Probability and Statistics for Engineers Homework Five
TMATH 390

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Contents

Problem 1

A college library has five copies of a certain text on reserve. Two copies (1 and 2) are first editions, and the other three (3, 4, and 5) are second editions. A student examines these books in random order, stopping only when a second printing has been selected. One outcome is 4, and another is 215.

(a) List all outcomes in the same space S.
(b) Let A denote the event that exactly one book must be examined. What outcomes are in A?
(c) Let B be the even that book 4 is the one selected. What outcomes are in B?
(d) Let c be the even that book 2 is not examined. What outcomes are in C?

Answer to a

Answer to b

Answer to c

Answer to d

Problem 2

An engineering firm is constructing power plants at three different sites.
Define the events E_1, E_2, and E_3 as follows:

- E_1 = the plant at site 1 is completed by the contract date.
- E_2 = the plant at site 2 is completed by the contract date.
- E_3 = the plant at site 3 is completed by the contract date.

Draw a Venn diagram that depicts these three events as intersecting circles. Shade the region on the Venn diagram corresponding to each of the following events (redraw the Venn diagram for each question):

(a) At least one plant is completed by the contract date.
(b) All plants are completed by the contract date.
(c) None of the plants are completed by the contract date.
(d) Only the plant at site 1 is completed by the contract date.
(e) Exactly one of the three plants is completed by the contract date.
(f) Either the plant at site 1 or site 2 or both of the two plants are completed by the contract date.

Answer to a

Answer to b

Answer to c

Answer to d

Answer to e

Answer to f

Problem 3

Let $A_i = i$th student got a perfect score on midterm 1, for $i = 1, ..., 18$.

(a) Interpret $\left(\bigcap_{i=1}^{18} A_i\right)'$ in English.

(b) Interpret $\bigcup_{i=1}^{18} A_i'$ in English. Is it equivalent to part a?

Answer to a

Answer to b

Problem 4

Using everything we have learned about events and probability, prove that

$$P(A|B) + P(A'|B) = 1$$

Do not assume that events A and B are independent (i.e. $P(A \cap B) \neq P(A) \cdot P(B)$). Explain each step of your calculation.

Answer to problem 4

Problem 5

Five companies (A, B, C, D, and E) that make electrical relays compete each year to be the sole supplier of relays to a major automobile manufacturer. The auto company's records show that the probabilities of choosing a company to be the sole supplier are

Supplier chose:	A	B	C	D	E
Probability:	0.30	0.20	0.10	0.25	0.15

(a) Suppose that supplier E goes out of business this year, leaving the remaining 4 companies to compete with one another. What are the new probabilities of companies A, B, C, and D being chosen ad the sole supplier this year?
(b) Suppose the auto company narrows the choice of suppliers to companies A and C. What is the probability that company A is chosen this year?

Answer to a

Answer to b

Problem 6

Number 5.16 in your book on page 214:
In Exercise 6, suppose that there is a probability of 0.01 that a digit is incorrectly sent over a communication channel (i.e., that a digit sent as a 1 is received as a 0, or vice versa). Consider a message that consists of exactly 60% 1s.

(a) What is the proportion of 1s received at the end of the channel?
(b) If a 1 is received, what is the probability that a 1 was sent? Hint: Use the tree diagram from Exercise 6.

Exercise 6 for reference: Information theory is concerned with the transmission of data, usually encoded as a stream of 0s and 1s, over communication channels. Because channels are "noisy", there is a chance that some 0s

sent through the channel are mistakenly received at the other end as 1s, and vice versa. The majority of digits sent, however, are not altered by the channel.

Draw a tree diagram that depicts the type of bit sent (either 0 or 1) and the type of bit received at the end of the channel.

Answer to a

Answer to b

Problem 7

Number 5.19 in your book on page 214:
In forensic science, the probability that any two people match with respect to a given characteristic (hair color, blood type, etc.) is called the *probability of a match*. Suppose that the frequencies of blood phenotypes in the population are as follows:

A	B	AB	O
0.42	0.10	0.04	0.44

(a) What is the probability that two randomly chosen people both have blood type A?
(b) Repeat the calculation in part (a) for the other three blood types.
(c) Find the probability that two randomly chosen people have matching blood types. Note: A person can have only one phenotype.
(d) The probability that two people do not match for a given characteristic is called *discriminating power*. What is the discriminating power for the comparison of two people's blood types in part (c)?

Answer to a

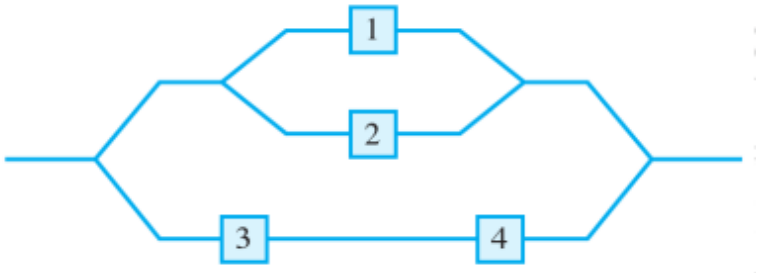
Answer to b

Answer to c

Answer to d

Problem 8

Number 5.21 in your book on page 214:
Consider a system of components connected as shown in the following figure:



Components 1 and 2 are connected in parallel, so that their subsystem functions correctly if either component 1 or 2 functions. Components 3 and 4 are connected in series, so their subsystem works only if both components work correctly.

If all components work independently of one another and P(a given component works) = 0.9, calculate the probability that the entire system works correctly.

Answer to problem 8

Problem 9

Let p be the sample proportion, written as $p = \frac{n_f}{n}$, where n = sample size, and n_f =, the number of females in the sample. Show that $E[p] = \pi$ and $V[p] = \frac{\pi(1-\pi)}{n}$, where π = proportion of females in population. Hint: Use what you know about Binomial distributions.

Answer to problem 9

Problem 10

A survey of the members of a large professional engineering society is conducted to determine their views on proposed changes to an ASTM measurement standard. Suppose that 80% of he entire membership favor the proposed changes. Hint: Use the result from the previous problem.

(a) Calculate the mean and standard deviation of the sampling distribution of the proportion of engineers in samples of size 25 who favor the proposed changes.
(b) Calculate the mean and standard deviation of the sampling distribution of the proportion of engineers in samples of size 100 who favor the proposed changes.

Answer to a

Answer to b

Problem 11

The lifetime of a certain battery is normally distributed with a mean value of 8 hours and a standard deviation of 1 hour. There are four such batteries in a package.

(a) What is the probability that the average lifetime of the four batteries exceeds 9 hours?
(b) If T denotes the average lifetime of the four batteries in a randomly selected package, find the numerical value of T_0 for which $P(T \geq T_0) = 0.95$.

Answer to a

Answer to b

Problem 12

The number of flaws x on an electroplated automobile grill is known to have the following probability mass function: $p(0) = 0.6$, $p(1) = 0.2$, $p(2) = 0.1$, $p(3) = 0.1$.

(a) Calculate the mean and standard deviation of x.
(b) What are the mean and standard deviation of the sampling distribution of the average number of flaws per grill in a random sample of 50 grills?
(c) For a random sample of 50 grills. Calculate the approximate probability that the average number of flaws per grill exceeds 0.8.

Answer to a

Answer to b

Answer to c