Probability and Statistics for Engineers Homework Six
TMATH 390

Ben Foster*
Instructor: Julia Eaton

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*Institute of Technology, University of Washington Tacoma

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Problem 1

Exercise 2 in 7.1 on page 297
A random sample of ten homes in a particular area, each heated with natural gas, is selected, and the amount of gas (therms) used during January is determined for each home. The resulting observations are: 103, 156, 118, 89, 125, 147, 122, 109, 138, and 99.

(a) Use an unbiased estimator to compute a point estimate of μ, the average amount of gas used by all houses in the area.
(b) Use an unbiased estimator to compute a point estimate of π, the proportion of all homes that use over 100 therms.

Answer to a We can use $\bar{x}$ as an unbiased estimator to compute a point estimate of μ since $\bar{x}$ is an unbiased estimator of μ.

$$\bar{x} = \frac{1}{n}\sum x_i$$

$$\bar{x} = \frac{103 + 156 + 118 + 89 + 125 + 147 + 122 + 109 + 138 + 99}{10}$$

$$\bar{x} = 120.6$$

Answer to b We know that p is an unbiased estimator of the population proportion π. Therefore, we just need to figure out the population proportion for all the houses using more than 100 therms. Since 8 houses use above 100 therms, then we get an answer of $p = 0.8$.

Problem 2

Exercise 4 in 7.1 on pages 297 and 298
Random samples of n trees are taken from a large area of forest, and the proportion of diseased trees in each sample is determined. The actual proportion of diseased trees, π, is unknown. (Hint: It'll help to use R in this exercise).

(a) For random samples of size $n = 10$, calculate the area under the sampling distribution curve for p between the points $\pi - 0.10$ and $\pi + 0.10$. That is, find the probability that the sample proportion lies within ± 0.10 (i.e, 10%) of the population proportion. Use the formula for the upper bound on the standard error of p (see Section 5.6) in your calculations.
(b) Repeat the probability calculation in part (a) for samples of size $n = 50$, $n = 100$, and $n = 1000$. (Use the normal approximation to the binomial.)
(c) Graph the probabilities you found in parts (a) and (b) versus their corresponding sample sizes, n. What can you conclude from this graph?

Answer to a This question is asking us to find $P(\pi - 0.10 \leq p \leq \pi + 0.10)$ when π is unknown and size n is known:

$$\begin{aligned} &P(\pi - 0.10 \leq p \leq \pi + 0.10) \\ &= P(-0.10 \leq p - \pi \leq 0.10) \\ &= P(\frac{-0.10}{\sigma_p} \leq \frac{p-\pi}{\sigma_p} \leq \frac{0.10}{\sigma_p} \end{aligned}$$

Since we know that $\sigma_p = \frac{1}{2\sqrt{n}}$ thanks to the formula for the upper bound on the standard error of p from section 5.6 and we know that $\frac{p-\pi}{\sigma_p} = z$, then:

$$\begin{aligned} &= P((-0.10)(2\sqrt{n}) \leq z \leq (0.10)(2\sqrt{n})) \\ &= P(-0.2\sqrt{n} \leq z \leq 0.2\sqrt{n}) \\ &= P(z \leq 0.2\sqrt{n}) - [1 - P(z \leq 0.2\sqrt{n})] \\ &= 2 \cdot P(z \leq 0.2\sqrt{n}) - 1 \\ &= 2(0.7357) - 1 \\ &= 0.4714 \end{aligned}$$

Therefore, for random samples of size 10, the proportion of diseased trees is 0.4714.

Answer to b For different sizes of n, we can use the formula we found in the previous part, $2 \cdot P(z \leq 0.2\sqrt{n}) - 1$, to find the new proportions of diseased trees with new sample sizes:
When $n = 50$

$$\begin{aligned} &2 \cdot P(z \leq 0.2\sqrt{50}) - 1 \\ &= 2(0.9207) - 1 \\ &= 0.8414 \end{aligned}$$

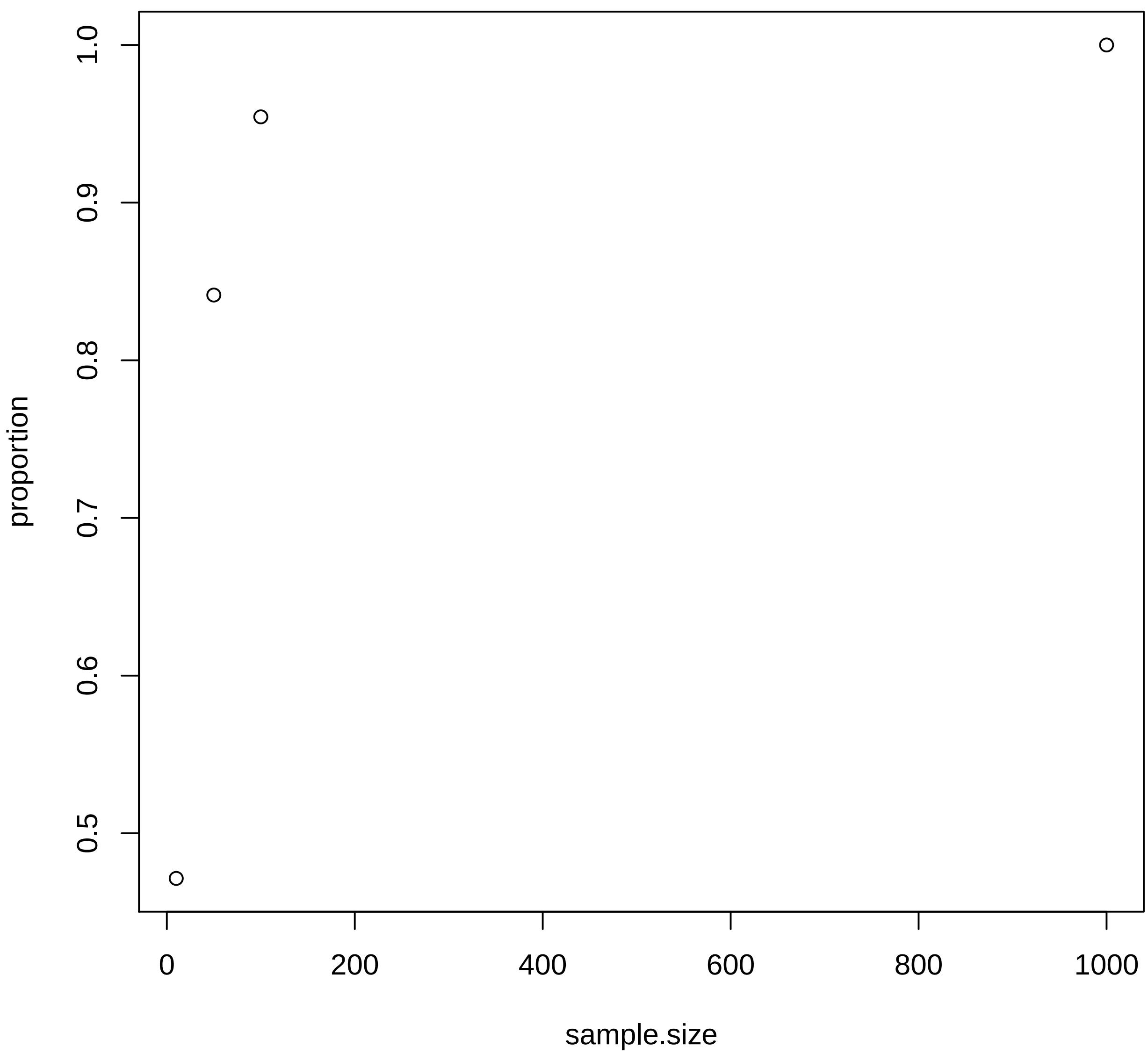
When $n = 100$

$$\begin{aligned} &2 \cdot P(z \leq 0.2\sqrt{100}) - 1 \\ &= 2(0.9772) - 1 \\ &= 0.9544 \end{aligned}$$

When $n = 1000$

$$\begin{aligned} &2 \cdot P(z \leq 0.2\sqrt{1000}) - 1 \\ &= 2(1) - 1 \\ &= 1 \end{aligned}$$

Answer to c



There isn't much we can determine from this graph besides the fact that as the sample size goes up, the proportion of diseased trees also goes up. The fact that at sample size 1000, we end up with a proportion of 1, then this is most likely an error since it seems weird that we suddenly have all diseased trees.

Problem 3

Exercise 7 in 7.2 on page 305
Assuming that n is large, determine the confidence level for each of the following two-sided confidence intervals:

(a) $\bar{x} \pm \frac{3.09s}{\sqrt{n}}$
(b) $\bar{x} \pm \frac{2.81s}{\sqrt{n}}$
(c) $\bar{x} \pm \frac{1.44s}{\sqrt{n}}$
(d) $\bar{x} \pm \frac{s}{\sqrt{n}}$

Note: the variable $z = \frac{(\bar{x}-\mu}{s/\sqrt{n}}$ also has approximately a standard normal distribution. To solve each of these, we would just go to the Appendix Table 1 on pages 582-583 and use the z values given as limits to the interval on the standard normal distribution so we may find our confidence level.

Answer to a

$$+3.09 = 0.999$$

$$-3.09 = 0.001$$

$$\text{difference } = 0.998$$

The confidence level for this two-sided confidence interval is 99.8%.

Answer to b

$$+2.81 = 0.9975$$

$$-2.81 = 0.0025$$

$$\text{difference } = 0.995$$

The confidence level for this two-sided confidence interval is 99.5%.

Answer to c

$$+1.44 = 0.9251$$

$$-1.44 = 0.0749$$

$$\text{difference } = 0.8502$$

The confidence level for this two-sided confidence interval is 85%.

Answer to d

$$+1.00 = 0.8413$$

$$-1.00 = 0.1587$$

$$\text{difference } = 0.6826$$

The confidence level for this two-sided confidence interval is 68.3%.

Problem 4

Exercise 8 in 7.2 on page 305
What z critical value in the large-sample two-sided confidence interval for μ should be used to obtain each of the following confidence levels?

(a) 98%
(b) 85%
(c) 75%
(d) 99.9%

We can find each of these by multiplying the percentage by 100, adding 1 and then dividing by two to get the value on the standard normal distribution table, and then doing a reverse lookup to find the z critical score.

Answer to a

$$\frac{0.98+1}{2} = 0.99$$

Doing a reverse lookup in the table, we get a z critical score of 2.33. Therefore:

$$\bar{x} \pm \frac{2.33s}{\sqrt{n}}$$

Answer to b

$$\frac{0.85+1}{2} = 0.925$$

Doing a reverse lookup in the table, we get a z critical score of 1.44. Therefore:

$$\bar{x} \pm \frac{1.44s}{\sqrt{n}}$$

Answer to c

$$\frac{0.75+1}{2} = 0.875$$

Doing a reverse lookup in the table, we get a z critical score of 1.15. Therefore:

$$\bar{x} \pm \frac{1.15s}{\sqrt{n}}$$

Answer to d

$$\frac{0.999+1}{2} = 0.9995$$

Doing a reverse lookup in the table, we get a z critical score of 3.29. Therefore:

$$\bar{x} \pm \frac{3.29s}{\sqrt{n}}$$

Problem 5

Exercise 11 in 7.2 on page 306
Suppose that a random sample of 50 bottles of a particular brand of cough syrup is selected, and the alcohol content of each bottle is determined. Let μ denote the average alcohol content for the population of all bottles of the brand under study. Suppose that the resulting 95% confidence interval is (7.8, 9.4).

(a) Would a 90% confidence interval calculated from this same sample have been narrower or wider than the given interval? Explain your reason.
(b) Consider the following statement: There is a 95% chance that μ is between 7.8 and 9.4. Is this statement correct? Why or why not?
(c) Consider the following statement: We can be highly confident that 95% of all bottles of this type of cough syrup have an alcohol content that is between 7.8 and 9.4. Is this statement correct? Why or why not? (d) Consider the following statement: if the process of selecting a sample size 50 and then computing the corresponding 95% interval is repeated 100 times, 95 of the resulting intervals will include μ. Is this statement correct? Why or why not?

Answer to a The 90% confidence interval calculated from this same sample would have been narrower than the given interval since the formula we use to determine the interval, $\bar{x} \pm \frac{(z)(s)}{\sqrt{n}}$, depends on the z critical value which is directly determined from the percentage of the confidence interval, then the smaller confidence we have, the smaller the interval will be. The price of a higher confidence level is a loss in precision.

Answer to b No, this statement is not correct because a confidence level of 95% implies that 95% of all samples would give an interval that includes μ, while the above statement is saying that the given interval will never change, which isn't true. The 95% refers to the long-run percentage of *all* possible samples resulting in an interval that includes μ.

Answer to c No, this statement is not correct because the confidence level is just telling us what proportion of samples have an interval that contains μ while the statement above is suggesting that the confidence level is telling us the proportion of values that lie within an interval.

Answer to d No, this statement is not correct because the 95% confidence we are using refers to the long-run percentage of *all* possible samples resulting in an interval that includes μ and in this statement, we only repeated the computation of the interval 100 times and not for every sample. It is very likely that less or more than 95 of the samples will contain the correct interval.

Problem 6

Exercise 16 in 7.2 on page 307
The article "Evaluating Tunnel Kiln Performance" (*Amer. Ceramic Soc. Bull.*, August 1997: 59-63) gave the following summary information for fracture strengths (MPa) of $n = 169$ ceramic bars fired in a particular kiln: $\bar{x} = 89.10$, $s = 3.73$.

(a) Calculate a two-sided confidence interval for true average fracture strength using a confidence level of 95%. Does it appear that true average fracture strength has been precisely estimated?
(b) Suppose the investigators had believed a priori that the population standard deviation was about 4 MPa. Based on this supposition, how large a sample would have been required to estimate μ to within 0.5 MPa with 95% confidence?

Answer to a

$$\bar{x} \pm \frac{(z)(s)}{\sqrt{n}}$$

Since we are dealing with a confidence level of 95%, then we use the formula from problem 4 to find the z critical value of 1.96.

$$89.10 \pm \frac{(1.96)(3.73)}{13}$$

$$(88.54, 89.66)$$

Answer to b In order to solve this, the only thing we can really change is the sample size n. We want the value we are adding and subtracting from $\bar{x}$ to be equal to 0.5, therefore:

$$0.5 = \frac{(1.96)(4)}{\sqrt{n}}$$

Solving for n, we get a sample size of 245.86, or 246 rounded up.

Problem 7

Exercise 18 in 7.2 on page 307
Determine the confidence level for each of the following large-sample one sided confidence bounds:

(a) Upper bound: $\bar{x} + \frac{0.84s}{\sqrt{n}}$
(b) Lower bound: $\bar{x} - \frac{2.05s}{\sqrt{n}}$
(c) Upper bound: $\bar{x} + \frac{0.67s}{\sqrt{n}}$

To answer all of these questions, all we have to do is use the Appendix Table 1 on pages 582 and 583 and use the z critical value to find the corresponding proportion.

Answer to a Corresponding proportion to z critical value of 0.84:
0.7995 or 80%.

Answer to b Corresponding proportion to z critical value of 2.05:
0.9798 or 98%.

Answer to c Corresponding proportion to z critical value of 0.67:
0.7486 or 75%.

Problem 8

Exercise 19 in 7.2 on page 307
The charge-to-tap time (min) for a carbon steel in one type of open hearth furnace was determined for each heat in a sample of size 36, resulting in a sample mean time of 382.1 and a sample standard deviation of 31.5. Calculate a 95% upper confidence bound for true average charge-to-tap time.

Answer Since we are dealing with a confidence level of 95%, then we use the table to find a z critical value of 1.645. To find the upper confidence bound, we would use the formula:

$$\bar{x} + \frac{(z)(s)}{\sqrt{n}}$$

$$382.1 + \frac{(1.645)(31.5)}{\sqrt{36}}$$

$$390.73625$$

With a confidence level of 95%, the value of μ lies in the interval (0, 390.74) min. Since we can't have a negative charge-to-tap time, then the lower bound was changed to 0.

Problem 9

Exercise 23 in 7.3 on page 315
TV advertising agencies face growing challenges in reaching audience members because viewing TV programs via digital streaming is increasingly popular. The Harris poll reported on November 13, 2012, that 53% of 2343 American adults surveyed said they have watched digitally streamed TV programming on some type of device.

(a) Calculate and interpret a confidence interval at the 99% confidence level for the proportion of all adult Americans who have watched streamed programming.
(b) What sample size would be required for the width of a 99% CI to be at most 0.05 irrespective of the value of p?

Answer to a In order to calculate this, we will use the formula $p \pm (z \text{ critical value }) \sqrt{p(1-p)/n}$ where $p = 0.53$, $z = 2.576$, and $n = 2343$.

$$0.53 \pm 2.576\sqrt{\frac{0.53(1-0.53)}{2343}}$$

$$(0.50, 0.56)$$

Given the above interval, we are 99% confident the proportion of all adult Americans who watched streamed programming is between about 50% and 56%.

Answer to b We will solve this problem in a similar way we solved problem 6.b. We just set $2.576\sqrt{\frac{0.53(1-0.53)}{n}}$ equal to 0.05 and then solve for n. By doing this, we get a sample size of 662.

Problem 10

Exercise 30 in 7.3 on page 317
A manufacturer of small appliances purchases plastic handles for coffeepots from an outside vendor. If a handle is cracked, it is considered defective and must be discarded. A very large shipment of handles is received. The proportion of defective handles, π, is of interest. How many handles from the shipment should be inspected to estimate π to within 0.1 with 99% confidence?

Answer In order to answer this question, we must use the formula

$$n = \pi(1-\pi)\left[\frac{z}{B}\right]^2$$

Where n is the sample size, z is the z critical score, π is the proportion of defective handles received which is going to be equal to 0.5 since $\pi(1-\pi)$ is largest when $\pi = 0.5$, and B is the bound which equals 0.1 in this case.

$$n = 0.5(0.5)\left[\frac{2.576}{0.1}\right]^2$$

$$n = 165.8944$$

Round up to the nearest integer and we get $n = 166$.