TMATH 390 Spring 2015

Homework Set 7

- 1. State whether each of the following assertions is a legitmate statistical hypothesis and why.
 - (a) H: $\sigma > 100$
 - (b) H: $\bar{x} = 45$
 - (c) H: $\tilde{\mu} \neq 2.0$
 - (d) H: $s \le 0.50$
 - (e) H: $\frac{\sigma_1}{\sigma_2} < 1$
 - (f) H: $\bar{x}_1 \bar{x}_2 = -5.0$
 - (g) H: $\lambda < 0.1$, where λ is the parameter of an exponential distribution used to model component lifetime.
 - (h) H: $\pi=0.10$, where π is the population proportion of components that need a warranty service.
 - (i) H: x =sound intensity of a certain source (in decibels) has a lognormal distribution.
- 2. Before agreeing to purchase a large order of polyetheylene sheaths for a particular type of high-pressure, oil-filled submarine power cable, a company wants to see conclusive evidence that the population standard deviation of sheath thickness is less than 0.05 mm. What hypotheses should be tested, and why? In this context, what are the type I and type II errors?
- 3. Lightbulbs of a certain type are advertised ahaving an average lifetime of 750 hours. The price of these bulbs is very favorable, so a potential customer has decided to go ahead with a purchase arrangement unless it can be conclusivelydemonstrated that the true average lifetime is smaller than what is advertized. A random sample of 50 bulbs was selected, the lifetime of each bulb determined, and the appropriate hypotheses were tested using computer software, which gave the following output.

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Variable n Mean StDev SEMean Z P-Value lifetime 50 738.44 38.20 5.40 -2.14 0.016
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- (a) Identify the form of the alternative hypothesis used for this problem and justify your answer.
- (b) What conclusion would be appropriate at the 0.05 significance level? At the 0.01 significance level?
- 4. To obtain information on the corrosion-resistance properties of a certain type of steel conduit, 35 specimens are buried in soil for an extended period. The maximum penetration (in mils) is then is then measured for each specimen, yielding a sample mean penetration of 52.7 and a sample standard deviation of 4.8. The conduits were manufactured with the specification that true average penetration be at most 50 mils. Does the sample data indicate that specifications have not been met? State the relevant hypotheses, calculate the value of the appropriate z statistic, determine the P-value (including a picture of it) and state your conclusion at the 0.05 significance level.
- 5. A certain pen has been designed so that true average writing lifetime under controlled conditions (involving the use of a writing machine) is at least 10 hours. A random sample of 18 pens is selected, the writing lifetime of each is determined, and a normal quantile plot of the resulting data supports the use of a one-sample t test.

- (a) What hypotheses should be tested if the investigators believe a priori that the design specification has been satisfied?
- (b) What conclusion is appropriate if the hypotheses of part (a) are tested, t=-2.3, and $\alpha=.05$?
- (c) What conclusion is appropriate if the hypotheses of part (a) are tested, t=-1.8, and $\alpha=.01$?
- 6. Data with n=26 observations on escape time (sec) for oil workers in a simulated exercise, from which the sample mean and sample standard deviation are 370.69 and 24.36, respectively. Suppose the investigators had believed a priori that true average escape time would be at most 6 minutes. Do the data contradict this prior belief? Assuming normality, state and test the appropriate hypotheses using a significance level of 0.05.
- 7. Shoveling is not exactly a high-tech activity but continues to be a required task in our information age. The article "A Shovel with a perforated blade reduces energy expenditure required for digging wet clay" (*Human Factors*, 2010:492-502) reported on an experiment in which each of 13 workers was provided with both a conventional shovel and a shovel whose blade was perforated with small holes. The authors of teh cited article provided the following data on stable energy expendeture (measured in kilocalories per kg of subject per pounds of clay):

| Worker | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 |
|--------------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|
| Conventional | 0.0011 | 0.0014 | 0.0018 | 0.0022 | 0.0010 | 0.0016 | 0.0028 | 0.0020 | 0.0015 | 0.0023 | 0.0017 | 0.0020 | 0.0014 |
| Perforated | 0.0011 | 0.0010 | 0.0019 | 0.0013 | 0.0011 | 0.0017 | 0.0024 | 0.0020 | 0.0013 | 0.0017 | 0.0020 | 0.0013 | 0.0013 |

Carry out a test of hypotheses at significance level 0.05 to see whether the true average energy expendeture using the conventional shovel exceeds that using the perforated shovel.

8. Which factors are relevant to the time a consumer spends looking at a product on the shelf prior to selection? The article "Effects of Base Price Upon Search Behavior of Consumers in a Supermarket" (*J. Econ. Psycho.*, 2003: 637-652) reported the following data on elapsed time (sec) for fabric softener purchasers and washing0up liquid purchasers; the former product is significantly more expensive than the latter. These products were chosen because they are similar with respect to allocated shelf space and number of alternative brands.

| Product | sample size | sample mean | sample st.dev. |
|-------------------|-------------|-------------|----------------|
| Fabric softener | 15 | 30.47 | 19.15 |
| Washing-up liquid | 18 | 26.53 | 15.37 |

- (a) What, if any, assumptions, are needed before the t inferential procedure can be used to compare true average elapsed times?
- (b) Carry out a test of hypotheses to decide whether the true average differences in elapsed time differs from zero.
- 9. Sample observations on stabilized viscosity of asphalt specimens are (2781, 2900, 3013, 2856, and 2888). For a particular application, it is required that true average viscosity be 3000. Does this requirement appear to have been satisfied? State and test the appropriate hypotheses at significance level 0.05.

10. Criminologists have long debated whether there is a relationship between weather and violent crime. The author of the article "Is There a Season for Homicide?") *Criminology*, 1988: 287-296) classified 1361 homicides according to season, resulting in the data below.

Season: Winter Spring Summer Autumn **Frequency:** 328 334 372 327

midsize

full-size

Type of

Do the data suggest that the homicide rate somehow depends on the season? State the relevant hypotheses, then carry out the hypothesis test at the 5% significance level.

11. A random sample of individuals who drive to work in a large metropolitan area was obtained, and each individual was categorized according to commuting distance (in miles).

| Commuting distance | | | | | | | | |
|--------------------|------------|-----------------|---------------------------------|--------------------------|--|--|--|--|
| | | Low | Medium | High | | | | |
| | | [0 mi, 10 mi) | $[10\mathrm{mi},20\mathrm{mi})$ | $[20\mathrm{mi},\infty)$ | | | | |
| vehicle | subcompact | 6 | 27 | 19 | | | | |
| | compact | 8 | 36 | 17 | | | | |

21

Do the data suggest that there is an association between type of vehicle and commuting distance? State the appropriate hypotheses and carry out the test using a significance level of 0.05.

45

18

33

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