Questions a to e

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Question a:
The adjacency matrix is:
[{}, {0=1}, {0=1, 1=1}, {0=1, 2=1}, {}, {4=1}, {5=1}, {6=1}, {}, {1=1, 2=1, 3=1, 8=1}, {3=1}, {2=1, 10=1}, {9=1, 10=1}, {}, {1=1, 2=1, 13=1}, {7=1, 13=1}, {13=1, 14=1}, {1=1,
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Question b:
The graph is connected
The median degree is 6
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Question c:
The number of triangle subgraph is 284
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Question d:
The minimum spanning tree(undirected graph) found by Kruskal’s algorithm is
[{1=1}, {0=1, 2=1, 91=1}, {1=1, 97=1, 3=1, 111=1}, {2=1, 9=1}, {5=1}, {4=1, 6=1}, {5=1, 7=1}, {6=1, 15=1}, {9=1}, {3=1, 8=1, 12=1}, {11=1, 12=1}, {10=1}, {17=1, 9=1, 10=1},
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 62=1}, {80=1, 66=1}, {68=1, 71=1, 76=1}, {29=1}, {76=1, 77=1}, {80=1, 67=1, 73=1, 75=1, 92=1}, {75=1}, {79=1}, {81=1, 83=1, 109=1, 78=1, 110=1}, {81=1, 72=1, 76=1}, {80=1,
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 {106=1, 92=1}, {2=1}, {81=1}, {89=1}, {110=1}, {60=1}, {70=1}, {104=1}, {84=1, 103=1, 105=1, 61=1}, {104=1, 108=1}, {96=1, 88=1}, {34=1}, {105=1}, {79=1}, {100=1, 79=1},
 {2=1}]
Runtime:0.016s
The optimal cost is: 1419
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Question e:
The minimum spanning tree(undirected graph) found by Kruskal’s algorithm is
[{1=1}, {0=1, 2=1, 91=1}, {1=1, 97=1, 3=1, 111=1}, {2=1, 9=1}, {5=1}, {4=1, 6=1}, {5=1, 7=1}, {6=1, 15=1}, {9=1}, {3=1, 8=1, 12=1}, {11=1, 12=1}, {10=1}, {17=1, 9=1, 10=1},
 {14=1, 15=1}, {16=1, 13=1}, {17=1, 7=1, 13=1}, {14=1}, {82=1, 21=1, 85=1, 12=1, 93=1, 15=1}, {21=1}, {20=1, 21=1, 22=1, 23=1, 24=1, 25=1, 26=1, 27=1}, {19=1}, {17=1, 18=1,
 19=1}, {19=1}, {19=1, 69=1}, {19=1}, {19=1, 30=1, 46=1}, {19=1}, {19=1}, {33=1, 34=1, 35=1, 29=1, 30=1, 31=1}, {74=1, 28=1}, {25=1, 28=1}, {32=1, 86=1, 41=1, 28=1}, {31=1},
 {28=1}, {107=1, 28=1}, {28=1}, {43=1}, {38=1, 58=1}, {37=1, 39=1, 40=1, 41=1}, {38=1}, {38=1}, {38=1, 42=1, 43=1, 31=1}, {41=1, 90=1}, {48=1, 36=1, 41=1, 44=1}, {43=1, 45=1},
  {44=1}, {25=1}, {48=1, 49=1}, {50=1, 43=1, 47=1}, {47=1}, {48=1, 51=1, 52=1, 53=1}, {50=1, 57=1, 59=1}, {50=1, 56=1}, {50=1, 54=1}, {53=1}, {70=1}, {52=1}, {51=1}, {37=1},
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 {106=1, 92=1}, {2=1}, {81=1}, {89=1}, {110=1}, {60=1}, {70=1}, {104=1}, {84=1, 103=1, 105=1, 61=1}, {104=1, 108=1}, {96=1, 88=1}, {34=1}, {105=1}, {79=1}, {100=1, 79=1},
 {2=1}]
Runtime:0.0s
The optimal cost is: 1419
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Question f

The running time, optimal cost, and the optimal solution are shown as above. The reason why the time is different is that the two algorithms have different time complexity

Question g

x_{ij} indicates the network flow on edge (i,j) is in the tree.

For each vertex i, ensure that exactly one incoming and one outgoing edge are present: Out degree constraint: $\sum_j x_{ij}$ = 1 for all vertices i. In degree constraint: $\sum_j x_{ji}$ = 1 for all vertices i

Objective function: Minimize the sum of the edge weights in the tree

$\sum_i \sum_j c_{ij}x_{ij}$ c_{ij} is the cost of edge (i,j), and x_{ij} is the network flow on edge (i,j) is in the tree.

Question h

The result is shown as below. The algorithm is optimized and costs less time than the algorithms as above.

```
time used:0.000997304916381836
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 (103, 104, {'weight': 1}), (104, 105, {'weight': 1}), (105, 108, {'weight': 3})]
The total cost is 1419
```