Assignment_3

Lava Kumar

2022-10-19

1)Formulation and solving this transportation problem using R.

Loading all the required packages.

```
library("lpSolveAPI")
```

```
## Warning: package 'lpSolveAPI' was built under R version 4.1.3
```

```
library("lpSolve")
```

```
## Warning: package 'lpSolve' was built under R version 4.1.3
```

```
library("tinytex")
```

Creation of table as per mentioned in the question.

```
tab <- matrix(c(22,14,30,600,100,
                16,20,24,625,120,
                80,60,70,"-","-"), ncol=5,byrow=T)
colnames(tab) <- c("Warehouse1", "Warehouse2", "Warehouse3", "ProductionCost", "ProductionCapacity")
rownames(tab) <- c("PlantA", "PlantB", "Monthly Demand")
tab <- as.table(tab)
tab
```

```
##                Warehouse1 Warehouse2 Warehouse3 ProductionCost
## PlantA         22         14         30         600
## PlantB         16         20         24         625
## Monthly Demand 80         60         70         -
##                ProductionCapacity
## PlantA         100
## PlantB         120
## Monthly Demand -
```

The Objective Function is to Minimize the Transportation cost

$$Z = 622X_{11} + 614X_{12} + 630X_{13} + 0X_{14} + 641X_{21} + 645X_{22} + 649X_{23} + 0X_{24}$$

Subject to the following constraints

$$Supply\ \ Constraints$$

$$X_{11} + X_{12} + X_{13} + X_{14} <= 100$$

$$X_{21} + X_{22} + X_{23} + X_{24} <= 120$$

$$Demand\ \ Constraints$$

$$X_{11} + X_{21} >= 80$$

$$X_{12} + X_{22} >= 60$$

$$X_{13} + X_{23} >= 70$$

$$X_{14} + X_{24} >= 10$$

$$\text{Non} - Negativity\ \ Constraints$$

$$X_{ij} >= 0 \quad \text{Where i = 1,2 and j = 1,2,3,4}$$

```
#Since demand is not equal to supply creating dummy variables
#Creating a matrix for the given objective function
costs <- matrix(c(622,614,630,0,
                  641,645,649,0), ncol=4, byrow=T)
#Defining the column names and row names
colnames(costs) <- c("Warehouse1", "Warehouse2", "Warehouse3", "Dummy")
rownames(costs) <- c("PlantA", "PlantB")
costs
```

```
##        Warehouse1 Warehouse2 Warehouse3 Dummy
## PlantA        622        614        630     0
## PlantB        641        645        649     0
```

```
#Defining the row signs and row values
row.signs <- rep("<=",2)
row.rhs <- c(100,120)
#Since it's supply function it cannot be greater than the specified units.
#Defining the column signs and column values
col.signs <- rep(">=",4)
col.rhs <- c(80,60,70,10)
#Since it's demand function it can be greater than the specified units.
#Running the lp.transport function
lptrans <- lp.transport(costs,"min",row.signs,row.rhs,col.signs,col.rhs)
```

```
#Getting the objective value
lptrans$objval
```

```
## [1] 132790
```

*The minimization value so obtained is **$132,790** This is the lowest total cost that may be obtained by including the costs of manufacturing and transporting the defibrillators.*

```r
#Getting the constraints value
lptrans$solution
```

```
##      [,1] [,2] [,3] [,4]
## [1,]    0   60   40    0
## [2,]   80    0   30   10
```

80X21 AEDs *in Plant B - Warehouse1*

60X12 AEDs *in Plant A - Warehouse2*

40X13 AEDs *in Plant A - Warehouse3*

30X23 AEDs *in Plant B - Warehouse3 It should be produced at each factory and then transported to each of the three wholesaler warehouses in order to lower the overall cost of manufacture and transportation.X24 is a "throw-away variable" since "10" appears in the fourth variable.*

2)Formulating the dual of the above transportation problem

Since the primal was to minimize the transportation cost the dual of it would be to maximize the value added(VA).

We are aware that the number of constants in dual is equal to the number of variables in primal. The primary of the LP is asked in the first question. We shall maximize in the dual because we choose to minimize in the primal. Let's utilize the dual problem's variables P and WH.

```r
disdu <- matrix(c(622,614,630,100,"P_A",
641,645,649,120,"P_B",
80,60,70,220,"-",
"WH_1","WH_2","WH_3","-","-"),ncol=5,nrow=4,byrow=TRUE)
colnames(disdu) <- c("W1","W2","W3","Prod Cap","Supply (Dual)")
rownames(disdu) <- c("PlantA","PlantB","Monthly Demand","Demand (Dual)")
disdu <- as.table(disdu)
disdu
```

```
##                W1   W2   W3   Prod Cap Supply (Dual)
## PlantA         622  614  630  100      P_A
## PlantB         641  645  649  120      P_B
## Monthly Demand 80   60   70   220      -
## Demand (Dual)  WH_1 WH_2 WH_3 -        -
```

$$Maximize \quad VA = \quad 80WH_1 + 60WH_2 + 70WH_3 - 100P_A - 120P_B$$

Subject to the following constraints

$$Total \;\; Payments \;\; Constraints$$

$$WH_1 - P_A >= 622$$

$$WH_2 - P_A >= 614$$

$$WH_3 - P_A >= 630$$

$$WH_1 - P_B >= 641$$
$$WH_2 - P_B >= 645$$
$$WH_3 - P_B >= 649$$

$$\text{Where}\ \ WH_1 = Warehouse\ \ 1$$
$$WH_2 = Warehouse\ \ 2$$
$$WH_3 = Warehouse\ \ 3$$
$$P_A = Plant\ \ 1$$
$$P_B = Plant\ \ 2$$

3)Economic Interpretation of the dual

$$WH_1 <= 622 + P_A$$
$$WH_2 <= 614 + P_A$$
$$WH_3 <= 630 + P_A$$
$$WH_1 <= 641 + P_B$$
$$WH_2 <= 645 + P_B$$
$$WH_3 <= 649 + P_B$$

$$\text{F}rom\ \ the\ \ above\ \ we\ \ get\ \ to\ \ see\ \ that\ \ WH_1 - P_A >= 622$$

$$that\ \ can\ \ be\ \ exponented\ \ as\ \ WH_1 <= 622 + P_A$$

$$\text{H}ere\ \ WH_1\ \ is\ \ considered\ \ as\ \ the\ \ price\ \ payments\ \ being\ \ received\ \ at\ \ the\ \ origin\ \ which\ \ is\ \ nothing\ \ else,$$
$$\text{b}ut\ \ the\ \ revenue,\ \ whereas\ \ P_A + 622\ \ is\ \ the\ \ money\ \ paid\ \ at\ \ the\ \ origin\ \ at\ \ Plant_A$$

$$\text{T}herefore\ \ the\ \ equation\ \ turns,\ \ out\ \ to\ \ be\ \ MR_1 >= MC_1.$$

$$\text{F}or\ \ a\ \ profit\ \ maximization,\ \ The\ \ Marginal\ \ Revenue(MR)\ \ should\ \ be\ \ equal\ \ to\ \ Marginal\ \ Costs(MC)$$

$$\text{T}herefore,\ \ MR_1 = MC_1$$

$$\text{B}ased\ \ on\ \ above\ \ interpretation,\ \ we\ \ can\ \ conclude\ \ that,$$
$$\text{P}rofit\ \ maximization\ \ happens\ \ place\ \ if\ \ MC\ \ is\ \ equal\ \ to\ \ MR.$$

If MR < MC, to obtain the marginal revenue, we must reduce the costs at the plants (MR).

If MR > MC, to obtain the Marginal Revenue (MR), we need to increase the production supply.