings/SuffixArray.h
 */
template <typename T> struct suffix_array {
    const int N;
    vector<int> sa, rank, lcp;
    /**
     * Assumes 0 <= s[i] < max_val.
     * @time O((nlogn) + max_val)
     * @memory O(n + max_val)
     */
    suffix_array(const T& s, int max_val) : N(ssize(s)), sa(N), rank(s.begin(), s.end()),
        ↪ lcp(N) {
        iota(sa.begin(), sa.end(), 0);
        vector<int> tmp(N + 1, -1);
        for (int len = 0; len < N; len = max(1, 2 * len)) {//suffix array
            iota(tmp.begin(), tmp.begin() + len, N - len);
            for (int i = 0, j = len; i < N; i++)
                if (sa[i] >= len)
                    tmp[j++] = sa[i] - len;
            vector<int> freq(max_val, 0);
            for (int val : rank) freq[val]++;
            partial_sum(freq.begin(), freq.end(), freq.begin());
            for (int i = N - 1; i >= 0; i--)
                sa[--freq[rank[tmp[i]]]] = tmp[i];
            copy(rank.begin(), rank.end(), tmp.begin());
            max_val = 1, rank[sa[0]] = 0;
            for (int i = 1; i < N; i++) {
                int a = sa[i - 1], b = sa[i];
                max_val += (tmp[a] != tmp[b] || tmp[a + len] != tmp[b + len]);
                rank[b] = max_val - 1;
            }
            if (max_val == N) break;
        }
        for (int i = 0, k = 0; i < N; i++) {//lcp array
            if (k > 0) k--;
            if (rank[i] == 0) continue;
            int j = sa[rank[i] - 1];
            while (max(i, j) + k < N && s[j + k] == s[i + k]) k++;
            lcp[rank[i] - 1] = k;
        }
    }
};
```

Suffix Array Related Queries

```cpp
//cat suffix_array_query.hpp | ./hash.sh
//ae4931
#pragma once
#include "suffix_array.hpp"
#include "../range_data_structures/rmq.hpp"
/**
```

```
 * Various queries you can do based on suffix array.
 * @see https://github.com/yosupo06/Algorithm/blob
 * /master/src/string/suffixarray.hpp
 */
template <typename T> struct sa_query {
    T s;
    suffix_array<T> info;
    RMQ<int> rmq_lcp, rmq_sa;
    /**
     * Assumes 0 <= s[i] < max_val.
     * @time O((nlogn) + max_val)
     * @memory O((nlogn) + max_val) An O(max_val) size freq array is used
     * temporarily during suffix array construction. Only O(n log n) memory is
     * permanently stored by this struct.
     */
    sa_query(const T& a_s, int max_val) :
        s(a_s),
        info(suffix_array(s, max_val)),
        rmq_lcp(info.lcp, [](int i, int j) -> int {return min(i, j);}),
        rmq_sa(info.sa, [](int i, int j) -> int {return min(i, j);}) {}
    /**
     * Returns length of longest common prefix of suffixes s[idx1...N),
     * s[idx2...N), 0-based indexing.
     *
     * To check if two substrings s[l1..r1), s[l2..r2) are equal:
     * r1-l1 == r2-l2 && get_lcp(l1, l2) >= r1-l1
     * @time O(1)
     */
    int get_lcp(int idx1, int idx2) const {
        if (idx1 == idx2) return ssize(s) - idx1;
        auto [le, ri] = minmax(info.rank[idx1], info.rank[idx2]);
        return rmq_lcp.query(le, ri);
    }
    /**
     * Returns 1 if suffix s[idx1 ... N) < s[idx2 ... N).
     * Returns 0 if idx1 == idx2.
     * @time O(1)
     */
    bool less(int idx1, int idx2) const {
        return info.rank[idx1] < info.rank[idx2];
    }
    /**
     * Returns range [le, ri) such that:
     * - for all i in [le, ri): t == s.substr(info.sa[i], ssize(t))
     * - `ri - le` is the # of matches of t in s.
     * @time O(|t| * log(|s|))
     */
    pair<int, int> find(const T& t) const {
        auto cmp = [&](int i, int cmp_val) -> bool {
            return s.compare(i, ssize(t), t) < cmp_val;
        };
        auto le = lower_bound(info.sa.begin(), info.sa.end(), 0, cmp);
        auto ri = lower_bound(le, info.sa.end(), 1, cmp);
        return {le - info.sa.begin(), ri - info.sa.begin()};
    }
    /**
     * Returns min i such that t == s.substr(i, ssize(t)) or -1. For example,
     * replace RMQ with kth-smallest PST/Wavelet to solve
     * https://open.kattis.com/problems/anothersubstringqueryproblem
     * @time O(|t| * log(|s|))
     */
    int find_first(const T& t) const {
        auto [le, ri] = find(t);
        if (le == ri) return -1;
        return rmq_sa.query(le, ri);
    }
};
```

Palindrome Query

```
//cat palindrome_query.hpp | ./hash.sh
//68c8e1
#pragma once
#include "../../kactl/content/strings/Manacher.h"
struct pal_query {
    const int N;
    array<vi, 2> pal_len;
    /**
     * @time O(n)
     * @memory O(n)
     */
    pal_query(const string& s) : N(ssize(s)), pal_len(manacher(s)) {}
    /**
     * Returns 1 if substring s[le...ri) is a palindrome.
     * Returns 1 when le == ri.
     * @time O(1)
     */
    bool is_pal(int le, int ri) const {
        assert(0 <= le && le <= ri && ri <= N);
        int len = ri - le;
        return pal_len[len & 1][le + len / 2] >= len / 2;
    }
};
```

Trie

```
//cat trie.hpp | ./hash.sh
//2aa8c6
#pragma once
//source: https://cp-algorithms.com/string/aho_corasick.html#construction-of-the-trie
const int K = 26;//alphabet size
struct trie {
    const char MIN_CH = 'A';//'a' for lowercase, '0' for digits
    struct node {
        int next[K], cnt_words = 0, par = -1;
        char ch;
        node(int a_par = -1, char a_ch = '#') : par(a_par), ch(a_ch) {
            fill(next, next + K, -1);
        }
    };
    vector<node> t;
    trie() : t(1) {}
    void insert(const string& s) {
        int v = 0;
        for (char ch : s) {
            int let = ch - MIN_CH;
```

```
            if (t[v].next[let] == -1) {
                t[v].next[let] = ssize(t);
                t.emplace_back(v, ch);
            }
            v = t[v].next[let];
        }
        t[v].cnt_words++;
    }
    int find(const string& s) const {
        int v = 0;
        for (char ch : s) {
            int let = ch - MIN_CH;
            if (t[v].next[let] == -1) return 0;
            v = t[v].next[let];
        }
        return t[v].cnt_words;
    }
};
```