7.8 Lab: Non-linear Modeling

- 7.8.1 Polynomial Regression and Step Functions
- 7.8.2 Splines
- 7.8.3 GAMs

In this lab, we re-analyze the **Wage** data considered in the examples throughout this chapter in order to illustrate the fact that many of the complex non-linear fitting procedures discussed can be easily implemented in R. We begin by loading the **ISLR** library, which contains the data.

```
library(ISLR)
```

```
## Warning: package 'ISLR' was built under R version 3.4.4
```

```
attach(Wage)
```

7.8.1 Polynomial Regression and Step Functions

We now examine how Figure 7.1 was produced. We first fit the model using the following command:

```
fit = lm(wage ~ poly(age, 4), data = Wage)
coef(summary(fit))
```

```
##                  Estimate Std. Error    t value     Pr(>|t|)
## (Intercept)    111.70361  0.7287409 153.283015 0.000000e+00
## poly(age, 4)1  447.06785 39.9147851  11.200558 1.484604e-28
## poly(age, 4)2 -478.31581 39.9147851 -11.983424 2.355831e-32
## poly(age, 4)3  125.52169 39.9147851   3.144742 1.678622e-03
## poly(age, 4)4  -77.91118 39.9147851  -1.951938 5.103865e-02
```

This syntax fits a linear model, using the **lm()** function, in order to predict **wage** using a fourth-degree polynomial in **age:poly(age,4)**. The **poly()** command allows us to avoid having to write out a long formula with powers of **age**. The function returns a matrix whose columns are a basis of *orthogonal polynomials*, which essentially means that each column is a linear combination of the variables **age**, **age$^{2,\ age}$3**, and **age^4**.

However, we can also use **poly()** to obtain **age**, **age$^{2,\ age}$3**, and **age^4** directly, if we prefer. We can do this by using the **raw=TRUE** argument to the **poly()** function. Later, we see that this change does not affect the model in a meaningful way – though the choice of basis clearly affects the coefficient estimates, it does not affect the fitted values obtained.

```
fit2 = lm(wage ~ poly(age, 4, raw = T), data = Wage)
coef(summary(fit2))
```

```
##                              Estimate   Std. Error   t value     Pr(>|t|)
## (Intercept)            -1.841542e+02 6.004038e+01 -3.067172 0.0021802539
## poly(age, 4, raw = T)1  2.124552e+01 5.886748e+00  3.609042 0.0003123618
## poly(age, 4, raw = T)2 -5.638593e-01 2.061083e-01 -2.735743 0.0062606446
## poly(age, 4, raw = T)3  6.810688e-03 3.065931e-03  2.221409 0.0263977518
## poly(age, 4, raw = T)4 -3.203830e-05 1.641359e-05 -1.951938 0.0510386498
```

There are several other equivalent ways of fitting this model, which showcase the flexibility of the formula language in R. For example:

```
fit2a = lm(wage ~age + I(age^2) + I(age^3) + I(age^4), data = Wage)
coef(summary(fit2a))
```

```
##                   Estimate   Std. Error   t value     Pr(>|t|)
## (Intercept) -1.841542e+02 6.004038e+01 -3.067172 0.0021802539
## age          2.124552e+01 5.886748e+00  3.609042 0.0003123618
## I(age^2)    -5.638593e-01 2.061083e-01 -2.735743 0.0062606446
## I(age^3)     6.810688e-03 3.065931e-03  2.221409 0.0263977518
## I(age^4)    -3.203830e-05 1.641359e-05 -1.951938 0.0510386498
```

This syntax simply creates the polynomial basis functions on the fly, taking care to protect terms like **age^2** via the *wrapper* function **I()** (the ^ symbol has a special meaning in formulas).

```
fit2b = lm(wage ~ cbind(age, age^2, age^3, age^4), data = Wage)
```

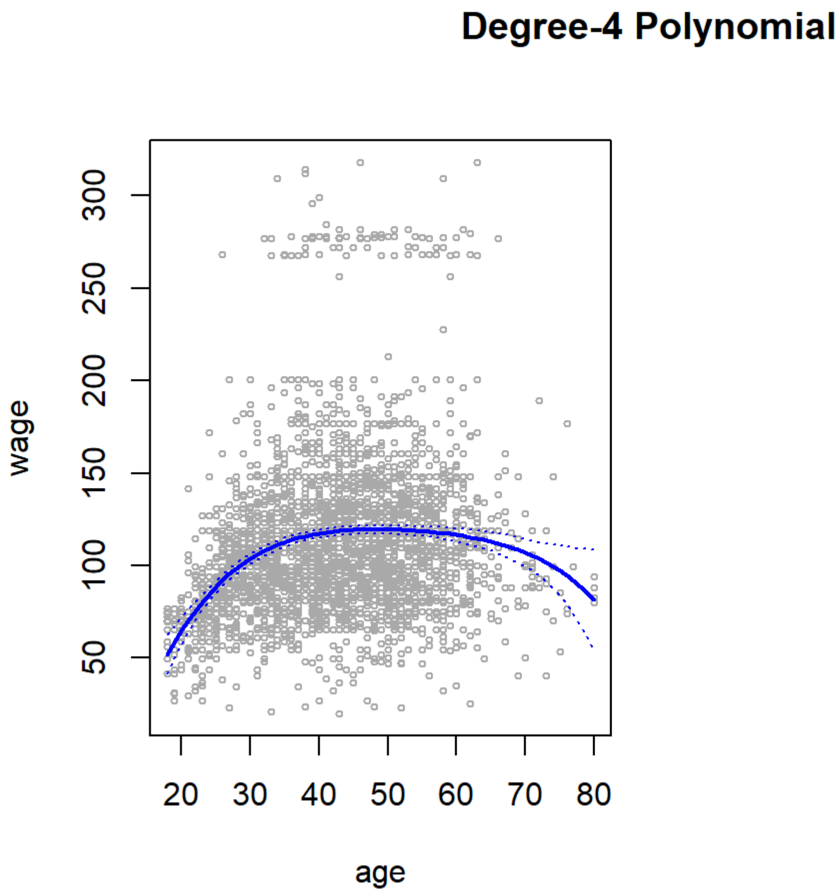
This syntax does the same more compactly, using the **cbind()** function for building a matrix from a collection of vectors; any function call such as **cbind()** inside a formula also serves as a wrapper.

We now create a grid of values for **age** at which we want predictions, and then we call the generic **predict()** function, specifying that we want standard errors as well.

```
agelims = range(age)
age.grid = seq(from = agelims[1], to = agelims[2])
preds = predict(fit, newdata = list(age = age.grid), se = TRUE)
se.bands = cbind(preds$fit + 2*preds$se.fit, preds$fit - 2*preds$se.fit)
```

Finally, we plot the data and add the fit from the degree-4 polynomial.

```
par(mfrow = c(1,2), mar=c(4.5, 4.5, 1, 1), oma = c(0, 0, 4, 0))
plot(age, wage, xlim = agelims, cex = .5, col = "darkgrey")
title("Degree-4 Polynomial", outer = T)
lines(age.grid, preds$fit, lwd = 2, col = "blue")
matlines(age.grid, se.bands, lwd = 1, col = "blue", lty = 3)
```



Here, the **mar** and **oma** arguments to **par()** allow us to control the margins of the plot, and the **title()** function creates a figure title that spans both subplots.

We mentioned earlier that whether or not an orthogonal set of basis functions is produced in the **poly()** function will not affect the model obtained in a meaningful way. What do we mean by this? The fitted values obtained in either case are identical:

```
preds2 = predict(fit2, newdata = list(age = age.grid), se = T)
max(abs(preds$fit - preds2$fit))
```

```
## [1] 7.81597e-11
```

In performing a polynomial regression we must decide on the degree of the polynomial to use. One way to do this by using hypothesis tests. We now fit models ranging from linear to a degree-5 polynomial and seek to determine the simplest model which is sufficient to explain the relationship between **wage** and **age**. We use the **anova()** function, which performs an *analysis of variance* (ANOVA, using an F-test) in order to test the null hypothesis that a model M_1 is sufficient to explain the data against the alternative hypothesis that a more complex model M_2 is required. In order to use the **anova()** function, M_1 and M_2 must be *nested* models: the predictors in M_1 must be a subset of the predictors in M_2. In this case, we fit five different models and sequentially compare the simpler model to the more complex model.

```
fit.1 = lm(wage ~ age, data = Wage)
fit.2 = lm(wage ~ poly(age, 2), data = Wage)
fit.3 = lm(wage ~ poly(age, 3), data = Wage)
fit.4 = lm(wage ~ poly(age, 4), data = Wage)
fit.5 = lm(wage ~ poly(age, 5), data = Wage)
anova(fit.1, fit.2, fit.3, fit.4, fit.5)
```

```
## Analysis of Variance Table
##
## Model 1: wage ~ age
## Model 2: wage ~ poly(age, 2)
## Model 3: wage ~ poly(age, 3)
## Model 4: wage ~ poly(age, 4)
## Model 5: wage ~ poly(age, 5)
##   Res.Df     RSS Df Sum of Sq        F    Pr(>F)
## 1   2998 5022216
## 2   2997 4793430  1    228786 143.5931 < 2.2e-16 ***
## 3   2996 4777674  1     15756   9.8888  0.001679 **
## 4   2995 4771604  1      6070   3.8098  0.051046 .
## 5   2994 4770322  1      1283   0.8050  0.369682
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

The p-value comparing the lienar **Model 1** to the quadratic **Model 2** is essentially zero ($< 10^{-15}$), indicating that a lienar fit is not sufficient. Similarly, the p-value comparing the quadratic **Model 2** to the cubic **Model 3** is very low (0.0017), so the quadratic fit is also insufficient. The p-value comparing the cubic and degree-4 polynomials, **Model 3** and **Model 4**, is approximately 5%, while the degree-5 polynomial **Model 5** seems unneccessary because its p-value is 0.37. Hence, either a cubic or a quartic pollynomial appear to provide a reasonable fit to the data, but lower- or higher-order models are not justified.

In this case, instead of using the **anova()** function, we could have obtained these p-values more succintcly by exploiting the fact that **poly()** creates orthogonal polynomials.

```
coef(summary(fit.5))
```

```
##                   Estimate Std. Error     t value     Pr(>|t|)
## (Intercept)      111.70361  0.7287647 153.2780243 0.000000e+00
## poly(age, 5)1   447.06785 39.9160847  11.2001930 1.491111e-28
## poly(age, 5)2 -478.31581 39.9160847 -11.9830341 2.367734e-32
## poly(age, 5)3  125.52169 39.9160847   3.1446392 1.679213e-03
## poly(age, 5)4  -77.91118 39.9160847  -1.9518743 5.104623e-02
## poly(age, 5)5  -35.81289 39.9160847  -0.8972045 3.696820e-01
```

Notice that the p-values are the same, and in fact the square of the t-statistics are equal to the F-statistics from the **anova()** function; for example:

```
(-11.983)^2
```

```
## [1] 143.5923
```

However, the ANOVA method works whether or not we used orthogonal polynomials; it also works when we have other terms in the model as well. For example, we can use **anova()** to compare these three models:

```
fit.1 = lm(wage ~ education + age, data = Wage)
fit.2 = lm(wage ~ education + poly(age, 2), data = Wage)
fit.3 = lm(wage ~ education + poly(age, 3), data = Wage)
anova(fit.1, fit.2, fit.3)
```

```
## Analysis of Variance Table
##
## Model 1: wage ~ education + age
## Model 2: wage ~ education + poly(age, 2)
## Model 3: wage ~ education + poly(age, 3)
##   Res.Df     RSS Df Sum of Sq        F Pr(>F)
## 1   2994 3867992
## 2   2993 3725395  1    142597 114.6969 <2e-16 ***
## 3   2992 3719809  1      5587   4.4936 0.0341 *
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

As an alternative to using hypothesis tests and ANOVA, we could choose the polynomial degree using cross-validation, as discussed in Chapter 5.

Next we consider the task of predicting whether an individual earns more than $250,000 per year. We proceed much as before, except that first we create the appropriate response vector, and then we apply the **glm()** function using **family=binomial** in order to fit a polynomial logistic regression model.

```
fit = glm(I(wage > 250) ~ poly(age, 4), data = Wage, family = binomial)
```

Note that we again use the wrapper **I()** to create this binary response variable on the fly. The expression **wage > 250** evaluates to a logical variable containg **TRUE**s and **FALSE**s, which **glm()** coerces to binary by setting the **TRUE**s to 1 and the **FALSE**s to 0.

Once again, we make predictions using the **predict()** function.

```
preds = predict(fit, newdata = list(age = age.grid), se = T)
```

However, calculating the confidence intervals is slightly more involved than in the linear regression case. The default prediction type for a **glm()** model is **type="link"**, which is what we use here. This means we get predictions for the *logit*: that is, we have fit a model of the form

$$\log(\frac{\Pr(Y=1|X)}{1-\Pr(Y=1|X)}) = X\beta$$

and the predictions given are of the form $X\hat{\beta}$. The standard errors given are also of this form. In order to obtain confidence intervals for $\Pr(Y=1|X)$, we use the transformation

$$\Pr(Y=1|X) = \frac{\exp(X\beta)}{1+\exp(X\beta)}$$

```
pfit = exp(preds$fit) / (1 + exp(preds$fit))
se.bands.logit = cbind(preds$ift + 2 * preds$se.fit, preds$fit - 2 * preds$se.fit)
se.bands = exp(se.bands.logit) / (1 + exp(se.bands.logit))
```

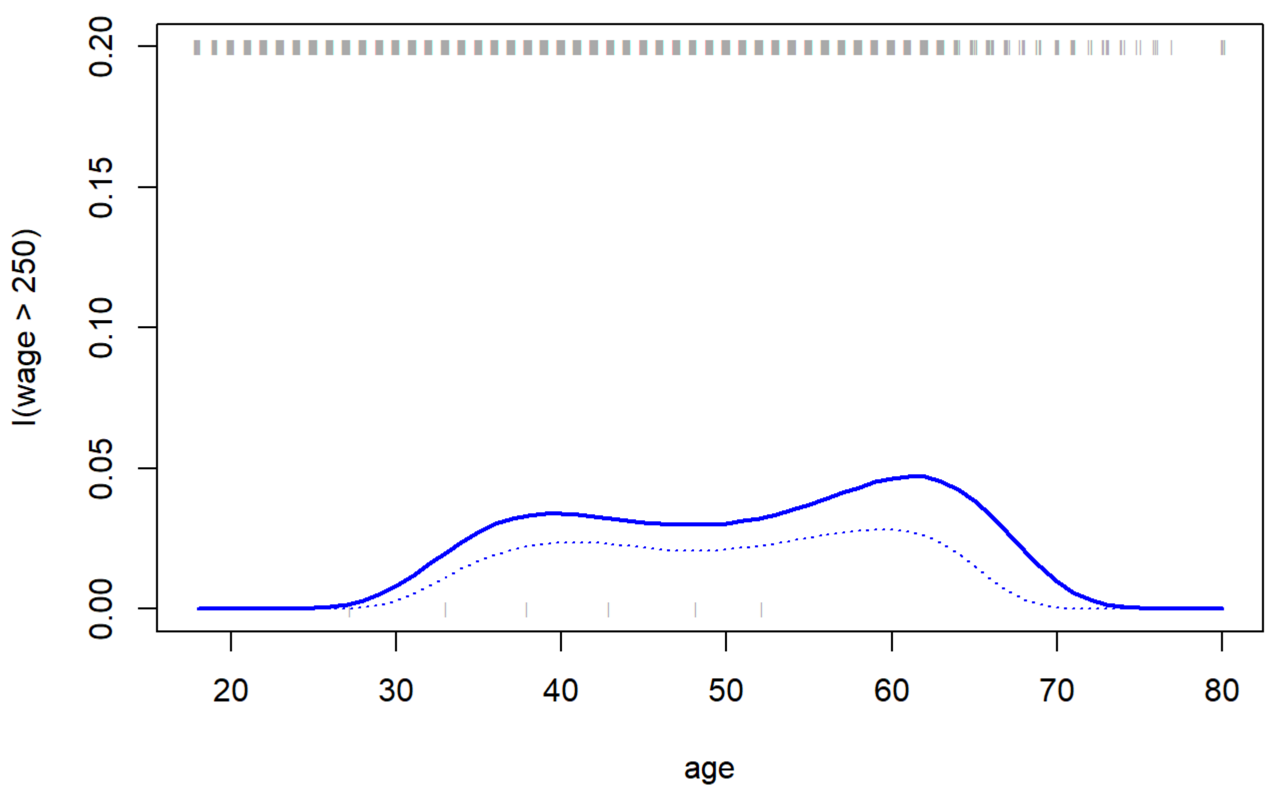
Note that we could ahve directly computed the probabilities by selecting the **type="response"** option in the **predict()** function.

```
preds = predict(fit, newdata = list(age = age.grid), type = "response", se = T)
```

However, the corresponding confidence intervals would not have been sensible because we would end up with negative probabilities!

Finally, the right-hand plot from Figure 7.1 was made as follows:

```
plot(age, I(wage > 250), xlim = agelims, type = "n", ylim = c(0, .2))
points(jitter(age), I((wage > 25) / 5), cex = .5, pch = "|", col = "darkgrey")
lines(age.grid, pfit, lwd = 2, col = "blue")
matlines(age.grid, se.bands, lwd = 1, col = "blue", lty = 3)
```



We have drawn the **age** values corresponding to the observations with **age** values above 250 as grey marks on top of the plot, and those with **wage** values below 250 are shown as grey marks on the bottom of the plot. We used the **jitter()** function to jitter the **age** values a bit so that the observations with the same **age** value do not cover each other up. This type of plot is often called a *rug plot*.

In order to fit a step function, as discussed in Section 7.2, we use the **cut()** function.

```
table(cut(age, 4))
```

```
##
## (17.9,33.5]   (33.5,49]   (49,64.5] (64.5,80.1]
##         750        1399         779          72
```

```
fit = lm(wage ~ cut(age, 4), data = Wage)
coef(summary(fit))
```

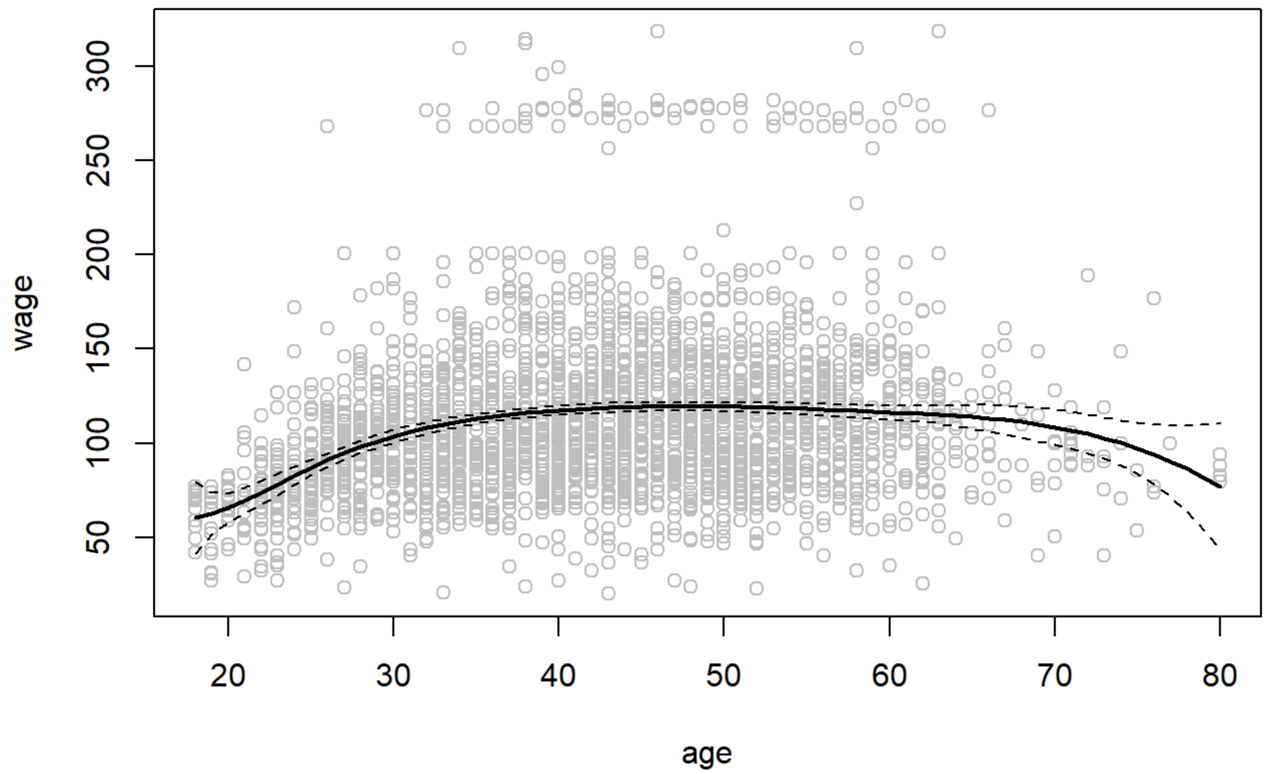
```
##                          Estimate Std. Error   t value     Pr(>|t|)
## (Intercept)             94.158392   1.476069 63.789970 0.000000e+00
## cut(age, 4)(33.5,49]    24.053491   1.829431 13.148074 1.982315e-38
## cut(age, 4)(49,64.5]    23.664559   2.067958 11.443444 1.040750e-29
## cut(age, 4)(64.5,80.1]   7.640592   4.987424  1.531972 1.256350e-01
```

Here, **cut()** automatically picked the cutpoints at 33.5, 49, and 64.5 years of age. We could also have specified our own cutpoints directly using the **breaks** options. The function **cut()** returns an ordered categorical variable; the **lm()** function then creates a set of dummy variables for use in the regression. The **age < 33.5** category is left out, so the intercept coefficient of $94,160 can be interpreted as the average salary for those under 33.5 years of age, and the other coefficients can be interpreted as the average additional salary for those in the other age groups. We can produce predictions and plots just as we did in the case of the polynomial fit.

7.8.2 Splines

In order to fit regression splines in R, we use the **splines** library. In Section 7.4, we saw that regression splines can be fit by constructing an appropriate matrix of basis functions. The **bs()** function generates the entire matrix of basis functions for splines with the specified set of knots. By default, cubic splines are produced. Fitting **wage** to **age** using a regression spline is simple:

```
library(splines)
fit = lm(wage ~ bs(age, knots = c(25, 40, 60)), data = Wage)
pred = predict(fit, newdata = list(age = age.grid), se = T)
plot(age, wage, col = 'grey')
lines(age.grid, pred$fit, lwd = 2)
lines(age.grid, pred$fit + 2 * pred$se, lty = 'dashed')
lines(age.grid, pred$fit - 2 * pred$se, lty = 'dashed')
```



Here we have predicted knots at ages 25, 40, and 60. This produces a spline with six basis functions. (Recall that a cubic spline with three knots has seven degrees of freedom; these degrees of freedom are used up by an intercept plus six basis functions.) We could also use the **df** option to produce a spline with knots at uniform quantiles of the data.

```
dim(bs(age, knots = c(25, 40, 60)))
```

```
## [1] 3000    6
```

```
dim(bs(age, df = 6))
```

```
## [1] 3000    6
```

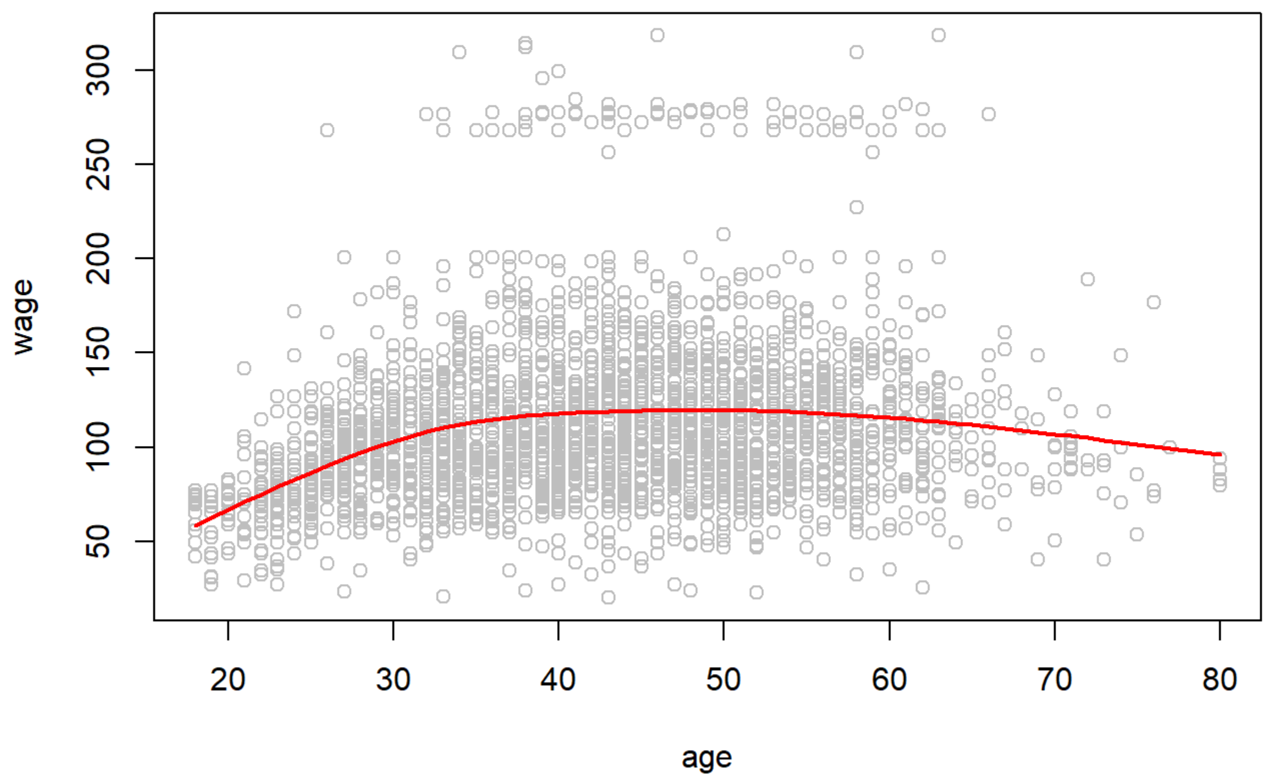
```
attr(bs(age, df = 6), 'knots')
```

```
##   25%   50%   75%
## 33.75 42.00 51.00
```

In this case R chooses knots at ages 33.8, 42.0, and 51.0, which correspond to the 25th, 50th, and 75th percentiles of **age**. The function **bs()** also has a **degree** argument, so we can fit splines of any degree, rather than the default degree of 3 (which yields a cubic spline).

In order to instead fit a natural spline, we use the **ns()** function. Here we fit a natural spline with four degrees of freedom.

```
fit2 = lm(wage ~ ns(age, df = 4), data = Wage)
pred2 = predict(fit2, newdata = list(age = age.grid), se = T)
plot(age, wage, col = 'grey')
lines(age.grid, pred2$fit, col = 'red', lwd = 2)
```



As with the **bs()** function, we could instead specify the knots directly using the **knots** option.

In order to fit a smoothing spline, we use the **smooth.spline()** function. Figure 7.8 was produced with the following code:

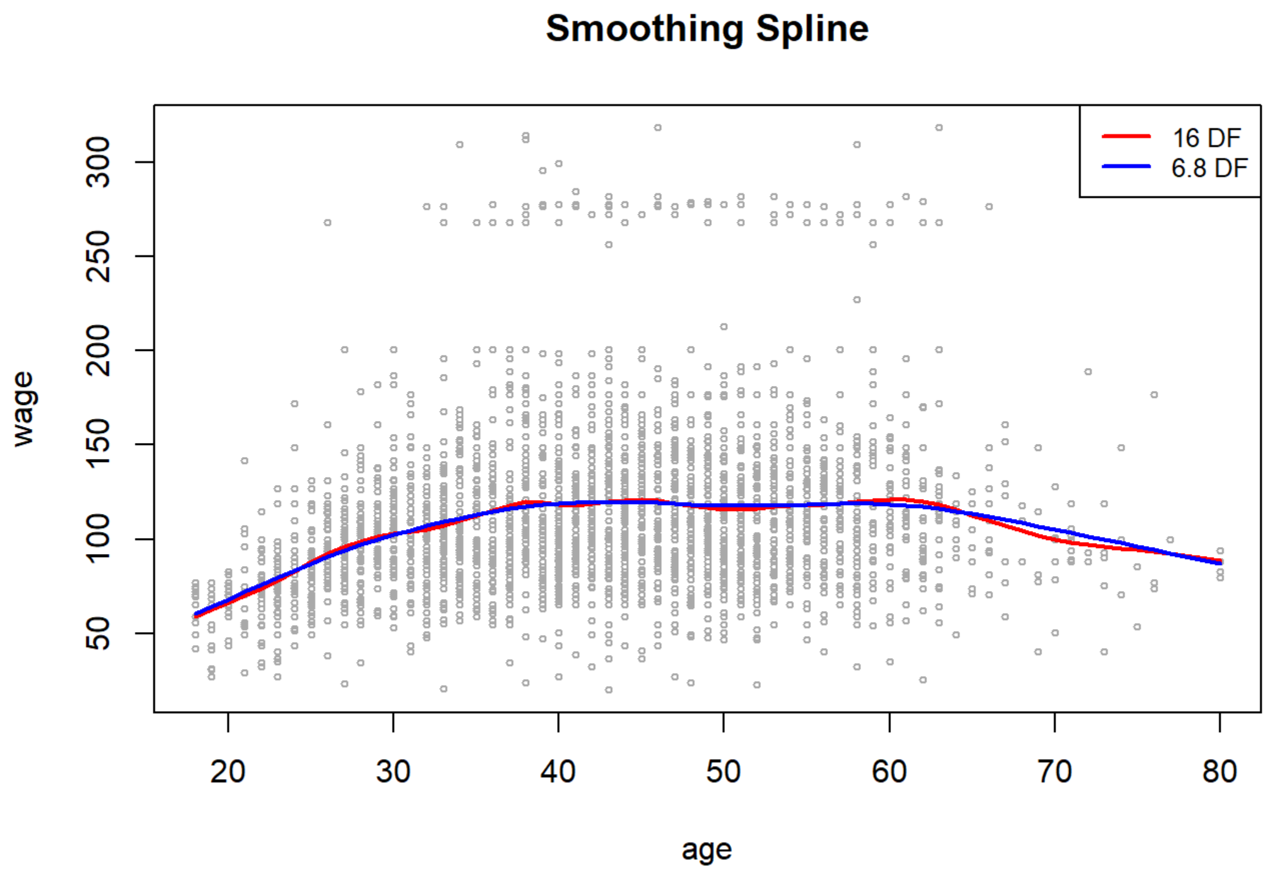
```
fit = smooth.spline(age, wage, df = 16)
fit2 = smooth.spline(age, wage, cv = TRUE)
```

```
## Warning in smooth.spline(age, wage, cv = TRUE): cross-validation with non-
## unique 'x' values seems doubtful
```

```
fit2$df
```

```
## [1] 6.794596
```

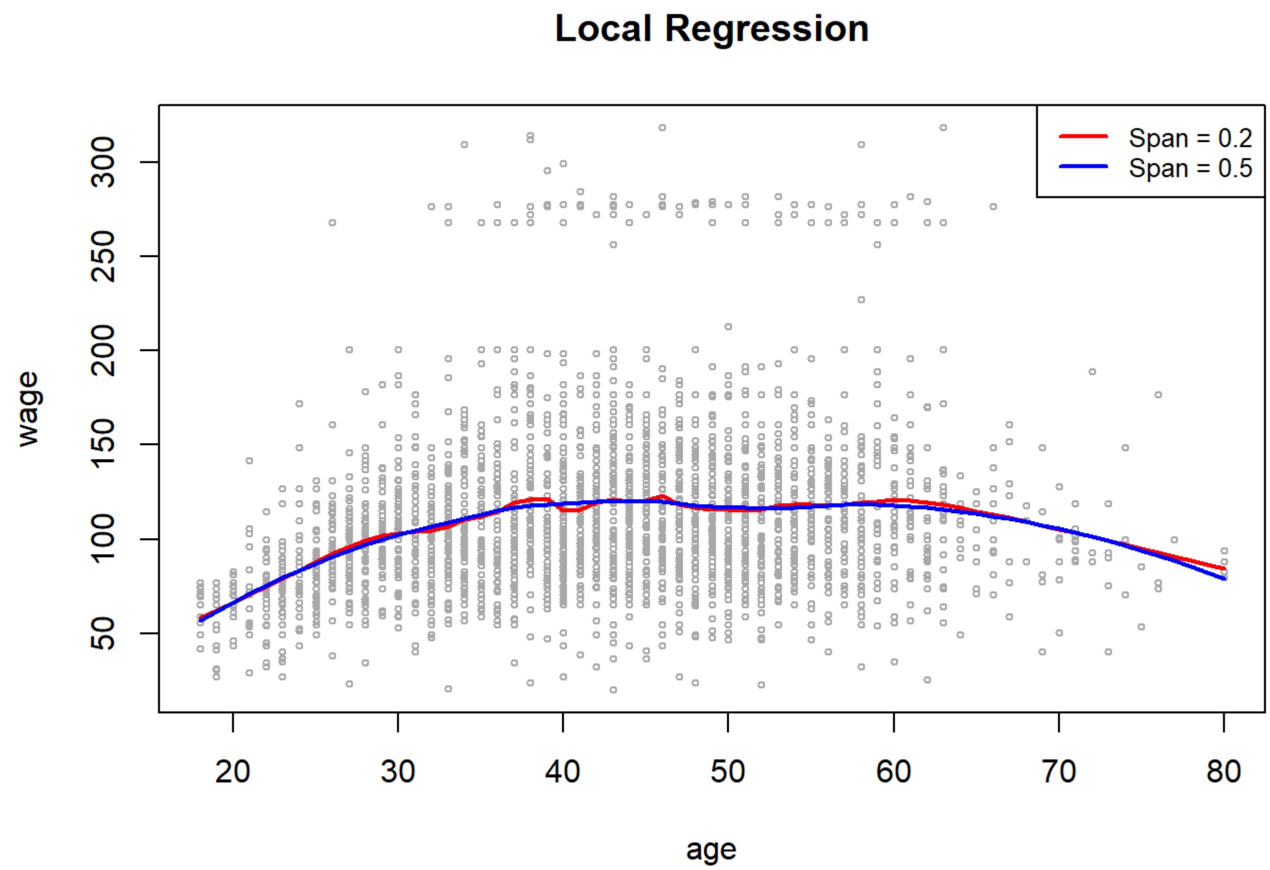
```
plot(age, wage, xlim = agelims, cex = .5, col = 'darkgrey')
title('Smoothing Spline')
lines(fit, col = 'red', lwd = 2)
lines(fit2, col = 'blue', lwd = 2)
legend('topright', legend = c('16 DF', '6.8 DF'), col = c('red', 'blue'), lty = 1, lwd =
2, cex = .8)
```



Notice that in the first call to **smooth.spline()**, we specified **df=16**. The function then determines which value of λ leads to 16 degrees of freedom. In the second call to **smooth.spline()**, we select the smoothness level by cross-validation; this results in a value of λ that yields 6.8 degrees of freedom.

In order to perform local regression, we use the **loess()** function.

```
plot(age, wage, xlim = agelims, cex = .5, col = 'darkgrey')
title('Local Regression')
fit = loess(wage ~ age, span = .2, data = Wage)
fit2 = loess(wage ~ age, span = .5, data = Wage)
lines(age.grid, predict(fit, data.frame(age = age.grid)), col = 'Red', lwd = 2)
lines(age.grid, predict(fit2, data.frame(age = age.grid)), col = 'blue', lwd = 2)
legend('topright', legend = c('Span = 0.2', 'Span = 0.5'), col = c('red', 'blue'), lty =
1, lwd = 2, cex = .8)
```



Here we have performed local linear regression using spans of 0.2 and 0.5: that is, each neighborhood consists 20% or 50% of the observations. The larger the span, the smoother the fit. The **locfit** library can also be used for fitting local regression models in R.

7.8.3 GAMs

We now fit a GAM to predict **wage** using natural spline functions of **year** and **age**, treating **education** as a qualitative predictor, as in (7.16). Since this is just a big linear regression model using an appropriate choice of basis functions, we can simply do this using the **lm()** function.

```
gam1 = lm(wage ~ ns(year, 4) + ns(age, 5) + education, data = Wage)
```

We now fit the model(7.16) using smoothing splines rather than natural splines. In orderto fit more general sorts of GAMs, using smoothing splines or other components that cannot be expressed in terms of basis functions and then fit using least squares regression, we will need to use the **gam** library in R.

The **s()** function, which is part of the **gam** library, is used to indicate that we would like to use a smoothing spline. We specify that the function of **year** should have 4 degrees of freedom, and that the function of **age** will have 5 degrees of freedom. Since **education** is qualitative, we leave it as is, and it is converted into four dummy variables. We use the **gam()** function in order to fit a GAM using these components. All of the terms in (7.16) are fit simultaneously, taking each other into account to explain the response.

```
library(gam)
```

```
## Warning: package 'gam' was built under R version 3.4.4
```

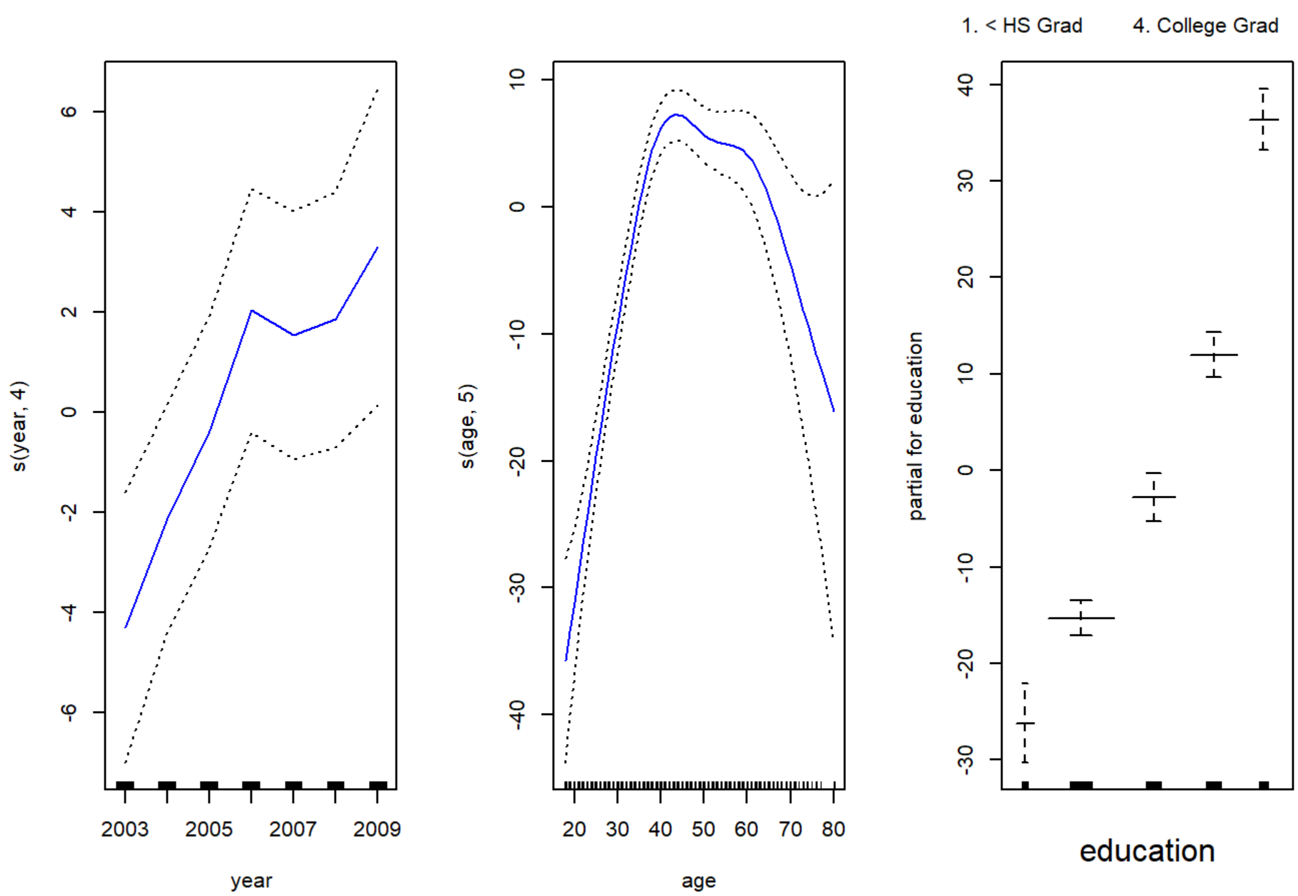
```
## Loading required package: foreach
```

```
## Warning: package 'foreach' was built under R version 3.4.4
```

```
## Loaded gam 1.15
```

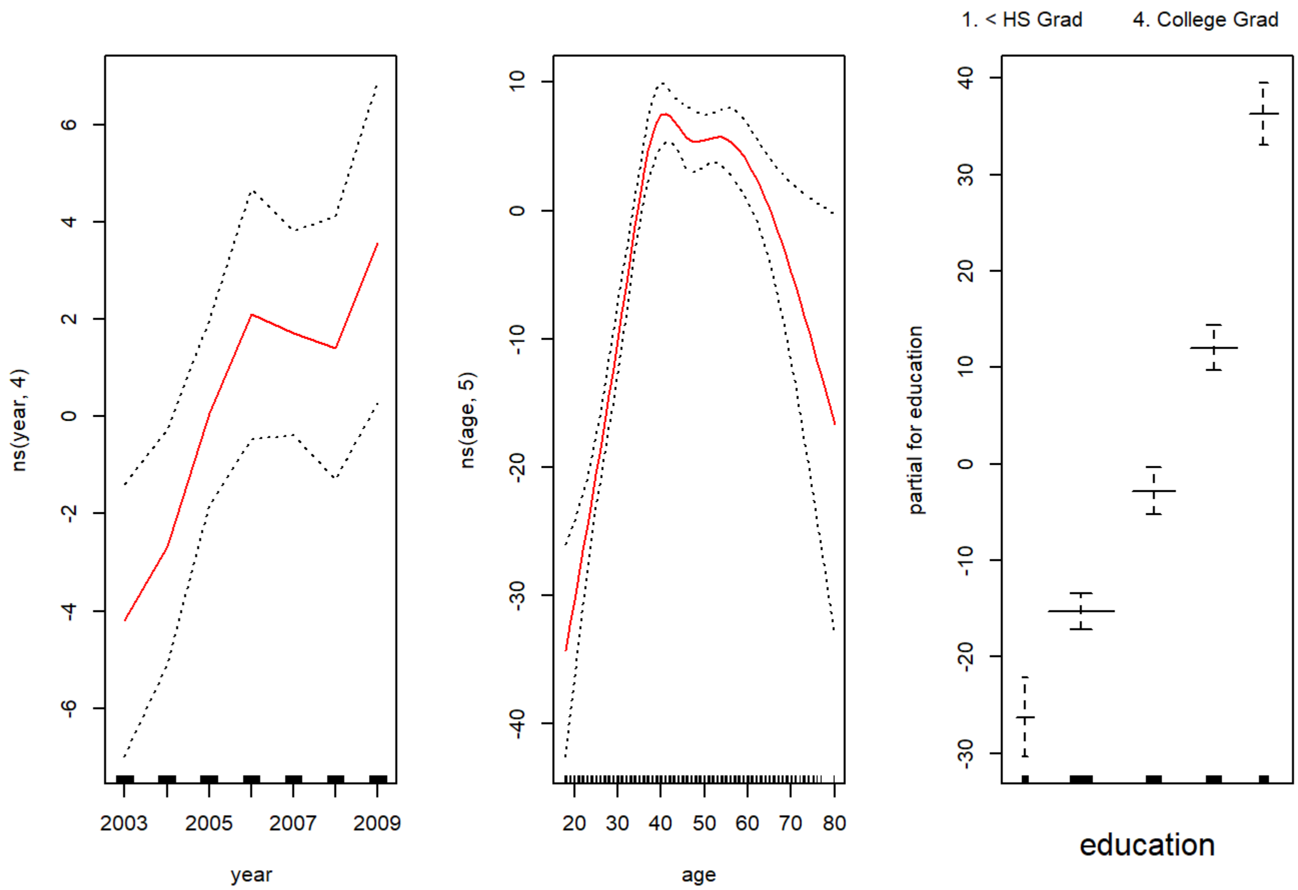
```
gam.m3 = gam(wage ~ s(year, 4) + s(age, 5) + education, data = Wage)
```

```
par(mfrow = c(1,3))
plot(gam.m3, se = T, col = 'blue')
```



The generic **plot()** function recognizes that **gam.m3** is an object of class **gam**, and invokes the appropriate **plot.gam()** method. Conveniently, even though **gam1** is not of class **gam**, but rather of class **lm**, we can *still* use **plot.gam()** on it. Figure 7.11 was produced using the following expression:

```
### Note: plot.gam() seems to have been changed to plot.Gam()
par(mfrow = c(1,3))
plot.Gam(gam1, se = T, col = 'red')
```



Notice here we had to use **plot.gam()** rather than the *generic* **plot()** function.

In these plots, the function of **year** look rather linear. We can perform a series of ANOVA tests in order to determine which of these three models is best: a GAM that excludes **year** (M_1), a GAM that uses a linear function of **year** (M_2), or a GAM that uses a spline function of **year** (M_3).

```
gam.m1 = gam(wage ~ s(age, 5) + education, data = Wage)
gam.m2 = gam(wage ~ year + s(age, 5) + education, data = Wage)
anova(gam.m1, gam.m2, gam.m3, test = 'F')
```

```
## Analysis of Deviance Table
##
## Model 1: wage ~ s(age, 5) + education
## Model 2: wage ~ year + s(age, 5) + education
## Model 3: wage ~ s(year, 4) + s(age, 5) + education
##   Resid. Df Resid. Dev Df Deviance       F    Pr(>F)
## 1      2990    3711731
## 2      2989    3693842  1  17889.2 14.4771 0.0001447 ***
## 3      2986    3689770  3   4071.1  1.0982 0.3485661
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

We find that there is compelling evidence that a GAM with a linear function of **year** is better than a GAM that does not include **year** at all (p-value = 0.00014). However, there is no evidence that a non-linear function of **year** is need (p-value = 0.349). In other words, based on the results of this ANOVA, M_2 is preferred.

The **summary()** function proudces a summary of the gam fit.

```
summary(gam.m3)
```

```
##
## Call: gam(formula = wage ~ s(year, 4) + s(age, 5) + education, data = Wage)
## Deviance Residuals:
##     Min      1Q  Median      3Q     Max
## -119.43  -19.70   -3.33   14.17  213.48
##
## (Dispersion Parameter for gaussian family taken to be 1235.69)
##
##     Null Deviance: 5222086 on 2999 degrees of freedom
## Residual Deviance: 3689770 on 2986 degrees of freedom
## AIC: 29887.75
##
## Number of Local Scoring Iterations: 2
##
## Anova for Parametric Effects
##               Df  Sum Sq Mean Sq F value    Pr(>F)
## s(year, 4)     1   27162   27162  21.981 2.877e-06 ***
## s(age, 5)      1  195338  195338 158.081 < 2.2e-16 ***
## education      4 1069726  267432 216.423 < 2.2e-16 ***
## Residuals   2986 3689770    1236
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Anova for Nonparametric Effects
##              Npar Df Npar F  Pr(F)
## (Intercept)
## s(year, 4)        3  1.086 0.3537
## s(age, 5)         4 32.380 <2e-16 ***
## education
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

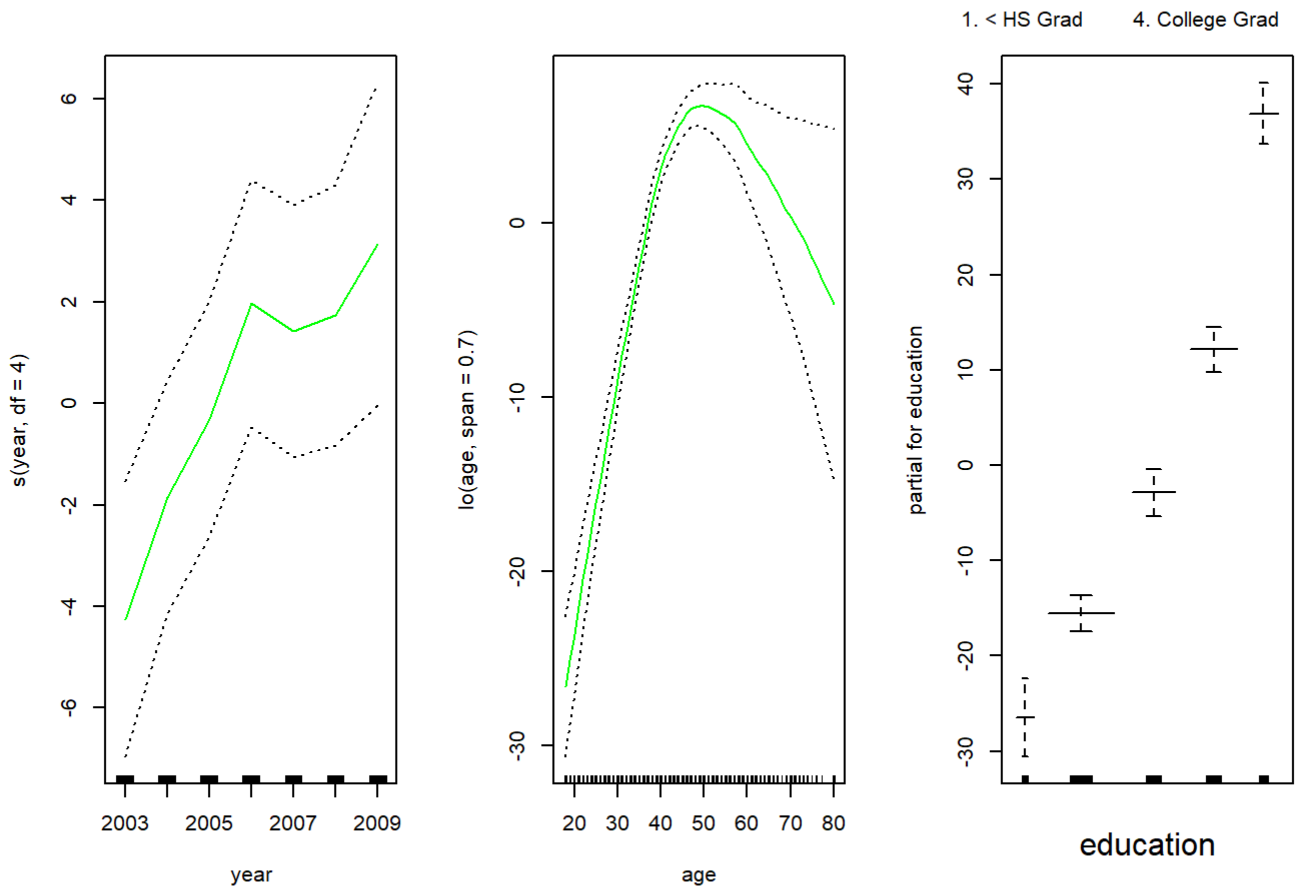
The p-values for **year** and **age** correspond to a null hypothesis of a linear relationship versus the alternative of a non-linear relationship. The large p-value for **year** reinforces our conclusion from the ANOVA test that a linear function is adequate for this term. However, there is very clear evidence that a non-linear term is required for **age**.

We can make predictions from **gam** objects, just like form **lm** objects, using the **predict()** method for the class **gam**. Here we make predictions on the training set.

```
preds = predict(gam.m2, newdata = Wage)
```

We can alos use local regression fits as building blocks in a GAM, using the **lo()** function.

```
gam.lo = gam(wage ~ s(year, df = 4) + lo(age, span = 0.7) + education, data = Wage)
par(mfrow = c(1, 3))
plot.Gam(gam.lo, se = T, col = 'green')
```



Here we have used local regression for the **age** term, with a span of 0.7. We can use the **lo()** function to create interactions before calling the **gam()** function. For example,

```
gam.lo.i = gam(wage ~ lo(year, age, span = 0.5) + education, data = Wage)
```

```
## Warning in lo.wam(x, z, wz, fit$smooth, which, fit$smooth.frame,
## bf.maxit, : liv too small. (Discovered by lowesd)
```

```
## Warning in lo.wam(x, z, wz, fit$smooth, which, fit$smooth.frame,
## bf.maxit, : lv too small. (Discovered by lowesd)
```

```
## Warning in lo.wam(x, z, wz, fit$smooth, which, fit$smooth.frame,
## bf.maxit, : liv too small. (Discovered by lowesd)
```

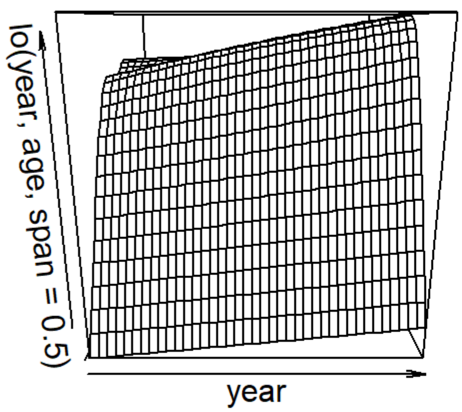
```
## Warning in lo.wam(x, z, wz, fit$smooth, which, fit$smooth.frame,
## bf.maxit, : lv too small. (Discovered by lowesd)
```

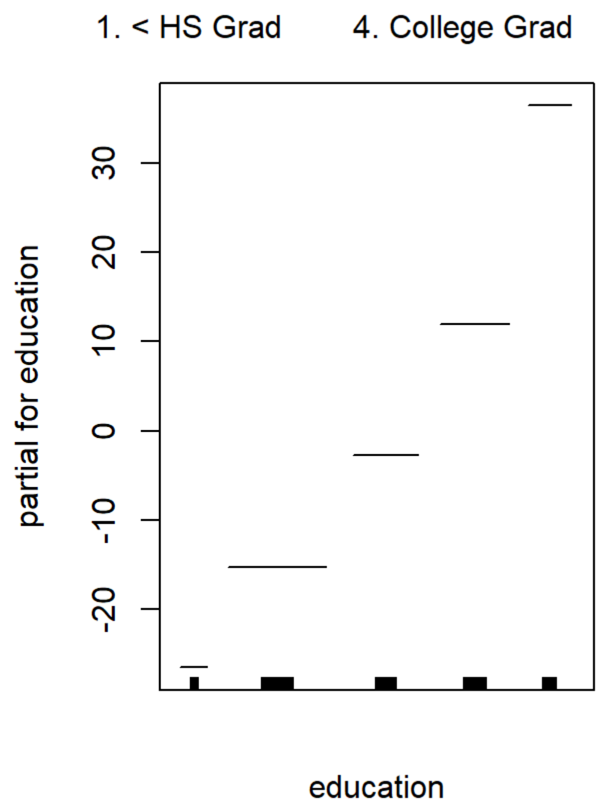
fits a two-term model, in which the first term is an interaction between **year** and **age**, fit by a local regression surface. We can plot the resulting two-dimensional surface if we first install the **akima** package.

```
library(akima)
```

```
## Warning: package 'akima' was built under R version 3.4.4
```

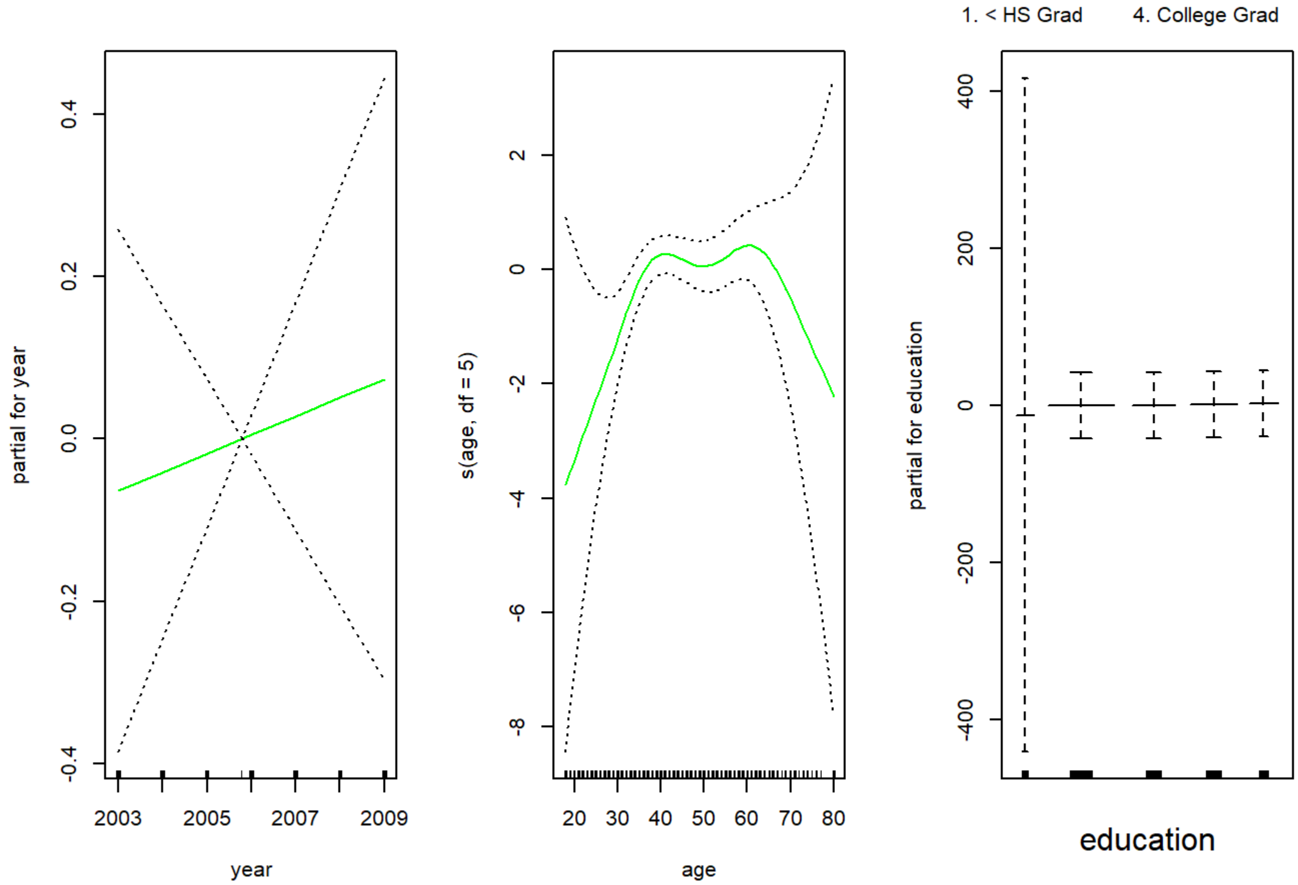
```
par(mfrow = c(1, 2))
plot(gam.lo.i)
```





In order to fit a logistic regression GAM, we once again use the **I()** function in constructing the binary response variable and set **family = binomial**.

```
gam.lr = gam(I(wage > 250) ~ year + s(age, df = 5) + education, family = binomial, data
 = Wage)
par(mfrow = c(1, 3))
plot(gam.lr, se = T, col = 'green')
```



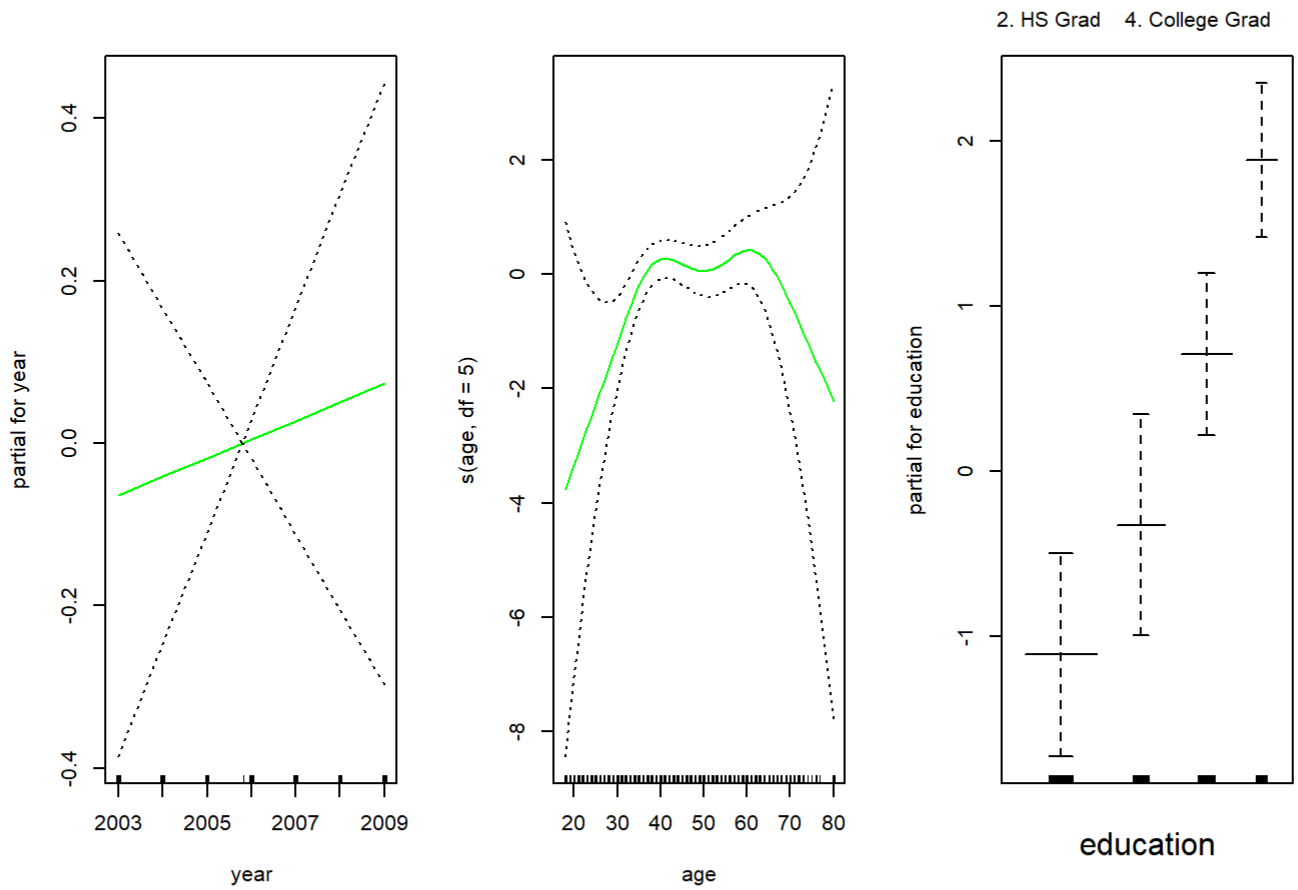
It is easy to see that there are no high earners in the **< HS** category:

```
table(education, I(wage > 250))
```

```
##
## education           FALSE TRUE
##   1. < HS Grad        268    0
##   2. HS Grad          966    5
##   3. Some College     643    7
##   4. College Grad     663   22
##   5. Advanced Degree  381   45
```

Hence, we fit a logistic regression GAM using all but this category. This provides more sensible results.

```
gam.lr.s = gam(I(wage > 250) ~ year + s(age, df = 5) + education, family = binomial, dat
a = Wage, subset = (education != "1. < HS Grad"))
par(mfrow = c(1, 3))
plot(gam.lr.s, se = T, col = 'green')
```



partial for year
0.4
0.2
0.0
-0.2
-0.4
2003
2005
2007
2009
year
s(age, df = 5)
2
0
-2
-4
-6
-8
20
30
40
50
60
70
80
age
2. HS Grad
4. College Grad
partial for education
2
1
0
-1
education