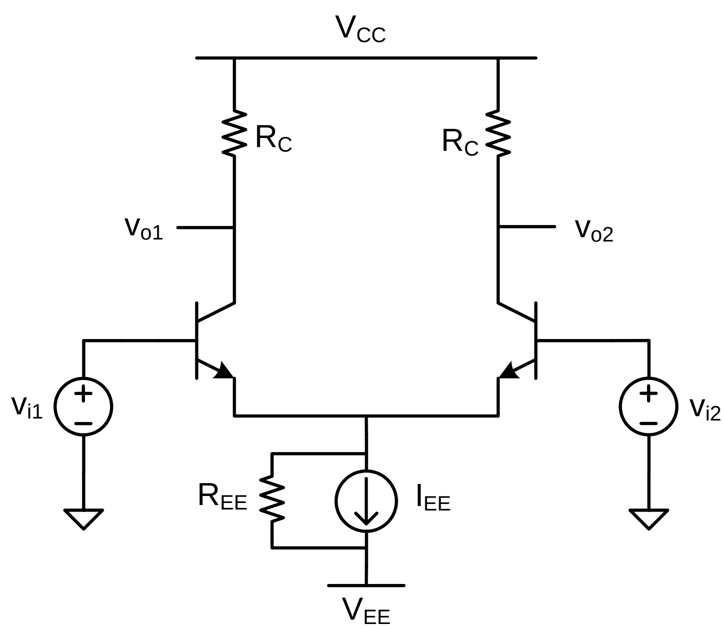
13 Differential Amplifiers

A differential amplifier accepts two input signals and provides an output that, ideally, is a response only to the difference between the input voltage levels. The output may be a single node referenced to ground or may be the difference between two nodes, neither of which is ground.

One common configuration of a single amplifier stage that acts as a differential amplifier consists of two BJTs with their emitters tied to the same node, called an emitter-coupled pair. A typical schematic is shown below.



Small signal inputs, v_{i1} and v_{i2}, are applied to the bases while the outputs, v_{o1} and v_{o2}, are taken at the collectors. Both are often DC coupled, as shown. The inputs can be DC coupled since there is no DC bias at the base and only the differential signal is amplified (ideally). The output can be DC coupled if a differential signal is used and the DC level is ignored. For non-differential, single-ended output, the signal is taken at one of the collectors, referenced to ground, and is often AC coupled.

The current source on the emitters is shown as a Norton equivalent. It may be just a resistor or it may involve one or more transistors and have a high output resistance.

The differential amplifier is constructed from matched transistors and matched collector resistors. At zero input, the current into the current source is equally divided between the left and right sides of the emitter-coupled pair. The output voltages at the collectors will then be the same and have zero differential output. For equal and non-zero inputs, $v_{i1} = v_{i2}$, the current remains equally divided with equal outputs and zero differential output. But the voltages at the outputs will rise or fall together if R_{EE} is finite. For different inputs, say, $v_{i1} > v_{i2}$, at least some collector current will be shifted from the right side to the left and $v_{o1} < v_{o2}$, producing a negative differential gain.

It is important that the two npn transistors as well as the collector resistors are <u>closely matched</u>. Transistor mismatch leads to different collector currents at the same base-emitter voltage. Similarly, R_C mismatch causes a difference in v_{o1} and v_{o2} even at identical collector currents.

We define the differential mode input as the difference between the two input voltages,

$$v_{i,dm} = v_{i1} - v_{i2}$$

and the common mode input as the average of the two input voltages

$$v_{i,cm} = \tfrac{1}{2} (v_{i1} + v_{i2})$$

from which we obtain

$$v_{i1} = v_{i,cm} + \tfrac{1}{2} v_{i,dm}$$

$$v_{i2} = v_{i,cm} - \tfrac{1}{2} v_{i,dm}$$

Similarly, the differential and common mode outputs are given by

$$v_{o,dm} = v_{o1} - v_{o2}$$

$$v_{o,cm} = \tfrac{1}{2} (v_{o1} + v_{o2})$$

from which we obtain

$$v_{o1} = v_{o,cm} + \tfrac{1}{2}\, v_{o,dm}$$

$$v_{o2} = v_{o,cm} - \tfrac{1}{2}\, v_{o,dm}$$

An analysis of the differential amplifier shown above is relatively simple for either pure differential mode, $v_{i1} = -\, v_{i2}$, or for pure common mode, $v_{i1} = v_{i2}$. We carry out the analysis for those pure modes below. To determine the response to arbitrary inputs v_{i1} and v_{i2}, we can resolve those inputs into differential and common mode components and simply superimpose our differential and common mode solutions. It is important to note that <u>any</u> input, v_{i1} and v_{i2}, can be written as a sum of differential and common mode components.

Example 13-1

Input signals to a differential amplifier are $v_{i1} = 3.2$ mV and $v_{i2} = 4.5$ mV.
The open circuit voltage gain of the amplifier in differential and common mode are $A_{v,dm} = 1480$ and $A_{v,cm} = 6.5$.

What are the output voltages, v_{o1} and v_{o2}?

Solution:

$$v_{i,dm} = v_{i1} - v_{i2} = 3.2\text{ mV} - 4.5\text{ mV} = -1.3\text{ mV}$$

$$v_{i,cm} = \tfrac{1}{2}(v_{i1} + v_{i2}) = \tfrac{1}{2}(3.2\text{ mV} + 4.5\text{ mV}) = 3.85\text{ mV}$$

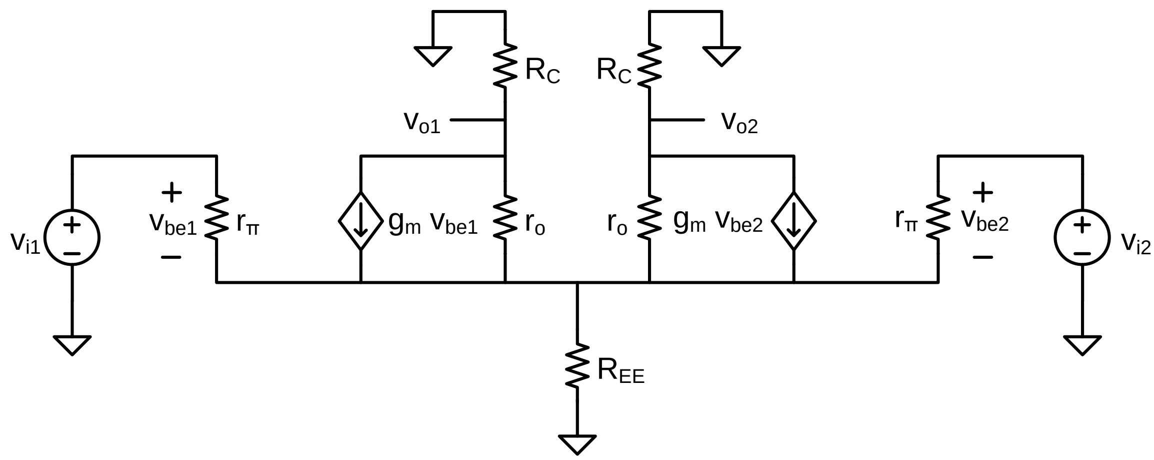
$$v_{o,dm} = A_{v,dm}\, v_{i,dm} = 1480\,(-1.3\text{ mV}) = -1.924\text{ V}$$

$$v_{o,cm} = A_{v,cm}\, v_{i,cm} = 6.5 \bullet 3.85\text{ mV} = 25\text{ mV}$$

$$v_{o1} = v_{o,cm} + \tfrac{1}{2}\, v_{o,dm} = 25\text{ mV} + \tfrac{1}{2} \bullet (-1.924\text{ V}) = -0.937\text{ V}$$

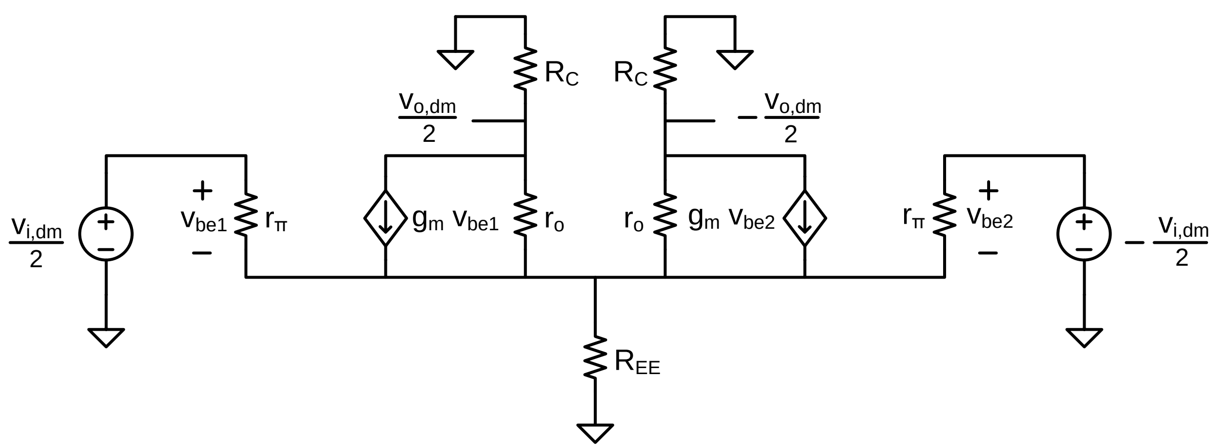
$$v_{o2} = v_{o,cm} - \tfrac{1}{2}\, v_{o,dm} = 25\text{ mV} - \tfrac{1}{2} \bullet (-1.924\text{ V}) = 0.987\text{ V}$$

The small signal circuit of our differential amplifier is shown below.



We have assumed identical transistors with equal bias currents so $r_{\pi 1} = r_{\pi 2} = r_\pi$, $g_{m1} = g_{m2} = g_m$ and $r_{o1} = r_{o2} = r_o$.

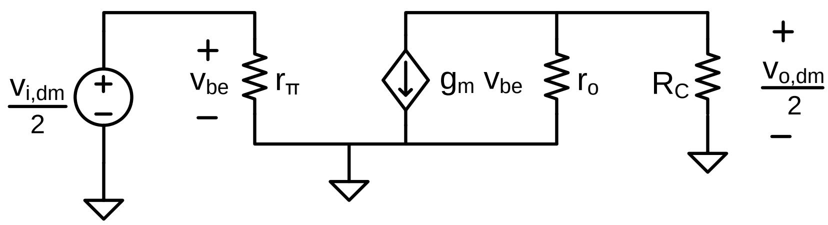
In pure differential mode, $v_{i1} = -v_{i2}$, and we have $v_{i,dm} = v_{i1} - v_{i2} = 2\,v_{i1} = -2\,v_{i2}$, giving us inputs $v_{i1} = ½\,v_{i,dm}$ and $v_{i2} = -½\,v_{i,dm}$. The outputs are pure differential and given by $v_{o1} = ½\,v_{o,dm}$ and $v_{o2} = -½\,v_{o,dm}$. The small signal circuit is then



This is a <u>linear</u> circuit with two independent sources. To find a solution, it is convenient to use superposition. We look at the response to each of the two inputs separately with the other set to zero and then add the results for the total response to simultaneous input signals.

We want to determine the behavior of the coupled emitter node, the non-ground side of R_{EE}. With the right source zeroed, the left source will cause some voltage at the coupled emitter node, say v′. Because the circuit is symmetrical, if we zero the left source, the right source will cause exactly – v′ at the coupled emitter node since the right source voltage is the negative of the left. When we superimpose these results, we get v′ – v′ = 0 at the coupled emitter node for both sources present. We say that the coupled emitter node is a virtual ground for pure differential input because it stays at ground potential without being physically grounded.

The virtual ground at the coupled emitter node divides the differential circuit into two separate half circuits. They are identical and independent since they are connected only by a ground connection. We now place a ground at the coupled emitter and look at just one of the half circuits.



We have a very familiar circuit, the single transistor common emitter amplifier. We have already seen the results for the common emitter for open circuit gain, input resistance and output resistance. We have the following.

For differential mode

$$A_{v,dm} = v_{o,dm} / v_{i,dm}$$
$$= (v_{o,dm} / 2) / (v_{i,dm} / 2)$$
$$= - g_m (R_C \,||\, r_o)$$
$$R_{in,dm} = v_{i,dm} / i_b$$
$$= 2 (v_{i,dm} / 2) / i_b$$
$$= 2\, r_\pi$$

where base current, i_b, flows to both inputs but in opposite directions and represents the total current from the differential input.

For $R_{out,dm}$, we apply a differential signal, v_x, to the output. Then

$$R_{out,dm} = v_x / i_x$$

But v_x is applied to the output of the full circuit and only $v_x / 2$ is applied to the half circuit. Then we have

$$\begin{aligned} R_{out,dm} &= v_x / i_x \\ &= 2 (v_x / 2) / i_x \\ &= 2 (R_C \| r_o) \end{aligned}$$

Finally

$$\begin{aligned} G_{m,dm} &= i_{sc} / v_{i,dm} \\ &= ½\, i_{sc} / (v_{i,dm} / 2) \\ &= - ½\, g_m \end{aligned}$$

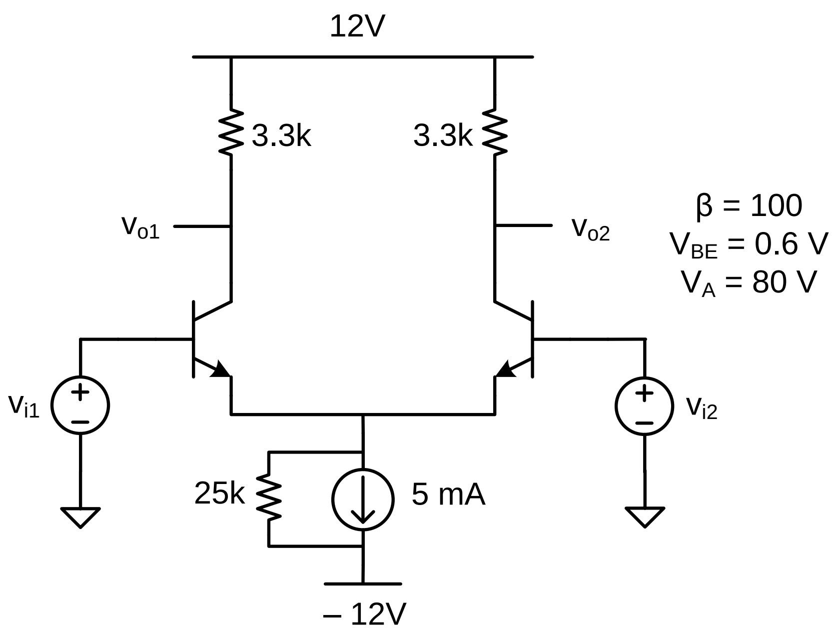
where i_{sc}, the short circuit current, is just i_c and, again, the same small signal collector current, i_c, flows in both collectors but in opposite directions.

As a check, we substitute into the relationship between transconductance and open circuit voltage gain, $G_{m,dm} = A_{v,dm} / R_{out,dm}$ and get

$$\begin{aligned} - ½\, g_m &= - g_m (R_C \| r_o) / 2 (R_C \| r_o) \\ &= - ½\, g_m \end{aligned}$$

For pure differential mode, the differential amplifier with an emitter-coupled pair looks very much like the familiar one-transistor common emitter amplifier. The open circuit voltage gain is the same while the input resistance is doubled and the output resistance is also doubled. The transconductance is halved.

Example 13-2



Find the differential open circuit voltage gain as well as the differential input and output resistance of this amplifier.

Solution:

The DC bias collector current in each transistor is ½ of the current source value.

$$I_C = \tfrac{1}{2} \{ 5\text{ mA} + [-0.6\text{ V} - (-12\text{ V})] / 25\text{ k}\Omega \} = 2.7\text{ mA}$$

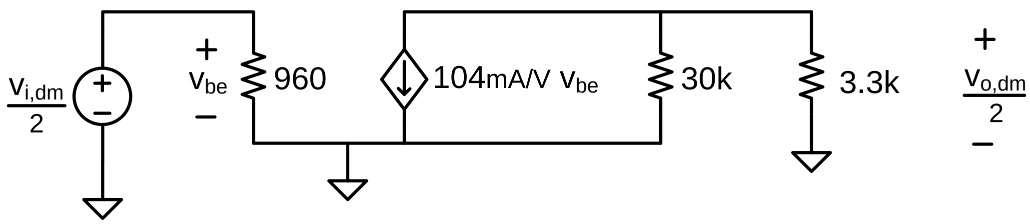
Then we get

$$g_m = I_C / V_t = 2.7\text{ mA} / .026\text{ V} = 104\text{ mA/V}$$

$$r_\pi = \beta / g_m = 100 / 104\text{ mA/V} = 960\ \Omega$$

$$r_o = V_A / I_C = 80\text{ V} / 2.7\text{ mA} = 30\text{ k}\ \Omega$$

The differential half circuit is

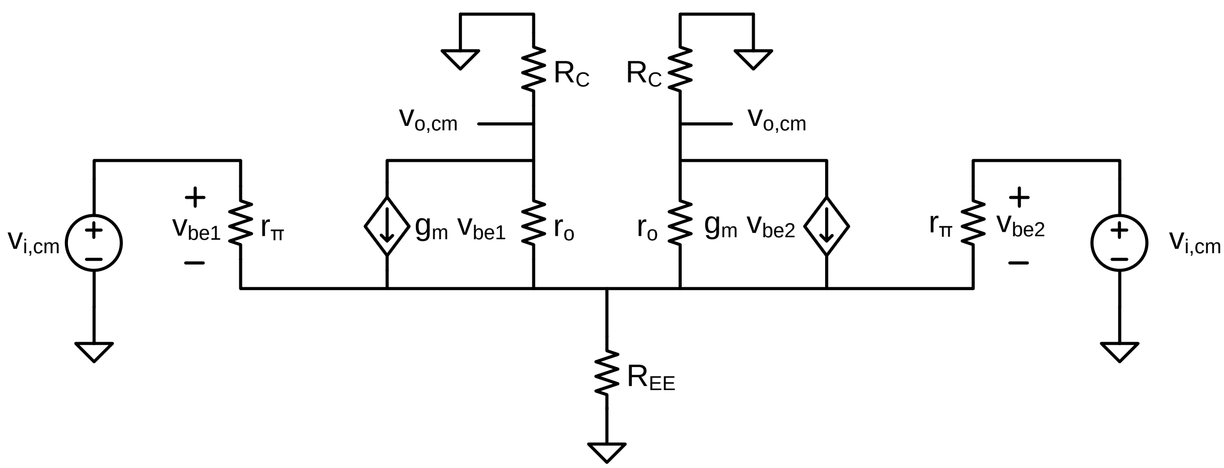


$A_{v,dm} = -g_m(R_C \| r_o) = -104\text{ mA/V }(3.3\text{ k}\Omega \| 30\text{ k}\Omega) = -310$

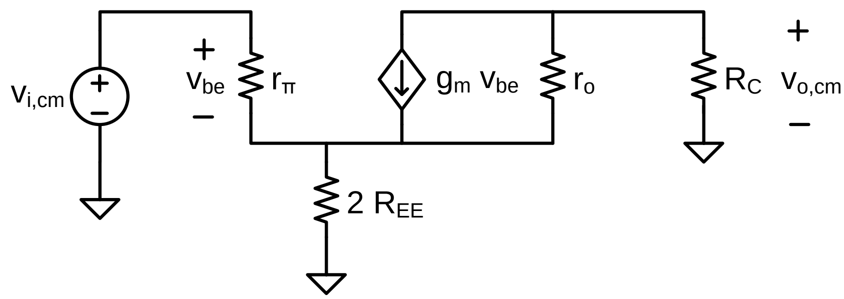
$R_{in,dm} = 2r_\pi = 2 \cdot 960\ \Omega = 1.9\text{ k}\Omega$

$R_{out,dm} = 2(R_C \| r_o) = 2(3.3\text{ k}\Omega \| 30\text{ k}\Omega) = 5.9\text{ k}\Omega$

Now for pure common mode input, the inputs are equal, $v_{i1} = v_{i2} = v_{i,cm}$ and the outputs are also equal, $v_{o1} = v_{o2} = v_{o,cm}$. We get the following small signal circuit.



We can again apply superposition to this circuit. But if we consider the two independent sources separately, both sources result in the <u>same</u> voltage at the coupled emitter node. Rather than the effect of the two sources cancelling as they did in differential mode, they now <u>add</u>. We can still identify a half circuit, but the coupled emitter node is no longer a virtual ground. The effect of the right source on the left half circuit is to <u>double</u> the current in R_{EE} for the half circuit. So we form a common mode half circuit that doubles the value of R_{EE} to account for the current from the other half circuit.



The analysis of this circuit we've also seen already. It is just the common emitter with emitter degeneration.

$$A_{v,cm} = v_{o,cm} / v_{i,cm}$$

$$= -g_m R_C / (1 + g_m 2 R_{EE}) \qquad \text{(ignoring } r_o\text{)}$$

$$R_{in,cm} = r_\pi + (\beta+1)\ 2 R_{EE}$$

$$\approx r_\pi (1 + g_m 2 R_{EE})$$

$$R_{out,cm} = R_C$$

$$G_{m,cm} = -g_m / (1 + g_m 2 R_{EE})$$

We see the reduction in voltage gain and transconductance, as well as the increase in input resistance, caused by emitter degeneration which now consists of the $2\ R_{EE}$ half circuit emitter resistance. In the case of an ideal current source with $R_{EE} = \infty$, $A_{v,cm} = 0$ and there is no common mode response.

We next define the <u>common</u> <u>mode</u> <u>rejection</u> <u>ratio</u>.

$$\text{CMRR} = \left| \frac{A_{v,dm}}{A_{v,cm}} \right|$$

$$= \frac{g_m (R_C \parallel r_o)}{\dfrac{g_m R_C}{1 + g_m 2 R_{EE}}}$$

$$\text{CMRR} \approx 1 + g_m 2 R_{EE} \qquad \text{if } R_C \ll r_o$$

As a ratio of differential mode gain to common mode gain, the CMRR is a measure of the ability of an amplifier to respond to an input signal difference and ignore the common mode component.

A large value for the CMRR is usually desirable for a differential amplifier. We are interested in the differential signal and not the common mode signal so we'd like the common mode suppressed as much as possible in the output. Note again that for an ideal current source ($R_{EE} = \infty$), we get an ideal differential amplifier with CMRR = ∞. There is no common mode gain since $R_{EE} = \infty$. We'll return to the differential amplifier after we've looked at active current sources (current sources with active components) that are capable of very large output resistance and ideal for differential amplifiers.

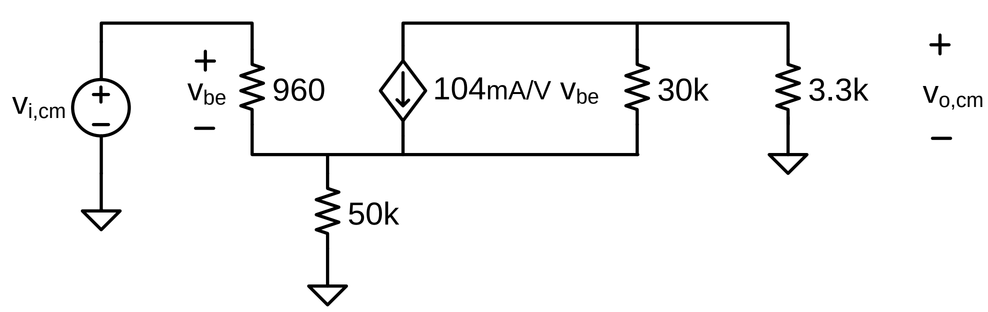
As with any voltage amplifier, high input resistance is a desirable and often necessary characteristic. For the BJT differential pair, $R_{in,dm} = 2\ r_\pi$ and we can use a small collector current or perhaps the Darlington configuration to push r_π up if needed.

Example 13-3

Find the common mode open circuit voltage gain, $R_{in,cm}$ and $R_{out,cm}$, and the CMRR for the differential amplifier of Example 13-2.

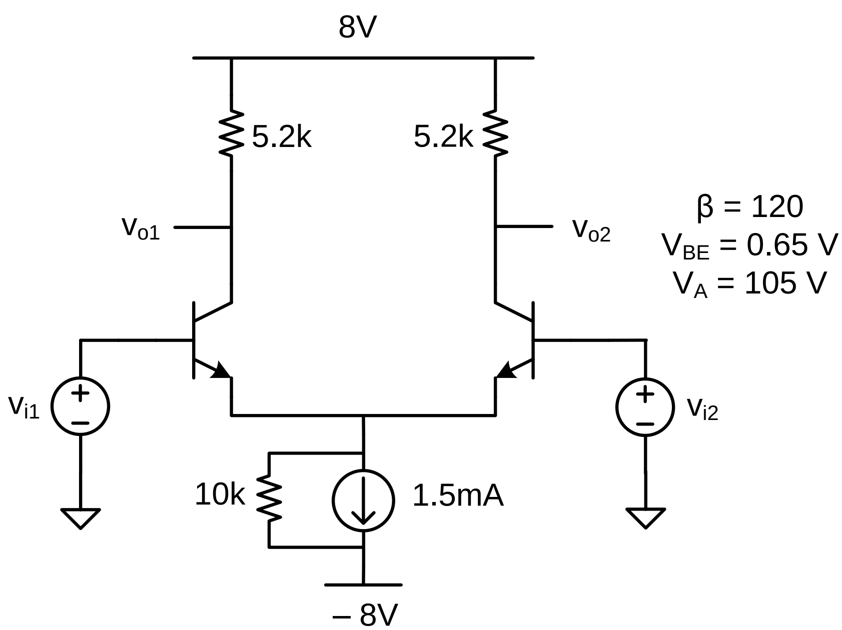
Solution:

The common mode half circuit is



$$A_{v,cm} = -g_m R_C / (1 + g_m 2 R_{EE})$$
$$= -104 \text{ mA/V } 3.3 \text{ k}\Omega / (1 + 104 \text{ mA/V} \bullet 2 \bullet 25 \text{ k}\Omega)$$
$$= -0.066$$
$$R_{in,cm} = r_\pi (1 + g_m 2 R_{EE})$$
$$= 960 \ \Omega (1 + 104 \text{ mA/V} \bullet 2 \bullet 25 \text{ k}\Omega) = 5.0 \text{ M}\Omega$$
$$R_{out,cm} = R_C = 3.3 \text{ k}\Omega$$
$$CMRR = 1 + g_m 2 R_{EE}$$
$$= 1 + 104 \text{ mA/V} \bullet 2 \bullet 25 \text{ k}\Omega = 5200$$
$$\approx A_{v,dm} / A_{v,cm}$$

Example 13-4



Find v_{o1} and v_{o2} for v_{i1} = 15.8 mV and v_{i2} = 18.4 mV.

Solution:

The DC collector current in each transistor is

$$I_C = \quad = \tfrac{1}{2} (1.5 \text{ mA} + 7.35 \text{ V} / 10 \text{ k}\Omega) = 1.1 \text{ mA}$$

$g_m = I_C / V_t = 1.1 \text{ mA} / .026 \text{ V} = 42 \text{ mA/V}$

$r_\pi = \beta / g_m = 120 / 42 \text{ mA/V} = 2.9 \text{ k}\Omega$

$r_o = V_A / I_C = 105 \text{ V} / 1.1 \text{ mA} = 95 \text{ k}\Omega$

$A_{v,dm} = -g_m (R_C \,||\, r_o)$

$\quad = -42 \text{ mA/V} (5.2 \text{ k}\Omega \,||\, 95 \text{ k}\Omega) = -207$

$A_{v,cm} = -g_m R_C / (1 + g_m 2 R_{EE})$

$\quad = -42 \text{ mA/V } 5.2 \text{ k}\Omega / (1 + 42 \text{ mA/V} \bullet 2 \bullet 10 \text{ k}\Omega) = -0.26$

$v_{i,dm} = v_{i1} - v_{i2} = 15.8 \text{ mV} - 18.4 \text{ mV} = -2.6 \text{ mV}$

$v_{i,cm} = \frac{1}{2} (v_{i1} + v_{i2}) = \frac{1}{2} (15.8 \text{ mV} + 18.4 \text{ mV}) = 17.1 \text{ mV}$

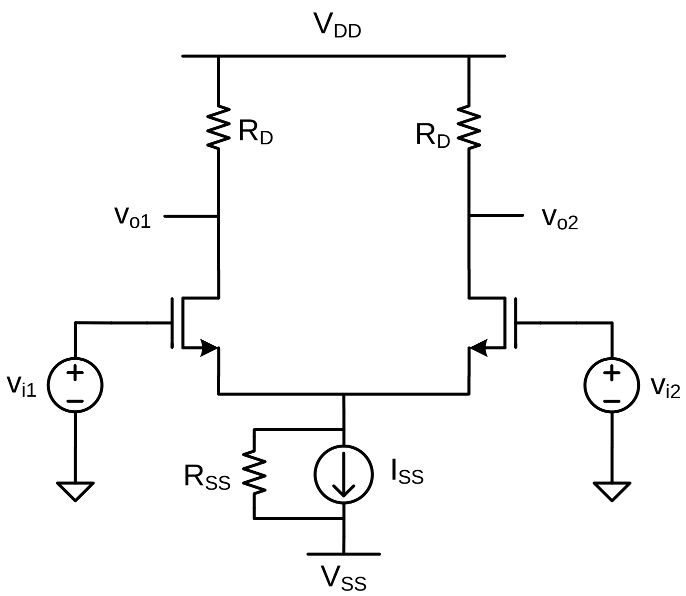
$v_{o,dm} = A_{v,dm} v_{i,dm} = -207 (-2.6 \text{ mV}) = 538 \text{ mV}$

$v_{o,cm} = A_{v,cm} v_{i,cm} = -0.26 \bullet 17.1 \text{ mV} = -4.4 \text{ mV}$

$v_{o1} = v_{o,cm} + \frac{1}{2} v_{o,dm} = -4.4 \text{ mV} + \frac{1}{2} \bullet 538 \text{ mV} = 0.265 \text{ V}$

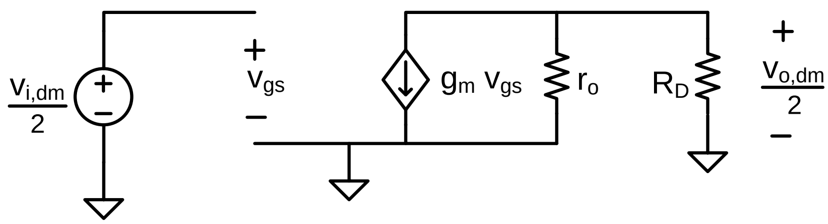
$v_{o2} = v_{o,cm} - \frac{1}{2} v_{o,dm} = -4.4 \text{ mV} - \frac{1}{2} \bullet 538 \text{ mV} = -0.273 \text{ V}$

Another approach to achieving high input resistance is to use MOSFETs for the differential pair.



The insulated gate of a MOSFET has a very large resistance which becomes the input resistance in a common source configuration. The two n-channel MOSFETs as well as the drain resistors are matched pairs providing an equally divided current when $v_{i1} = v_{i2}$ so that $v_{o1} = v_{o2}$.

For MOSFETs we can form differential and common mode half circuits just as we did in the bipolar case. The differential half circuit is



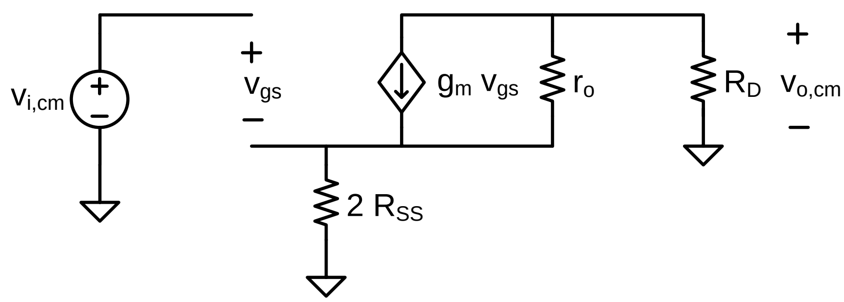
$$A_{v,dm} = -g_m \, (R_D \,||\, r_o)$$

$$R_{in,dm} = \infty$$

$$R_{out,dm} = 2 \, (R_D \,||\, r_o)$$

$$G_{m,dm} = -\tfrac{1}{2} \, g_m$$

The factor of 2 in the output resistance and the transconductance come again from half voltage combined with the current, i_d or i_x, for the half circuit. The common mode half circuit is



$$A_{v,cm} = -g_m R_D \, / \, (1 + g_m \, 2 R_{SS}) \quad \text{(ignoring } r_o\text{)}$$

$$R_{in,cm} = \infty$$

$$R_{out,cm} = R_D \qquad \text{(again ignoring } r_o\text{)}$$

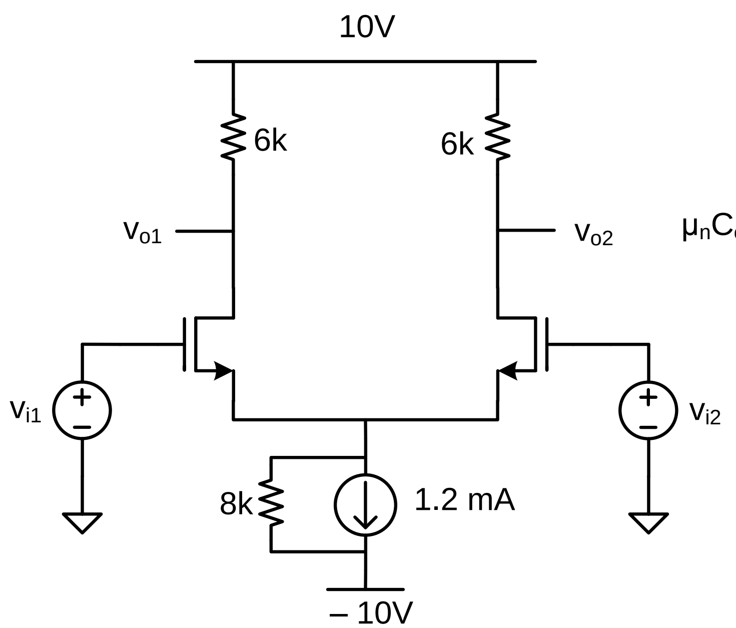
$$G_{m,cm} = -g_m \, / \, (1 + g_m \, 2 R_{SS})$$

Then we get

$$CMRR = 1 + g_m\, 2\, R_{SS}$$

The advantage with MOSFETs is greater input resistance, but the generally lower transconductance and voltage gain continue to be a disadvantage for the MOSFET differential amplifier.

Example 13-5



$\mu_n C_{ox} = 100\ \mu A/V^2$
$W/L = 15$
$V_t = 0.9\ V$
$V_A = 85\ V$

Find the open circuit differential and common mode voltage gain as well as the CMRR for this MOSFET differential amplifier.

Solution:

The drain current is found from the MOSFET relationship for saturation mode and the total current through the current source at the common source node voltage. First, the drain current is

$$\begin{aligned} I_D &= \mu_n C_{ox} (W / L)\ ½\ (V_{GS} – V_t)^2 \\ &= 0.1\ mA/V^2\ 15 \cdot ½\ (0 – V_S – 0.9\ V)^2 \\ &= 0.75\ mA/V^2\ (V_S + 0.9\ V)^2 \end{aligned}$$

The total current through the current source is 2 I_D.

$$2\ I_D\ =\ I_{SS} + (\ V_S - V_{SS}\ )\ /\ R_{SS}$$
$$=\ 1.2\ mA + [\ V_S - (\ -10\ V\ )\ ]\ /\ 8\ k\Omega$$
$$=\ 2.45\ mA + V_S\ /\ 8\ k\Omega$$
$$I_D\ =\ 1.22\ mA + V_S\ /\ 16\ k\Omega$$

Now we have two equations in I_D and V_S.

$$I_D\ =\ 0.75\ mA/V^2\ (\ V_S + 0.9\ V\ )^2$$
$$I_D\ =\ 1.22\ mA + V_S\ /\ 16\ k\Omega$$

These two equations are solved simultaneously to obtain these two solutions.

$$V_S\ =\ 0.39\ V \text{ and } -2.1\ V$$

Only the second value, – 2.1 V, is possible since the source voltage must be negative.

$$I_D\ =\ 0.75\ mA/V^2\ (\ -2.1\ V + 0.9\ V\ )^2\ =\ 1.1\ mA$$

Note that the current source has total current 2 I_D = 2.2 mA = 1.2 mA + 1.0 mA where 1.0 mA ≈ 7.9 V / 8 kΩ is just the current through the output resistance of the current source (within round-off error).

Now we have for the small signal parameters

$$g_m\ =\ 2\ I_D\ /\ (\ V_{GS} - V_t\ )$$
$$=\ 2 \bullet 1.1\ mA\ /\ (\ 2.1\ V - 0.9\ V\ )\ =\ 1.8\ mA/V$$
$$r_o\ =\ V_A\ /\ I_D\ =\ 85\ V\ /\ 1.1\ mA\ =\ 77\ k\Omega$$

Finally, the differential and common mode gain are

$$A_{v,dm} = -g_m\ (R_D \,||\, r_o)$$
$$= -1.8\ \text{mA/V}\ (6\ \text{k}\Omega \,||\, 77\ \text{k}\Omega) = -10$$
$$A_{v,cm} = -g_m R_D / (1 + g_m 2 R_{SS})$$
$$= -1.8\ \text{mA/V} \cdot 6\ \text{k}\Omega / (1 + 1.8\ \text{mA/V} \cdot 2 \cdot 8\ \text{k}\Omega)$$
$$= -0.36$$

and the CMRR is

$$\text{CMRR} = A_{v,dm} / A_{v,cm} = 1 + g_m 2 R_{SS}$$
$$= 1 + 1.8\ \text{mA/V} \cdot 2 \cdot 8\ \text{k}\Omega = 30$$