(6.) a) $54n^3 + 17 \Rightarrow \Theta(n^3)$

$O(n^3)$: $54n^3 + 17 \leq cn^3, \ \forall \ n \geq n_0$

let $c = 71$

since $17n^3 \geq 17$ for $n \geq 1 = n_0$

$\therefore \ 54n^3 + 17 \leq 71n^3 = 54n^3 + 17n^3$, for $n \geq 1 = n_0$

so $T(n) \in O(n^3)$

$\Omega(n^3)$: $54n^3 + 17 \geq cn^3, \ \forall \ n \geq n_0$

let $c = 53$

since $54n^3 > 53n^3 \ \forall n \geq 1 = n_0$

so, $54n^3 + 17 \geq 53n^3$, for $n \geq 1 = n_0$

$\therefore \ T(n) \in \Omega(n^3)$

So, since $T(n) \in O(n^3) \wedge T(n) \in \Omega(n^3)$

$\Rightarrow T(n) \in \Theta(n^3)$

(6.) b) Show that $54n^3 + 17 \notin \Theta(n^2)$

By contradiction.

Assume $T(n) \in \Theta(n^2)$

This implies $T(n) \in O(n^2) \wedge T(n) \in \Omega(n^2)$

since $\lim\limits_{n \to \infty} \left| \frac{T(n)}{g(n)} \right| = \lim\limits_{n \to \infty} \left| \frac{54n^3 + 17}{n^2} \right| = \infty \Rightarrow T(n) \in \omega(n^2)$

$\therefore$ $T(n) \notin O(n^2)$ since $T(n) \in \omega(n^2)$ ※

$\Rightarrow T(n) \notin \Theta(n^2)$

c) $T(n) = a_d n^d + a_{d-1} n^{d-1} + \dots + a_1 n + a_0$, a_i = constant $\forall i = 0, \dots, d$, $a_d > 0$

show $T(n) \in \Theta(n^d)$

$\Omega(n^d)$: $T(n) \geq a_d n^d$, $n \geq 1 = n_0$, $c = a_d$

$\therefore T(n) \in \Omega(n^d)$

$O(n^d)$: $T(n) \leq \sum_{i=0}^{d} (a_i) \cdot n^d$, $n \geq 1 = n_0$, $c = \sum_{i=0}^{d} (a_i)$

$\therefore T(n) \in O(n^d)$

$\therefore$ since $T(n) \in \Omega(n^d) \wedge T(n) \in O(n^d)$

$\Rightarrow T(n) \in \Theta(n^d)$