

1. A college library has five copies of a certain text on reserve. Two copies (1 and 2) are first editions, and the other three (3, 4, and 5) are second editions. A student examines these books in random order, stopping only when a second printing has been selected. One outcome is 4, and another is 215.

- (a) List all outcomes in the sample space  $S$ .
- (b) Let  $A$  denote the event that exactly one book must be examined. What outcomes are in  $A$ ?
- (c) Let  $B$  be the event that book 4 is the one selected. What outcomes are in  $B$ ?
- (d) Let  $C$  be the event that book 2 is not examined. What outcomes are in  $C$ ?

2. An engineering firm is constructing power plants at three different sites.

Define the events  $E_1$ ,  $E_2$ , and  $E_3$  as follows:

$E_1$  = the plant at site 1 is completed by the contract date.

$E_2$  = the plant at site 2 is completed by the contract date.

$E_3$  = the plant at site 3 is completed by the contract date.

Draw a Venn diagram that depicts these three events as intersecting circles. Shade the region on the Venn diagram corresponding to each of the following events (redraw the Venn diagram for each question):

- (a) At least one plant is completed by the contract date.
- (b) All plants are completed by the contract date.
- (c) None of the plants is completed by the contract date.
- (d) Only the plant at site 1 is completed by the contract date.
- (e) Exactly one of the three plants is completed by the contract date.
- (f) Either the plant at site 1 or site 2 or both of the two plants are completed by the contract date.

3. Let  $A_i = i$ th student got a perfect score on midterm 1, for  $i = 1, \dots, 18$ .

- (a) Interpret  $\left(\bigcap_{i=1}^{18} A_i\right)'$  in English.

- (b) Interpret  $\bigcup_{i=1}^{18} A_i'$  in English. Is it equivalent to part (a)?

4. Using everything we have learned about events and probability, prove that

$$P(A|B) + P(A'|B) = 1.$$

Do not assume that events  $A$  and  $B$  are independent (i.e.  $P(A \cap B) \neq P(A) \cdot P(B)$ ). Explain each step of your calculation.

5. Five companies (A, B, C, D, and E) that make electrical relays compete each year to be the sole supplier of relays to a major automobile manufacturer. The auto company's records show that the probabilities of choosing a company to be the sole supplier are

Supplier chose:	A	B	C	D	E
Probability:	0.30	0.20	0.10	0.25	0.15

- (a) Suppose that supplier E goes out of business this year, leaving the remaining 4 companies to compete with one another. What are the new probabilities of companies A, B, C, and D being chosen as the sole supplier this year?
  - (b) Suppose the auto company narrows the choice of suppliers to companies A and C. What is the probability that company A is chosen this year?
6. #5.16 in your book
7. #5.19 in your book
8. #5.21 in your book
9. Let  $p$  be the sample proportion, written as  $p = n_f/n$ , where  $n$  = sample size, and  $n_f$  = the number of females in the sample. Show that  $\mathbf{E}[p] = \pi$  and  $\mathbf{V}[p] = \pi(1 - \pi)/n$ , where  $\pi$  = proportion of female in population. Hint: Use what you know about Binomial distributions.
10. A survey of the members of a large professional engineering society is conducted to determine their views on proposed changes to an ASTM measurement standard. Suppose that 80% of the entire membership favor the proposed changes. Hint: Use the result from the previous problem.
- (a) Calculate the mean and standard deviation of the sampling distribution of the proportion of engineers in samples of size 25 who favor the proposed changes.
  - (b) Calculate the mean and standard deviation of the sampling distribution of the proportion of engineers in samples of size 100 who favor the proposed changes.
11. The lifetime of a certain battery is normally distributed with a mean value of 8 hours and a standard deviation of 1 hour. There are four such batteries in a package.
- (a) What is the probability that the average lifetime of the four batteries exceeds 9 hours?
  - (b) If  $T$  denotes the average lifetime of the four batteries in a randomly selected package, find the numerical value of  $T_0$  for which  $P(T \geq T_0) = .95$ .
12. The number of flaws  $x$  on an electroplated automobile grill is known to have the following probability mass function:  $p(0) = .6$ ,  $p(1) = .2$ ,  $p(2) = .1$ ,  $p(3) = .1$ .
- (a) Calculate the mean and standard deviation of  $x$ .
  - (b) What are the mean and standard deviation of the sampling distribution of the average number of flaws per grill in a random sample of 50 grills?
  - (c) For a random sample of 50 grills, calculate the approximate probability that the average number of flaws per grill exceeds .8.