South 1	Dakota Mines			Pa	age 1
Listings		38	Iterate Chooses	12	
			39	Iterate Submasks	12
1	CONTEST	1	40	Iterate Supermasks	12
2	Tips and Tricks	1	41	Number of Distinct Subsequences DP	13
3	Hash codes	1	42	PBDS	
4	Test on random inputs	2	43	Random	
5	MAX FLOW	2	44	Safe Hash	
6	Hungarian	2	45	RANGE DATA STRUCTURES	
7	Min Cost Max Flow	2	46	Number Distinct Elements	
8	GRAPHS	3	47	Implicit Lazy Segment Tree	
9	Block Vertex Tree	3	48	Kth Smallest	
10	Bridge Tree	3	49	Merge Sort Tree	
11	Bridges and Cuts	3	50	BIT	
12	Centroid Decomp	4	51	RMQ	
13	Frequency Table of Tree Distance	5	52	Lazy Segment Tree	
14	Count Paths Per Node	5	53	STRINGS	
15	Dijkstra	6	54	Binary Trie	
16	HLD	6	55	KMP	
17	Hopcroft Karp	6	56	Longest Common Prefix Query	
18	Kth Node on Path	7	57	Palindrome Query	
19	LCA	7	58	Trie	
20	Rooted Tree Isomorphism	8	59	Suffix Array and LCP Array	
21	MATH	8	00		10
22	Derangements	8			
23	Binary Exponentiation MOD	8			
24	Fibonacci	9			
25	Matrix Multiplication	9			
26	Mobius Inversion	9			
27	N Choose K MOD	9			
28		10			
29	Prime Sieve				
30	Row Reduce				
31	Solve Linear Equations MOD				
32	Euler's Totient Phi Function				
33	MISC				
34	Cartesian Tree				
35	Count Rectangles				
36	Max Rectangle in Histogram				
37	Monotonic Stack				

CONTEST

Tips and Tricks

Tips and Tricks

```
- [C++ tips and tricks](https://codeforces.com/blog/entry/74684)
- use '#pragma GCC optimize "trapy" to invoke RTE (Run Time Error) upon integer overflow
- use 'mt19937 rng(chrono::steady_clock::now().time_since_epoch().count());' to generate
   - 'shuffle(arr.begin(), arr.end(), rng);'
- use pramgas for C++ speed boost
#pragma GCC optimize("03,unroll-loops")
#pragma GCC target("avx2,bmi,bmi2,lzcnt,popcnt")
### Troubleshooting
...
/* stuff you should look for
   * int overflow, array bounds
   * special cases (n=1?)
   * do smth instead of nothing and stay organized
   * WRITE STUFF DOWN
   * DON'T GET STUCK ON ONE APPROACH
""
Author: Beng
- refer to [KACTL Troubleshoot](../../kactl/content/contest/troubleshoot.txt)
## Sources
- [[Tutorial] GCC Optimization Pragmas] (https://codeforces.com/blog/entry/96344)
- [Don't use rand(): a guide to random number generators in
```

Hash codes

```
#!/usr/bin/env bash
#Hashes a file, ignoring all:
# - whitespace
  - comments
# - asserts
# - includes
# - pragmas
#Use to verify that code was correctly typed.
#usage:
# chmod +x hash.sh
# cat a.cpp / ./hash.sh
#or just copy this command:
# cat a.cpp | sed -r '/(assert|include|pragma)/d' | cpp -fpreprocessed -P | tr -d
    \hookrightarrow '[:space:]' | md5sum | cut -c-6
sed -r '/(assert|include|pragma)/d' | cpp -fpreprocessed -P | tr -d '[:space:]' | md5sum
    \hookrightarrow | cut -c-6
```

Test on random inputs

```
#!/usr/bin/env bash
#runs 2 programs against each other on random inputs until they output different results
#source: https://github.com/Errichto/youtube/blob/master/testing/s.sh
#usage:
# chmod +x test.sh
# ./test.sh
for((i = 1; ; ++i)); do
    echo $i
    ./test.out > in
    diff --ignore-all-space <(./a.out < in) <(./brute.out < in) || break
done</pre>
```

MAX FLOW

Hungarian

```
//cat hungarian.hpp / ./hash.sh
//625431
#pragma once
//source: https://e-maxx.ru/algo/assignment_hungary
//input: cost[1...n][1...m] with 1 <= n <= m
//n workers, indexed 1, 2, ..., n
//m jobs, indexed 1, 2, ..., m
//it costs 'cost[i][j]' to assign worker i to job j (1<=i<=n, 1<=j<=m)
//this returns *min* total cost to assign each worker to some distinct job
//0(n^2 * m)
//trick 1: set 'cost[i][j]' to INF to say: "worker 'i' cannot be assigned job 'j'"
//trick 2: 'cost[i][j]' can be negative, so to instead find max total cost over all
     \hookrightarrow matchings: set all 'cost[i][j]' to '-cost[i][j]'.
//Now max total cost = - hungarian(cost).min_cost
const long long INF = 1e18;
struct match {
    long long min_cost;
    vector<int> matching; //worker 'i' (1<=i<=n) is assigned to job 'matching[i]'
         \hookrightarrow (1<=matching[i]<=m)
};
match hungarian(const vector<vector<long long>>& cost) {
    int n = cost.size() - 1, m = cost[0].size() - 1;
    assert(n <= m):</pre>
    vector<int> p(m + 1), way(m + 1);
    vector<long long> u(n + 1), v(m + 1);
    for (int i = 1; i <= n; i++) {
        p[0] = i;
        int j0 = 0;
        vector<long long> minv(m + 1, INF);
        vector<bool> used(m + 1, 0);
            used[j0] = 1;
            int i0 = p[j0], j1 = 0;
            long long delta = INF;
            for (int j = 1; j <= m; j++)
                if (!used[j]) {
                    long long cur = cost[i0][j] - u[i0] - v[j];
```

```
if (cur < minv[i])</pre>
                     minv[j] = cur, way[j] = j0;
                if (minv[j] < delta)</pre>
                     delta = minv[j], j1 = j;
            }
        for (int j = 0; j \le m; j++)
            if (used[i])
                u[p[i]] += delta, v[i] -= delta;
                minv[i] -= delta:
        j0 = j1;
    } while (p[j0] != 0);
    do {
        int j1 = way[j0];
        p[j0] = p[j1];
        j0 = j1;
    } while (j0);
vector < int > ans(n + 1);
for (int j = 1; j <= m; j++)
    ans[p[i]] = i;
return {-v[0], ans};
```

Min Cost Max Flow

```
//cat min_cost_max_flow.hpp | ./hash.sh
//579c56
#pragma once
//source: https://e-maxx.ru/algo/min_cost_flow
const long long INF = 1e18;
struct min_cost_max_flow {
   using 11 = long long;
   struct edge {
       int a, b;
       11 cap, cost, flow;
       int back;
   };
   const int N;
    vector<edge> e;
    vector<vector<int>> g;
    min_cost_max_flow(int a_n) : N(a_n), g(N) {}
    void add_edge(int a, int b, ll cap, ll cost) {
        edge e1 = {a, b, cap, cost, 0, (int)g[b].size() };
        edge e2 = {b, a, 0, -cost, 0, (int)g[a].size() };
       g[a].push_back(e.size());
       e.push back(e1):
       g[b].push_back(e.size());
        e.push back(e2);
   }
   pair<11, 11> get_flow(int s, int t, 11 total_flow) {
       11 \text{ flow} = 0. \text{ cost} = 0:
       while (flow < total_flow) {</pre>
            vector<ll> d(N, INF);
            vector<int> p_edge(N), id(N, 0), q(N), p(N);
            int qh = 0, qt = 0;
            q[qt++] = s;
            d[s] = 0;
```

```
while (qh != qt) {
                int v = q[qh++];
                id[v] = 2;
                if (ah == N) ah = 0:
                for (int i = 0; i < (int)g[v].size(); i++) {</pre>
                    const edge& r = e[g[v][i]];
                    if (r.flow < r.cap && d[v] + r.cost < d[r.b]) {
                        d[r.b] = d[v] + r.cost;
                        if (id[r.b] == 0) {
                            q[qt++] = r.b;
                            if (qt == N) qt = 0;
                        } else if (id[r.b] == 2) {
                            if (--qh == -1) qh = N - 1;
                            q[qh] = r.b;
                        id[r.b] = 1;
                        p[r.b] = v;
                        p_{edge}[r.b] = i;
                }
            if (d[t] == INF) break:
            11 addflow = total flow - flow:
            for (int v = t; v != s; v = p[v]) {
                int pv = p[v], pr = p_edge[v];
                addflow = min(addflow, e[g[pv][pr]].cap - e[g[pv][pr]].flow);
            for (int v = t; v != s; v = p[v]) {
                int pv = p[v], pr = p_edge[v], r = e[g[pv][pr]].back;
                e[g[pv][pr]].flow += addflow;
                e[g[v][r]].flow -= addflow;
                cost += e[g[pv][pr]].cost * addflow;
            flow += addflow;
        return {flow, cost};
    }
};
```

GRAPHS

Block Vertex Tree

```
//cat block_vertex_tree.hpp | ./hash.sh
//ea8ef1
#pragma once
#include "bridges_and_cuts.hpp"
//returns adjacency list of block vertex tree
//usage:
// info cc = bridge_and_cut(adj, m);
// vector<vector<int>> bvt = block_vertex_tree(adj, cc);
//to loop over each *unique* bcc containing a node v:
// for (int bccid : bvt[v]) {
// bccid -= n;
// ...
// }
//to loop over each *unique* node inside a bcc:
```

```
// for (int v : bvt[bccid + n]) {
// }
vector<vector<int>> block_vertex_tree(const vector<vector<pair<int, int>>>& adj, const
    \hookrightarrow info% cc) {
   int n = adj.size();
   vector<vector<int>> bvt(n + cc.num bccs):
    vector<bool> vis(cc.num_bccs, 0);
   for (int v = 0; v < n; v++) {
       for (auto [_, e_id] : adj[v]) {
            int bccid = cc.bcc_id[e_id];
            if (!vis[bccid]) {
                vis[bccid] = 1;
                bvt[v].push_back(bccid + n); //add edge between original node, and bcc
                bvt[bccid + n].push_back(v);
       }
        for (int bccid : bvt[v]) vis[bccid - n] = 0;
   }
    return bvt;
```

Bridge Tree

Bridges and Cuts

```
//cat bridges_and_cuts.hpp | ./hash.sh
//5f0c60
#pragma once
//0(n+m) time & space
//2 edge cc and bcc stuff doesn't depend on each other, so delete whatever is not needed
//handles multiple edges
//
//example initialization of 'adj':
//for (int i = 0; i < m; i++) {
// int u, u;
// cin >> u >> v;
// u--, v--;
```

```
// adj[u].emplace_back(v, i);
// adj[v].emplace_back(u, i);
1/3
struct info {
    //2 edge connected component stuff (e.g. components split by bridge edges)
         \hookrightarrow https://cp-algorithms.com/graph/bridge-searching.html
    int num 2 edge ccs:
    vector<bool> is_bridge;//edge id -> 1 iff bridge edge
    vector<int> two_edge_ccid;//node -> id of 2 edge component (which are labeled 0, 1,
         \hookrightarrow .... 'num 2 edge ccs'-1)
    //bi connected component stuff (e.g. components split by cut/articulation nodes)
         \hookrightarrow https://cp-algorithms.com/graph/cutpoints.html
    int num_bccs;
    vector<bool> is_cut;//node -> 1 iff cut node
    vector<int> bcc_id; //edge id -> id of bcc (which are labeled 0, 1, ..., 'num_bccs'-1)
};
info bridge_and_cut(const vector<vector<pair<int/*neighbor*/, int/*edge id*/>>>&

    → adj/*undirected graph*/, int m/*number of edges*/) {

    //stuff for both (always keep)
    int n = adj.size(), timer = 1;
    vector<int> tin(n, 0);
    //2 edge cc stuff (delete if not needed)
    int num_2_edge_ccs = 0;
    vector<bool> is_bridge(m, 0);
    vector<int> two_edge_ccid(n), node_stack;
    node_stack.reserve(n);
    //bcc stuff (delete if not needed)
    int num_bccs = 0;
    vector<bool> is_cut(n, 0);
    vector<int> bcc_id(m), edge_stack;
    edge_stack.reserve(m);
    auto dfs = [&](auto self, int v, int p_id) -> int {
         int low = tin[v] = timer++, deg = 0;
         node_stack.push_back(v);
         for (auto [to, e_id] : adj[v]) {
             if (e_id == p_id) continue;
             if (!tin[to]) {
                 edge_stack.push_back(e_id);
                 int low_ch = self(self, to, e_id);
                 if (low_ch >= tin[v]) {
                     is cut[v] = 1:
                     while (1) {
                         int edge = edge_stack.back();
                         edge_stack.pop_back();
                         bcc_id[edge] = num_bccs;
                         if (edge == e_id) break;
                     }
                     num_bccs++;
                 }
                 low = min(low, low_ch);
                 deg++;
             } else if (tin[to] < tin[v]) {</pre>
                 edge_stack.push_back(e_id);
                 low = min(low, tin[to]);
        }
         if (p_id == -1) is_cut[v] = (deg > 1);
         if (tin[v] == low) {
             if (p_id != -1) is_bridge[p_id] = 1;
```

```
while (1) {
      int node = node_stack.back();
      node_stack.pop_back();
      two_edge_ccid[node] = num_2_edge_ccs;
      if (node == v) break;
    }
    num_2_edge_ccs++;
}
return low;
};
for (int i = 0; i < n; i++)
    if (!tin[i])
      dfs(dfs, i, -1);
return {num_2_edge_ccs, is_bridge, two_edge_ccid, num_bccs, is_cut, bcc_id};
}</pre>
```

Centroid Decomp

```
//cat centroid_decomp.hpp | ./hash.sh
//31dd4e
#pragma once
// Time and Space complexity are given in terms of n where n is the number of nodes in
// Time complexity O(n \log n)
// Space complexity O(n)
// Given an unweighted, undirected tree and a function,
// centroid_decomp runs the function on every decomposition
// see count_paths_per_node for example usage
struct centroid_decomp {
   vector<vector<int>> adi:
   function<void(const vector<vector<int>>&, int)> func;
    vector<int> sub_sz;
    centroid_decomp(const vector<vector<int>>& a_adj,
                    const function<void(const vector<vector<int>>&, int)>& a_func)
        : adj(a_adj), func(a_func), sub_sz(adj.size()) {
        decomp(find_centroid(0));
   }
   void calc_subtree_sizes(int u, int p = -1) {
        sub sz[u] = 1:
       for (int v : adj[u]) {
            if (v == p) continue;
            calc subtree sizes(v. u):
            sub_sz[u] += sub_sz[v];
   }
   int find centroid(int root) {
        calc_subtree_sizes(root);
       int u = root, p = -1;
       while (1) {
            int big_ch = -1;
           for (int v : adj[u]) {
                if (v == p) continue;
```

Frequency Table of Tree Distance

```
//cat count_paths_per_length.hpp / ./hash.sh
//272170
#pragma once
#include "../../kactl/content/numerical/FastFourierTransform.h"
#include "centroid_decomp.hpp"
//returns array 'num_paths' where 'num_paths[i]' = # of paths in tree with 'i' edges
//0(n \log^2 n)
vector<long long> count_paths_per_length(const vector<vector<int>>& a_adj/*unrooted,
     \hookrightarrow connected tree*/) {
    vector<long long> num_paths(a_adj.size(), 0);
    auto func = [&](const vector<vector<int>>& adj, int root) -> void {
        vector<double> total_depth(1, 1.0);
        for (int to : adj[root]) {
            vector<double> cnt_depth(1, 0.0);
            for (queue<pair<int, int>> q({{to, root}}); !q.empty();) {
                cnt_depth.push_back(q.size());
                queue<pair<int, int>> new_q;
                while (!q.empty()) {
                    auto [curr, par] = q.front();
                    q.pop();
                    for (int ch : adj[curr]) {
                        if (ch == par) continue;
                        new_q.emplace(ch, curr);
                }
                swap(q, new_q);
                vector<double> prod = conv(total_depth, cnt_depth);
                for (int i = 1; i < (int)prod.size(); i++) num_paths[i] +=</pre>
                     if (total_depth.size() < cnt_depth.size())</pre>

    total_depth.resize(cnt_depth.size(), 0.0);

            for (int i = 1; i < (int)cnt_depth.size(); i++) total_depth[i] +=</pre>

    cnt_depth[i];
```

```
};
centroid_decomp decomp(a_adj, func);
return num_paths;
}
```

Count Paths Per Node

```
//cat count_paths_per_node.hpp | ./hash.sh
//8811b2
#pragma once
#include "centroid_decomp.hpp"
//0-based nodes
//returns array 'num_paths' where 'num_paths[i]' = number of paths with k edges where
    \hookrightarrow node 'i' is on the path
//0(n \log n)
vector<long long> count_paths_per_node(const vector<vector<int>>& a_adj/*unrooted
    \hookrightarrow tree*/. int k) {
   vector<long long> num_paths(a_adj.size());
    auto func = [&](const vector<vector<int>>& adj, int root) -> void {
        vector<int> pre_d(1, 1), cur_d(1);
        auto dfs = [&] (auto self, int u, int p, int d) -> long long {
            if (d > k)
                return 0;
            if (int(cur_d.size()) <= d)</pre>
                cur_d.push_back(0);
            cur_d[d]++;
            long long cnt = 0;
            if (k - d < int(pre_d.size()))</pre>
                cnt += pre_d[k - d];
            for (int v : adj[u]) {
                if (v != p)
                     cnt += self(self, v, u, d + 1):
            num_paths[u] += cnt;
            return cnt;
        };
        auto dfs_child = [&](int child) -> long long {
            long long cnt = dfs(dfs, child, root, 1);
            pre_d.resize(cur_d.size());
            for (int i = 1; i < int(cur_d.size()) && cur_d[i]; i++) {</pre>
                pre_d[i] += cur_d[i];
                \operatorname{cur} d[i] = 0:
            }
            return cnt;
        };
        for (int child : adj[root])
            num_paths[root] += dfs_child(child);
        pre_d = vector<int>(1);
        cur_d = vector<int>(1);
        for (auto it = adj[root].rbegin(); it != adj[root].rend(); it++)
            dfs_child(*it);
   };
    centroid_decomp decomp(a_adj, func);
```

```
return num_paths;
}
```

Dijkstra

```
//cat dijkstra.hpp | ./hash.sh
//8fe9d3
#pragma once
//returns array 'len' where 'len[i]' = shortest path from node 'start' to node 'i'
//For example 'len[start]' will always = 0
const long long INF = 1e18;
vector<long long> dijkstra(const vector<vector<pair<int, long long>>>& adj /*directed or
    \hookrightarrow undirected, weighted graph*/, int start) {
    using node = pair<long long, int>;
    vector<long long> len(adj.size(), INF);
    len[start] = 0;
    priority_queue<node, vector<node>, greater<node>> q;
    q.emplace(0, start);
    while (!q.empty()) {
        auto [curr_len, v] = q.top();
        q.pop();
        if (len[v] < curr_len) continue;//important check: TLE without it</pre>
        for (auto [to, weight] : adj[v])
            if (len[to] > weight + len[v]) {
                len[to] = weight + len[v];
                q.emplace(len[to], to);
   return len:
```

HLD

```
//cat hld.hpp / ./hash.sh
//ba76ea
#pragma once
//source: https://codeforces.com/blog/entry/53170
//mnemonic: Heavy Light Decomposition
//NOLINTNEXTLINE(readability-identifier-naming)
struct HLD {
    struct node {
        int sub_sz = 1, par = -1, time_in = -1, next = -1;
    vector<node> tree;
    HLD(vector<vector<int>>& adj/*forest of unrooted trees*/) : tree(adj.size()) {
        for (int i = 0; i < (int)adj.size(); i++) {</pre>
            if (tree[i].next == -1) {//lowest indexed node in each tree becomes root
                tree[i].next = i;
                dfs1(i, adj);
                dfs2(i, adj, timer);
            }
        }
    }
    void dfs1(int v, vector<vector<int>>& adj) {
        auto par = find(adj[v].begin(), adj[v].end(), tree[v].par);
        if (par != adj[v].end()) adj[v].erase(par);
```

```
for (int& to : adi[v]) {
            tree[to].par = v:
            dfs1(to, adj);
            tree[v].sub sz += tree[to].sub sz:
            if (tree[to].sub_sz > tree[adj[v][0]].sub_sz)
                swap(to, adj[v][0]);
        }
    }
    void dfs2(int v, const vector<vector<int>>& adj, int& timer) {
        tree[v].time_in = timer++;
        for (int to : adj[v]) {
            tree[to].next = (timer == tree[v].time_in + 1 ? tree[v].next : to);
            dfs2(to, adj, timer);
   }
    // Returns inclusive-exclusive intervals (of time_in's) corresponding to the path
         \hookrightarrow between u and v, not necessarily in order
    // This can answer queries for "is some node 'x' on some path" by checking if the
         \hookrightarrow tree[x].time_in is in any of these intervals
    // u, v must be in the same component
    vector<pair<int, int>> path(int u, int v) const {
        vector<pair<int, int>> res;
        for (;; v = tree[tree[v].next].par) {
            if (tree[v].time_in < tree[u].time_in) swap(u, v);</pre>
            if (tree[tree[v].next].time_in <= tree[u].time_in) {</pre>
                res.emplace_back(tree[u].time_in, tree[v].time_in + 1);
                return res;
            res.emplace_back(tree[tree[v].next].time_in, tree[v].time_in + 1);
    }
    // Returns interval (of time_in's) corresponding to the subtree of node i
    // This can answer queries for "is some node 'x' in some other node's subtree" by
         \hookrightarrow checking if tree[x].time_in is in this interval
    pair<int, int> subtree(int i) const {
        return {tree[i].time_in, tree[i].time_in + tree[i].sub_sz};
   }
    // Returns lca of nodes u and v
    // u, v must be in the same component
    int lca(int u, int v) const {
        for (:: v = tree[tree[v].next].par) {
            if (tree[v].time_in < tree[u].time_in) swap(u, v);</pre>
            if (tree[tree[v].next].time_in <= tree[u].time_in) return u;</pre>
    }
};
```

Hopcroft Karp

```
//cat hopcroft_karp.hpp | ./hash.sh
//de75d7
#pragma once
//source: https://github.com/foreverbell/acm-icpc-cheat-sheet/
// blob/master/src/graph-algorithm/hopcroft-karp.cpp
//Worst case O(E*sqrt(V)) but faster in practice
struct match {
    //# of edges in matching (which = size of min vertex cover by öKnig's theorem)
    int size_of_matching;
```

```
//an arbitrary max matching is found. For this matching:
    //if l to r[node left] == -1:
    // node_left is not in matching
    //else:
    // the edge 'node_left' <=> l_to_r[node_left] is in the matching
    //similarly for r to l with edge r to l \lceil node \ right \rceil \iff node \ right \ in \ matching \ if
         \hookrightarrow r_to_l[node_right] != -1
    //matchings stored in l_to_r and r_to_l are the same matching
    //provides way to check if any node/edge is in matching
    vector<int> l_to_r, r_to_l;
    //an arbitrary min vertex cover is found. For this muc: muc_l[node_left] is 1 iff

→ node_left is in the min vertex cover (same for mvc_r)
    //if muc_l[node_left] is 0, then node_left is in the corresponding maximal
         \hookrightarrow independent set
    vector<bool> mvc_l, mvc_r;
};
//Think of the bipartite graph as having a left side (with size lsz) and a right side
     \hookrightarrow (with size rsz).
//Nodes on left side are indexed 0.1....lsz-1
//Nodes on right side are indexed 0,1,...,rsz-1
//'adj' is like a directed adjacency list containing edges from left side -> right side:
//To initialize 'adj': For every edge node_left <=> node_right, do:
     \hookrightarrow adj[node_left].push_back(node_right)
match hopcroft_karp(const vector<vector<int>>& adj/*bipartite graph*/, int rsz/*number
     \hookrightarrow of nodes on right side*/) {
    int size_of_matching = 0, lsz = adj.size();
    vector<int> l_to_r(lsz, -1), r_to_l(rsz, -1);
    while (1) {
        queue<int> q;
        vector<int> level(lsz, -1);
        for (int i = 0; i < lsz; i++)</pre>
             if (l_to_r[i] == -1)
                level[i] = 0, q.push(i);
        bool found = 0:
        vector<bool> mvc_l(lsz, 1), mvc_r(rsz, 0);
        while (!a.emptv()) {
            int u = q.front();
            q.pop();
            mvc 1[u] = 0:
            for (int x : adj[u]) {
                mvc_r[x] = 1;
                int v = r_{to_1[x]};
                if (v == -1) found = 1;
                 else if (level[v] == -1) {
                     level[v] = level[u] + 1;
                     q.push(v);
                }
             }
        if (!found) return {size_of_matching, l_to_r, r_to_l, mvc_l, mvc_r};
        auto dfs = [&](auto self, int u) -> bool {
            for (int x : adi[u]) {
                int v = r_to_1[x];
                if (v == -1 || (level[u] + 1 == level[v] && self(self, v))) {
                     1 to r[u] = x:
                     r_{to_1[x]} = u;
                     return 1:
```

Kth Node on Path

```
//cat kth_node_on_path.hpp / ./hash.sh
//c59307
#pragma once
#include "lca.hpp"
struct kth_node_on_path {
    LCA lca:
    kth_node_on_path(const vector<vector<pair<int, long long>>>& adj/*forest of weighted
         \hookrightarrow trees*/) : lca(adj) {}
    //consider path \{u, u's par, \ldots, LCA(u, v), \ldots, v's par, v\}. This returns the node
         \hookrightarrow at index k
    //assumes 0 <= k <= number of edges on path from u to v
    // u. v must be in the same component
    int query(int u, int v, int k) const {
        int lca uv = lca.get lca(u, v);
        int u_lca = lca.tree[u].depth - lca.tree[lca_uv].depth;
        int v_lca = lca.tree[v].depth - lca.tree[lca_uv].depth;
        assert(0 <= k && k <= u lca + v lca):
        return k <= u_lca ? lca.kth_par(u, k) : lca.kth_par(v, u_lca + v_lca - k);</pre>
    }
};
```

LCA

```
//cat lca.hpp / ./hash.sh
//f2cd84
#pragma once
//https://codeforces.com/blog/entry/74847
//mnemonic: Least/Lowest Common Ancestor
//NOLINTNEXTLINE(readability-identifier-naming)
struct LCA {
       int jmp = -1, jmp_edges = 0, par = -1, depth = 0;
       long long dist = OLL;
    vector<node> tree;
   LCA(const vector<vector<pair<int, long long>>>& adj/*forest of weighted trees*/):
        \hookrightarrow tree(adj.size()) {
       for (int i = 0; i < (int)adj.size(); i++) {</pre>
            if (tree[i].imp == -1) {//lowest indexed node in each tree becomes root
                tree[i].jmp = i;
                dfs(i, adj);
            }
       }
   }
   void dfs(int v, const vector<vector<pair<int, long long>>>& adj) {
```

```
int jmp, jmp_edges;
        if (tree[v].jmp != v && tree[v].jmp_edges == tree[tree[v].jmp].jmp_edges)
            jmp = tree[tree[v].jmp].jmp, jmp_edges = 2 * tree[v].jmp_edges + 1;
            jmp = v, jmp_edges = 1;
        for (auto [ch, w] : adj[v]) {
            if (ch == tree[v].par) continue;
            tree[ch] = {
                jmp,
                jmp_edges,
                1 + tree[v].depth,
                w + tree[v].dist
            };
            dfs(ch, adj);
    }
    //traverse up k edges in O(\log(k)). So with k=1 this returns 'v''s parent
    int kth_par(int v, int k) const {
        k = min(k, tree[v].depth);
        while (k > 0) {
            if (tree[v].jmp_edges <= k) {</pre>
                k -= tree[v].jmp_edges;
                v = tree[v].jmp;
            } else {
                k--;
                v = tree[v].par;
        }
        return v;
    }
    // x, y must be in the same component
    int get_lca(int x, int y) const {
        if (tree[x].depth < tree[y].depth) swap(x, y);</pre>
        x = kth_par(x, tree[x].depth - tree[y].depth);
        while (x != v) {
            if (tree[x].jmp != tree[y].jmp)
                x = tree[x].jmp, y = tree[y].jmp;
                x = tree[x].par, y = tree[y].par;
        }
        return x;
    }
    int dist_edges(int x, int y) const {
        return tree[x].depth + tree[y].depth - 2 * tree[get_lca(x, y)].depth;
    long long dist_weight(int x, int y) const {
        return tree[x].dist + tree[y].dist - 2 * tree[get_lca(x, y)].dist;
    }
};
```

Rooted Tree Isomorphism

```
//cat subtree_isomorphism.hpp | ./hash.sh
//64d64a
#pragma once
// Complexity given in terms of n where n is the number of nodes in the tree
```

```
// Time complexity O(n \log n)
// Space complexity O(n)
// Given an undirected or directed rooted tree
// rooted_subtree_isomorphism classifies each rooted subtree
struct info {
    int num distinct subtrees: //0 <= id[i] < num distinct subtrees for all i
    vector<int> id; //id[u] == id[v] iff subtree u is isomorphic to subtree v
};
info subtree_iso(const vector<vector<int>>& adj, int root) {
    vector<int> id(adj.size());
    map<vector<int>, int> hashes;
    auto dfs = [&](auto self, int u, int p) -> int {
        vector<int> ch_ids;
        ch_ids.reserve(adj[u].size());
        for (int v : adj[u]) {
            if (v != p)
                ch_ids.push_back(self(self, v, u));
        sort(ch_ids.begin(), ch_ids.end());
        auto it = hashes.find(ch ids):
        if (it == hashes.end())
            return id[u] = hashes[ch_ids] = hashes.size();
        return id[u] = it->second;
    };
    dfs(dfs, root, root);
    return {(int)hashes.size(), id};
```

MATH

Derangements

Binary Exponentiation MOD

```
//cat exp_mod.hpp | ./hash.sh
```

```
//3be256
//returns (base^pw)/mod in O(log(pw)), but returns 1 for 0^0
//What if base doesn't fit in long long?
//Since (base^pw) %mod == ((base%mod)^pw) %mod we can calculate base under mod of 'mod'
//What if pw doesn't fit in long long?
//case 1: mod is prime
//(base^pw)%mod == (base^(pw%(mod-1)))%mod (from Fermat's little theorem)
//so calculate pw under mod of 'mod-1'
//note 'mod-1' is not prime, so you need to be able to calculate 'pw%(mod-1)' without
     \hookrightarrow division
//case 2: non-prime mod
//let t = totient(mod)
//if pw >= log2(mod) then (base^pw)%mod == (base^(t+(pw%t)))%mod (proof)
     \hookrightarrow https://cp-algorithms.com/algebra/phi-function.html#generalization)
//so calculate pw under mod of 't'
//incidentally, totient(p) = p - 1 for every prime p, making this a more generalized
     \hookrightarrow version of case 1
int pow(long long base, long long pw, int mod) {
    assert(0 <= pw && 0 <= base && 1 <= mod);</pre>
    int res = 1;
    base %= mod;
    while (pw > 0) {
        if (pw & 1) res = res * base % mod;
        base = base * base % mod;
        pw >>= 1;
    return res;
```

Fibonacci

Matrix Multiplication

```
//cat matrix_mult.hpp | ./hash.sh

//e4e421

#pragma once

// source: https://codeforces.com/blog/entry/80195
```

```
// generic matrix multiplication (not overflow safe)
// will RTE if the given matricies are not compatible
// Time: O(n * m * inner)
// Space: 0(n * m)
template<typename T> vector<vector<T>> operator * (const vector<vector<T>>& a, const

    vector<vector<T>>& b) {
    assert(a[0].size() == b.size());
    int n = a.size(), m = b[0].size(), inner = b.size();
    vector<vector<T>> c(n, vector<T>(m));
    for (int i = 0; i < n; i++) {
        for (int k = 0; k < inner; k++) {
            for (int j = 0; j < m; j++)
                c[i][j] = c[i][j] + a[i][k] * b[k][j];
        }
    }
    return c;
```

Mobius Inversion

N Choose K MOD

```
//cat n_choose_k_mod.hpp | ./hash.sh
//f3a1a9
#pragma once
//for mod inverse
#include "exp_mod.hpp"
// usage:
       n_{choose} = k \ nk(n, 1e9+7) to use 'choose', 'inv' with inputs strictly < n
// or:
    n_choose_k nk(mod, mod) to use 'choose_with_lucas_theorem' with arbitrarily large
    \hookrightarrow inputs
struct n_choose_k {
   n_choose_k(int n, int a_mod) : mod(a_mod), fact(n, 1), inv_fact(n, 1) {
        //this implementation doesn't work if n > mod because n! % mod = 0 when n > =
             \hookrightarrow mod. So 'inv_fact' array will be all 0's
        assert(max(n, 2) <= mod);</pre>
        //assert mod is prime. mod is intended to fit inside an int so that
        //multiplications fit in a longlong before being modded down. So this
        //will take sqrt(2^31) time
        for (int i = 2; i * i <= mod; i++) assert(mod % i);</pre>
```

```
for (int i = 2: i < n: i++)
             fact[i] = 1LL * fact[i - 1] * i % mod:
        inv_fact.back() = pow(fact.back(), mod - 2, mod);
        for (int i = n - 2: i >= 2: i--)
             inv_fact[i] = inv_fact[i + 1] * (i + 1LL) % mod;
    //classic n choose k
    //fails when n \ge mod
    int choose(int n, int k) const {
        if (k < 0 \mid k > n) return 0:
        //now we know 0 <= k <= n so 0 <= n
        return 1LL * fact[n] * inv_fact[k] % mod * inv_fact[n - k] % mod;
    //lucas theorem to calculate n choose k in O(log(k))
    //need to calculate all factorials in range [0,mod), so O(mod) time&space, so need
         \hookrightarrow smallish prime mod (< 1e6 maybe)
    //handles n >= mod correctly
    int choose_with_lucas_theorem(long long n, long long k) const {
        if (k < 0 \mid | k > n) return 0;
        if (k == 0 | | k == n) return 1;
        return 1LL * choose_with_lucas_theorem(n / mod, k / mod) * choose(n % mod, k %
             \hookrightarrow mod) % mod:
    //returns \ x \ such \ that \ x * n % \ mod == 1
    int inv(int n) const {
        assert(1 <= n); //don't divide by 0 :)</pre>
        return 1LL * fact[n - 1] * inv_fact[n] % mod;
    }
    int mod;
    vector<int> fact, inv_fact;
};
```

Partitions

```
//cat partitions.hpp / ./hash.sh
//3356f6
#pragma once
//https://oeis.org/A000041
//0 (n sqrt n) time, but small-ish constant factor (there does exist a 0 (n log n)
     \hookrightarrow solution as well)
vector<int> partitions(int n, int mod) {
    vector<int> dp(n, 1);
    for (int i = 1; i < n; i++) {
        long long sum = 0:
        for (int j = 1, pent = 1, sign = 1; pent <= i; j++, pent += 3 * j - 2, sign =
             \hookrightarrow -sign) {
            if (pent + j <= i) sum += dp[i - pent - j] * sign + mod;</pre>
            sum += dp[i - pent] * sign + mod;
        dp[i] = sum % mod;
    }
    return dp;
```

Prime Sieve

//cat prime_sieve.hpp | ./hash.sh

```
//45fc23
#pragma once
//a_prime[val] = some random prime factor of 'val'
//to check if 'val' is prime:
// if (a_prime[val] == val)
//to get all prime factors of a number 'val' in O(log(val)):
// while (val > 1) {
       int p = a prime[val]:
       //p is some prime factor of val
        val /= p;
// }
const int N = 1e6 + 10;
int a_prime[N];
void calc_seive() {
    iota(a_prime, a_prime + N, 0);
    for (int i = 2; i * i < N; i++)</pre>
        if (a_prime[i] == i)
            for (int j = i * i; j < N; j += i)
                a_prime[j] = i;
```

Row Reduce

```
//cat row_reduce.hpp | ./hash.sh
//1d7c3e
#pragma once
//for mod inverse
#include "exp_mod.hpp"
//First 'cols' columns of mat represents a matrix to be left in reduced row echelon form
//Row operations will be performed to all later columns
//example usage:
// row_reduce(mat, mat[0].size(), mod) //row reduce matrix with no extra columns
pair<int/*rank*/, int/*determinant*/> row_reduce(vector<vector<int>>& mat, int cols, int
    int n = mat.size(), m = mat[0].size(), rank = 0, det = 1;
    assert(cols <= m);</pre>
    for (int col = 0; col < cols && rank < n; col++) {</pre>
        //find arbitrary pivot and swap pivot to current row
        for (int i = rank; i < n; i++)</pre>
            if (mat[i][col] != 0) {
                if (rank != i) det = det == 0 ? 0 : mod - det;
                swap(mat[i], mat[rank]);
                break;
        if (mat[rank][col] == 0) {
            det = 0:
            continue;
        det = (1LL * det * mat[rank][col]) % mod;
        //make pivot 1 by dividing row by inverse of pivot
        int a_inv = pow(mat[rank][col], mod - 2, mod);
        for (int j = 0; j < m; j++)
            mat[rank][j] = (1LL * mat[rank][j] * a_inv) % mod;
        //zero-out all numbers above & below pivot
        for (int i = 0; i < n; i++)
```

```
if (i != rank && mat[i][col] != 0) {
    int val = mat[i][col];
    for (int j = 0; j < m; j++) {
        mat[i][j] -= 1LL * mat[rank][j] * val % mod;
        if (mat[i][j] < 0) mat[i][j] += mod;
    }
    rank++;
}
assert(rank <= min(n, cols));
return {rank, det};
}</pre>
```

Solve Linear Equations MOD

```
//cat solve_linear_mod.hpp / ./hash.sh
//44cc6e
#pragma once
#include "row_reduce.hpp"
struct matrix_info {
    int rank. det:
    vector<int> x;
};
//Solves\ mat\ *\ x\ =\ b\ under\ prime\ mod.
//mat is a n (rows) by m (cols) matrix, b is a length n column vector, x is a length m
    \hookrightarrow vector.
//assumes n,m >= 1, else RTE
//Returns rank of mat, determinant of mat, and x (solution vector to mat * x = b).
//x is empty if no solution. If rank < m, there are multiple solutions and an arbitrary
    \hookrightarrow one is returned.
//Leaves mat in reduced row echelon form (unlike kactl) with b appended.
//0(n * m * min(n,m))
matrix_info solve_linear_mod(vector<vector<int>& mat, const vector<int>& b, int mod) {
    assert(mat.size() == b.size());
    int n = mat.size(), m = mat[0].size();
    for (int i = 0; i < n; i++)
        mat[i].push_back(b[i]);
    auto [rank, det] = row_reduce(mat, m, mod);//row reduce not including the last column
    //check if solution exists
    for (int i = rank; i < n; i++) {</pre>
        if (mat[i].back() != 0) return {rank, det, {} }; //no solution exists
    //initialize solution vector ('x') from row-reduced matrix
    vector<int> x(m, 0);
    for (int i = 0, j = 0; i < rank; i++) {
        while (mat[i][j] == 0) j++; //find pivot column
        x[j] = mat[i].back();
    return {rank, det, x};
```

Euler's Totient Phi Function

```
//cat totient.hpp | ./hash.sh
//36bd41
#pragma once
//Euler's totient function counts the positive integers
```

```
//up to a given integer n that are relatively prime to n.
//
//To improve, pre-calc prime factors or use Pollard-rho to find prime factors.
int totient(int n) {
   int res = n;
   for (int i = 2; i * i <= n; i++) {
      if (n % i == 0) {
        while (n % i == 0) n /= i;
        res -= res / i;
      }
   if (n > 1) res -= res / n;
   return res;
}
```

MISC

Cartesian Tree

```
//cat cartesian_tree.hpp | ./hash.sh
//0b95bc
#pragma once
#include "monotonic_stack.hpp"
//min cartesian tree
vector<int> cartesian_tree(const vector<int>& arr) {
   int n = arr.size();
   auto rv /*reverse*/ = [&](int i) -> int {
       return n - 1 - i;
   }:
   vector<int> left = monotonic_stack<int>(arr, greater());
    vector<int> right = monotonic_stack<int>(vector<int>(arr.rbegin(), arr.rend()),
        \hookrightarrow greater()):
   vector<int> par(n);
   for (int i = 0; i < n; i++) {
        int l = left[i], r = rv(right[rv(i)]);
       if (1 >= 0 \&\& r < n) par[i] = arr[1] > arr[r] ? 1 : r;
       else if (1 >= 0) par[i] = 1;
       else if (r < n) par[i] = r;</pre>
        else par[i] = i;
   }
   return par;
```

Count Rectangles

//cat count_rectangles.hpp | ./hash.sh

```
vector<vector<int>> cnt(n + 1, vector<int>(m + 1, 0));
vector<int> arr(m, 0):
auto rv /*reverse*/ = [&](int j) -> int {
    return m - 1 - j;
for (int i = 0; i < n; i++) {</pre>
    for (int j = 0; j < m; j++)
        arr[j] = grid[i][j] * (arr[j] + 1);
    vector<int> left = monotonic_stack<int>(arr, greater());
    vector<int> right = monotonic_stack<int>(vector<int>(arr.rbegin(), arr.rend()),
        for (int j = 0; j < m; j++) {
        int l = j - left[j] - 1, r = rv(right[rv(j)]) - j - 1;
        cnt[arr[j]][l + r + 1]++;
        cnt[arr[j]][1]--;
        cnt[arr[j]][r]--;
}
for (int i = 1; i <= n; i++)
    for (int k = 0; k < 2; k++)
        for (int j = m; j > 1; j--)
            cnt[i][j - 1] += cnt[i][j];
for (int j = 1; j <= m; j++)</pre>
   for (int i = n; i > 1; i--)
        cnt[i - 1][j] += cnt[i][j];
return cnt;
```

Max Rectangle in Histogram

```
//cat max_rect_histogram.hpp | ./hash.sh
//4e6291
#pragma once
#include "monotonic_stack.hpp"
long long max_rect_histogram(const vector<int>& arr) {
    int n = arr.size():
    auto rv /*reverse*/ = [&](int i) -> int {
        return n - 1 - i;
    vector<int> left = monotonic_stack<int>(arr, greater_equal());
    vector<int> right = monotonic_stack<int>(vector<int>(arr.rbegin(), arr.rend()),

    greater_equal());
    long long max_area = 0;
    for (int i = 0; i < n; i++) {
        int 1 = left[i], r = rv(right[rv(i)]);//arr[i] is the max of range (l, r)
        \max_{\text{area}} = \max(\max_{\text{area}}, 1LL * arr[i] * (r - 1 - 1));
    return max_area;
```

Monotonic Stack

```
//cat monotonic_stack.hpp | ./hash.sh

//4c7a40

#pragma once

//usages:

// vector<int> left = monotonic_stack<int>(arr, less()); //(or replace 'less' with:

$\to$ less_equal, greater, greater_equal
```

Iterate Chooses

```
//cat iterate_chooses.hpp | ./hash.sh
//c79083
#pragma once

// source: https://github.com/kth-competitive-programming/
// kactl/blob/main/content/various/chapter.tex
// iterates all bitmasks of size n with k bits set
// Time Complexity: O(n choose k)
// Space Complexity: O(1)

int next_subset(int mask) {
   int c = mask & -mask, r = mask + c;
   return r | (((r ^ mask) >> 2) / c);
}

void iterate_chooses(int n, int k, const function<void(int)>& func) {
   for (int mask = (1 << k) - 1; mask < (1 << n); mask = next_subset(mask))
      func(mask);
}</pre>
```

Iterate Submasks

```
//cat iterate_submasks.hpp | ./hash.sh
//084c05
#pragma once

// iterates all submasks of mask
// Time Complexity: 0(3^n) to iterate every submask of every mask of size n
// Space Complexity: 0(1)

void iterate_submasks(int mask, const function<void(int)>& func) {
   for (int submask = mask; submask; submask = (submask - 1) & mask)
        func(submask);
}
```

Iterate Supermasks

```
//cat iterate_supermasks.hpp | ./hash.sh
//76b38f
#pragma once

// iterates all supermasks of mask
// Time Complexity: 0(3^n) to iterate every supermask of every mask of size n
// Space Complexity: 0(1)

void iterate_supermasks(int mask, int n, const function<void(int)>& func) {
   for (int supermask = mask; supermask < (1 << n); supermask = (supermask + 1) | mask)
        func(supermask);
}</pre>
```

Number of Distinct Subsequences DP

```
//cat num_distinct_subsequences.hpp | ./hash.sh
//9542f5
#pragma once
//returns number of distinct subsequences
//the empty subsequence is counted
int num_subsequences(const vector<int>& arr, int mod) {
    int n = arr.size();
    vector < int > dp(n + 1, 1);
    map<int, int> last;
    for (int i = 0; i < n; i++) {</pre>
        int& curr = dp[i + 1] = 2 * dp[i]:
        if (curr >= mod) curr -= mod;
        auto it = last.find(arr[i]);
        if (it != last.end()) {
            curr -= dp[it->second];
            if (curr < 0) curr += mod;</pre>
            it->second = i:
       } else last[arr[i]] = i;
    return dp[n];
```

PBDS

```
//cat policy_based_data_structures.hpp / ./hash.sh
//807de9
#pragma once
//place these includes *before* the '#define int long long' else compile error
//not using <br/> <br/>bits/extc++.h> as it compile errors on codeforces c++20 compiler
#include <ext/pb_ds/assoc_container.hpp>
#include <ext/pb_ds/tree_policy.hpp>
using namespace __gnu_pbds;
//BST with extra functions https://codeforces.com/bloq/entry/11080
//order_of_key - # of elements *strictly* less than given element
//find_by_order - find kth largest element, k is 0 based so find_by_order(0) returns min
template < class T > using indexed_set = tree < T, null_type, less < T >, rb_tree_tag,

    tree_order_statistics_node_update>;

//example initialization:
indexed_set<pair<long long, int>> is;
//hash table (apparently faster than unordered_map):
    \hookrightarrow https://codeforces.com/blog/entry/60737
```

```
//example initialization:
gp_hash_table<string, long long> ht;
```

Random

```
//cat random.hpp | ./hash.sh
//61293c
#pragma once
//source: https://codeforces.com/blog/entry/60442
//MUCH RANDOM!!!
seed sea seed{
    (uint32_t)chrono::duration_cast<chrono::nanoseconds>
    (chrono::high_resolution_clock::now().time_since_epoch()).count(),
    (uint32_t)random_device()(),
    (uint32_t)(uintptr_t)make_unique<char>().get(),
    (uint32_t)__builtin_ia32_rdtsc()
mt19937 rng(seed);
//intended types: int, unsigned, long long
//returns a random number in range [l, r)
template<class T> inline T get_rand(T 1, T r) {
    assert(1 < r);</pre>
    return uniform_int_distribution<T>(1, r - 1)(rng);
```

Safe Hash

```
//cat safe_hash.hpp | ./hash.sh
//d9ea53
#pragma once
//source: https://codeforces.com/blog/entry/62393
struct custom_hash {
    static uint64_t splitmix64(uint64_t x) {
        // http://xorshift.di.unimi.it/splitmix64.c
       x += 0x9e3779b97f4a7c15;
       x = (x ^ (x >> 30)) * 0xbf58476d1ce4e5b9;
       x = (x ^ (x >> 27)) * 0x94d049bb133111eb;
       return x ^ (x >> 31);
   }
   size_t operator()(uint64_t x) const {
        static const uint64_t FIXED_RANDOM =

    chrono::steady_clock::now().time_since_epoch().count();

       return splitmix64(x + FIXED_RANDOM);
   }
};
//usage:
unordered_map<long long, int, custom_hash> safe_map;
#include "policy_based_data_structures.hpp"
gp_hash_table<long long, int, custom_hash> safe_hash_table;
```

Number Distinct Elements

```
//cat distinct_query.hpp | ./hash.sh
//d0d922
#pragma once
//source: https://cp-algorithms.com/data_structures/segment_tree.html
// #preserving-the-history-of-its-values-persistent-segment-tree
//works with negatives
//O(n \log n) time and space
struct distinct_query {
    struct node {
        int sum;
        int lch, rch;//children, indexes into 'tree'
        node(int a_sum, int a_lch, int a_rch) : sum(a_sum), lch(a_lch), rch(a_rch) {}
    };
    const int N;
    vector<int> roots;
    deque<node> tree;
    distinct_query(const vector<int>& arr) : N(arr.size()), roots(N + 1, 0) {
        tree.emplace_back(0, 0, 0); //acts as null
        map<int, int> last_idx;
        for (int i = 0; i < N; i++) {</pre>
            roots[i + 1] = update(roots[i], 0, N, last_idx[arr[i]]);
            last_idx[arr[i]] = i + 1;
    }
    int update(int v, int tl, int tr, int idx) {
        if (tr - tl == 1) {
            tree.emplace_back(tree[v].sum + 1, 0, 0);
            return tree.size() - 1;
        int tm = tl + (tr - tl) / 2;
        int lch = tree[v].lch;
        int rch = tree[v].rch:
        if (idx < tm)
            lch = update(lch, tl, tm, idx);
            rch = update(rch, tm, tr, idx);
        tree.emplace_back(tree[lch].sum + tree[rch].sum, lch, rch);
        return tree.size() - 1:
    //returns number of distinct elements in range [l,r)
    int query(int 1, int r) const {
        assert(0 \le 1 \&\& 1 \le r \&\& r \le N);
        return query(roots[1], roots[r], 0, N, 1 + 1);
    int query(int vl, int vr, int tl, int tr, int idx) const {
        if (tree[vr].sum == 0 || idx <= tl)</pre>
            return 0;
        if (tr <= idx)
            return tree[vr].sum - tree[vl].sum;
        int tm = tl + (tr - tl) / 2;
        return query(tree[v1].lch, tree[vr].lch, tl, tm, idx) +
               query(tree[v1].rch, tree[vr].rch, tm, tr, idx);
};
```

```
//cat implicit_seg_tree.hpp | ./hash.sh
//cbc0c0
#pragma once
//example initialization:
// implicit_seg_tree<10,000,000> ist(l, r);
template <int N> struct implicit_seg_tree {
   using dt = array<long long, 2>;//min, number of mins
   using ch = long long;
   static dt combine(const dt& 1, const dt& r) {
       if (1[0] == r[0]) return {1[0], 1[1] + r[1]};
       return min(1, r);
   }
   static constexpr dt UNIT{(long long)1e18, OLL);
   struct node {
       dt val;
       ch lazy;
       int lch, rch; // children, indexes into 'tree', -1 for null
       node(const dt& a_val) : val(a_val), lazy(0), lch(-1), rch(-1) {}
   int ptr = 0, root_1, root_r;//[root_1, root_r) defines range of root node; handles
   implicit_seg_tree(int 1, int r) : root_l(1), root_r(r) {
       tree[ptr++] = node(dt{0, r - 1});
   void apply(int v, ch add) {
       tree[v].val[0] += add;
       tree[v].lazy += add;
   }
   void push(int v, int tl, int tr) {
       if (tr - tl > 1 && tree[v].lch == -1) {
           int tm = tl + (tr - tl) / 2;
            assert(ptr + 1 < N);</pre>
            tree[v].lch = ptr;
            tree[ptr++] = node(dt{0, tm - tl});
            tree[v].rch = ptr;
            tree[ptr++] = node(dt{0, tr - tm});
       }
       if (tree[v].lazy) {
            apply(tree[v].lch, tree[v].lazy);
            apply(tree[v].rch, tree[v].lazy);
            tree[v].lazy = 0;
   }
   //update range [l,r)
   void update(int 1, int r, ch add) {
       update(0, root_1, root_r, 1, r, add);
   }
   void update(int v, int tl, int tr, int l, int r, ch add) {
       if (r <= tl || tr <= 1)
           return:
       if (1 <= t1 && tr <= r)
           return apply(v, add);
       push(v, tl, tr);
       int tm = tl + (tr - tl) / 2;
       update(tree[v].lch, tl, tm, l, r, add);
       update(tree[v].rch, tm, tr, 1, r, add);
       tree[v].val = combine(tree[tree[v].lch].val,
```

Kth Smallest

```
//cat kth_smallest.hpp | ./hash.sh
//023ee8
#include <bits/stdc++.h>
using namespace std;
#pragma once
//source: https://cp-algorithms.com/data_structures/segment_tree.html
// #preserving-the-history-of-its-values-persistent-segment-tree
struct kth smallest {
   struct node {
        int sum:
        int lch, rch;//children, indexes into 'tree'
        node(int a_sum, int a_lch, int a_rch) : sum(a_sum), lch(a_lch), rch(a_rch) {}
   int mn = INT_MAX, mx = INT_MIN;
   vector<int> roots;
   deque<node> tree;
    kth_smallest(const vector<int>& arr) : roots(arr.size() + 1, 0) {
        tree.emplace_back(0, 0, 0); //acts as null
        for (int val : arr) mn = min(mn, val), mx = max(mx, val + 1);
        for (int i = 0; i < (int)arr.size(); i++)</pre>
            roots[i + 1] = update(roots[i], mn, mx, arr[i]);
    int update(int v, int tl, int tr, int idx) {
        if (tr - tl == 1) {
            tree.emplace_back(tree[v].sum + 1, 0, 0);
            return tree.size() - 1:
        int tm = tl + (tr - tl) / 2;
        int lch = tree[v].lch:
        int rch = tree[v].rch;
        if (idx < tm)</pre>
            lch = update(lch, tl, tm, idx);
            rch = update(rch, tm, tr, idx);
        tree.emplace_back(tree[lch].sum + tree[rch].sum, lch, rch);
        return tree.size() - 1;
    /* find (k+1)th smallest number in range [l, r)
```

```
# k is 0-based, so query(l,r,0) returns the min

*/

int query(int l, int r, int k) const {
    assert(0 <= k && k < r - l); //note this condition implies l < r
    assert(0 <= 1 && r < (int)roots.size());
    return query(roots[l], roots[r], mn, mx, k);
}

int query(int vl, int vr, int tl, int tr, int k) const {
    assert(tree[vr].sum > tree[vl].sum);
    if (tr - tl == 1)
        return tl;
    int tm = tl + (tr - tl) / 2;
    int left_count = tree[tree[vr].lch].sum - tree[tree[vl].lch].sum;
    if (left_count > k) return query(tree[vl].lch, tree[vr].lch, tl, tm, k);
    return query(tree[vl].rch, tree[vr].rch, tm, tr, k - left_count);
}
};
```

Merge Sort Tree

```
//cat merge_sort_tree.hpp | ./hash.sh
//c885bd
#pragma once
//For point updates: either switch to policy based BST, or use sart decomposition
struct merge_sort_tree {
    const int N, S/*smallest power of 2 >= N*/;
    vector<vector<int>> tree;
    merge_sort_tree(const vector<int>& arr) : N(arr.size()), S(N ? 1 << __lg(2 * N - 1)
        \hookrightarrow : 0), tree(2 * N) {
        for (int i = 0; i < N; i++)</pre>
            tree[i + N] = {arr[i]};
        rotate(tree.rbegin(), tree.rbegin() + S - N, tree.rbegin() + N);
        for (int i = N - 1; i >= 1; i--) {
            const auto& 1 = tree[2 * i];
            const auto& r = tree[2 * i + 1]:
            tree[i].reserve(l.size() + r.size());
            merge(1.begin(), 1.end(), r.begin(), r.end(), back_inserter(tree[i]));
        }
    }
    int value(int v. int x) const {
        return lower_bound(tree[v].begin(), tree[v].end(), x) - tree[v].begin();
    int to_leaf(int i) const {
        i += S:
        return i < 2 * N ? i : 2 * (i - N);
    //How many values in range [l, r) are \langle x?
    //0(log^2(n))
    int query(int 1, int r, int x) const {
        int res = 0:
        for (1 = to_leaf(1), r = to_leaf(r); 1 < r; 1 >>= 1, r >>= 1) {
            if (1 & 1) res += value(1++, x);
            if (r & 1) res += value(--r, x):
        return res;
   }
};
```

BIT

```
//cat bit.hpp / ./hash.sh
//bc6b34
#pragma once
//mnemonic: Binary Indexed Tree
//NOLINTNEXTLINE(readability-identifier-naming)
template<class T> struct BIT {
    const int N;
    vector<T> bit;
    BIT(int a_n) : N(a_n), bit(N, 0) {}
    BIT(const vector<T>& a) : N(a.size()), bit(a) {
        for (int i = 0; i < N; i++) {</pre>
            int i = i \mid (i + 1):
            if (j < N) bit[j] += bit[i];</pre>
        }
    }
    void update(int i, const T& d) {
        assert(0 <= i && i < N);
        for (; i < N; i |= i + 1) bit[i] += d;
    T sum(int r) const {//sum of range [0, r)
        assert(0 <= r && r <= N);
        T ret = 0:
        for (; r > 0; r \&= r - 1) ret += bit[r - 1];
        return ret:
    T sum(int 1, int r) const {//sum of range [l, r)
        assert(0 <= 1 && 1 <= r && r <= N);
        return sum(r) - sum(1);
    }
    //Returns\ min\ pos\ (0 <= pos <= N+1)\ such\ that\ sum\ of\ [0,\ pos)\ >=\ sum
    //Returns N + 1 if no sum is >= sum, or 0 if empty sum is.
    //Doesn't work with negatives
    int lower bound(T sum) const {
        if (sum <= 0) return 0:
        int pos = 0:
        for (int pw = 1 << __lg(N | 1); pw; pw >>= 1)
            if (pos + pw <= N && bit[pos + pw - 1] < sum)</pre>
                pos += pw. sum -= bit[pos - 1]:
        return pos + 1;
    }
};
```

RMQ

```
//cat rmq.hpp | ./hash.sh
//082180

#pragma once
//source: https://github.com/kth-competitive-programming/
// kactl/blob/main/content/data-structures/RMQ.h
//usage:
// vector<long long> arr;
// ...
// RMQ<long long> rmq(arr, [0](auto x, auto y) { return min(x,y); });
//
//to also get index of min element, do:
// RMQ<pair<T, int>> rmq(arr, [0](auto x, auto y) { return min(x,y); });
//and initialize arr[i].second = i (0<=i<n)
```

```
//If there are multiple indexes of min element, it'll return the smallest
//(left-most) one
//mnemonic: Range Min/Max Query
//NOLINTNEXTLINE(readability-identifier-naming)
template <class T> struct RMQ {
    vector<vector<T>> dp;
    function<T(const T&, const T&)> op:
    RMQ(const vector<T>& arr, const function<T(const T&, const T&)>& a_op) : dp(1, arr),
         \hookrightarrow op(a_op) {
        for (int pw = 1, k = 1, n = arr.size(); 2 * pw <= n; pw *= 2, k++) {
            dp.emplace_back(n - 2 * pw + 1);
            for (int j = 0; j < n - 2 * pw + 1; j++)
                dp[k][j] = op(dp[k - 1][j], dp[k - 1][j + pw]);
        }
    }
    //inclusive-exclusive range [l, r)
    T query(int 1, int r) const {
        assert(0 <= 1 && 1 < r && r <= (int)dp[0].size());</pre>
        int \lg = \_\lg(r - 1);
        return op(dp[lg][1], dp[lg][r - (1 << lg)]);</pre>
   }
};
```

Lazy Segment Tree

```
//cat seg_tree.hpp | ./hash.sh
//98767a
#pragma once
//source: https://codeforces.com/blog/entry/18051,

→ https://qithub.com/ecnerwala/cp-book/blob/master/src/seg_tree.hpp,

     \hookrightarrow https://qithub.com/yosupo06/Alqorithm/blob/master/src/datastructure/seqtree.hpp
//rotating leaves makes it a single complete binary tree (instead of a set of perfect
     \hookrightarrow binary trees)
//so standard implementations of
// - recursive seg tree
// - tree walks AKA binary search
//still work
struct seg_tree {
    using dt = long long;
    using ch = long long;
    static dt combine(const dt& 1, const dt& r) {
        return min(1, r);
   }
    static const dt INF = 1e18:
    struct node {
        dt val:
        ch lazv:
        int 1, r;//[l, r)
    const int N, S/*smallest power of 2 >= N*/;
    vector<node> tree;
    seg\_tree(const\ vector<dt>\&\ arr): N(arr.size()), S(N ? 1 << __lg(2 * N - 1): 0),
         \hookrightarrow tree(2 * N) {
        for (int i = 0; i < N; i++)
            tree[i + N] = \{arr[i], 0, i, i + 1\};
        rotate(tree.rbegin(), tree.rbegin() + S - N, tree.rbegin() + N);
        for (int i = N - 1; i >= 1; i--) {
            tree[i] = {
```

```
combine(tree[2 * i].val, tree[2 * i + 1].val),
            tree[2 * i].1,
            tree[2 * i + 1].r
       };
   }
void apply(int v, ch change) {
    tree[v].val += change;
    tree[v].lazy += change;
void push(int v) {
    if (tree[v].lazy) {
        apply(2 * v, tree[v].lazy);
        apply(2 * v + 1, tree[v].lazy);
        tree[v].lazy = 0;
   }
}
void build(int v) {
    tree[v].val = combine(tree[2 * v].val, tree[2 * v + 1].val);
int to_leaf(int i) const {
    i += S:
    return i < 2 * N ? i : 2 * (i - N);
}
//update range [l, r)
void update(int 1, int r, ch change) {
    assert(0 <= 1 && 1 <= r && r <= N);
    1 = to_leaf(1), r = to_leaf(r);
    int lca_l_r = __lg((1 - 1) ^ r);
    for (int lg = __lg(l); lg > __builtin_ctz(l); lg--) push(l >> lg);
    for (int lg = lca_l_r; lg > __builtin_ctz(r); lg--) push(r >> lg);
    for (int x = 1, y = r; x < y; x >>= 1, y >>= 1) {
        if (x & 1) apply(x++, change);
        if (y & 1) apply(--y, change);
    for (int lg = __builtin_ctz(r) + 1; lg <= lca_l_r; lg++) build(r >> lg);
    for (int lg = \_builtin\_ctz(1) + 1; lg <= \_lg(1); lg++) build(1 >> lg);
void update(int v/* = 1*/, int 1, int r, long long add) {
    if (r <= tree[v].1 || tree[v].r <= 1)</pre>
        return;
    if (1 <= tree[v].1 && tree[v].r <= r)</pre>
        return apply(v, add);
    push(v);
    update(2 * v, 1, r, add);
    update(2 * v + 1, 1, r, add);
    build(v);
//query range [l, r)
dt query(int 1, int r) {
    assert(0 \le 1 \&\& 1 \le r \&\& r \le N):
   1 = to_leaf(1), r = to_leaf(r);
    int lca_l_r = __lg((1 - 1) ^ r);
    for (int lg = __lg(1); lg > __builtin_ctz(1); lg--) push(1 >> lg);
    for (int lg = lca_l_r; lg > __builtin_ctz(r); lg--) push(r >> lg);
    dt resl = INF. resr = INF:
    for (; 1 < r; 1 >>= 1, r >>= 1) {
        if (1 & 1) resl = combine(resl, tree[l++].val);
```

```
if (r & 1) resr = combine(tree[--r].val, resr);
}
    return combine(resl, resr);
}
dt query(int v/* = 1*/, int 1, int r) {
    if (r <= tree[v].1 || tree[v].r <= 1)
        return INF;
    if (1 <= tree[v].1 && tree[v].r <= r)
        return tree[v].val;
    push(v);
    return combine(query(2 * v, 1, r), query(2 * v + 1, 1, r));
};</pre>
```

STRINGS

Binary Trie

```
//cat binary_trie.hpp / ./hash.sh
//33aa3a
#pragma once
struct binary_trie {
   const int MX_BIT = 62;
   struct node {
       long long val = -1;
       int sub_sz = 0;//number of inserted values in subtree
       array<int, 2> next = {-1, -1};
   };
    vector<node> t;
   binary_trie() : t(1) {}
    //delta = 1 to insert val, -1 to remove val, 0 to get the # of val's in this data
        \hookrightarrow structure
   int update(long long val, int delta) {
       int c = 0;
       t[0].sub sz += delta:
       for (int bit = MX_BIT; bit >= 0; bit--) {
            bool v = (val >> bit) & 1;
            if (t[c].next[v] == -1) {
                t[c].next[v] = t.size();
                t.emplace_back();
            c = t[c].next[v];
            t[c].sub_sz += delta;
       t[c].val = val:
       return t[c].sub sz:
   }
   int size() const {
       return t[0].sub_sz;
   }
   //returns x such that:
   // x is in this data structure
    // value of (x ^val) is minimum
   long long min_xor(long long val) const {
       assert(size() > 0);
       int c = 0:
       for (int bit = MX_BIT; bit >= 0; bit--) {
```

```
bool v = (val >> bit) & 1;
    int ch = t[c].next[v];
    if (ch != -1 && t[ch].sub_sz > 0)
        c = ch;
    else
        c = t[c].next[!v];
}
return t[c].val;
}
```

KMP

```
//cat kmp.hpp / ./hash.sh
//73f1be
#pragma once
//mnemonic: Knuth Morris Pratt
#include "../../kactl/content/strings/KMP.h"
//usage:
// string needle;
// ...
// KMP kmp(needle);
//or
// vector<int> needle;
// ...
// KMP kmp(needle);
//kmp doubling trick: to check if 2 arrays are rotationally equivalent: run kmp
//with one array as the needle and the other array doubled (excluding the first
//8 last characters) as the haystack or just use kactl's min rotation code
//NOLINTNEXTLINE(readability-identifier-naming)
template <class T> struct KMP {
    KMP(const T& a_needle) : needle(a_needle), pf(pi(needle)) {}
    // if haustack = "bananas"
    // needle = "ana"
    //
    // then we find 2 matches:
    // bananas
    // _ana___
    // ___ana_
    // 0123456 (indexes)
    // and KMP::find returns {1,3} - the indexes in haystack where
    // each match starts.
    // You can also pass in 0 for "all" and KMP::find will only
    // return the first match: {1}. Useful for checking if there exists
    // some match:
    // KMP::find(<haystack>,0).size() > 0
    vector<int> find(const T& haystack, bool all = 1) const {
        vector<int> matches;
        for (int i = 0, j = 0; i < (int)haystack.size(); i++) {</pre>
            while (j > 0 \&\& needle[j] != haystack[i]) j = pf[j - 1];
            if (needle[j] == haystack[i]) j++;
            if (j == (int)needle.size()) {
                matches.push_back(i - (int)needle.size() + 1);
                if (!all) return matches;
                j = pf[j - 1];
```

```
return matches:
}
T needle:
vector<int> pf;//prefix function
```

```
bool is_pal(int 1, int r) const {
        assert(0 <= 1 && 1 <= r && r <= N);
        int len = r - 1;
        return pal_len[len & 1][l + len / 2] >= len / 2;
};
```

Longest Common Prefix Query

```
//cat lcp_query.hpp / ./hash.sh
//c44f14
#pragma once
#include "../../ac-library/atcoder/string.hpp"
#include "../range_data_structures/rmq.hpp"
//computes suffix array, lcp array, and then sparse table over lcp array
//0(n \log n)
template<typename T> struct lcp_query {
    const int N:
    vector<int> sa, lcp, inv_sa;
    RMQ<int> rmq;
    lcp_query(const T& s) : N(s.size()), sa(atcoder::suffix_array(s)),
        return min(x, y);
   }) {
        for (int i = 0; i < N; i++) inv_sa[sa[i]] = i;</pre>
    //length of longest common prefix of suffixes s[idx1 \ldots n), s[idx2 \ldots n), 0-based
         \hookrightarrow indexing
    //You can check if two substrings s[l1..r1), s[l2..r2) are equal in O(1) by:
    //r1-l1 == r2-l2 \ \theta\theta \ longest\_common\_prefix(l1, l2) >= r1-l1
    int longest_common_prefix(int idx1, int idx2) const {
       if (idx1 == idx2) return N - idx1;
       idx1 = inv sa[idx1]:
       idx2 = inv_sa[idx2];
       if (idx1 > idx2) swap(idx1, idx2);
       return rmq.query(idx1, idx2);
   }
    //returns \ 1 \ if \ suffix \ s[idx1 ... n) < s[idx2 ... n)
    //(so \ 0 \ if \ idx1 == idx2)
    bool less(int idx1, int idx2) const {
       return inv_sa[idx1] < inv_sa[idx2];</pre>
};
```

Palindrome Query

```
//cat palindrome_query.hpp / ./hash.sh
//7326d0
#pragma once
#include "../../kactl/content/strings/Manacher.h"
struct pal_query {
   const int N;
   array<vi, 2> pal_len;
   pal_query(const string& s) : N(s.size()), pal_len(manacher(s)) {}
   //returns 1 if substring s[l...r) is a palindrome
    //(returns\ 1\ when\ l\ ==\ r)
```

Trie

```
//cat trie.hpp | ./hash.sh
//fd9c8d
#pragma once
//source: https://cp-algorithms.com/string/aho_corasick.html#construction-of-the-trie
const int K = 26;//alphabet size
struct trie {
    const char MIN_CH = 'A';//'a' for lowercase, '0' for digits
    struct node {
        int next[K], cnt_words = 0, par = -1;
        char ch;
        node(int a_par = -1, char a_ch = '#') : par(a_par), ch(a_ch) {
            fill(next, next + K, -1);
       }
   };
    vector<node> t:
    trie() : t(1) {}
    void add_string(const string& s) {
        int v = 0;
        for (char ch : s) {
            int let = ch - MIN CH:
            if (t[v].next[let] == -1) {
                t[v].next[let] = t.size();
                t.emplace_back(v, ch);
            v = t[v].next[let];
        t[v].cnt_words++;
    bool find_string(const string& s) const {
        int v = 0:
        for (char ch : s) {
            int let = ch - MIN_CH;
            if (t[v].next[let] == -1) return 0;
            v = t[v].next[let];
        return t[v].cnt_words;
   }
};
```

Suffix Array and LCP Array

```
//cat string.hpp | ./hash.sh
//67378f
#ifndef ATCODER_STRING_HPP
#define ATCODER_STRING_HPP 1
#include <algorithm>
#include <cassert>
```

```
#include <numeric>
#include <string>
#include <vector>
namespace atcoder {
namespace internal {
std::vector<int> sa_naive(const std::vector<int>& s) {
    int n = int(s.size());
    std::vector<int> sa(n);
    std::iota(sa.begin(), sa.end(), 0);
    std::sort(sa.begin(), sa.end(), [&](int 1, int r) {
        if (l == r) return false;
        while (1 < n \&\& r < n) {
            if (s[1] != s[r]) return s[1] < s[r];
            r++;
        return 1 == n;
   });
    return sa:
std::vector<int> sa_doubling(const std::vector<int>& s) {
    int n = int(s.size());
    std::vector<int> sa(n), rnk = s, tmp(n);
    std::iota(sa.begin(), sa.end(), 0);
    for (int k = 1; k < n; k *= 2) {
        auto cmp = [&](int x, int y) {
            if (rnk[x] != rnk[y]) return rnk[x] < rnk[y];</pre>
            int rx = x + k < n ? rnk[x + k] : -1;
            int ry = y + k < n ? rnk[y + k] : -1;
            return rx < ry;</pre>
        }:
        std::sort(sa.begin(), sa.end(), cmp);
        tmp[sa[0]] = 0;
        for (int i = 1; i < n; i++) {
            tmp[sa[i]] = tmp[sa[i-1]] + (cmp(sa[i-1], sa[i]) ? 1 : 0);
        }
        std::swap(tmp, rnk);
   }
    return sa;
// SA-IS, linear-time suffix array construction
// Reference:
// G. Nong, S. Zhang, and W. H. Chan,
// Two Efficient Algorithms for Linear Time Suffix Array Construction
template <int THRESHOLD_NAIVE = 10, int THRESHOLD_DOUBLING = 40>
std::vector<int> sa_is(const std::vector<int>& s, int upper) {
    int n = int(s.size());
    if (n == 0) return {};
    if (n == 1) return {0};
    if (n == 2) {
        if (s[0] < s[1]) {</pre>
            return {0, 1}:
       } else {
            return {1, 0};
```

```
}
if (n < THRESHOLD_NAIVE) {</pre>
    return sa naive(s):
if (n < THRESHOLD_DOUBLING) {</pre>
    return sa doubling(s):
std::vector<int> sa(n);
std::vector<bool> ls(n);
for (int i = n - 2; i \ge 0; i--) {
    ls[i] = (s[i] == s[i + 1]) ? ls[i + 1] : (s[i] < s[i + 1]);
std::vector<int> sum_l(upper + 1), sum_s(upper + 1);
for (int i = 0; i < n; i++) {
    if (!ls[i]) {
        sum_s[s[i]]++;
   } else {
        sum_l[s[i] + 1]++;
for (int i = 0; i <= upper; i++) {</pre>
    sum_s[i] += sum_l[i];
    if (i < upper) sum_l[i + 1] += sum_s[i];</pre>
auto induce = [&](const std::vector<int>& lms) {
    std::fill(sa.begin(), sa.end(), -1);
    std::vector<int> buf(upper + 1);
    std::copy(sum_s.begin(), sum_s.end(), buf.begin());
    for (auto d : lms) {
        if (d == n) continue;
        sa[buf[s[d]]++] = d;
    std::copy(sum_l.begin(), sum_l.end(), buf.begin());
    sa[buf[s[n-1]]++] = n-1;
    for (int i = 0; i < n; i++) {
        int v = sa[i];
        if (v >= 1 && !ls[v - 1]) {
            sa[buf[s[v - 1]] ++] = v - 1:
    std::copy(sum_l.begin(), sum_l.end(), buf.begin());
    for (int i = n - 1; i \ge 0; i--) {
        int v = sa[i]:
        if (v >= 1 && ls[v - 1]) {
            sa[--buf[s[v-1]+1]] = v-1;
   }
}:
std::vector<int> lms_map(n + 1, -1);
int m = 0:
for (int i = 1; i < n; i++) {
    if (!ls[i - 1] && ls[i]) {
        lms_map[i] = m++;
```

South Dakota Mines

```
std::vector<int> lms:
    lms.reserve(m);
    for (int i = 1; i < n; i++) {</pre>
        if (!ls[i - 1] && ls[i]) {
            lms.push_back(i);
        }
    }
    induce(lms);
    if (m) {
        std::vector<int> sorted_lms;
        sorted_lms.reserve(m);
        for (int v : sa) {
            if (lms_map[v] != -1) sorted_lms.push_back(v);
        std::vector<int> rec_s(m);
        int rec_upper = 0;
        rec_s[lms_map[sorted_lms[0]]] = 0;
        for (int i = 1: i < m: i++) {
            int l = sorted_lms[i - 1], r = sorted_lms[i];
            int end_1 = (lms_map[1] + 1 < m) ? lms[lms_map[1] + 1] : n;</pre>
            int end_r = (lms_map[r] + 1 < m) ? lms[lms_map[r] + 1] : n;
            bool same = true;
            if (end_1 - 1 != end_r - r) {
                same = false;
            } else {
                while (1 < end_1) {
                    if (s[1] != s[r]) {
                        break:
                    }
                    1++;
                    r++:
                if (1 == n \mid | s[1] != s[r]) same = false:
            if (!same) rec_upper++;
            rec_s[lms_map[sorted_lms[i]]] = rec_upper;
        auto rec sa =
            sa_is<THRESHOLD_NAIVE, THRESHOLD_DOUBLING>(rec_s, rec_upper);
        for (int i = 0; i < m; i++) {</pre>
            sorted_lms[i] = lms[rec_sa[i]];
        induce(sorted_lms);
   }
    return sa;
} // namespace internal
std::vector<int> suffix_array(const std::vector<int>& s, int upper) {
    assert(0 <= upper);</pre>
    for (int d : s) {
        assert(0 <= d && d <= upper);</pre>
   }
    auto sa = internal::sa_is(s, upper);
```

```
return sa:
template <class T> std::vector<int> suffix_array(const std::vector<T>& s) {
    int n = int(s.size());
    std::vector<int> idx(n);
    iota(idx.begin(), idx.end(), 0);
    sort(idx.begin(), idx.end(), [&](int 1, int r) { return s[1] < s[r]; });
    std::vector<int> s2(n);
    int now = 0:
    for (int i = 0; i < n; i++) {
        if (i && s[idx[i - 1]] != s[idx[i]]) now++;
        s2[idx[i]] = now;
    return internal::sa_is(s2, now);
std::vector<int> suffix_array(const std::string& s) {
    int n = int(s.size());
    std::vector<int> s2(n):
    for (int i = 0; i < n; i++) {
        s2[i] = s[i]:
    return internal::sa_is(s2, 255);
// Reference:
// T. Kasai, G. Lee, H. Arimura, S. Arikawa, and K. Park,
// Linear-Time Longest-Common-Prefix Computation in Suffix Arrays and Its
// Applications
template <class T>
std::vector<int> lcp_array(const std::vector<T>& s,
                           const std::vector<int>& sa) {
    int n = int(s.size());
    assert(n >= 1):
    std::vector<int> rnk(n):
    for (int i = 0; i < n; i++) {
        rnk[sa[i]] = i:
    std::vector<int> lcp(n - 1);
    int h = 0:
    for (int i = 0; i < n; i++) {
        if (h > 0) h--;
        if (rnk[i] == 0) continue;
        int j = sa[rnk[i] - 1];
        for (; j + h < n && i + h < n; h++) {
            if (s[j + h] != s[i + h]) break;
        lcp[rnk[i] - 1] = h;
    }
    return lcp;
std::vector<int> lcp_array(const std::string& s, const std::vector<int>& sa) {
    int n = int(s.size());
    std::vector<int> s2(n):
    for (int i = 0; i < n; i++) {</pre>
        s2[i] = s[i];
```

```
return lcp_array(s2, sa);
// Reference:
// D. Gusfield,
// Algorithms on Strings, Trees, and Sequences: Computer Science and
// Computational Biology
template <class T> std::vector<int> z_algorithm(const std::vector<T>& s) {
    int n = int(s.size());
    if (n == 0) return {};
    std::vector<int> z(n);
   z[0] = 0;
    for (int i = 1, j = 0; i < n; i++) {
        int & k = z[i];
        k = (j + z[j] \le i) ? 0 : std::min(j + z[j] - i, z[i - j]);
        while (i + k < n \&\& s[k] == s[i + k]) k++;
        if (j + z[j] < i + z[i]) j = i;
    z[0] = n;
    return z;
std::vector<int> z_algorithm(const std::string& s) {
   int n = int(s.size());
    std::vector<int> s2(n);
    for (int i = 0; i < n; i++) {</pre>
        s2[i] = s[i];
   }
    return z_algorithm(s2);
} // namespace atcoder
#endif // ATCODER_STRING_HPP
```