~~7. a) If the structure was an array~~

5. a) $T(n) = 47$, answer: $\Theta(1)$.

b) $T(n) = (4n+12)(6n+2)$

$= 24n^2 + 8n + 72n + 24$

$= 24n^2 + 80n + 24$

$\therefore\ T(n) \in \Theta(n^2)$

c) $T(n) = \sum_{i=0}^{n} 2^{i+c}$, c is a constant

since: $\sum_{j=0}^{m-1} ar^j = a\frac{1-r^m}{1-r}$, let $n = m-1 \Rightarrow m = n+1$
& $a = 1$, $r = 2$

$\therefore\ \sum_{i=0}^{n} 2^{i+c} = \sum_{i=0}^{n} 2^i 2^c = 2^c \sum_{i=0}^{n} 2^i$

$\Rightarrow 2^c\left[\frac{1-2^{n+1}}{1-2}\right] = 2^c(2^{n+1}-1) = T(n)$

$\therefore\ T(n) \in \Theta(2^n)$

⑤ d) $T(n) = \sum_{i=1}^{n} \sum_{j=i^2}^{n^2} c$, where c is a constant.

$$= \sum_{i=1}^{n} cn^2 - ci^2 + C = \frac{1}{6} c(4n^3 - 3n^2 + 5)$$

$\therefore$ $T(n) \in \Theta(n^3)$

(5.) e) $T(0)=1,\ T(1)=1$

$T(n) = 2T(n-2)+4,\ n \geq 2$

Solving Recurrence:

$T(n) = 2T(n-2)+4$

$= 2(2T(n-4)+4)+4$ sub

$= 4T(n-4)+8+4$

$= 4T(n-4)+12$

$= 4(2T(n-6)+4)+12$

$= 8T(n-6)+16+12$

$= 8T(n-6)+28$ sub

$= 2^k T(n-2k)+4(2^k-1)$ extrapolating $k \geq 1$

$= 2^{n/2} T(1) + 4(2^{n/2}-1)$ $\quad n = 2k \Rightarrow k = n/2$

$= 2^{n/2} T(1) + 2^2 2^{n/2} - 4$

$= 2^{n/2} + 2^2 2^{n/2} - 4 = 2^{n/2}(1+4) - 4$

$= 2^{n/2}(5) - 4$

$T(n) \in \Theta(2^{n/2})$

5. f) $T(1)=1$

$T(n) = T(\lceil n/2 \rceil) + 3$ for $n \geq 2$

Solving Recurrence:

$$T(n) = T(\lceil n/2 \rceil) + 3$$

$$= T(\lceil n/4 \rceil) + 3 + 3 \quad \text{sub}$$

$$= T(\lceil n/8 \rceil) + 3 + 3 + 3 \quad \text{sub}$$

$$= T(\lceil n/2^k \rceil) + 3k \quad \text{extrapolating } k > 0$$

$$= T(1) + 3k \qquad n = 2^k, \quad \lg n = k$$

$$= T(1) + 3\lg n$$

$$= 1 + 3\lg n$$

$\therefore\ T(n) \in \Theta(\lg n)$