Matrix Algebra Homework 2.4 Applications
TMATH 308

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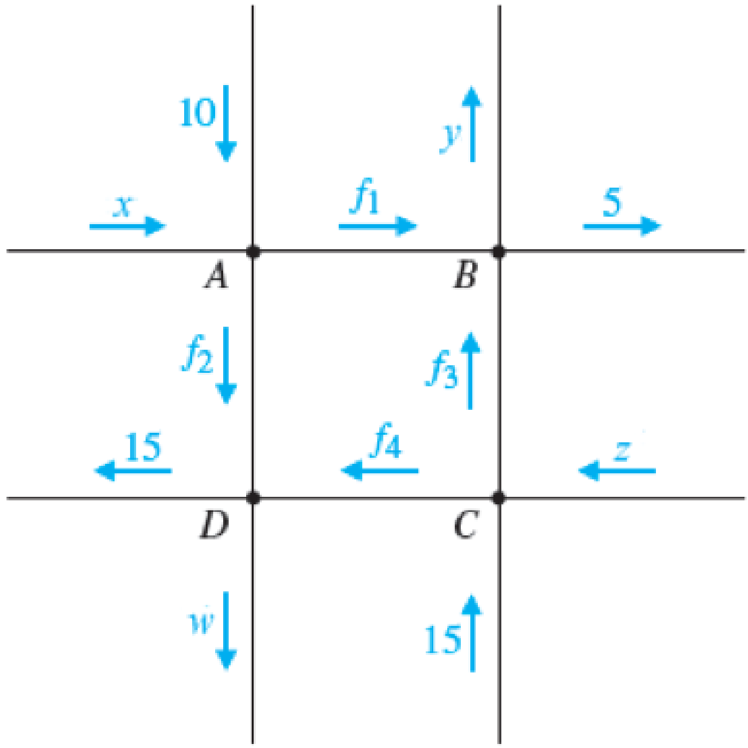
Contents

Problem 10

The downtown core of Gotham City consists of one-way streets, and the traffic flow has been measured at each intersection. For the city block shown in the figure, the numbers represent the average numbers of vehicles per minute entering and leaving intersections A, B, C, and D during business hours. Let $w = 20$, $x = 20$, $y = 30$, and $z = 25$.



(a) Set up and solve a system of linear equations to find the possible flows $f_1, ..., f_4$. (Use the parameter t as necessary.)

Answer

$$10 + 20 = f_1 + f_2$$
$$30 + 5 = f_1 + f_3$$
$$15 + 25 = f_3 + f_4$$
$$15 + 20 = f_2 + f_4$$

$$f_1 + f_2 = 30$$
$$f_1 + f_3 = 35$$
$$f_3 + f_4 = 40$$
$$f_2 + f_4 = 35$$

Now we can make a table from the equations:

$$\left.\begin{array}{cccc|c} 1 & 1 & 0 & 0 & 30 \\ 1 & 0 & 1 & 0 & 35 \\ 0 & 0 & 1 & 1 & 40 \\ 0 & 1 & 0 & 1 & 35 \end{array}\right.$$

In Row Reduced Echelon Form:

$$\left[\begin{array}{cccc|c} 1 & 0 & 0 & -1 & -5 \\ 0 & 1 & 0 & 1 & 35 \\ 0 & 0 & 1 & 1 & 40 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right.$$

$$(f_1, f_2, f_3, f_4) = (t - 5, 35 - t, 40 - t, t)$$

(b) If traffic is regulated on CD so that $f_4 = 15$ vehicles per minute, what will the average flows on the other streets be?

Answer

$$f_4 = t = 15$$
$$f_1 = t - 5 = 10$$
$$f_2 = 35 - t = 20$$
$$f_3 = 40 - t = 25$$

(c) What are the minimum and maximum possible flows on each street?

Answer Since the streets are one way, then we can't have any negative numbers. After doing a little analysis, the minimum that f_4 could be is zero, and we'll have to go to one of the other equations to see what the maximum will be. Looking at the equation for f_1, we can see that the minimum for t or f_4 can actually be 5 since anything less would cause a negative number. We can't tell the maximum from this equation. Looking at the equation for f_2, if t were larger than 35, then we would get a negative number. Looking at the equation for f_3, we can see that if t were larger than 40, then we would have a negative number, but we also already know that it can't be larger than 35. Therefore:

$$0 \leq f_1 \leq 30$$
$$0 \leq f_2 \leq 30$$
$$5 \leq f_3 \leq 35$$
$$5 \leq f_4 \leq 35$$

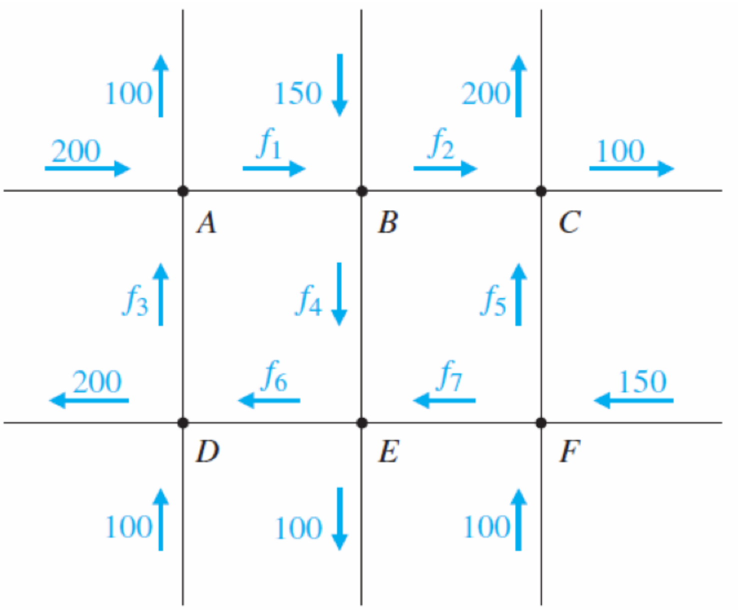
(d) How would the solution change if all of the directions were reversed?

Answer

If all the directions were reversed, then there would be no effect because they are all one way streets and for each intersection, the two vectors that are going into a specific intersection and the two that are coming out of that same intersection will still either be leaving or going in with the same relation.

Problem 11

Consider the following figure.



(a) Set up and solve a system of linear equations to find the possible flows in the network shown in the figure. (Use the parameters s and t as necessary.)

Answer

$$200 - 100 = f_1 - f_3$$
$$f_1 + 150 = f_2 + f_4$$
$$100 + 200 = f_2 + f_5$$
$$100 - 200 = f_3 - f_6$$
$$f_4 + f_7 = f_6 + 100$$
$$100 + 150 = f_5 + f_7$$

$$f_1 - f_3 = 100$$
$$f_1 - f_2 - f_4 = -150$$
$$f_2 + f_5 = 300$$
$$f_3 - f_6 = -100$$
$$f_4 - f_6 + f_7 = 100$$
$$f_5 + f_7 = 250$$

Now we can make a table from the equations:

$$\left[\begin{array}{ccccccc|c} 1 & 0 & -1 & 0 & 0 & 0 & 0 & 100 \\ 1 & -1 & 0 & -1 & 0 & 0 & 0 & -150 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 & 300 \\ 0 & 0 & 1 & 0 & 0 & -1 & 0 & -100 \\ 0 & 0 & 0 & 1 & 0 & -1 & 1 & 100 \\ 0 & 0 & 0 & 0 & 1 & 0 & 1 & 250 \end{array}\right]$$

In Row Reduced Echelon Form:

$$\left[\begin{array}{ccccccc|c} 1 & 0 & 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & -1 & 50 \\ 0 & 0 & 1 & 0 & 0 & -1 & 0 & -100 \\ 0 & 0 & 0 & 1 & 0 & -1 & 1 & 100 \\ 0 & 0 & 0 & 0 & 1 & 0 & 1 & 250 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

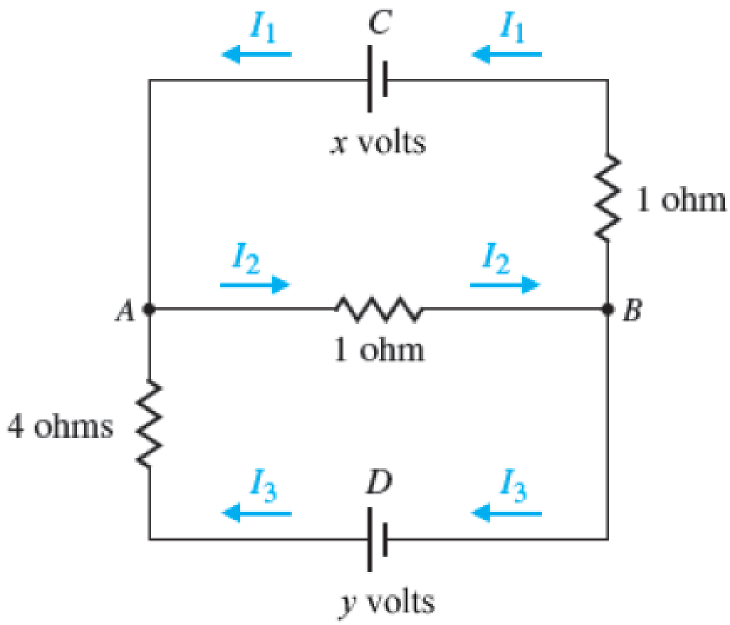
$$(f_1, f_2, f_3, f_4, f_5, f_6, f_7) = (s, 50 + t, s - 100, 100 - t + s, 250 - t, s, t)$$

(b) Is it possible for $f_1 = 130$ and $f_6 = 140$? [Answer this question first with reference to your solution in part (a) and then directly from the figure.]

Answer
Looking at the solution from part a, we can see that it is not possible to have $f_1 = 130$ and $f_6 = 140$ because $f_1 = f_6$. If we look at the figure though, we are unable to tell.

Problem 12

Determine the current for the given electrical network. (Assume x = 8 and y = 31.)



Answer
From analyzing the picture we get the equations:

$$I_1 - I_2 + I_3 = 0$$

$$I_1 + I_2 = 6$$

$$I_2 + 4I_3 = 12$$

From this we can put these into a matrix:

$$\begin{array}{ccc|c} 1 & -1 & 1 & 0 \\ 1 & 1 & 0 & 6 \\ 0 & 1 & 4 & 12 \end{array}$$

From this, we get the Row Reduced Echelon form:

$$\begin{array}{ccc|c} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & 4 \\ 0 & 0 & 1 & 2 \end{array}$$

And so:
$I_1 = 2$
$I_2 = 4$
$I_3 = 2$

Problem 5

Balance the chemical equation for the reaction.

$$FeS_2 + O_2 \rightarrow Fe_2O_3 + SO_2$$

First we get our equations:
$Fe : 1 + 0 - 2 = 0$
$S : 2 + 0 - 0 = 1$
$O : 0 + 2 - 3 = 2$
And from this we can get a table:

$$\left.\begin{array}{ccc|c} 1 & 0 & -2 & 0 \\ 2 & 0 & 0 & 1 \\ 0 & 2 & -3 & 2 \end{array}\right.$$

After we get the Row Reduced Echelon Form:

$$\left.\begin{array}{ccc|c} 1 & 0 & 0 & \frac{1}{2} \\ 0 & 1 & 0 & \frac{11}{8} \\ 0 & 0 & 1 & \frac{1}{4} \end{array}\right.$$

Now we just have to multiply each of the answers by 8 so that we don't end up with fractions.
first coefficient: 4
second coefficient: 11
third coefficient: 2
fourth coefficient: 8

Therefore:

$$4(FeS_2) + 11(O_2) \rightarrow 2(Fe_2O_3) + 8(SO_2)$$