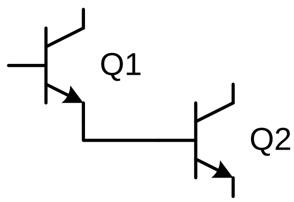
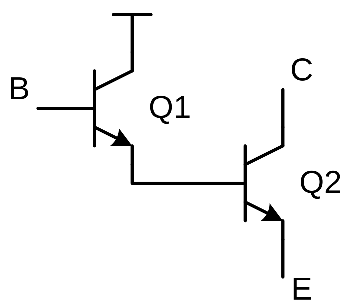
11 Darlington

The Darlington configuration is a two-transistor combination that can be used to greatly increase the current gain and the input resistance of the single transistor common emitter and emitter follower amplifiers. The most common Darlington configuration is this.



The essential element of the Darlington pair is that the emitter of transistor Q1 feeds the base of transistor Q2. This means that the base current of Q1 is multiplied by β+1 at the base of Q2 and by β(β+1) at the collector of Q2. The two-transistor combination has an effective current gain of $\beta\,(\beta+1) \approx \beta^2$.

We can think of the two-transistor Darlington configuration as a single transistor with the terminals indicated below. It has a current gain of β^2 and a base-emitter voltage of 1.2 − 1.4 V when both transistors are in active mode. The collector of Q1 is usually tied to the high supply.



The small signal parameters of the two transistors are related. First we have

$$I_{C1} = \beta\, I_{B1}$$

$$I_{B2} = (\beta+1)\, I_{B1} = (\beta+1)\, I_{C1} / \beta$$

$$I_{C2} = \beta\, I_{B2} = (\beta+1)\, I_{C1}$$

From this we get

$$g_{m1} = I_{C1} / V_t$$

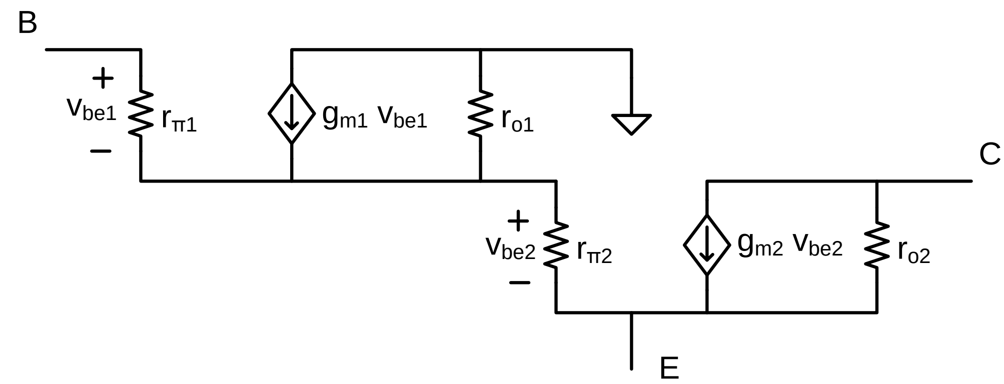
$$g_{m2} = I_{C2} / V_t = (\beta+1)\, I_{C1} / V_t = (\beta+1)\, g_{m1}$$

$$r_{\pi 1} = \beta / g_{m1}$$

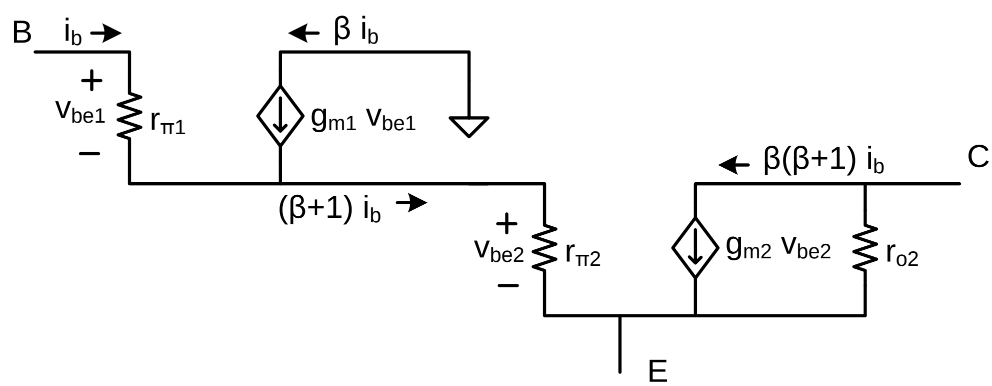
$$r_{\pi 2} = \beta / g_{m2} = \beta / (\beta+1)\, g_{m1} = r_{\pi 1} / (\beta+1)$$

We see that Q2 has a transconductance that is β+1 times <u>larger</u> than Q1 and an input resistance that is β+1 times <u>smaller</u> than Q1.

The small signal circuit for the Darlington configuration is



We can ignore r_{o1} in this circuit. Then we identify currents below.



Now we find r_π, g_m, r_o and β_C as the small signal model parameters of the <u>composite</u> transistor as follows.

$$r_\pi = v_{be} / i_b = (v_{be1} + v_{be2}) / i_b$$
$$= v_{be1} / i_b + (\beta+1)\, v_{be2} / (\beta+1)\, i_b$$
$$= r_{\pi1} + (\beta+1)\, r_{\pi2}$$

where we have used, from the above circuit, that $v_{be2} = (\beta+1)\, i_b\, r_{\pi2}$.

Then we have

$$r_\pi = r_{\pi1} + (\beta+1)\, r_{\pi2} = 2(\beta+1)\, r_{\pi2}$$

since $r_{\pi1} = (\beta+1)\, r_{\pi2}$.

For the transconductance, we get

$$g_m = \frac{i_c}{v_{be}} = \frac{\beta\,(\beta+1)\, i_b}{i_b\, r_{\pi1} + (\beta+1)\, i_b\, r_{\pi2}}$$
$$= \frac{\beta\,(\beta+1)}{r_{\pi1} + (\beta+1)\, r_{\pi2}}$$
$$= \frac{\beta\,(\beta+1)}{2\,(\beta+1)\, r_{\pi2}}$$
$$= \beta / 2\, r_{\pi2}$$

or we can write

$$g_m = \tfrac{1}{2}\, g_{m2}$$

For the current gain of the composite transistor

$$\beta_C = \beta\,(\beta+1) \approx \beta^2$$

and finally

$$r_o = r_{o2}$$

Note that we still have the relationship, $r_\pi = \beta_C / g_m$, for the composite transistor. From this we get

$$r_\pi = \beta\,(\beta+1) / \tfrac{1}{2}\, g_{m2} = 2\,(\beta+1)\, r_{\pi2}$$

and is just the result we obtained above.

Example 11-1

Consider an npn Darlington transistor pair with single transistor parameters of $\beta = 120$, $V_{BE} = 0.65$ V, and $V_A = 105$ V. If the Q1 (low current) transistor has a base current of 0.2 μA, find the small signal parameters of the composite transistor.

Solution:

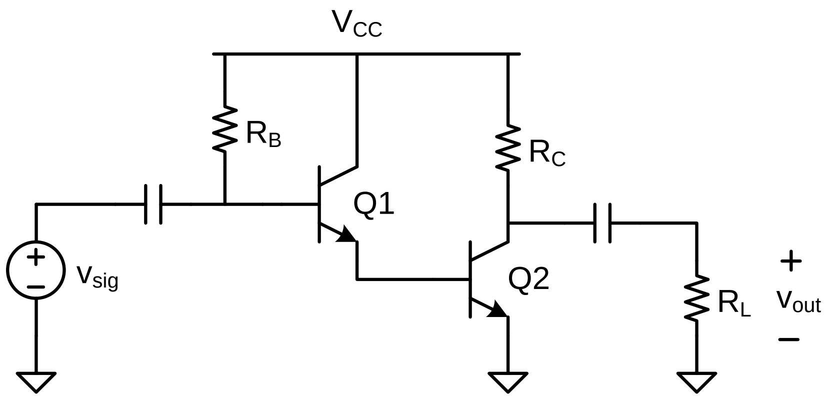
$$I_{B1} = 0.2\ \mu A$$
$$I_{B2} \approx I_{C1} = \beta\, I_{B1} = 120 \bullet 0.2\ \mu A = 24\ \mu A$$
$$I_{C2} = \beta\, I_{B2} = 120 \bullet 24\ \mu A = 2.9\ mA$$

For the composite transistor

$$\beta_C \approx \beta^2 = 120 \bullet 120 = 14400$$
$$g_m = \tfrac{1}{2}\, g_{m2} = \tfrac{1}{2}\, I_{C2} / V_t = \tfrac{1}{2}\, 2.9\ mA / .026\ V = 56\ mA/V$$
$$r_\pi \approx 2\, \beta\, r_{\pi 2} = 2\, \beta^2 / g_{m2} = 2 \bullet 14400 / 112\ mA/V = 257\ k\Omega$$
(we could also use $r_\pi = \beta_C / g_m$)
$$r_o = V_A / I_{C2} = 105\ V / 2.9\ mA = 36\ k\Omega$$

Notice from this example that for a collector current of a few mA, in addition to a <u>very large</u> β_C, we get a <u>very</u> <u>large</u> value of r_π for the Darlington pair. The value of g_m is reduced but still comparable to single transistors. The resistance, r_o, is also comparable. The current gain is large because the base current required at Q1 is only $1/\beta$ of what Q2 needs. This very small base current leads to both a large current gain and a large base-emitter resistance.

The common emitter amplifier with a Darlington pair of transistors is shown below.



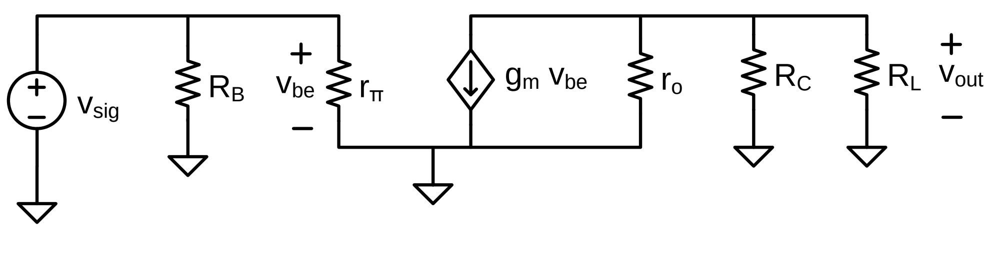
Remembering that the composite transistor base is the base of Q1 and the composite collector is the collector of Q2, we can write the following two equations to calculate the bias point.

$$V_{CC} - I_B R_B - V_{BE1} - V_{BE2} = 0$$

$$V_C = V_{CC} - \beta_C I_B R_C$$

The first equation determines the base current from which we get the collector current, $I_C = \beta_C I_B$. The second equation is used to confirm that we are in active mode for Q2, $V_C > 0.3$ V. Note that the collector connection to V_{CC} precludes saturation of Q1.

The small signal circuit for this amplifier looks just like the single transistor common emitter using Darlington parameters for the single transistor. And it is entirely equivalent to using the two-transistor small signal model for the Darlington.



The linear amplifier model parameters are just what we found for the common emitter amplifier, here using the composite transistor quantities.

$$R_{in} = R_B \,||\, r_\pi = R_B \,||\, 2\,(\beta+1)\, r_{\pi 2}$$

$$R_{out} = R_C \,||\, r_o = R_C \,||\, r_{o2}$$

$$A_v = -\,g_m\, R_{out} = -\,½\, g_{m2}\, R_{out}$$

where

$$g_{m2} = I_{C2} / V_t$$

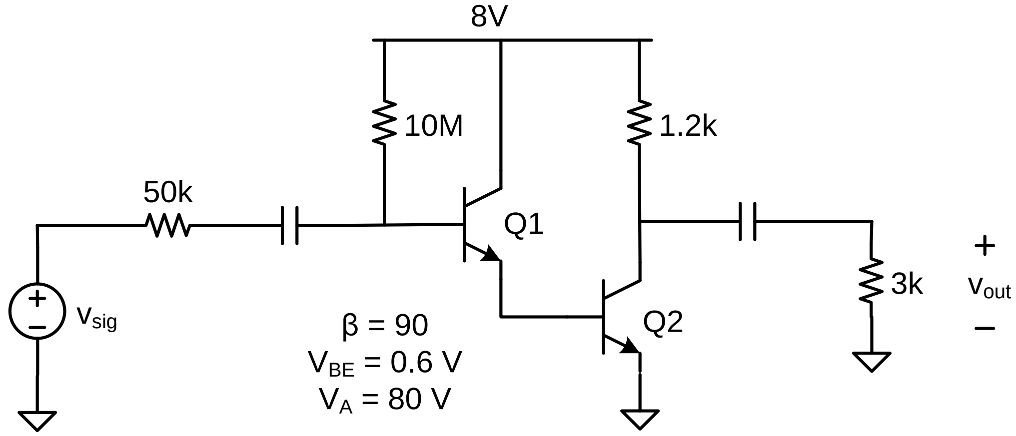
$$r_{\pi 2} = \beta / g_{m2}$$

$$r_{o2} = V_A / I_{C2}$$

This solution for the Darlington pair common emitter amplifier is completely analogous to the single transistor case. The Darlington case, however, has a very large current gain and very large base resistance, typically very favorable properties, but no serious deficiencies.

There are other forms of the Darlington common emitter. Emitter degeneration, with or without bypass, is used for the same reason as in the single transistor version, to facilitate DC biasing when β is uncertain. Using a voltage divider for biasing the base is also used for that reason. In the Darlington case, a voltage divider on the base has the further advantage that larger resistors can be used since avoiding loading the divider is easier with such a small base current.

Example 11-2



Find the linear, unilateral amplifier model parameters and the overall gain.

Solution:

$$I_{B1} = (8\text{ V} - 2 \cdot 0.6\text{ V}) / 10\text{ M}\Omega = 0.68\ \mu\text{A}$$

$$I_{C2} = \beta_C I_{B1} \approx 8100 \cdot 0.68\ \mu\text{A} = 5.5\text{ mA}$$

$$g_m = \tfrac{1}{2} I_{C2} / V_t = \tfrac{1}{2} \cdot 5.5\text{ mA} / .026\text{ V} = 106\text{ mA/V}$$

$$r_\pi = \beta_C / g_m \approx 8100 / 106\text{ mA/V} = 76\text{ k}\Omega$$

$$r_o = V_A / I_{C2} = 80\text{ V} / 5.5\text{ mA} = 15\text{ k}\Omega$$

Then the linear amplifier model parameters are

$$R_{in} = R_B \,||\, r_\pi = 10\text{ M}\Omega \,||\, 76\text{ k}\Omega = 75\text{ k}\Omega$$

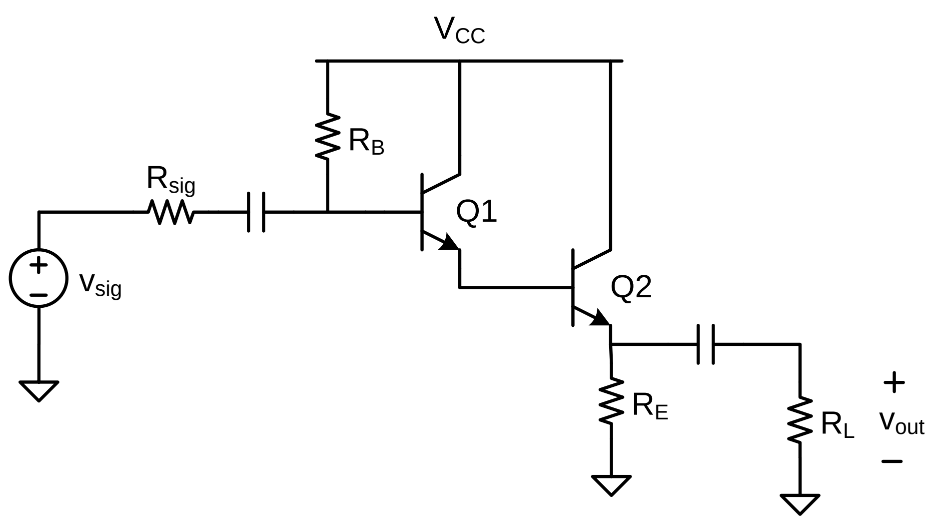
$$R_{out} = R_C \,||\, r_o = 1.2\text{ k}\Omega \,||\, 15\text{ k}\Omega = 1.1\text{ k}\Omega$$

$$A_v = -g_m R_{out} = -106\text{ mA/V}\ 1.1\text{ k}\Omega = -117$$

The overall gain is

$$\begin{aligned} A_{overall} &= A_v [\, R_{in} / (R_{sig} + R_{in}) \,] [\, R_L / (R_{out} + R_L) \,] \\ &= -117 [\, 75\text{ k}\Omega / (50\text{ k}\Omega + 75\text{ k}\Omega) \,] [\, 3\text{ k}\Omega / (1.1\text{ k}\Omega + 3\text{ k}\Omega) \,] \\ &= -51 \end{aligned}$$

The emitter follower amplifier with a Darlington pair is shown below.



where an input signal resistance has been included.

Using the composite transistor parameters for the Darlington pair, we can use our previous results for the emitter follower to write down the Darlington emitter follower results. Recall that the emitter follower is not unilateral, so the results do not apply to that amplifier model.

$$R_{in} = R_B \,||\, [\, r_\pi + (\beta_C+1)\,(R_E \,||\, R_L \,||\, r_o)\,]$$

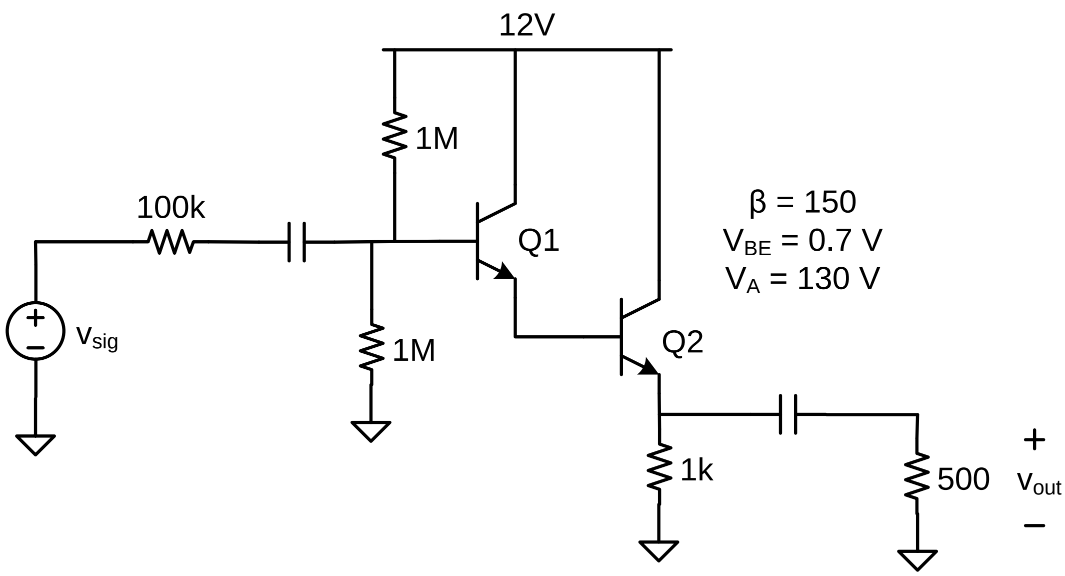
$$R_{out} = \left(\frac{r_\pi + R_{sig} \,||\, R_B}{\beta_C+1}\right) \,||\, (R_E \,||\, r_o)$$

$$A_{overall} = g_m\,(R_E \,||\, R_L \,||\, r_o \,||\, 1/g_m\,)\,\frac{R_{in}}{R_{sig} + R_{in}}$$

where r_π, g_m, r_o and β_c are composite parameters.

The voltage gain is again less than one. The primary advantages are the input resistance which can be very, very large because r_π is so large, and the relatively low output resistance. Applications requiring large input resistance are typical.

Example 11-3



Find the overall gain for this emitter follower.

Solution:

First we will make the assumption that the base current is much smaller that the base divider current. Then we can try an approximate DC solution.

$$V_{B1} = 12\text{ V} \cdot 1\text{ M}\Omega / 2\text{ M}\Omega = 6\text{ V}$$
$$V_{E2} = 6\text{ V} - 2 \cdot 0.7\text{ V} = 4.6\text{ V}$$
$$|I_E| = 4.6\text{ V} / 1\text{ k}\Omega = 4.6\text{ mA}$$

Then we get

$$I_{B1} \approx 4.6\text{ mA} / \beta^2 = 0.20\ \mu\text{A}$$

while

$$I_{divider} = 12\text{ V} / 2\text{ M}\Omega = 6\ \mu\text{A}$$

and we see

$$I_{B1} << I_{divider}$$

So our assumption about I_{B1} and $I_{divider}$ is justified and our calculation of V_{B1} is very nearly correct.

Now for the composite small signal parameters

$$g_m = \tfrac{1}{2} g_{m2} = \tfrac{1}{2} I_{C2} / V_t = 88\text{ mA/V}$$
$$r_\pi \approx 2\beta r_{\pi 2} = 2\beta^2 / g_{m2} = 250\text{ k}\Omega$$
$$r_o = r_{o2} = V_A / I_{C2} = 28\text{ k}\Omega$$

Then for the input resistance

$$R_{in} = R_B \,||\, [\, r_\pi + (\beta_C + 1)(R_E \,||\, R_L \,||\, r_o)\,] = 470\text{ k}\Omega$$

where we have used

$$R_B = R_{B1} \,||\, R_{B2} = 1\text{ M}\Omega \,||\, 1\text{ M}\Omega = 500\text{ k}\Omega.$$

Finally, the overall gain is

$$A_{overall} = g_m \, (R_E \,||\, R_L \,||\, r_o \,||\, 1/g_m) \, R_{in} \,/\, (R_{sig} + R_{in})$$
$$= 0.80$$

Note that without the follower amplifier, the signal delivered to the load directly would have been about 500 Ω / 100 kΩ = 0.5% rather than 80%. The input resistance of 470 kΩ makes a very substantial difference even when the voltage gain of the emitter follower is less than one.
