EE 400D

Electrical Engineering Design Seminar and Project

Control Algorithm
Stable Walking of Biped Robots

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Abstract

This paper deals with the theory behind the control algorithm for the stability of walking biped robots. Many control algorithms have been implemented to perform stable walking without using any pre-computed trajectories. Controlling the direction of balance for biped walking robot such as ROFI would essentially imitate the human walking form. The goal of ROFI's next generation robot is to maintain an upright torso while advancing one leg in front of the other in a continuous motion. When designing the control of the robot, the mechanical structure has to be analyzed carefully because of factors such as the number of degrees of freedom, weight of legs, and appropriate velocity and acceleration. By increasing the degrees of freedom, the robot becomes smoother but the complexity to control the robot increases as well. However, in order to fully develop a proper walking control algorithm, one has to consider the uneven and inclined floor. The current walking control techniques available that have been developed assume that the walking surface is perfectly flat with no inclination. Any slight unevenness of a floor can cause the robot to become unstable. In this paper, the author examines and suggests a control algorithm that has been developed by engineers from the research center Humanoid Robot Research Center stationed in South Korea. Also, the biped robot is modeled using the classical control model, the inverted pendulum.

Introduction

In our senior design course, EE 400D, our group assembled a biped robot with 12 degrees of freedom. Many similar biped robots have been developed, such as the Honda's humanoid robot Asimo. The goal of each robot is to have a stable control system so that the robot can perform a stable and reliable walking motion that resembles the human form. The currents robots available demonstrate an unnatural walking motion. The unnatural walking can be due to the weight of the robot. Understanding where to place the center of mass of the robot is important in establishing a robust system. Also, the number of degrees of freedom improve the steadiness and smoothness of the walking as well. As explained earlier, the complexity increases with the increase in degrees of freedom. Therefore, the control of a biped robot poses several challenging problems to designers. Most biped designs have a high center of mass which is supported by two feet. Many believe this is the essential reason why balancing and stabilizing a robot continues to create problems. Even after achieving an acceptable control system, several if not most robots appear to walk awkwardly with slow velocities. The walking control is performed on two levels: computation of the gait trajectories and local control along them. The overall stability or balancing should be considered on both levels [1]. On the first level, the synergy method and methods based on optimization are often used [7] [8]. By using the synergy method, most of the joint's trajectories are created and the remaining trajectories are calculated relative to the stability of the robot. The second level controllers calculate pre-computed trajectories for the robot's actuators. Being able to establish a coexisting relationship between the two levels should implement an acceptable human-like walking motion. If the dynamic balance can be maintained, dynamic walking is smoother and more active. Inertial forces must be controlled. If the inertial forces generated from the acceleration of the robot's body are not controlled, the robot will fall down. By modeling the robot with a 3D application like Solidworks, the masses and torques can be calculated.

Control Algorithm

1. Inverted Pendulum:

The inverted pendulum is a classical problem in dynamics and control theory and is widely used as a benchmark for testing control algorithms. The inverted pendulum is modeled using linear methods even though the system is essentially non-linear. The physical system consists of a cart, driven by a DC motor, and a pendulum attached to the cart as shown in the figure below [2]. The pendulum cannot maintain balance and will fall towards its downward position.

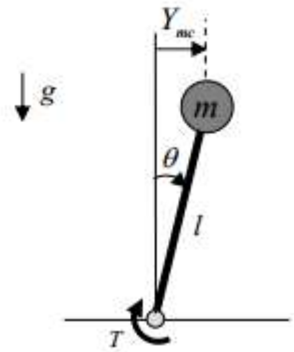

Figure 1 – Inverted Pendulum Model (Kim, Park, and Oh).

The objective of the inverted pendulum experiment is to keep the pendulum in an upright position when impulses or fluctuations in the horizontal direction occur. This objective has to be achieved with the understanding that the equilibrium at $\Theta = 0°$ is marginally stable. As the DC motor has only limited power and the track has finite length, there exist certain states of the physical system from which the pendulum cannot be steered back to the upright position. Therefore, certain parameters characterize the system state for which the system stability can always be maintained.

When a biped robot is supporting its body on one leg, it can be represented by a single inverted pendulum. The pendulum is composed of a grouped-mass center of gravity and a massless shaft with the length *l* [3]. The shaft links the center of gravity with the ankle joint of the supporting leg [4]. In the inverted pendulum model, the potential energy is at its maximum when the center of gravity is passing the normal axis. This is useful information because when our robot is walking, the robot keeps stationary when it reaches its maximum potential energy. This characteristic allows biped robots to be modeled with the simple inverted pendulum. When the biped robot reaches its maximum potential energy, this state is called *Zero-State-Point.* In order to have the robot dynamically walk, the figure below illustrates the dynamics of the biped robot [3].

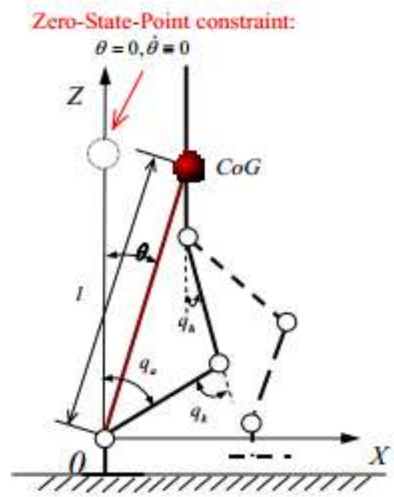

Figure 2 – Zero-State-Point (Li and Chen).

From the figure above, two simple assumptions can be derived. First, the contact between a robot and the surface is point-contact. Second, the robot's motions in sagittal and lateral planes are independent. From the first assumption, the point-contact essentially means the ankle joint of the supporting foot is passive. The second assumption shows the effect of the lateral motion on the sagittal dynamics can be ignored because the lateral side to side shaking motion is very small. This allows the dynamics in the sagittal and lateral planes to be handled separately. The motions of the inverted pendulum for the sagittal and lateral planes can be represented with the following equations.

$$\ddot{\theta} = \frac{g}{l} \sin \theta \tag{1}$$

$$\ddot{\emptyset} = \frac{g}{l} \sin \emptyset \tag{2}$$

θ and $\emptyset$ represent the pendulum's angles in the sagittal and lateral planes respectively and $g \ and \ l$ represent the acceleration of gravity and the rod's length respectively.

In the sagittal plane, the position of the center of gravity and the origin of the coordinates system lie in the ankle of the supporting foot as seen in the figure above. The position of the center of gravity can be calculated by:

$$x_c = l \sin \theta \tag{3}$$

$$z_c = l \cos \theta \tag{4}$$

Now, in order for the robot to move steadily, the robot must be able to demonstrate a walking patter, therefore, the pattern must be continuous and repeatable. An understanding of symmetry and periodic should be made. Through research, the periodicity and the symmetry of the gait contain the following restraints [4]:

$$x_c\big((k+1)T_s\big) - x_c(kT_s) = S_s \tag{5}$$

$$z_c\big((k+1)T_s\big) = z_c(kT_s) \tag{6}$$

To find the Zero State Point constraint, apply the following equation when the center of gravity passes the normal axis at t_l:

$$\theta(t_1) = \theta(kT_s + t_1) = 0 \tag{7}$$

$$\dot{\theta}(t_1) = \dot{\theta}(kT_s + t_1) = 0 \tag{8}$$

When walking, the switching of the gait occurs at the midway point of a leg swing, which occurs when $t_1$is equals to $\frac{T_s}{2}$. By using that relationship, the following equations can be derived:

$$\theta\big((k+1)T_s\big) = \theta(kT_s) = a\sin\frac{S_s}{l} \tag{9}$$

$$\theta\left(kT_s + \frac{T_s}{2}\right) = 0 \tag{10}$$

Using the equations for the motion of the inverted pendulum, the general differential equation can be expressed as:

$$\theta(t) = C_1 e^{-kt} + C_2 e^{-kt} \tag{11}$$

$$k = \sqrt{\frac{g}{l}} \text{ ; } C_1, C_2 \text{ are the undetermined coefficients.} \tag{12}$$

By using substitution, the equation of the pendulum angle can be derived as:

$$\theta(t) = K_q(e^{-kt} + e^{-kt}) \tag{13}$$

$$K_q = \frac{\mathrm{a}\sin(\frac{S_s}{l})}{e^{\frac{-KT_s}{2}} - e^{\frac{KT_s}{2}}} \tag{14}$$

These equations reveal that the trajectory of the center of gravity motion of the biped robot can be determined by the walking parameters in the sagittal plane. The equations for the lateral plane's trajectory of the center of gravity derived in a similar fashion as well. The trajectory of the center of gravity constraint equations are:

$$\emptyset(kT_s) = \emptyset\big((k+1)T_s\big) = a\sin\left(\frac{A_y}{l}\right) \tag{15}$$

$$\emptyset\left(kT_s + \frac{T_s}{2}\right) = \emptyset\left(kT_s + \frac{T_s}{2}\right) = 0 \tag{16}$$

A_y is the lateral swing of the center of gravity in the cycle. As specified before, in order to have the robot walk in a dynamic motion, being able to keep the body of the robot in an upright position is critical [5].

Also, to implement the walking cycle of our robot one can use the inverted pendulum model to set the frequency in order to produce natural swings. The equation to find the frequency can be found by using the following equation,

$$f_n = \frac{1}{2\Pi}\sqrt{\frac{g}{l}} \tag{17}$$

The lateral swing amplitude of the pelvis can be derived via ZMP of the inverted pendulum. The equation of motion for a simple inverted pendulum can be expressed as:

$$T = mgl\theta - ml^2\ddot{\theta} \tag{18}$$

$Where$:

$$T = torque\ at\ the\ joint$$
$$m = point\ mass$$
$$\Theta = angular\ displacement$$

By multiplying both sides by $\frac{1}{mg}$, the equations becomes:

$$\frac{T}{mg} = l\theta - \frac{l}{g}\left(l\ddot{\theta}\right) \tag{19}$$

Since $F = mg\ and\ Y = l\theta$, the equation can now be expressed as:

$$\frac{T}{F} = Y - \frac{l}{g}\left(\ddot{Y}\right) \tag{20}$$

Where:

$$F = ground\ reaction\ force$$
$$Y = lateral\ displacement\ of\ mass\ center$$

Therefore, the lateral displacement of the mass center is expressed as:

$$Y = A(1 + \frac{l}{g}\omega^2)\sin\omega t \tag{21}$$

The sinusoidal component is obtained from the relation between the robot walking and the lateral displacement of the mass center.

Another model of a biped robot can be demonstrated as a sequence of seven rigid bodies. These bodies include the following parts, both feet, lower legs, upper legs, and the trunk/torso. The head is included in the torso. From the figure below found in a reference document, one can see the rigid bodies and their respective angels [1].

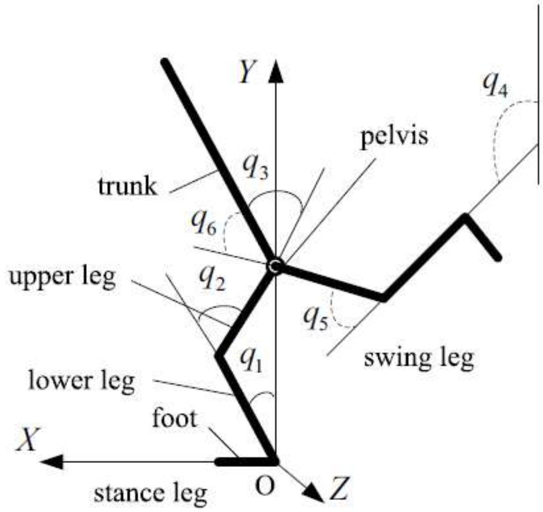

Figure 3 – Rigid Biped Model (Kondak and Hommel).

By using Kane's formalism [6], the dynamical equations can be derived. Here is the form of the derived equation.

$$\boldsymbol{M}(\boldsymbol{q})\dot{\omega} = \boldsymbol{f}(\boldsymbol{q},\omega) + \boldsymbol{T} \qquad (22)$$

In the *f* function, $\boldsymbol{q}$ is the vector of generalized coordinates, which are angles in the ankles, knees, hip joints, and ω is the corresponding generalized velocities. The matrix function, **M(q),** takes into account the mass distribution and the vector function, **f(q,ω),** describes the effect of both inertial forces and gravity. This model allows us to describe the torques of the robot's joints. The kinematic equation can be modeled with the following equation,

$$\dot{\boldsymbol{q}} = \omega \qquad (23)$$

2. Possible Controllers

Various designers have suggested suitable controllers to manipulate a walking cycle [2]. Before continuing, a description of a walking pattern must first be described. For the purpose of this report, let the following set of descriptions describe a complete walking cycle:

a. Stage 1: lift the right leg to its maximum flexion and height
b. Stage 2: lower the right leg until it makes complete contact with the ground
c. Stage 3: lift the left leg to its maximum flexion and height
d. Stage 4: lower the right leg until it makes complete contact with the ground

e. Stage 5: This stage follows stage 1 or stage 2, and brings the robot to a sanding equilibrium point.

Designers used the above walking cycle description to propose a control algorithm that is composed of three control strategies [9]. These strategies are: a real time balance control strategy, a walking pattern control strategy, and a predicted motion control strategy. Each control strategy is composed of several controllers that correspond to the objective of the desired strategy. Essentially, this method divides the walking cycle into several walking stages and uses a controller switching strategy to obtain its desired objective. Listed below is a table of each strategy and it describes what stage it will operate in and its objective as well [9].

Figure 4 – Summary of Possible Controllers (Kim, Park, and Oh 2006).

Control Scheme	Online Controller (working period)	Objective
Real-Time Balance Control	Damping controller (1st and 3rd Stages, SSPs of 2nd and 4th Stages)	Eliminate the upper body oscillations in single support phase by imposing damping at the ankle joints
	ZMP compensator (1st and 3rd Stages, SSPs of 2nd and 4th Stages)	Maintain dynamic balance by horizontal motions of the pelvis
	Soft landing controllers (DSPs of 2nd and 4th Stages)	Absorb landing impact and adapt the foot to the ground surface
Walking Pattern Control	Pelvis swing amplitude controller (DSPs of 2nd and 4th Stages)	Compensate the lateral swing amplitude of the pelvis by considering the amplitude of the ZMP
	Torso pitch/roll controller (DSPs of 2nd and 4th Stages)	Compensate the center position of the pelvis swing to balance the pitch & roll inclinations of the torso
Predicted Motion Control	Tilt over controller (1st and 3rd Stages)	Compensate the ankle joint trajectories to prevent "tilt over" in roll direction
	Landing position controller (2nd and 4th Stages)	Compensate the landing position to prevent unstable landing

These controllers could possibly work well for surfaces with a slight uneven floor; however, the robot is subject to fall down when the inclinations exceed a certain threshold. Designers have begun researching possible solutions to the uneven floors that present problems to the existing control algorithms. These solutions are but are not limited to: upright pose controller, landing position controller, and vibration reduction controller [2]. These solutions would then be added to the existing algorithm presented in the above table. Each controller is designed through mathematical models and experiments.

Conclusion

This paper presented the theory for a control algorithm of a biped robot. The theory associated with the algorithm is largely based on the classical control inverted pendulum problem. By modeling the biped robot as a simple inverted pendulum, the walking pattern can make the robot walk more human like compared to the unnatural slow motion that most robots tend to display. The use of a feedback controller such as a sensor can be used so that the robot can adapt to the unevenness of the surface plane. An uneven surface can cause the robot's system to become uneven even by a small fraction of a few degrees. Possible controllers were presented from past and present designers. Each aspect of the robot must be analyzed carefully when implementing a control algorithm. These elements include: hardware, software, and gait development. Designers and researching are connecting present algorithms to create an algorithm where a robot could possibly display large amounts of robustness with negligible interference from the floors inclination.

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