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Balancing user preferences for aircraft schedule recovery during irregular operations

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In this paper, the irregular operations problem is approached for the first time in a way that allows an airline to provide for schedule recovery with minimal deviations from the original aircraft routings. A network model with side constraints is presented in which delays and cancellations are used to deal with aircraft shortages in a way that ensures a significant portion of the original aircraft routings remain intact. The model is flexible, allowing user preferences in the objective, and thereby reflecting the immediate concerns of the decision-maker in the recovery schedule. The model can be tailored by airline operations personnel to emphasize differing solution characteristics. Fleet data provided by Continental Airlines are used to demonstrate the approach. Extensive testing indicates that optimal or near-optimal solutions are routinely obtained from the LP relaxation of the network formulation. When integrality is not achieved, a rounding heuristic is provided that finds feasible solutions within a small fraction of the optimum.

1. Introduction

On a daily basis, airlines are faced with resource shortages due to reductions in flight or station capacity that cause disruptions to planned flight schedules. Resources in the airline industry are capital intensive and hence are tightly coupled; therefore, the removal of even a single aircraft from the flight schedule for a short period will likely cause flight delays or cancellations. The most common reasons for these disruptions are mechanical problems, crew unavailability, bad weather, station congestion, fuel shortages, airport facility restrictions and security problems. When any of these problems occur, operations personnel must make real-time decisions that return the airline to its original schedule as soon as possible. Because these situations can be very costly for airlines, resulting in substantial loss of revenue and customer goodwill, it is imperative that cost-effective solutions be enacted quickly.

To restore the original schedule, operations personnel use a combination of flight delays, flight cancellations, diversions, spare aircraft and ferried aircraft. Spare aircraft are sometimes maintained at major hubs for use in emergencies to maintain an operable schedule. Ferrying involves flying an empty aircraft to a point of need to service a flight. However, the usage of spare or ferried aircraft is unusual due to the excessive costs of either option. In the majority of situations, a combination of flight delays and cancellations is used to recover the original schedule. A recovery period is defined as the time allotted for an airline to get back to its original schedule following a disruption.

When problems occur, passengers are inconvenienced and various costs are incurred. For example, crew reassignments often involve monetary penalties, gate changes frustrate and confuse passengers, and flight delays frequently result in a loss of customer goodwill. In addition, disruptions directly affect an airline's on-time performance, a factor that strongly influences customer choice.

A common complaint of many managers when using optimization-based decision support systems is that small changes in input often translate into drastically different solutions [1]. In the airline industry, many issues are taken into account in the construction of the original flight schedule. Changes to aircraft routings inconvenience passengers and affect crew schedules, gate assignments and maintenance. Reassigning more than a few aircraft when a single plane is grounded is unacceptable for most airlines. Consequently, dealing with irregular operations in a way that results in minimal deviation from the original aircraft routings is very desirable. Inclusion of this factor has not been addressed by the research community.

The purpose of this paper is to provide solution techniques to deal with minor day-to-day disruptions in the original flight schedule. These solutions will include options to delay or cancel flights while keeping intact as large a portion of the original aircraft routings as possible. In essence, the decision-maker is faced with the multiobjective problem of simultaneously minimizing delays, cancellations and deviations. Because of the inherent conflict in these objectives, any solution will necessarily reflect a tradeoff among them. One of our goals is to provide the decision-maker with a set of Pareto-optima that embodies these tradeoffs. The actual objective function value is not as important as the quality of the solutions as measured by the number of cancellations, the number of delays and the amount of deviation from the original aircraft routings. The amount of deviation will be gauged primarily by the number of intact flight paths and the number of swaps. A swap is defined as any flight leg flown by a plane not originally assigned to it. The issue of major disruptions is addressed in a companion paper [2] that deals with hub closures.

The problem addressed here is modeled as an Integer Linear Program (ILP) and solved using standard optimization software. It will be seen that in the great majority of cases it is sufficient to solve the LP relaxation of the original formulation. When integrality is not achieved, an adaptive rounding heuristic provides near-optimal feasible solutions.

At first glance, it might appear that simply delaying or canceling a subset of the original flights is all that is required. However, more often than not, such an approach can be costly for the airlines and produce infeasible solutions. When rescheduling, operations personnel must take into account the physical and regulatory constraints of the system. These include station curfews, end of period aircraft balance, FAA imposed maintenance and crew feasibility. Many airports maintain curfews, not allowing arrivals or departures between certain hours. When considering flight delays these curfews must be addressed. Having the correct number of planes at each station at the end of the recovery period is termed aircraft balance. This constraint ensures aircraft are positioned to fly all the scheduled flights when normal operations are resumed. Maintenance constraints imposed by the FAA require aircraft to be serviced at certain predetermined time intervals.

In practice, the problems of aircraft recovery and crew reassignment are handled separately [3]. When a disruption occurs, a new aircraft routing is proposed and a check is made to see if a feasible crew assignment exists. If not, the routing is sent back for modification. This process is repeated until an aircraft routing with a feasible crew assignment is obtained. This paper focuses only on the aircraft routing side of this process.

A special case of the irregular operations problem, known as the *ground delay program*, arises when the FAA imposes a reduction in capacity at airports during events such as inclement weather. In this situation, a reduced number of flight arrivals and departures are mandated for a specific duration. The designation of flight delays and cancellations is determined locally with respect to the affected airport [4,5]. Recently, the FAA announced that the current ground delay program is undergoing dramatic revision but new regulations have yet to be issued. For this reason, it is not considered here.

The remainder of the paper is organized as follows. Related work is outlined in Section 2. The network structure and methodology are introduced in Section 3 along with notation and the mathematical formulation. Section 4 contains extensive experimental results for two separate fleets. A heuristic for finding feasible solutions from the LP relaxation of the ILP model is given in Section 5. We conclude with a discussion of the limitations of the proposed approach along with several suggestions for future work.

2. Background

Many aspects of airline operations have long been the focus of operations research practitioners. Flight and crew scheduling, though, have received most of the attention in the literature. While recent years have seen an increased focus on problems surrounding irregular operations, the body of research in this area is still relatively small.

Teodorovic and Guberinic [6] were one of the first to model several aspects of the problem. They developed a graph construct in which nodes represent flights and arcs represent delays. A solution procedure based on branch and bound was proposed for minimizing total customer delay. Computations were illustrated with a small example, but many important real-world aspects of the problem were overlooked including station curfews, maintenance requirements and end of period aircraft balance. In addition, flight cancellations were not considered.

Teodorovic and Stojkovic [7] extended the aforementioned model by incorporating cancellations and station curfews; however, they did not address the aircraft balance constraint. Solutions were derived hierarchically by first minimizing the number of canceled flights and then minimizing passenger delay times. The computations were performed with a sequential dynamic programming-based heuristic that assigns as many flights as possible to an aircraft. Then the path with the least amount of delay covering the same number of flights is found and assigned to that aircraft. Experimentation showed that the approach produces solutions exhibiting massive deviations from the original schedule and excessive delays (5–10 hours).

Jarrah *et al.* [3] have presented a thorough overview of the problem along with two models based on a time-space network, one for delays and the other for cancellations. An attractive aspect of these models is that they have no side constraints; that is, they are pure minimum-cost network models. The cost structure used to determine flight values is complex taking into account the number of passengers on a flight, the number of passengers connecting downstream, downstream delays, downstream cancellations, lost crew time, and disruption of aircraft maintenance. Results are given for experiments conducted on sample data provided by United Airlines. While delays and cancellations are not addressed simultaneously, the approach was considered practical enough to be implemented by United [8].

Subsequently, Yu [9] has proposed an augmented framework for integrating delays and cancellations. His model provides special substitution nodes and arcs that allow aircraft swaps to be tracked directly. To some extent, swaps can be viewed as a measure of deviation from the original schedule. The model presented is a multicommodity network with side constraints. It is notationally burdensome with many different node and arc types. No solution techniques or examples are given.

Yan and Yang [10] were the first to incorporate flight cancellations, delays and ferry flights in a single model. They developed a basic time-space network representation of the problem which can be extended to include options to ferry aircraft and delay flights. A simple decision rule is used to minimize the schedule-perturbed time after an incident. A case study using real flight data from China Airlines is presented. The most comprehensive model investigated included delays, cancellations and ferry flights, and was solved using Lagrangian relaxation. The paper assumes a single fleet, all non-stop flights, and only considers one aircraft out of service at a time.

Argüello *et al.* [11] have presented a Greedy Randomized Adaptive Search Procedure (GRASP) to reconstruct aircraft routings in response to groundings and delays experienced over the course of the day. In the procedure, the neighbors of an incumbent solution are evaluated, and the most desirable are placed on a restricted candidate list. One candidate is selected randomly and becomes the incumbent. The approach was tested on data provided by Continental Airlines. Empirical results demonstrated the ability of the GRASP to quickly explore a wide range of scenarios and, in most cases, to produce an optimal or near-optimal solution.

Argüello *et al.* [12] also presented a time-band optimization model to solve the same problem. The model is constructed by transforming the routing problem into a time-based network in which the time horizon is discretized. The resulting formulation is an integral minimum-cost network flow problem with side constraints. Computational results demonstrate that the solutions obtained are either provably optimal or no more than a few percentage points from the lower bound.

Cao and Kanafani [13,14] have presented a 0-1 Quadratic Programming (QP) model for addressing cancellations and delays. The work is based on the delay model of Jarrah *et al.* [3] which they extended to include both delays and cancellations simultaneously, as well as the entire network of stations. All side constraints are moved to the objective using a quadratic penalty term. A complex system of assigning delay costs to flights is developed which includes a consideration of downstream connections. One difficulty with this scheme is its increasing complexity as more legs are added to the flight path. Nevertheless, the authors show that a problem in which the integrality requirement has been relaxed will return an integer solution. An algorithm similar to successive linear programming is used to solve the QP, though global optimality is not guaranteed since the problem is nonconvex. Hence, the solution quality is uncertain. To confront this issue, the authors recommend making a series of random starts and re-solving the relaxed problem. The effectiveness of the approach was demonstrated on randomly generated flight schedules containing up to four flight legs per aircraft.

To date, the work of Argüello *et al.* [11,12] and Yan *et al.* [10,15,16] represents the most comprehensive and practical approaches to rerouting aircraft in response to irregular operations. All past research, however, overlooks a critical factor in the acceptance of a recovery schedule by an airline; i.e., the degree of departure from the original aircraft routing. One of the main contributions of this paper is the extension of previous work to include a measure of this deviation.

3. Network structure and mathematical model

In this section, we present a model for aircraft recovery under irregular operations derived from the work of Yan and his co-authors [10,15,16]. The model solves the aircraft recovery problem for a single fleet with the objective of maximizing a modified "profit" function over the entire flight schedule. As we will see, the proposed objective function accounts for flight revenues, delays and cancellations. In addition, it includes an incentive to minimize deviation from the original schedule. The benefits of a particular flight along with an incentive to maintain it as a part of the flight path of a particular aircraft are incorporated in the objective function coefficients. The result is an integer single-commodity network model with side constraints, which is NP-hard due to the addition of side constraints to the otherwise pure network [17].

3.1. *Basic modeling constructs*

The proposed model is based on a simple time-space network as illustrated in Fig. 1. Flows on the network are aircraft. The vertical arcs represent aircraft on the ground which are waiting for a flight or are finished for the day. In the diagram, all flow is from top to bottom. Arrows are omitted on the ground arcs to avoid clutter. The sloping diagonal arcs represent flights from one station to

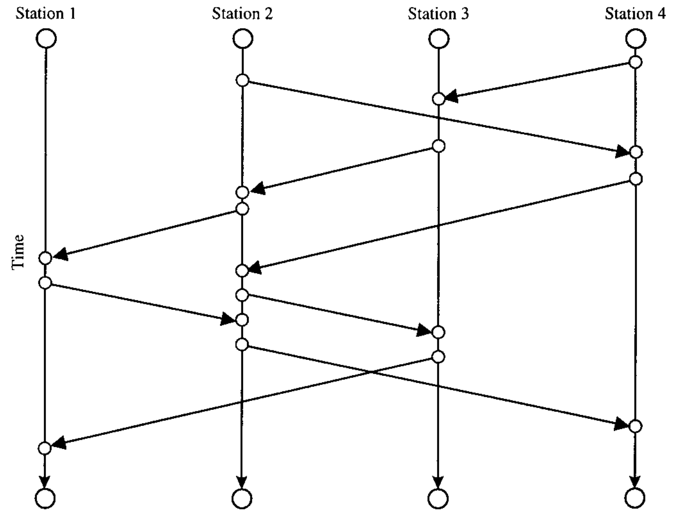

Fig. 1. Time space network representation.

another. These arcs begin at the flight's scheduled departure time and end at the scheduled arrival time plus the turnaround time for that type of aircraft. The turnaround time is the time it takes to prepare the plane for another flight. These flight arcs are restricted to binary values while ground arcs, which are only required to be nonnegative, will contain integer flow by construction. This can be seen from Fig. 1. Beginning with integer values on the vertical arcs, because only binary flow can enter or leave each station via flight arcs, integrality will always be maintained on the vertical arcs. Upper bounds on ground arcs may be included if there is an aircraft capacity limit at a given station. In some instances, not all flights will be covered. Flight arcs with no flow in the final solution are assumed to be canceled.

The larger nodes shown in Fig. 1 are the stations at the beginning and end of the recovery period. The top nodes may supply aircraft at the beginning of the recovery period and the bottom nodes force the proper balance of aircraft at the end. The smaller nodes are balance of flow (intermediate) nodes at each arc endpoint. These nodes are present at each flight departure or arrival. Intermediate nodes could also be used to supply or demand aircraft during the recovery period. In the diagram, the vertical axis represents time and the horizontal axis space.

The model presented can accommodate recovery periods of arbitrary length that begin and end at arbitrary time points in the day. If the recovery period begins in the middle of the day, supply nodes will contain all aircraft on the ground at that time. Planes in the air can be added to the model when they reach their current destination through intermediate supply nodes. The recovery period used in this study is 1 day. This means that one or more aircraft are taken out of service for an entire day. This is the convention used in Argüello *et al.* [12] and is adopted solely for convenience in testing the model. Recovery for many other irregular operations situations can be addressed using this model by simply choosing a recovery period and incorporating the appropriate supply and demand points and the relevant flights in the model.

3.1.1. *Incorporating delays*

To account for delays on a particular flight leg, a series of delay arcs are created to represent the available options for taking the flight at a later time. In Fig. 2, two delay options are shown for each of the four flight legs representing, say, 20 and 60 minute delays. To ensure that each leg is flown at most once, a side constraint must be added to the basic model requiring that the sum of flows on all arcs (variables) representing the same flight be less than or equal to one. The revenue for delayed flights is adjusted to include a cost per minute delayed. Intermediate nodes are omitted in this and the following figures to avoid clutter.

3.1.2. *Discouraging deviation in aircraft routings*

To "protect" original flight paths additional flight arcs are incorporated in the general model. This new construct, represented by the circular arc in Fig. 3a, will be called a *protection* arc. Protection arcs can cover any contiguous portion of the set of flights originally assigned to an aircraft. The side constraint introduced for delayed flights must now be expanded to include the extra flight option shown here. The new arc shown in Fig. 3a corresponds to one plane flying both flight legs, rather than a single new flight. By providing an incentive on the new arc a single plane is encouraged to fly both legs, thereby reducing the attractiveness of flying the two legs with separate aircraft.

The use of protection arcs to discourage schedule deviation can be extended to a flight path containing any number of flight legs. As shown in Fig. 3b, two extra

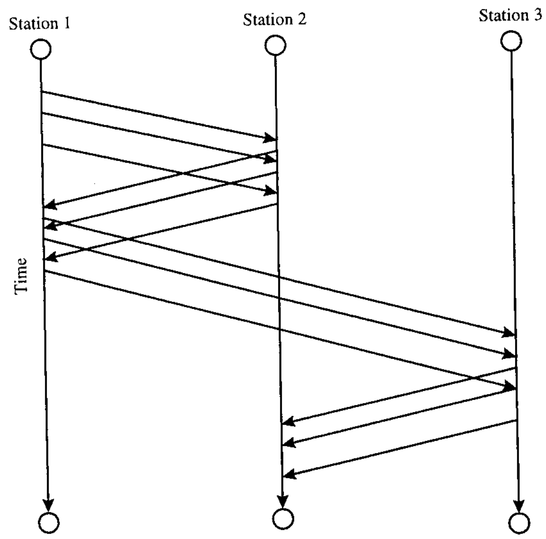

Fig. 2. Incorporating delays.

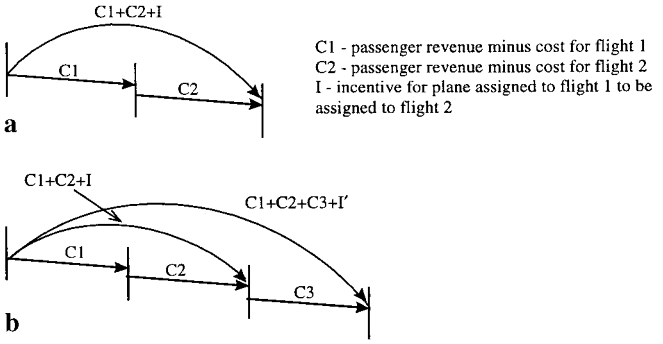

Fig. 3. (a) Protection arcs (b) Minimizing deviation from original schedule.

protection arcs are added for a plane originally assigned to three legs. Here, C3 is the net revenue of flight 3. The incentive on the arc that covers all three flights, I', is greater than the incentive to cover the first two flight legs together, I. In our formulation, all continuous combinations of the original flight path that include the first leg are given some incentive, but the greatest payoff is received if the entire flight path is maintained. This approach can be extended to any number of flight legs by adding the appropriate protection arcs. Of course, each side constraint that ensures a flight is assigned at most once must be expanded to include all flight and protection arcs that represent that flight.

As demonstrated presently, the primary strength of this modeling approach is its ability to generate solutions that reflect changing user preferences. By adjusting the number of delay options, the cost of delaying flights and the bonuses awarded for protecting flights, schedules with different average solution properties can be obtained. An airline that is the lone provider in a particular market may prefer lengthy delays to a flight cancellation, while an airline in a competitive market may cancel a flight rather than delay it more than an hour. With simple parameter adjustments, preferences like these can be easily included. This adds to the versatility and power of the model. In real-time control, it is essential to provide the decision-maker with a set of diverse, high quality solutions from which he or she can choose the most preferred, depending on the current situation.

3.1.3. *Through-flight considerations*

Through flights are mechanisms to serve long-haul markets with one or more intermediate stops, e.g., Boston–Newark–Los Angeles represents a through flight from Boston to Los Angeles. Passengers traveling from Boston to Los Angeles are able to stay on the same plane throughout the trip and often pay a premium to do so. The revenues associated with flight legs that make up through flights are not independent. If one of the legs is canceled, the through flight passengers will not make it to their destination as scheduled. To address these special cases another new construct is added to the model as shown in Fig. 4. An additional arc goes from the through flight's origin to its destination. It does not represent a new flight option, but rather one aircraft assigned to all of the flight legs in the through flight. Again, this additional arc must be included in the flight cover side constraint to ensure flights are not duplicated in the final solution. This construct can be applied to through flights with any number of intermediate stops.

Assume that the last two of the three flight legs shown in Fig. 4 represent a through flight. Let $0 < \pi < 1$ represent the proportion of passengers that are taking both the second and third flight leg. If either leg is flown individually or if the legs are flown by different aircraft, the revenue gained will be discounted by the factor $(1 - \pi)$. If the new arc, shown on the bottom, is taken the full revenue will be received for the through flight. The figure includes the protection arcs from the previous section to demonstrate how the two constructs are used together and how the arcs are assigned revenue. It is not possible to track every individual passenger connection because the model generated would be too unwieldy. However, when a large percentage of passengers are traveling the same set of flight legs a through flight can be designated.

3.2. *Mathematical model*

The model presented below takes as input a full flight schedule, plus information on aircraft groundings and returns a feasible recovery schedule with routes for each available plane. The following notation will be used in the development of the model.

Indices and Sets

Arcs:

g = index for set of ground arcs G;
f = index for set of flight arcs F;
p = index for set of protection and through flight arcs P;

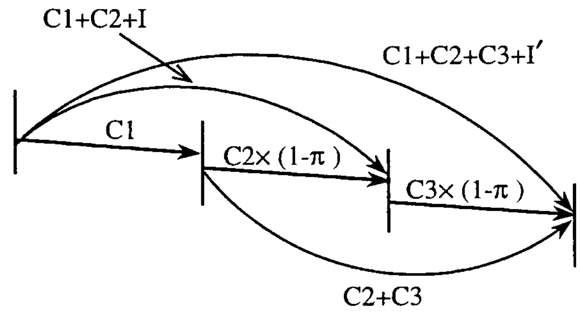

Fig. 4. Through flight arcs.

Nodes:

s = index for set of supply nodes S;
t = index for set of termination nodes T;
i = index for set of intermediate nodes I;
η = unique flight number representing each flight leg;
$O(i)$ = set of arcs originating at node i;
$T(i)$ = set of arcs terminating at node i;
$F(\eta)$ = set of arcs covering flight $\eta; F(\eta) \subset F \cup P$;
C_f = passenger revenue minus flight cost minus delay cost for flight arc f; if part of a through flight, revenues discounted by (1 - proportion of through-flight customers);
C_p = for protection arcs: passenger revenues minus flight costs of all flights covered minus delay costs of all flights covered plus the appropriate incentive, or for through-flight arcs: passenger revenues minus flight costs of all flights covered;
B_s = initial supply of aircraft at supply node s;
B_t = number of aircraft required for end-of-period balance at termination node t;
U_g = upper bound for ground arc g;

Variables

x_f = flow on flight arc f (binary);
y_p = flow on protection or through-flight arc p (binary);
z_g = flow on ground arc g (integer by construction).

Maximize

$$\sum_{f\in F} C_f x_f + \sum_{p\in P} C_p y_p, \tag{1a}$$

subject to:

$$\sum_{g\in O(s)} z_g = B_s \quad \forall s \in S, \tag{1b}$$

$$\sum_{g\in O(i)} z_g - \sum_{g\in T(i)} z_g + \sum_{f\in O(i)} x_f - \sum_{f\in T(i)} x_f + \sum_{p\in O(i)} y_p - \sum_{p\in T(i)} y_p = 0 \quad \forall i \in I, \tag{1c}$$

$$-\sum_{g\in T(t)} z_g = B_t \quad \forall t \in T, \tag{1d}$$

$$\sum_{f\in F(\eta)} x_f + \sum_{p\in F(\eta)} y_p \le 1 \quad \forall \eta \in N, \tag{1e}$$

$$x_f \in \{0,1\} \quad \forall f \in F, \tag{1f}$$

$$y_p \in \{0,1\} \quad \forall p \in P, \tag{1g}$$

$$0 \le z_g \le U_g \quad \forall g \in G, \tag{1h}$$

The objective function (1a) maximizes the modified profit function associated with the recovery period schedule. The first term captures revenue and delay cost on individual flight legs. The second term duplicates these figures for protection and through flight arcs. Equation (1e), the flight cover constraint, assures that at most one arc associated with a particular flight receives positive flow. The aircraft balance constraints, (1b) and (1d), account for the flow into and out of the network at the beginning and end of the recovery period, respectively. Flow balance at intermediate nodes is maintained by constraint (1c). All flight arcs (1f) and protection arcs (1g) are binary variables. Ground arcs (1h) are required only to be nonnegative but will be integral in any feasible solution. Note that if a flight is canceled, no benefit is received or cost incurred. To account explicitly for cancellations all that is necessary is to introduce a slack variable, s_η, in Equation (1e), a corresponding cost coefficient, C_η, and a third term in the objective function of the form $\Sigma_\eta C_\eta s_\eta$.

It is important to note that the objective function does not record true revenues and costs even if actual figures are used for each flight leg. (This is one of the reasons why cancellations are not explicitly addressed.) The addition of incentive values makes the objective function artificial. Including this factor, though, allows us to weight the arcs so that the accompanying solution has favorable properties in terms of the number of cancellations, number of delays, and amount of deviation to the original schedule. Through parametric analysis, we are then able to generate a wide range of solutions as a function of the relative importance of these criteria, given the current operating conditions (see Section 4.2.3).

More to the point, though, it is virtually impossible for airlines to provide accurate cost figures in real time, so maximizing net revenue is problematic in any case. With regard to delay costs, the best that can be obtained are rough estimates without regard to such intangibles as passenger inconvenience and loss of customer goodwill. Cancellation costs are even more difficult to estimate, varying dynamically with passenger, crew and equipment disruptions.

4. Computational experience

All programs were written using a combination of AMPL and C++ in conjunction with CPLEX 6.0. In solving the LP relaxation of model (1), a network simplex algorithm was applied to all pure network constraints; then the network solution was used as a starting point for solving the full LP via the dual simplex algorithm. When the solutions were not integral, CPLEX's IP (Integer Programming) solver was called. A heuristic for obtaining feasible integer solutions from the LP relaxation is given in Section 5.

Two data sets provided by Continental Airlines were used in the evaluation. The first involved the daily schedule of their Boeing 757 fleet. This is a relatively small fleet consisting of 16 aircraft servicing 42 daily flights between 13 cities in the United States, Peru, Columbia and the United Kingdom. Table 1 displays the problem data for each flight path. The schedule contains ten two-leg flight paths, two three-leg flight paths and four four-leg flight paths. Each flight leg has an origin and destination city, a departure and arrival time, and a unique flight number, η. All times in the schedule are reported in minutes past midnight eastern standard time. A 40-minute turnaround time was enforced between each flight leg on a path, i.e., all planes must spend 40 minutes on the ground between flights.

Table 1. Boeing 757 flight schedule

Aircraft	*Origin*	*Destination*	*Departure*	*Arrival*	*Flight,* η
101	LAX	EWR	660	970	170
101	EWR	SAN	1040	1398	203
102	EWR	LAX	660	1025	239
102	LAX	EWR	1080	1385	184
103	EWR	FLL	435	607	285
103	FLL	EWR	655	823	392
103	EWR	LIM	895	1356	703
103	LIM	EWR	1435	1885	704
104	EWR	IAH	630	858	711
104	IAH	BOG	925	1212	712
105	SAN	EWR	660	955	176
105	EWR	SNA	1035	1395	197
106	SEA	EWR	645	940	192
106	EWR	SFO	1005	1385	189
107	EWR	SFO	630	1005	173
107	SFO	EWR	1080	1390	174
108	EWR	IAH	360	583	150
108	IAH	SFO	640	887	151
108	SFO	IAH	940	1150	488
108	IAH	LAX	1260	1467	225
109	SFO	EWR	660	970	240
109	EWR	SEA	1050	1405	195
110	EWR	LAX	480	845	123
110	LAX	IAH	934	1120	133
110	IAH	EWR	1190	1386	134
111	SNA	EWR	660	945	196
111	EWR	LAS	1130	1462	1685
111	LAS	EWR	1545	1828	1684
112	EWR	MIA	455	636	233
112	MIA	EWR	720	900	236
112	EWR	LAX	965	1325	63
112	LAX	EWR	1530	1830	186
113	EWR	LAS	465	797	1641
113	LAS	EWR	880	1160	1640
113	EWR	LAS	1230	1556	1643
113	LAS	EWR	1615	1898	1642
114	LAX	EWR	780	1095	73
114	EWR	MAN	1200	1610	74
115	MAN	EWR	370	830	75
115	EWR	LAX	1065	1425	241
116	BOG	IAH	540	845	709
116	IAH	EWR	920	1120	710

The second set of test problems involved Continental's 737-100 fleet. This is a medium-size fleet consisting of 27 aircraft operating at 30 stations and servicing 162 flights daily in the continental United States. Of the 27 fight paths flown each day, six contain five flight legs, fifteen contain six flight legs, and six contain seven flight legs. For all problems solved, a 25-minute turnaround time was enforced. In all instances tested, a feasible solution was returned by the program.

An algorithm was developed to transform the arc-based solutions to path-based assignments for the available aircraft. This algorithm is not included in this paper but may be obtained from the authors. Determining the assignments is largely a matter of tracking the flows through the network solution.

4.1. *Results for 757 fleet*

For the first data set, nine options were allowed for each flight representing delays of 0, 10, 20, 30, 40, 50, 60, 90 and 120 minutes. The profit assigned to each flight arc was $(1000 + 2 \times \text{flight time} - 0.2 \times \text{minutes delayed})$. The profit was weighted by flight time, as longer flights were assumed more profitable. Protection arcs were assigned the sum of the values of the legs they covered plus 10 for each leg protected.

The model was coded and tested under 108 disruptive situations consisting of all combinations of grounding one, two and three aircraft for the day. The resulting LP formulation had 639 rows, 1196 columns and 3364 nonzero entries. The constraint matrix for each problem was very sparse, having a density of 0.0044. It was only necessary to solve the LP relaxation of (1) because CPLEX always returned solutions that satisfied the integrality requirement.

None of the 108 problem instances generated by grounding all combinations of one, two and three aircraft took more than 1 s of CPU time on a Pentium 100. For each problem instance the following statistics were recorded: number of delayed flights, total delay minutes, number of cancellations, number of swaps, and number of intact flight paths. A flight leg is counted as a swap if it is flown by any aircraft other than the one to which it was originally assigned. Averages for these statistics over the problems are shown in Table 2. The rows represent results by number of planes grounded.

Note that on average more flights are canceled than delayed when one, two or three aircraft are grounded. This is due in large part to the nature of the 757 flight schedule. This fleet operates in a nearly pure hub and spoke system. Out of 44 flight legs, 38 either originate or terminate at Newark (EWR). In most cases, if a plane is grounded at one of the spokes, two flight legs must be canceled. The second data set provides increased flexibility and allows for more experimentation with the model.

One objective of the model is to avoid deviation from the original aircraft routings. Two primary statistics for

Table 2. Results from 757 data

Number grounded	*Delayed flights*	*Total delay minutes*	*Canceled flights*	*Swaps*	*Intact flight paths*
One aircraft	0.5	42.5	1.9	2.1	14
Two aircraft	1.1	98	3.7	3.8	12.3
Three aircraft	1.7	149.7	5.6	5.1	10.8

measuring deviation are the number of swaps and intact flight paths. The minimum number of intact flight paths maintained when one aircraft was grounded was 14, when two aircraft were grounded was 12, and when three aircraft were grounded was 10. This means that at most one flight path was disrupted for each aircraft taken out of the schedule. Overall, 83.2% of all possible flight paths were maintained intact.

4.2. *Results for 737-100 fleet*

4.2.1. *Grounding a single plane*

Twenty of the 30 stations serviced by the 737-100 fleet have one or more aircraft grounded there overnight. To begin testing this data set, a single aircraft was grounded at each of these stations. Individual flight legs have unique revenues. The following parameters were used to solve all 20 instances: bonus per flight leg covered ($b = 300$) and delay cost per minute ($d = 5$). Delayed flights incur the delay cost multiplied by minutes delayed. Flight legs that are flown together via a protection arc are valued as the sum of the individual leg revenues minus the delay costs for the flight legs covered plus the appropriate bonus. Broken through flights lose a proportion of their revenue in accordance with the proportion of passengers disrupted.

The analysis considered eight delay options of lengths 0, 10, 20, 30, 40, 50, 60 and 90 minutes. The LP model had 2181 rows, 4485 columns with a matrix density of 0.0016. All 20 cases returned integer solutions initially in about 3 s on a Pentium 200. Summary statistics are reported in Table 3.

Looking first at the results for the number of intact flight paths, we see that, on average, 23 of 26 original flight paths were maintained (88%), and in the worst case 20 of 26 were maintained (77%). Other important statistics related to disruptions are the maximum number of delays and the maximum number of swaps. A delay occurs anytime that an aircraft takes a flight leg after the originally scheduled departure time. A swap occurs each time that an aircraft flies a leg that was not originally assigned to it. In the case with the most delays, eight of 162 (4.9%) flight legs were delayed. In the case with the most swaps, 15 of 162 (9.3%) flight legs were flown by planes not originally assigned to them.

4.2.2. *Grounding two planes*

Next, all combinations of grounding two aircraft were examined. This produced 192 instances. The same parameter values used for the single grounding case were used here; i.e., bonus per flight leg covered = 300 and delay cost per minute = 5.

For the same eight delay options, CPLEX returned integer solutions in 188 of the 192 instances (97.9%). The remaining instances were solved with CPLEX's IP solver. All solution times reported are in seconds and come from runs on a Pentium 200. Summary statistics resulting from integer solutions to the LP relaxation are reported in Table 4. For the four IP solutions, summary statistics are found in Table 5.

Table 3. Results from grounding a single plane for the 737-100 fleet

	Delays	*Delay minutes*	*Cancellations*	*Swaps*	*Intact flight paths*
Average	3.6	174.5	4.5	7.9	23.3
Maximum	8	410	6	15	25
Minimum	0	0	2	1	20

Table 4. Properties of feasible LP solutions

	Delays	*Delay minutes*	*Cancellations*	*Swaps*	*Intact flight paths*	*Solution time (s)*	*Objective value*
Average	7.8	397	9.0	15.1	20.0	2.5	414 831
Maximum	25	1220	12	34	24	4	425 885
Minimum	0	0	4	1	16	2	409 735

Table 5. Properties of IP solutions

	Delays	*Delay minutes*	*Cancellations*	*Swaps*	*Intact flight paths*	*Solution time (s)*	*Objective value*
Average	14.0	865	8.0	15.0	20.3	5.3	414 300
Maximum	17	1000	10	21	22	6	417 035
Minimum	11	770	6	9	18	4	411 900

Table 6. Properties of all solutions

	Delays	*Delay minutes*	*Cancellations*	*Swaps*	*Intact flight paths*	*Solution time (s)*	*Objective value*
Average	7.9	406.8	9.0	15.1	20.0	2.5	414 820
Maximum	25	1220	12	34	24	6	425 885
Minimum	0	0	4	1	16	2	409 735

Comprehensive summary statistics are displayed in Table 6. Examining the number of intact flight paths, on average, 20 of 25 original flight paths were maintained (80%), and in the worst case 16 of 25 were maintained (64%). Looking at the number of delays and swaps gives us another measure of deviation from schedule. In the worst case for delays, 25 of 162 flights (15%) were delayed. In the worst case for swaps, 34 of 162 flights (21%) were swapped.

Testing was also conducted in more extreme circumstances grounding up to six planes for the entire day. The model behaved in a manner similar to the cases where one and two planes were grounded. Of course, the solutions with more aircraft removed from the flight schedule had more delays, swaps and cancellations along with fewer intact flight paths. Summary results for these instances can be obtained from the authors.

Of the 192 cases, four required the IP solver. It is interesting to note that the average percent difference between the LP and IP solution values was 0.007% and in the worst case 0.013%. Thus the LP bound was very tight.

The decision-making capabilities of the model when canceling flights can be seen by comparing the average value of flights canceled to the average value of flights overall. A flight with a low revenue should be considered for cancellation before a flight with a higher revenue. The average revenue assigned to a flight leg was 2448. The maximum and minimum values were 4305 and 825, respectively. The average revenue of a canceled flight over all the recovery schedules generated was 2306, or 142 less than the average flight overall. As expected, flights chosen for cancellation, on average, had revenue less than that of the average flight.

4.2.3. *Tradeoff curves*

The next set of experiments was designed to demonstrate the ability of the model to generate solutions that reflect changing user preferences. By adjusting the bonuses awarded for protecting flights, the cost of delaying flights and the number of delay options, solutions with different properties can be obtained. Selected results are presented in graphical format. As one parameter is varied, the other two parameters are held constant at their default values. The parameters to be varied along with their default values are bonus per leg covered (default = 150), delay cost per minute (default = 2), and delay options (default = 0, 10, 20, 30, 40, 50, 60, 90 minutes).

All values reported are averages of 60 problem instances associated with the 737-100 fleet. These consist of all 20 instances of grounding a single aircraft and 40 randomly selected instances of grounding two aircraft. For the cases where delay options are varied, problem sizes range from 1254 rows and 2250 columns with a corresponding matrix density 0.0026 to 4001 rows and 9452 columns with a corresponding matrix density 0.0009.

Figure 5(a–d) shows how solution properties change as the bonus value is progressively increased in increments of 50 from 0 to 350. Recall that the bonus value is the

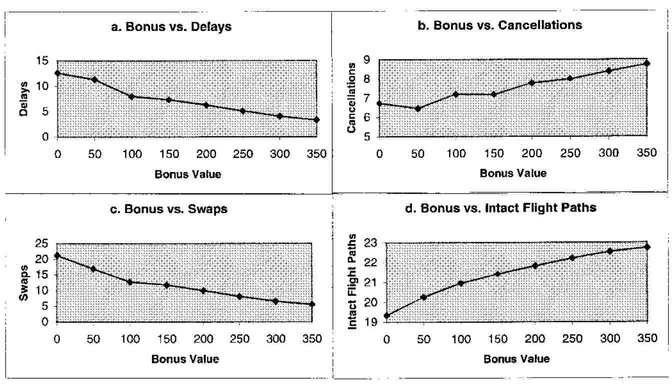

Fig. 5. Tradeoff curves for bonus variation.

incentive added to each leg in an original flight path that is kept intact. A number of important observations were made:

- the number of delays (Fig. 5a), total delay minutes, and swaps (Fig. 5c) all decrease steadily (~linearly) as the bonus value is increased;
- the number of cancellations (Fig. 5b) and intact flight paths (Fig. 5d) both increase steadily as the bonus value is increased; and
- tracking the average delay time and the number of noninteger solutions produces no clear pattern.

These results have the following logical interpretation. As the bonus for keeping flight paths intact increases, so does the number of intact flight paths. To obtain more intact flight paths more flight legs must be canceled. Since the only two options for flights not flown as scheduled are delay and cancellation, as cancellations rise, delays and total delay minutes will decrease.

The most important observation to be made here is that by raising the bonus value in the model the number of intact flight paths and the number of swaps found in the solution can be altered. Swaps are the number of flight legs flown by planes not originally assigned to them. Intact flight paths and swaps are the primary measures of deviation from the schedule. Therefore, by increasing the bonus, we can decrease schedule perturbation.

The results displayed in Fig. 6(a–d) show how selected solution properties change as the cost per minute of delays is progressively increased from 0 to 20. Again, a number of important observations are noted:

- the number of delays (Fig. 6a), total delay minutes and swaps (Fig. 6c) all decrease steadily as the cost per minute of delays is increased;
- the number of cancellations (Fig. 6b) and intact flight paths (Fig. 6d) both increase as the cost per minute of delays is increased; and
- tracking the average delay time and the number of noninteger solutions produces no clear pattern.

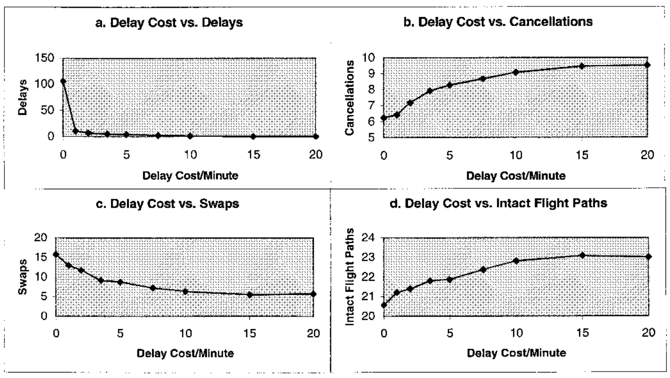

Fig. 6. Tradeoff curves for delay cost variation.

As the cost of delaying flights increases more flights are canceled than delayed. A decline in the number of swaps and an increase in the number of intact flight paths accompany the decrease in the number of delays. We note that in a manner similar to that of adjusting the bonus value, changing the cost per minute of delay also allows us to change the properties of the derived solutions. Figure 6(a–d) illustrates how higher delay costs, while increasing cancellations, increase the number of intact flight paths and decrease the number of swaps, thereby reducing schedule perturbation.

The results in Fig. 7(a–d) demonstrate how the solution properties change under the five sets of delay options shown in Table 7. Observed patterns are:

- the number of delays (Fig. 7a), cancellations (Fig. 7b), swaps (Fig. 7c) and intact flight paths (Fig. 7d) maintain nearly uniform values under the different delay options; and
- the total delay minutes and the average delay time both decrease steadily as the delay interval is decreased while the number of noninteger solutions increases.

The only clear benefit of reducing the delay interval from 30 minutes to 5 minutes is a decrease in total delay minutes and average time of delay. However, as the delay interval is decreased the computation time increases due to a noticeable increase in model size. The instances with four delay options solved in a few seconds, while the in-

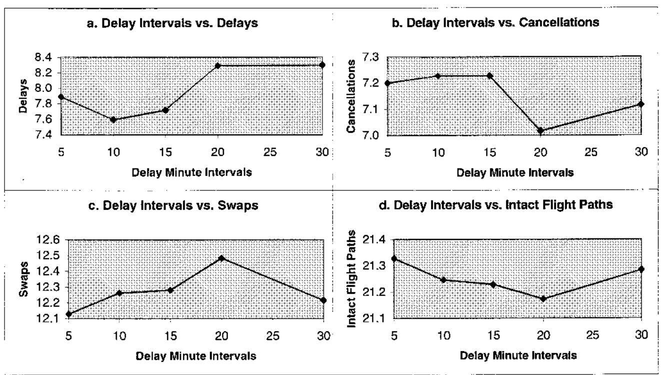

Fig. 7. Tradeoff curves for delay option variation.

Table 7. Delay options tested

Intervals tested	*Delay options*
30 min intervals	0, 30, 60, 90 minutes
20 min intervals	0, 20, 40, 60, 80, 90 minutes
15 min intervals	0, 15, 30, 45, 60, 75, 90 minutes
10 min intervals	0, 10, 20, 30, 40, 50, 60, 70, 80, 90 minutes
5 min intervals	0, 5, 10, 15, 20, 25, 30, 35, 40, 45, 50, 55, 60, 65, 70, 75, 80, 85, 90 minutes

stances with 19 delay options took about 45 seconds on a Pentium 100.

These results are noteworthy because they show that we can solve a much smaller problem and get very similar solutions. This bodes well for extension of the model to much larger instances. In addition, it should be noted that the total delay times and average delay times for the problems are generally overstated. For example, a flight may only need to be delayed 12 minutes, but if the only option in the model is for a 30 minute delay that is the option chosen. Post-processing could shorten the delay statistics and give a more accurate picture of the new schedule. This observation holds true for all the delay results presented.

5. Heuristic to obtain feasible solutions

Fractional solutions to model (1) imply that aircraft divide and then recombine at the end of the day. As problem sizes increase the time to solve integer programs increases exponentially. Because the irregular operations problem demands real-time solutions, it is not practical, in the case of much larger problem instances, to use an IP solver when the LP relaxation fails to deliver a feasible solution. Thus a rounding heuristic has been developed to obtain feasible integer solutions from the LP relaxation. A flow chart is shown in Fig. 8 for clarification.

Rounding Heuristic

Input: Noninteger solution to the LP relaxation of model (1).

Output: A feasible assignment consisting of purely integer flows for each aircraft.

A. Separate all paths with purely integer flow.

B. For a path with a split point, if no contingencies exist (see below), round flights on path with higher revenue to 1 and flights on all other paths to 0. Go to E.
If contingencies exist, make choice accordingly and go to E. If no feasible assignment exists, go to C. Contingencies take one of the following three forms:
   1. Aircraft balance must be maintained.
   2. Flight legs may only be flown once.
   3. When paths split or combine one of the options must be chosen.

C. No option is feasible with already assigned variables so reverse decisions using contingencies until a feasible assignment is reached. If successful go to E, else go to D.

D. Repeated flight legs are causing infeasibility. Look for a loop to cancel that will return feasibility. Go to E.

E. If paths with fractional values remain go to B, else feasible 0-1 schedule has been obtained.

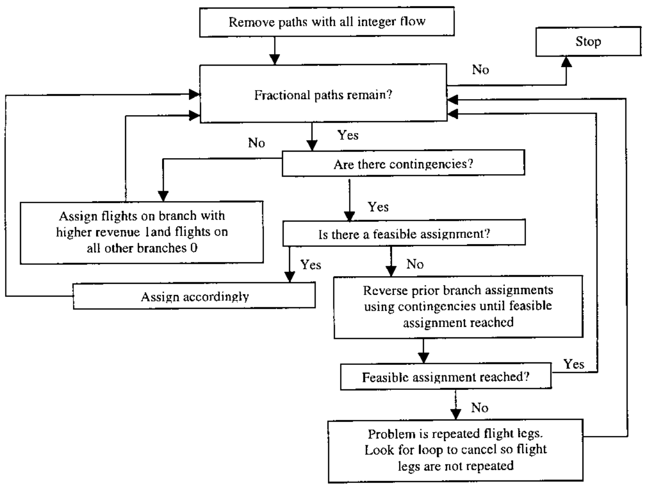

Fig. 8. Flow chart for rounding heuristic.

To demonstrate how the heuristic works, consider the network in Fig. 9 consisting of three split paths. Three aircraft begin the day at stations MKE (Milwaukee), MIA (Miami) and EWR (Newark) and must finish the day at BUF (Buffalo), CLE (Cleveland) and TPA (Tampa). In a fractional solution, the path for each plane splits at some point during the day. Assume that when the paths split they split in half. Applying the heuristic, there are no paths with purely integer flow (Step A) and we begin with the plane at MKE. Assume the path ending at BUF has higher revenue. Since no moves are forced by contingencies, the flights on the path ending at BUF are assigned flow of 1 and the flights on the path ending at CLE are assigned flow of 0 (Step B). As fractional paths remain, we move to the plane starting at MIA, where we must choose the path on the left. As the flow to CLE on the first aircraft was rounded to 0, to maintain aircraft balance this plane must finish at CLE (Step B, contingency 1). Finally, the third plane, which begins in EWR,

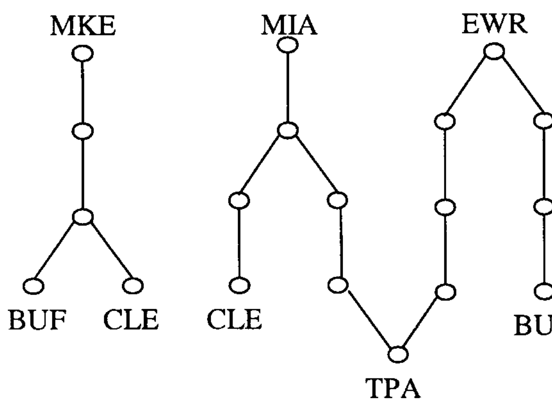

Fig. 9. Example to clarify heuristic.

must take the left path and finish at TPA. Two contingencies force this move: (1) either the right path of plane 2 or the left path of plane 3 must be taken (plane 2 took its left path); and (2) the first plane finished at BUF so the third plane cannot end there.

The heuristic was tested on eight fractional solutions obtained from runs done using the 737-100 data. Near-optimal solutions were found in all cases. A comparison of delays, cancellations, swaps and intact flight paths for those solutions found with the rounding heuristic versus the IP solutions is shown in Table 8. The values in the table are averages over the eight problems. Using these four statistics to measure the quality of the solutions, we find them very similar. The solutions obtained using the rounding heuristic have more cancellations and intact flight paths on average, but fewer delays and swaps than the IP solutions. On average, when looking at the objective value, the heuristic returned solutions 0.59% from the known optimal solution.

6. Summary and conclusions

In this paper, a network flow model with side constraints was introduced to help airlines better manage irregular operations in real time. The proposed model not only includes options for delaying and canceling flights, but also incorporates a measure of deviation from the original aircraft routings. For most airlines, major perturbations to the original flight schedule are unacceptable when only one or two aircraft are grounded.

Extensive testing demonstrated the robustness and flexibility of the model. For the 757 test problems, solving the LP relaxation of the network formulation provided integer solutions quickly. Moreover, these solutions maintained a large portion of the original schedule. The tradeoff curves presented in Section 4 demonstrate the flexibility of the model in its ability to generate solutions that reflect changing user preferences. In all cases tested, when the LP relaxation did not produce integer solutions, the rounding heuristic found near-optimal feasible solutions in negligible time. This heuristic is likely to prove valuable in any implementation covering major hub and spoke systems when solving the true IP problem may be too time consuming.

One weakness of the model is that it does not track individual passengers and thus does not consider passenger connections. The size of such a model, however, would render it impossible to solve in a reasonable time frame. The use of through-flight arcs helps bridge this gap by approximating the cost of breaking these special-case passenger connections. A second factor not robustly addressed at this time is maintenance. Our simple way of handling this requirement is to remove aircraft scheduled for maintenance at the end of the day from consideration when delaying and canceling flights. In this manner, aircraft requiring service are assured to reach their proper destinations.

Table 8. Performance of heuristic

	Delays	*Cancellations*	*Swaps*	*Intact flight paths*
Heuristic solutions	16.1	8.5	25.4	17.9
IP solutions	18.6	7.0	29.0	17.3

The work in this paper is intended to provide a stepping stone to more comprehensive approaches to rerouting aircraft in irregular operations situations. In research currently underway, we are expanding this model to include maintenance requirements while allowing some freedom to use aircraft bound for maintenance in the recovery schedule. In addition, for large airlines, this problem must be addressed in the context of multiple fleets. The approach is now being extended to allow swaps and reassignments between different fleets for the case of hub closures [2].

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