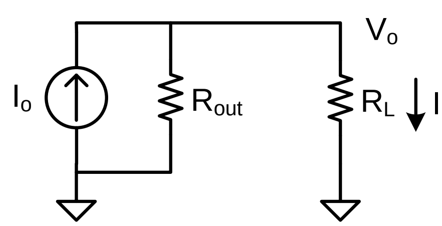
14 Current Mirrors

An ideal current source is a device that provides a fixed current into any load. That means the same current is delivered regardless of the voltage at the current source output necessary to deliver it.

$V_o = I_o R_L$

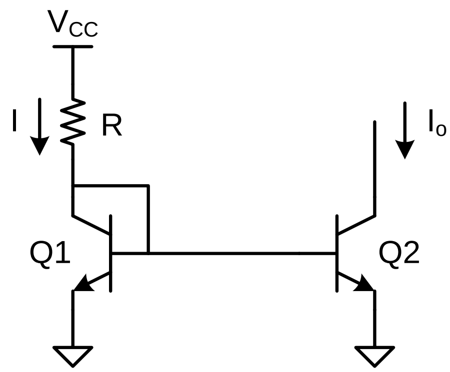
V_o becomes arbitrarily large for larger and larger load resistance.

A real current source has finite output resistance and will deliver a smaller current as the load resistance is increased. The current source is conveniently modeled as a Norton equivalent circuit where R_{out} is the output resistance of the current source.



$$I = I_o [R_{out} / (R_{out} + R_L)] < I_o$$
$$I = I_o \text{ for } R_{out} \rightarrow \infty$$

A current mirror is one common implementation of a current source. There are a number of versions, the simplest of which is shown below. It is made from a pair of matched npn transistors.



This common form of a current source might more correctly be called a current sink because it draws current from a load rather than provide current to a load. We'll see later another current source that does, indeed, deliver current to a load. In either case, these devices are referred to as current sources.

For the circuit above, the current source output at the collector of Q2 sinks current I_o. As long as Q2 remains in the active mode, the collector current, I_o, is relatively insensitive to collector voltage. That tells us this current source has a relatively high output resistance.

The output current, I_o, is controlled by transistor, Q1, and resistor, R. Q1 is in active mode although it is near saturation with $V_{CE} = V_{BE} = 0.6 – 0.7$ V.

The current, I, is found from

$$V_{CC} - I\,R - V_{BE} = 0$$
$$I = (V_{CC} - V_{BE}) / R$$

Q1 and Q2 have the same base-emitter voltage and, because they are matched transistors, will have the same collector current except for the small effects of transistor output resistance, r_o. The collector voltages, V_{C1} and V_{C2}, are, in general, different. $V_{C1} = V_{BE}$ while $V_{C2} > 0.3$ V. But note also that I is the total of the collector current of Q1 and the base current of both transistors while I_o is just the collector current of Q2.

$$I = I_C + 2\,I_B$$

where I_C is the collector current of Q1. Since the collector currents are very nearly equal, we write $I_C = I_o$ and get

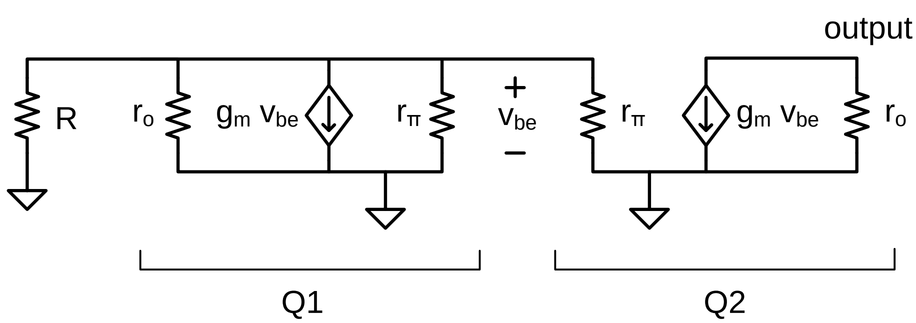
$$I = I_o + 2\,I_B = I_o + 2\,I_o / \beta$$

Solving for I_o, we have

$$I_o = \frac{I}{1 + 2/\beta} \approx I$$

The current on the left side of the circuit, I, is <u>mirrored</u> to the right side and the output, I_o.

The small signal circuit for the current mirror is shown here.

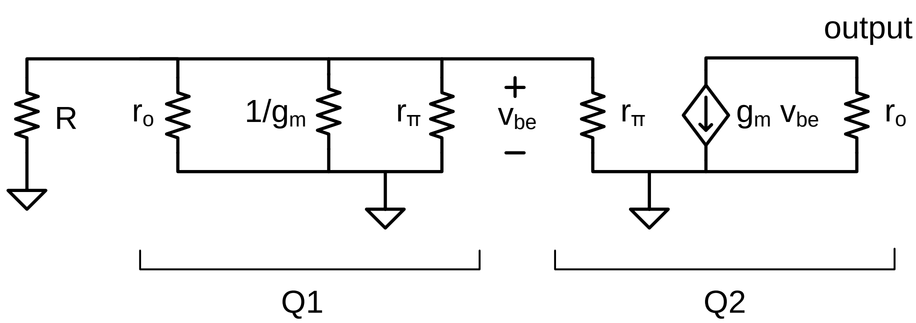


Transistor parameters are matched with the same collector current, and v_{be} is common to the two devices. The output is a DC current and does not appear in the small signal circuit although the output node is indicated. We are interested in the response to changes in the voltage at the output node, that is, in the output resistance of the current source. That output resistance is just the resistance seen at the output node.

$$R_{out} = r_o = V_A / I_C$$

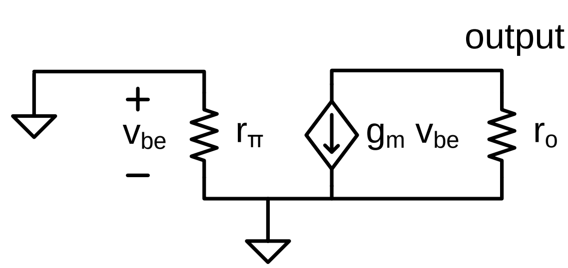
where a test voltage applied to the output node sees r_o to ground and does not activate the current source. This output resistance can typically be as large as 50 – 100 kΩ, depending on output current, making the two-transistor current mirror a reasonable current source in many applications.

We note that the base-collector connection of Q1 is a common configuration for current source transistors. Such a configuration can be approximated as a ground connection at the transistor base. The reasoning for this is as follows. The current source, $g_m v_{be}$, of Q1 is controlled by the voltage across r_π which is also the voltage across the current source itself because of the base-collector short. The equation, $i_c = g_m v_{be}$, then takes on a new meaning. If v_{be} is the voltage across the current source carrying current, i_c, then this is just an Ohm's Law relationship for a resistor, $1/g_m$. So we replace the current source with a resistor, $1/g_m$, when this base-collector short is present.



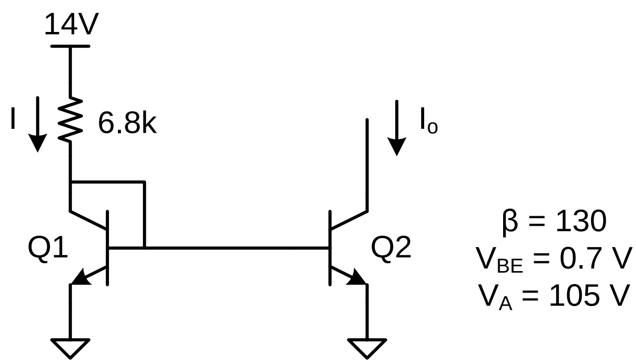
The value of $1/g_m$ is usually quite small, often on the order of 10 Ω. The path through Q1 from the base of Q2 is then a low resistance and can be approximated by a ground connection. This will often simplify our analysis of current mirror small signal circuits.

Our two-transistor current mirror now has the approximate small signal circuit shown below where Q1 is replaced by a ground connection at the Q2 base.



Example 14-1

Find I, I_o, and the percent difference between I and I_o for the current source shown below. Ignore the effects of different collector voltage. Determine the value of R_{out}.



Solution:

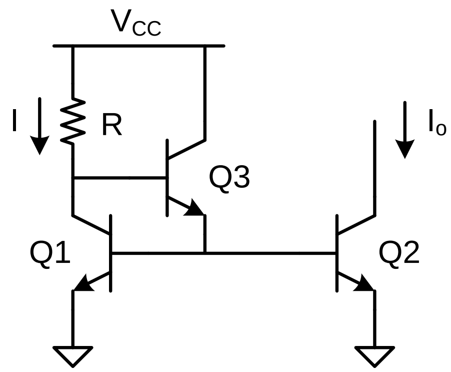
$I = (V_{CC} - V_{BE}) / R = 1.956$ mA

$I_o = I / (1 + 2/\beta) = 1.926$ mA

Percent difference = 100 (0.03 mA / 1.956 mA) = 1.5 %

$R_{out} = r_o = V_A / I_C = 55\ k\Omega$

Another version of the current mirror is shown below.



In this circuit, a third npn transistor is added to deliver most of the base current to the two mirror transistors directly from the supply voltage rather than taking it through resistor R. A small amount of base current for Q3 is still drawn through resistor R, but only 1/ (β+1) of the current required at the bases of Q1 and Q2. The current I is now given by

$$I = (V_{CC} - 2\, V_{BE}) / R$$

since the collector of Q1 is held at 2 V_{BE}. Then we have, taking the collector current of Q1 equal to that of Q2 = I_o,

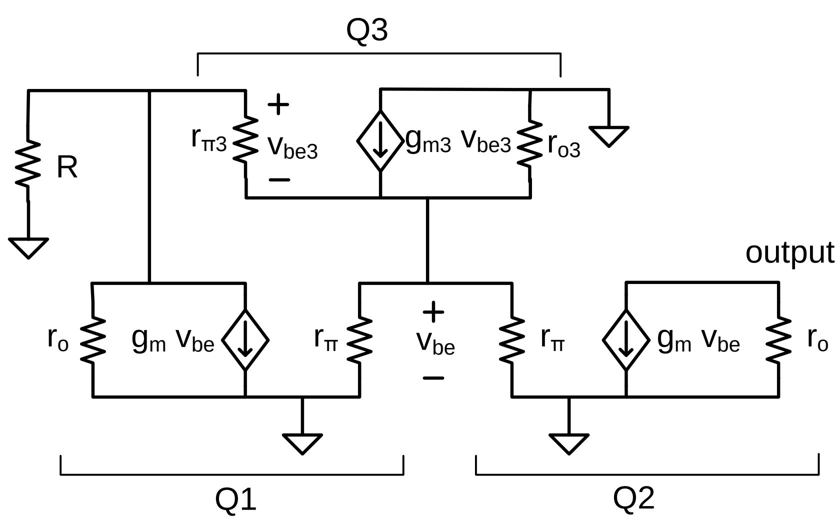
$$\begin{aligned} I &= I_o + I_{B3} \\ &= I_o + [\, 1 / (\beta+1)\,]\, |I_{E3}| \\ &= I_o + [1 / (\beta+1)]\, 2\, I_{B1,2} \\ &= I_o + [1 / (\beta+1)]\, 2\, I_o / \beta \end{aligned}$$

Finally, solving for I_o,

$$I_o = \frac{I}{1 + 2 / \beta\, (\beta+1)} \approx \frac{I}{1 + 2 / \beta^2} \approx I$$

The output current, I_o, more closely matches the left-side current, I, than for our previous two-transistor current mirror where 1 + 2 / β in the previous expression has been replaced by 1 + 2 / β^2 in this relationship. This means we have less sensitivity to β which can be important under close matching requirements.

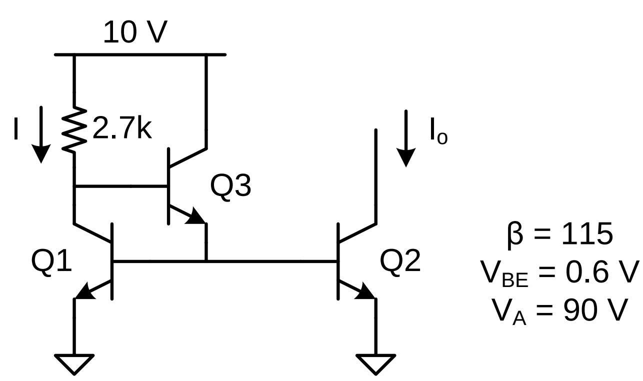
The small signal circuit for this current mirror is shown below.



In this circuit, $g_{m1} = g_{m2} = g_m$, $r_{o1} = r_{o2} = r_o$, and $r_{\pi 1} = r_{\pi 2} = r_\pi$. We easily see that the output resistance is again just r_o.

But we note, for future purposes, that the circuit can again be approximated with a ground connection at the Q2 base, this time replacing both Q1 and Q3. Although the Q1 base-collector short does not exist in this configuration, the base-emitter of Q3 is a relatively low resistance path to ground for Q2 and we get a similar result. Specifically, a small voltage applied to the node that connects the Q1 base and Q3 emitter results in a relatively large current into that node. The magniduce of the resistance at that node can be roughly calculated as follows. An initial voltage, v_x, applied to the Q1 base results in Q1 base current, v_x / r_π, and Q1 collector current, $\beta\, v_x / r_\pi$. This results in a fall in the Q1 collector voltage, $- R\, \beta\, v_x / r_\pi$, as that collector current is drawn through resistor, R. But that, in turn, would impose a reverse voltage across $r_{\pi 3}$ and a Q3 collector current, $\beta\, R\, (\beta\, v_x / r_\pi) / r_{\pi 3}$, delivered to the gounded Q3 collector node. The result is an approximate effective resistance at the original Q1 base node of $v_x / [\, \beta\, R\, (\beta\, v_x / r_\pi) / r_{\pi 3}\,] = r_\pi\, r_{\pi 3} / R\, \beta^2$. The β^2 term in the denominator makes this a very small resistance and justifies the virtual ground approximation.

Example 14-2



Find the output current, I_o, and the output resistance, R_{out}. What is the percent difference between I and I_o?

Solution:

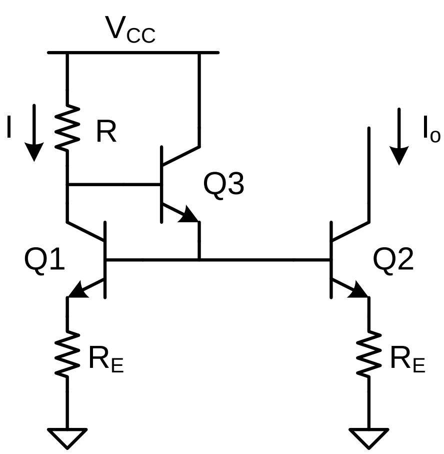
$$I_o \approx I = (V_{CC} - 2V_{BE}) / R = 8.8\text{ V} / 2.7\text{ k}\Omega = 3.26\text{ mA}$$

$$R_{out} = r_o = V_A / I_C = 90\text{ V} / 3.26\text{ mA} = 28\text{ k}\Omega$$

The relative difference between I and I_o is

$$(I - I_o) / I = 1 - 1 / (1 + 2/\beta^2) \approx 2/\beta^2 = .00015 = 0.015\%$$

Another implementation of the current mirror that has higher output resistance than either of the versions so far considered is shown below. Matched resistors have been added in the emitters of transistors, Q1 and Q2.



The relationship between I and I_o is identical to that for the previous three-transistor version.

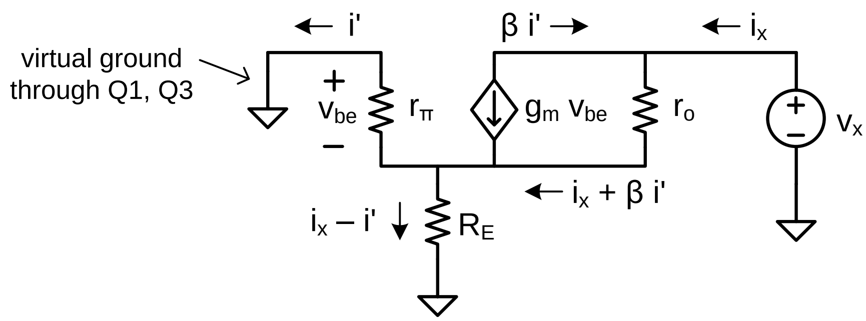
$$I_o \approx \frac{I}{1 + 2/\beta^2} \approx I$$

But in this case, I_o is given by

$$I_o \approx (V_{CC} - 2V_{BE}) / (R + R_E)$$

For the output resistance we can again approximate the small signal circuit by using the fact that, at the base of Q2, looking into the base of Q1 and the emitter of Q3, there is effectively a low resistance path to ground. For the previous three-transistor current mirror, we made an approximate calculation of that resistance. For the present case, we have the addition of the emitter resistor, R_E. This is typically a relatively small resistance and does not substantially change the situation.

The small signal circuit for Q2 then is approximated by the following, including a test source, v_x, at the output.



$$v_x = (i_x + \beta i') r_o + (i_x - i') R_E$$

$$(i_x - i') R_E = i' r_\pi$$

Combining these to eliminate i', results in

$$v_x = i_x [r_o (1 + g_m r_\pi \| R_E) + r_\pi \| R_E]$$

$$R_{out} = v_x / i_x = r_o (1 + g_m r_\pi \| R_E) + r_\pi \| R_E$$

$$\approx r_o (1 + g_m r_\pi \| R_E)$$

where we have used $\beta = r_\pi g_m$. This can be a very large output resistance, potentially an order of magnitude, or more, larger than r_o. With this design we are closer to the ideal case where $R_{out} \rightarrow \infty$.

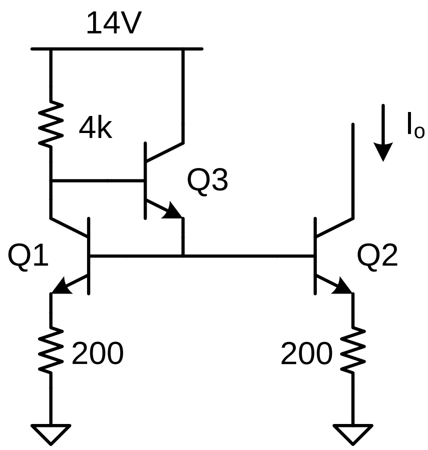
The addition of emitter resistors to the three-transistor current mirror is documented below using B2spice with V_{CC} = 10 V, I_o = 1 mA, R_E = 1 kΩ, and 2N2222 transistors.

Output Resistance

	calculated	simulation
without R_E	102 kΩ	102 kΩ
with R_E	3.5 MΩ	3.4 MΩ

Example 14-3

Find the output current and output resistance of this current mirror.



$\beta = 120$
$V_{BE} = 0.65$ V
$V_A = 110$ V

Solution:

$I_o = (14\text{ V} - 2 \cdot 0.65\text{ V}) / 4.2\text{ k}\Omega = 3.0\text{ mA}$

$g_m = 3.0\text{ mA} / .026\text{ V} = 115\text{ mA/V}$

$r_\pi = 120 / 115\text{ mA/V} = 1.0\text{ k}\Omega$

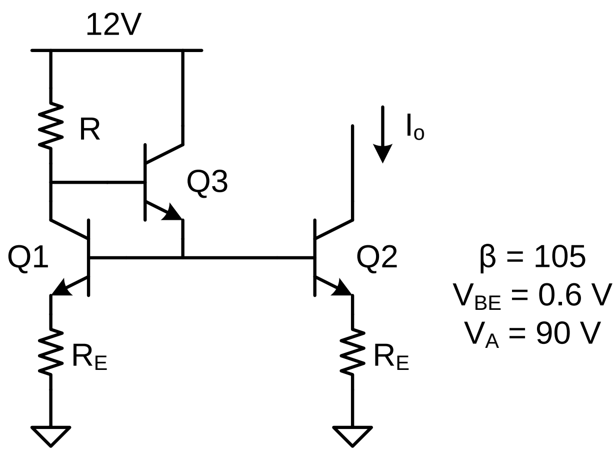
$r_o = 110\text{ V} / 3.0\text{ mA} = 37\text{ k}\Omega$

$R_{out} = r_o (1 + g_m\, r_\pi \,||\, R_E)$

$= 37\text{ k}\Omega\,[\,1 + 115\text{ mA/V}\;(1.0\text{ k}\Omega \,||\, 200\,\Omega)\,]$

$= 750\text{ k}\Omega$

Example 14-4



Design this circuit to have $I_o = 1.8$ mA and $R_{out} \geq 3$ MΩ.

Solution:

$$I_o = (12\text{ V} - 2 \cdot 0.6\text{ V}) / (R + R_E) = 1.8\text{ mA}$$

$$R + R_E = 10.8\text{ V} / 1.8\text{ mA} = 6\text{ k}\Omega$$

This determines the sum of the resistors. Next we find R_E. R_{out} is given by $r_o (1 + g_m\, r_\pi \,||\, R_E)$ but for $I_C = 1.8$ mA, g_m, r_π, r_o can all be calculated. We only have R_E to adjust to make our specification for R_{out}.

$$g_m = I_C / V_t = 69\text{ mA/V}$$

$$r_\pi = \beta / g_m = 1.5\text{ k}\Omega$$

$$r_o = V_A / I_C = 50\text{ k}\Omega$$

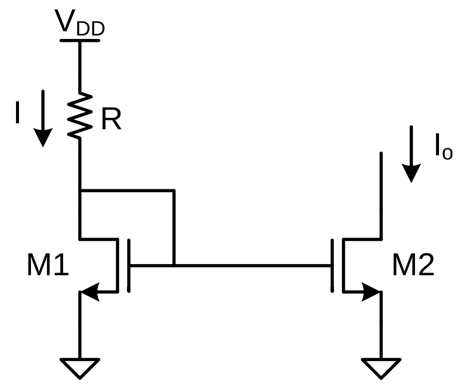
$$R_{out} = r_o (1 + g_m\, r_\pi \,||\, R_E)$$

$$= 50\text{ k}\Omega\, (1 + 69\text{ mA/V } 1.5\text{ k}\Omega \,||\, R_E) \geq 3\text{ M}\Omega$$

$$1.5\text{ k}\Omega \,||\, R_E \geq 855\ \Omega$$

$R_E \geq 2.0$ kΩ. If $R_E = 2$ kΩ, then $R = 4$ kΩ

A MOSFET current mirror is shown below. The variation using a third transistor to supply the gate of M1 and M2 is of no value here since there is no gate current.



The transistors are matched and we have $I_o = I$ without any gate current. Only the effect of output resistance of M2 will cause I_o to differ from I.

Transistor M1 remains in the correct operating region (saturation) since $V_{GD} = 0 < V_{th}$.

$$I = \mu_n C_{ox} (W / L) \tfrac{1}{2} (V_G - V_{th})^2$$

$$I = (V_{DD} - V_G) / R$$

Combining these

$$I = \mu_n C_{ox} (W / L) \tfrac{1}{2} (V_{DD} - I R - V_{th})^2$$

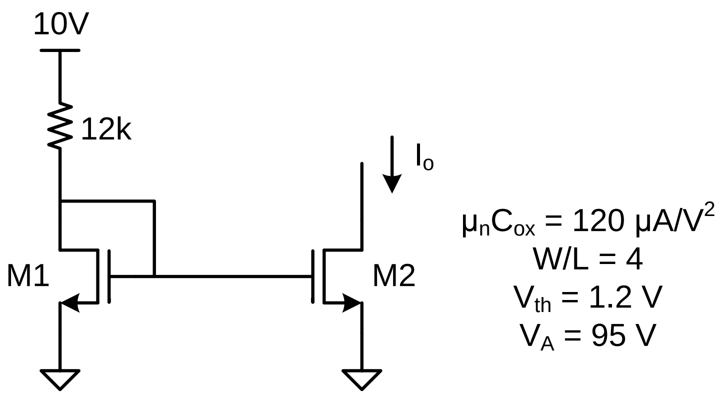
We then solve this quadratic equation for $I = I_o$ and choose the solution that is consistent with the transistor turned on in saturation mode. Output resistance is again $R_{out} = r_o$.

Like the BJT case, we can add source resistors to increase output resistance. The result is very similar to what we saw in that case.

$$R_{out} \approx r_o (1 + g_m R_S)$$

but the generally smaller value of g_m for MOSFETs provides a considerably smaller increase in output resistance than for BJTs.

Example 14-5



Find the current output and the output resistance of the MOSFET current source.

Solution:

For M1, we have

$$V_G = V_{DD} - I_D R_D = 10\text{ V} - I_D\, 12\text{ k}\Omega$$

$$I_D = \mu_n C_{ox} (W/L)\, ½\, (V_{GS} - V_{th})^2$$

$$= 0.120\text{ mA/V}^2\, 4 \bullet ½\, (10\text{ V} - I_D\, 12\text{ k}\Omega - 1.2\text{ V})^2$$

$$= 0.24\text{ mA/V}^2\, (8.8\text{ V} - I_D\, 12\text{ k}\Omega)^2$$

$$0 = 77 - 215\, I_D + 144\, I_D^2$$

Solving this quadratic, we get

$$I_D = 0.60\text{ mA and } 0.89\text{ mA}$$

The second solution is not valid since, in that case,

$$V_{GS} = 10\text{ V} - 12\text{ k}\Omega\, 0.89\text{ mA} = -0.7\text{ V}$$

and the transistor not conducting.

For the first solution we get

$$V_G = 10\text{ V} - 12\text{ k}\Omega\ 0.60\text{ mA} = 2.8\text{ V} > V_{th}$$

The transistor is turned on in this case. And it must be in saturation mode since $V_{GD} = 0$. The output resistance is

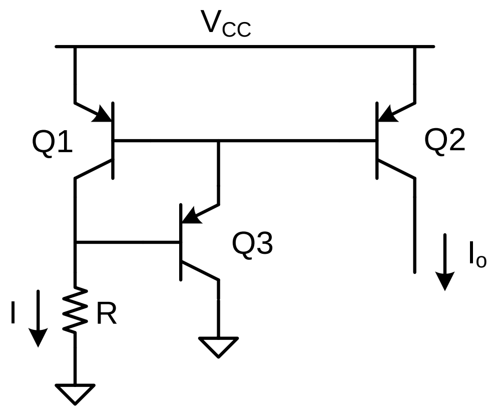
$$R_{out} = V_A / I_D = 95\text{ V} / 0.6\text{ mA} = 158\text{ k}\Omega$$

To construct a current mirror that will source current rather than sink current, we must use pnp bipolar transistors or p-channel MOSFETs. In the bipolar case, pnp transistors conduct a current of majority holes from the emitter injected into the base and then swept to the collector. Voltages are reversed compared to npn transistors and the emitter-base junction is biased with the emitter about 0.6 – 0.7V higher than the base. The collector has a lower voltage than the emitter to reverse bias the collector-base junction. Emitter-base voltage, as in the npn case, controls the main current flow from emitter to collector where I_C and V_{BE} have the familiar exponential relationship, and $I_C = \beta\ I_B$.

The p-channel MOSFET has much the same relationship to the n-channel device. Majority hole current flows from the source (at a higher voltage) to the drain (at a lower voltage). The gate voltage must be at least a threshold voltage below the source for the transistor to be turned on and less than a threshold voltage below the drain to be in the saturation mode.

In both the pnp BJT case and the p-channel MOSFET case, the “opposite” direction of current flow in these devices makes them appropriate to source rather than sink current.

The BJT version of a current source using pnp transistors with a third transistor to supply base currents is shown below.

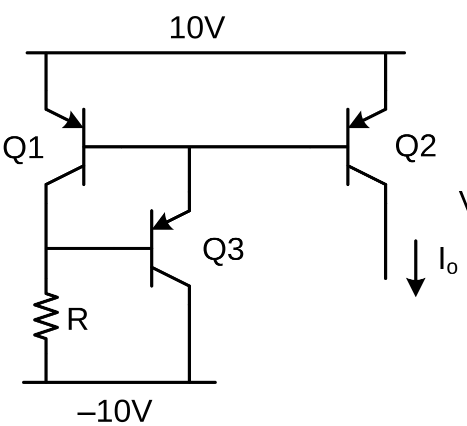


The voltage across R is 2 V_{EB} less than V_{CC}, and I_o is very nearly I.

$$I_o \approx I = (V_{CC} - 2V_{EB}) / R$$

The output resistance is r_o of Q2. Matched emitter resistors added to this circuit will again greatly increase the output resistance and results are similar to what we found for the npn version.

Example 14-6



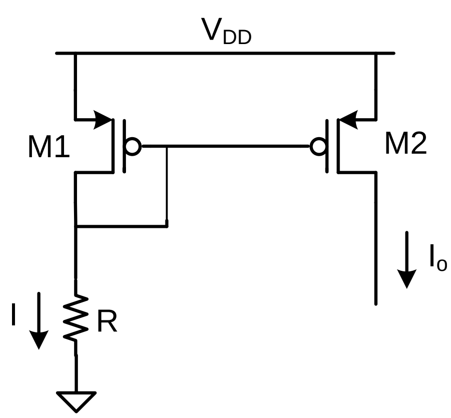
Find the value of R for I_o = 1.5 mA. What is R_{out}?

Solution:

$$R = (V_{CC} - 2\,V_{EB} - V_{EE}) / I_o$$
$$= [\,10\text{ V} - 1.4\text{ V} - (-10\text{ V})\,] / 1.5\text{ mA} = 12.4\text{ k}\Omega$$

$$R_{out} = r_o = V_A / I_C$$
$$= 85\text{ V} / 1.5\text{ mA} = 57\text{ k}\Omega$$

The p-channel MOSFET version of a current source, using two transistors, is a direct extension of the n-channel MOSFET current sink and is constructed like the pnp current source.



The current, I, is calculated as the drain current of M1 and the output resistance is r_o of M2.