

8.4 Exercises

- Exercise 7
- Exercise 8
- Exercise 9
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- Exercise 12

```
# load all libraries needed for these exercises
library(MASS)
library(randomForest)
```

```
## Warning: package 'randomForest' was built under R version 3.4.4
```

```
## randomForest 4.6-14
```

```
## Type rfNews() to see new features/changes/bug fixes.
```

```
library(tree)
```

```
## Warning: package 'tree' was built under R version 3.4.4
```

```
library(ISLR)
```

```
## Warning: package 'ISLR' was built under R version 3.4.4
```

```
library(gbm)
```

```
## Warning: package 'gbm' was built under R version 3.4.4
```

```
## Loading required package: survival
```

```
## Loading required package: lattice
```

```
## Loading required package: splines
```

```
## Loading required package: parallel
```

```
## Loaded gbm 2.1.3
```

```
library(leaps)
```

```
## Warning: package 'leaps' was built under R version 3.4.4
```

```
library(class)  
library(datasets)
```

Exercise 7

In the lab, we applied random forests to the **Boston** data using **mtry=6** and using **ntree=25** and **ntree=500**. Create a plot displaying the test error resulting from random forests on this data set for a more comprehensive range of values for **mtry** and **ntree**. You can model your plot after Figure 8.10. Describe the results obtained.

```

# Boston data set is in the MASS library
library(MASS)

# create a training set index with half the data
set.seed(1)
train = sample(1:nrow(Boston), size = nrow(Boston)/2, replace = FALSE)

# per Figure 8.10, we will test 1 to 500 trees, with  $m = p$ ,  $m = p/2$ , and  $m = \sqrt{p}$ 
ntrees = 1:500
test.mse = data.frame(p = rep(NaN, length(ntrees)), p2 = rep(NaN, length(ntrees)), sqrt.
p = rep(NaN, length(ntrees)))

# one of the columns is the response and therefore not a valid predictor
p = ncol(Boston) - 1
columns = c('p', 'p2', 'sqrt.p')
m.values = c(p, p/2, sqrt(p))

# Load the randomForest() function from the randomForest library
library(randomForest)

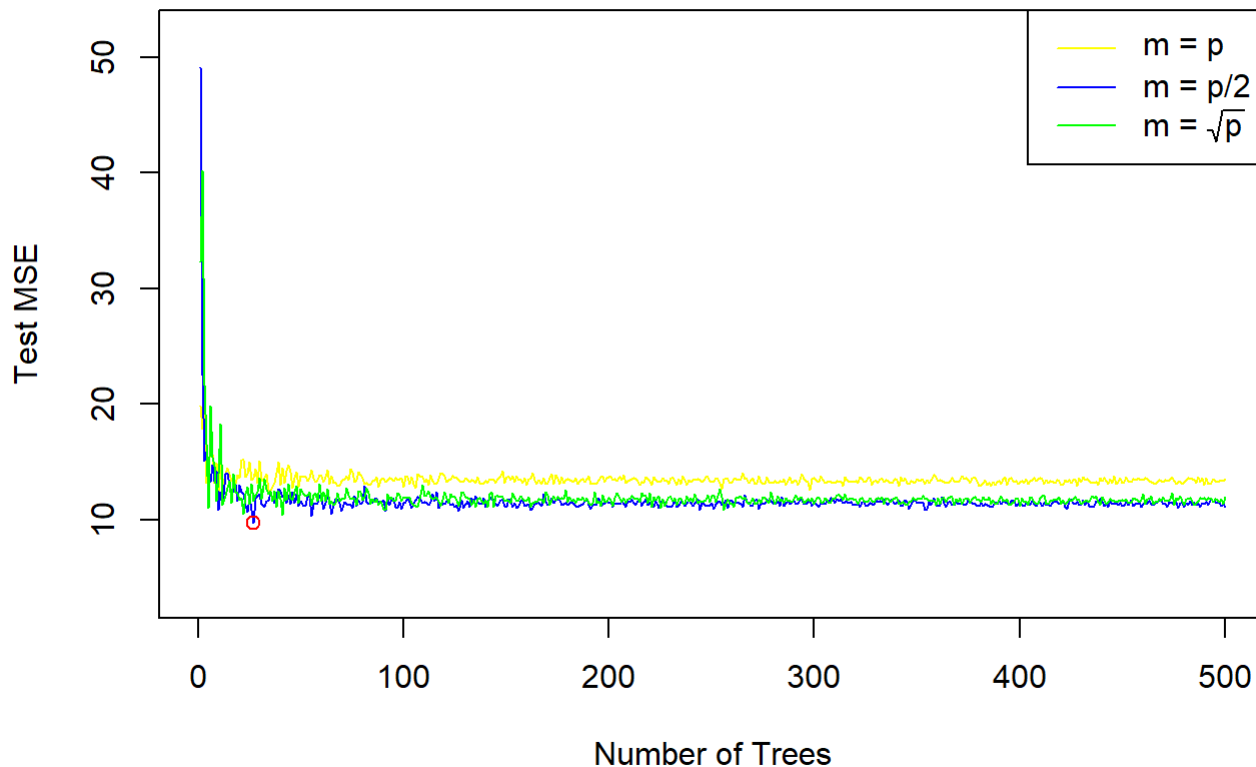
getTestMSE = function(ntree, mtry){
  rf.model = randomForest(medv ~ ., data = Boston, subset = train, mtry = mtry, ntree =
ntree)
  rf.pred = predict(rf.model, newdata = Boston[-train,])
  return(mean((rf.pred - Boston[-train, 'medv'])^2))
}

# loop through different numbers of trees and find the three test errors
# record the minimum test error
min.error = Inf
for (ntree in ntrees){
  for (i in 1:length(m.values)){
    this.col = columns[i]
    m = m.values[i]
    this.error = getTestMSE(ntree, m)
    test.mse[ntree, this.col] = this.error
    if (this.error < min.error){
      min.error = this.error
      min.ntree = ntree
      min.m = m
    }
  }
}

```

```
# Bound the y-axis limits of the plot to avoid cutting off the lines
y.min = min.m - 3
y.max = max(test.mse) + 3

# plot test errors in the same fasion as Figure 8.10
plot(ntrees, test.mse$p, xlab = 'Number of Trees', ylab = 'Test MSE', col = 'yellow', type = 'l', ylim = c(y.min, y.max))
lines(ntrees, test.mse$p2, col = 'blue')
lines(ntrees, test.mse$sqrt.p, col = 'green')
points(min.ntree, min.error, col = 'red')
legend('topright', col = c('yellow', 'blue', 'green'), legend = c('m = p', 'm = p/2', expression(paste('m = ', sqrt(p)))), lty = 1)
```



In this plot, it's hard to see much of a difference between the different values of m , but we can clearly see that additional trees after about 50 are not providing additional test accuracy.

```
# print the minimum error and the associated ntree and mtry
min.error
```

```
## [1] 9.782963
```

```
min.ntree
```

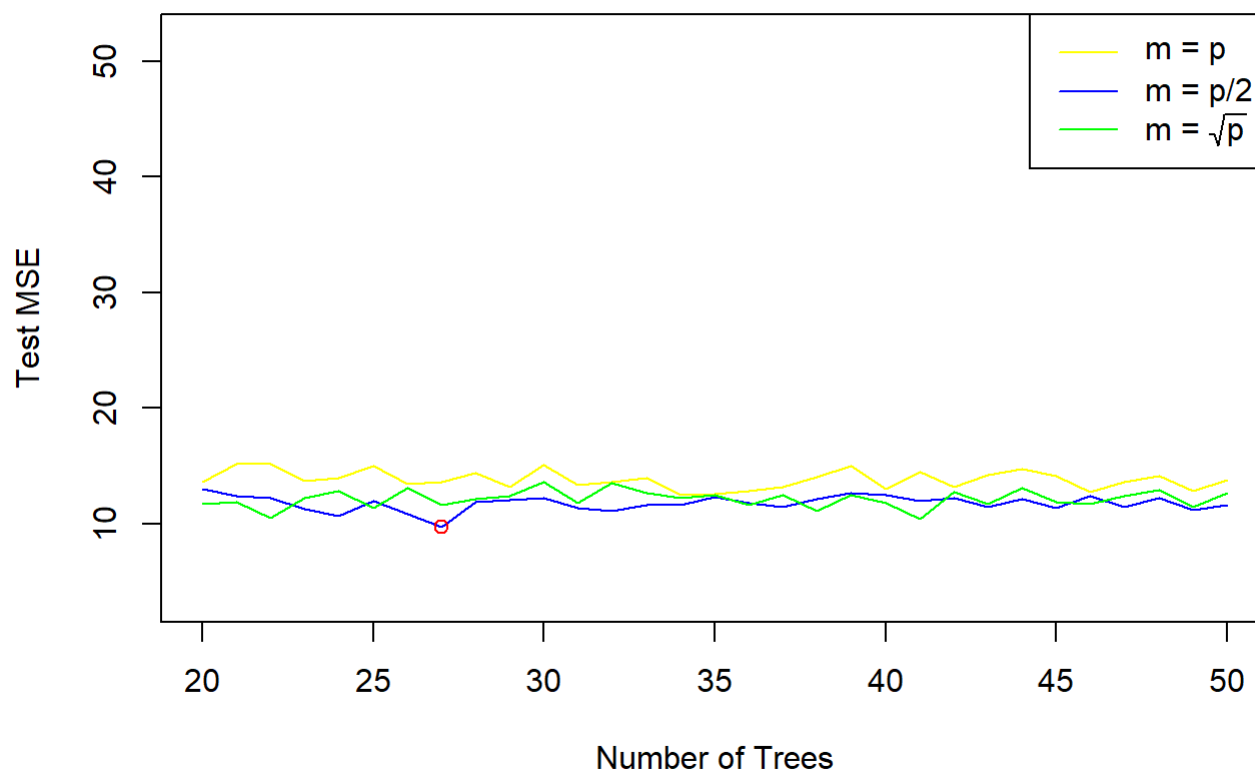
```
## [1] 27
```

```
min.m
```

```
## [1] 6.5
```

The lowest test MSE resulted from aggregating 27 trees and considering $p/2$ predictors at each branch. Let's zoom in our plot around here to get a better look at what's going on.

```
zoom.index = 20:50
plot(zoom.index, test.mse[zoom.index, 'p'], xlab = 'Number of Trees', ylab = 'Test MSE',
     col = 'yellow', type = 'l', ylim = c(y.min, y.max))
lines(zoom.index, test.mse[zoom.index, 'p2'], col = 'blue')
lines(zoom.index, test.mse[zoom.index, 'sqrt.p'], col = 'green')
points(min.ntree, min.error, col = 'red')
legend('topright', col = c('yellow', 'blue', 'green'), legend = c('m = p', 'm = p/2', ex-
pression(paste('m = ', sqrt(p)))), lty = 1)
```



So here we can see that in this range of numbers of trees, there really isn't much difference between the different values for m or the number of trees being used. In fact, our selection of this particular pair of m and n trees seems like it could just be a bit of noise in the variations in test error. In conclusion, for this data set, it seems like the only important thing is to use a sufficient number of trees in the bagging and/or random forests. 25 or more trees seems sufficient based on this data.

Exercise 8

In the lab, a classification tree was applied to the **Carseats** data set after converting **Sales** into a qualitative response variable. Now we will seek to predict **Sales** using regression trees and related approaches, treating the response as a quantitative variable.

```
# the Carseats data set resides in the ISLR library
library(ISLR)
```

a. Split the data set into a training set and a test set.

```
# we'll go 50-50 on training and test data
train = sample(nrow(Carseats), nrow(Carseats)/2, replace = F)
```

b. Fit a regression tree to the training set. Plot the tree and interpret the results. What test MSE do you obtain?

```
# the tree() function is in the tree library
library(tree)

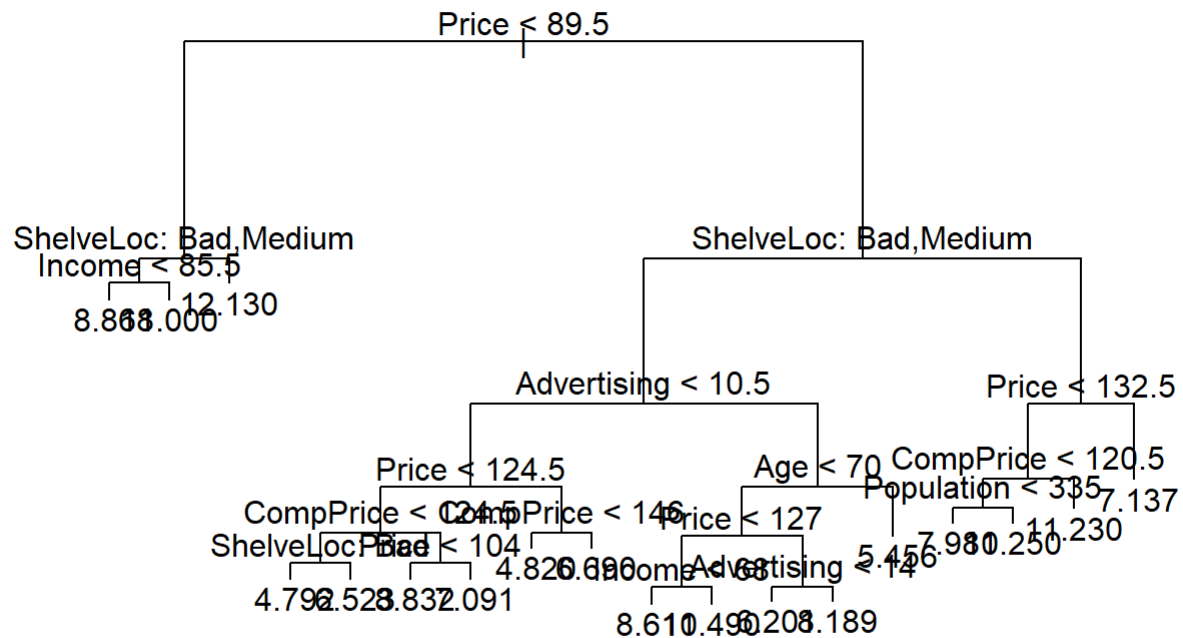
# Make a reusable function for getting the test mse on this data set
getTestMSE = function(model){
  model.pred = predict(model, newdata = Carseats[-train,])
  return(mean((model.pred - Carseats[-train, 'Sales'])^2))
}

tree.model = tree(Sales ~ ., data = Carseats, subset = train)
tree.mse = getTestMSE(tree.model)
tree.mse
```

```
## [1] 4.730486
```

A simple regression tree yields a test MSE of 4.73.

```
plot(tree.model)
text(tree.model, pretty = 0)
```



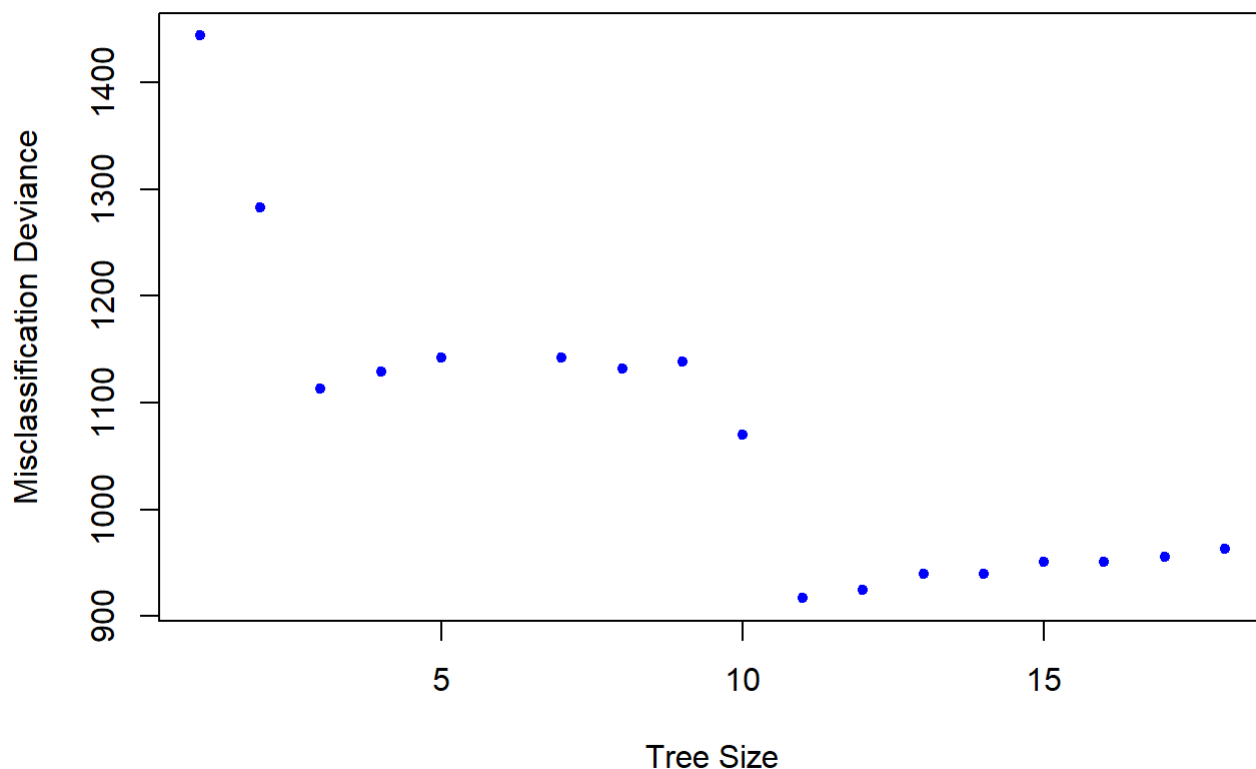
In this tree, the most important predictors are considered to be **Price** and **ShelveLoc**.

- c. Use cross-validation in order to determine the optimal level of tree complexity. Does pruning the tree improve the test MSE?

```

set.seed(4)
cv.carseats = cv.tree(tree.model)
plot(cv.carseats$size, cv.carseats$dev, pch = 20, col = 'blue', xlab = 'Tree Size', ylab = 'Misclassification Deviance')

```



```
best.size = cv.carseats$size[which.min(cv.carseats$dev)]
best.size
```

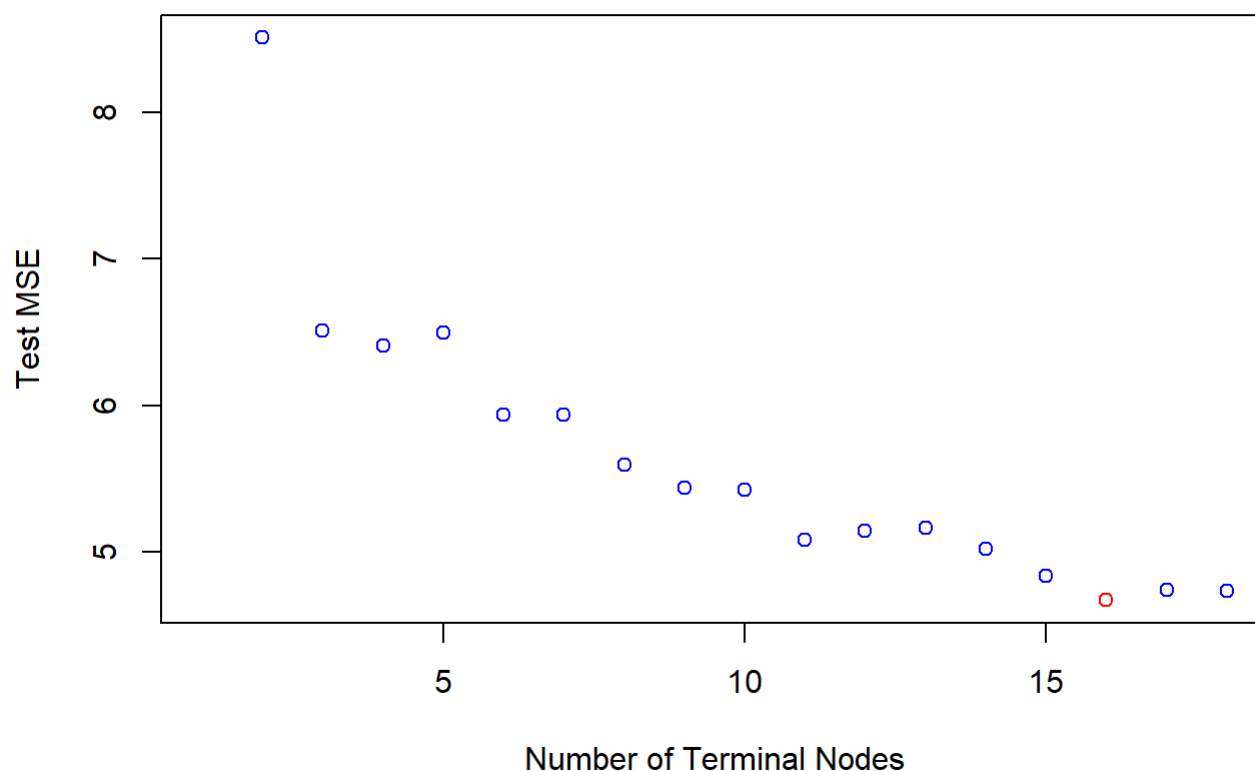
```
## [1] 11
```

```
prune.model = prune.tree(tree.model, best = best.size)
getTestMSE(prune.model)
```

```
## [1] 5.081844
```

The pruned model with 11 terminal nodes does not yield a lower test MSE. Note, I played with the seed value I set before taking the CV so that I would get a smaller tree. Seeds 1-3 actually did not result in a smaller tree.

```
# Let's plot the test MSE of different sizes to see if any of them yield a smaller test
MSE
max.size = length(cv.carseats$size) + 1
test.mse = rep(NA, max.size)
for (this.size in 2:max.size){
  test.mse[this.size] = getTestMSE(prune.tree(tree.model, best = this.size))
}
best.size = which.min(test.mse)
plot(test.mse, xlab = "Number of Terminal Nodes", ylab = "Test MSE", col = "blue")
points(best.size, test.mse[best.size], col = "red")
```

```
best.size
```

```
## [1] 16
```

```
test.mse[best.size]
```

```
## [1] 4.670888
```

The pruned tree with only 16 terminal nodes has a slightly lower test MSE than the fully grown tree in this case.

- d. Use the bagging approach in order to analyze this data. What test MSE do you obtain? Use the **importance()** function to determine which variables are most important.

```
set.seed(1)
```

```
# We need to set mtry equal to the number of predictors in the model in order to perform
# bagging instead of random forests
p = ncol(Carseats) - 1
bag.carseats = randomForest(Sales ~ ., data = Carseats, subset = train, mtry = p, import
ance = TRUE)
bag.mse = getTestMSE(bag.carseats)
bag.mse
```

```
## [1] 3.158778
```

Wow! It's a huge performance improvement. The test MSE obtained from bagging is much smaller than the test MSE from the pruned tree.

```
importance(bag.carseats)
```

```
##           %IncMSE IncNodePurity
## CompPrice 20.77899423    130.943589
## Income    7.25172186     80.581428
## Advertising 21.54915861    185.810892
## Population 0.41671466     55.003974
## Price     54.13927377    431.922698
## ShelfLoc  41.38863827    248.451451
## Age       12.63004181    111.973059
## Education  3.32618162     35.487637
## Urban      0.01734356      6.507844
## US         2.33948140      7.191645
```

In terms of training MSE, the bagging model lists **Price** and **ShelveLoc** as the two most important predictors. **Advertising** and **CompPrice** are also considered important, but they don't impact the training MSE as much. Recall that the fully grown tree also considered **Price** and **ShelveLoc** to be the most important factors in predicting Sales.

- e. Use random forests to analyze this data. What test MSE do you obtain? Use the **importance()** function to determine which variables are most important. Describe the effect of m , the number of variables considered at each split, on the error rate obtained.

```
set.seed(1)

# First we'll use  $m = p/2$ 
m = p/2
rf.carseats = randomForest(Sales ~ ., data = Carseats, subset = train, mtry = m, importance = TRUE)
rf.mse = getTestMSE(rf.carseats)
rf.mse
```

```
## [1] 3.475288
```

Random forests with $m = p/2$ doesn't yield quite as low a test MSE as bagging, but it is still a significant improvement over the full tree and pruned tree test MSE.

```
importance(rf.carseats)
```

##	%IncMSE	IncNodePurity
## CompPrice	14.1636754	119.055120
## Income	4.4456966	93.357926
## Advertising	19.3332935	169.700848
## Population	-0.9103591	77.596018
## Price	43.1857963	387.705189
## ShelveLoc	34.1445934	232.120164
## Age	10.4461913	118.779148
## Education	0.9741664	51.301962
## Urban	0.7789178	8.517634
## US	4.8030455	20.208178

Nothing too different about the order of predictor importance. It may be worth noting that the predictors which were considered much more important in the bagging model are a bit closer in importance to the other predictors.

```
set.seed(1)

# Now we'll try  $m = \sqrt{p}$ 
m = sqrt(p)
rf.carseats = randomForest(Sales ~ ., data = Carseats, subset = train, mtry = m, importance = TRUE)
rf.mse = getTestMSE(rf.carseats)
rf.mse
```

```
## [1] 4.107447
```

Random forests with $m = \sqrt{p}$ yields a test MSE similar to the pruned tree test MSE - not great.

```
importance(rf.carseats)
```

##	%IncMSE	IncNodePurity
## CompPrice	9.8053219	111.37676
## Income	1.5185003	102.50976
## Advertising	16.4317019	156.84286
## Population	0.2753494	101.21260
## Price	36.0495211	336.83466
## ShelveLoc	29.1520438	196.30048
## Age	9.1381909	129.59990
## Education	3.6112924	68.37053
## Urban	0.1102256	11.87917
## US	6.6298204	36.73006

The importance table yielded with $m = \sqrt{p}$ is similar to the one with $m = p/2$ with respect to predictor importance on training MSE. However, predictor importance has become even more “flat”.

Overall, random forests yielded the best performance on this data set. All of the models observed considered **Price** and **ShelveLoc** to be the best indicators of **Sales**.

Exercise 9

This problem involves the **OJ** data set, which is part of the **ISLR** package.

- a. Create a training set containing a random sample of 800 observations and a test set containing the remaining observations.

```
set.seed(1)
train = sample(nrow(OJ), 800)
```

- b. Fit a tree to the training data, with **Purchase** as the response and the other variables as predictors. Use the **summary()** function to produce summary statistics about the tree, and describe the results obtained.

```
tree.model = tree(Purchase ~ ., data = OJ, subset = train)
summary(tree.model)
```

```
##
## Classification tree:
## tree(formula = Purchase ~ ., data = OJ, subset = train)
## Variables actually used in tree construction:
## [1] "LoyalCH"      "PriceDiff"    "SpecialCH"    "ListPriceDiff"
## Number of terminal nodes: 8
## Residual mean deviance: 0.7305 = 578.6 / 792
## Misclassification error rate: 0.165 = 132 / 800
```

The resulting tree has 8 terminal nodes and only uses 4 predictors from the data set, which has 17 available predictors to use. The training misclassification error rate was 0.165.

- c. Type in the name of the tree object in order to get a detailed text output. Pick one of the terminal nodes and interpret the information displayed.

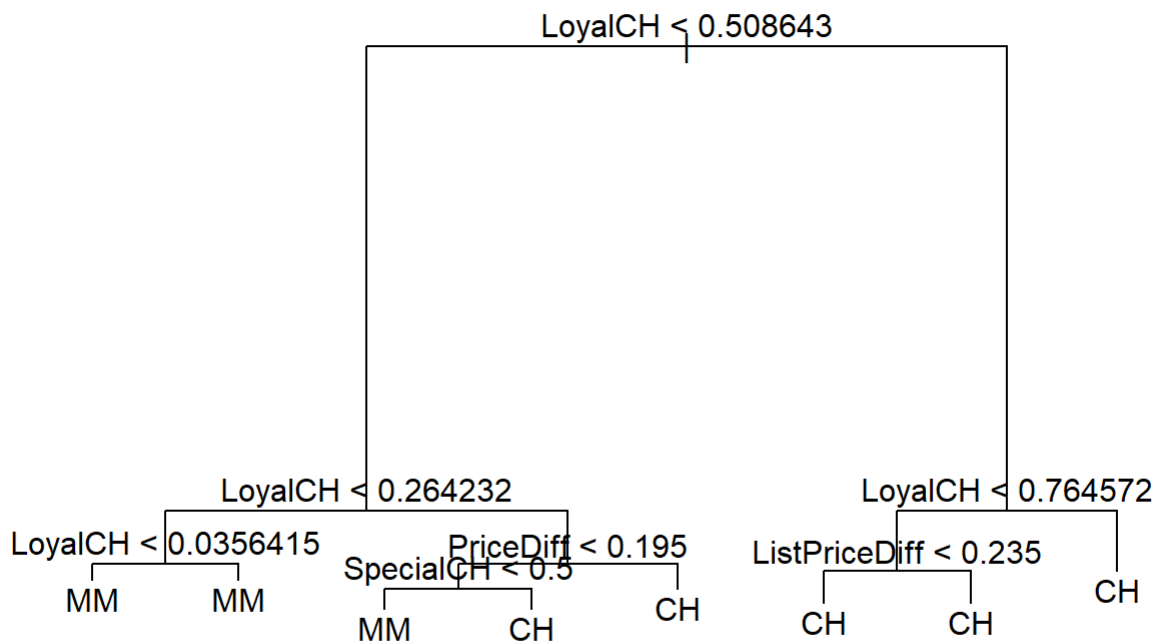
```
tree.model
```

```
## node), split, n, deviance, yval, (yprob)
##      * denotes terminal node
##
## 1) root 800 1064.00 CH ( 0.61750 0.38250 )
##    2) LoyalCH < 0.508643 350 409.30 MM ( 0.27143 0.72857 )
##      4) LoyalCH < 0.264232 166 122.10 MM ( 0.12048 0.87952 )
##        8) LoyalCH < 0.0356415 57 10.07 MM ( 0.01754 0.98246 ) *
##        9) LoyalCH > 0.0356415 109 100.90 MM ( 0.17431 0.82569 ) *
##      5) LoyalCH > 0.264232 184 248.80 MM ( 0.40761 0.59239 )
##        10) PriceDiff < 0.195 83 91.66 MM ( 0.24096 0.75904 )
##          20) SpecialCH < 0.5 70 60.89 MM ( 0.15714 0.84286 ) *
##          21) SpecialCH > 0.5 13 16.05 CH ( 0.69231 0.30769 ) *
##        11) PriceDiff > 0.195 101 139.20 CH ( 0.54455 0.45545 ) *
##    3) LoyalCH > 0.508643 450 318.10 CH ( 0.88667 0.11333 )
##      6) LoyalCH < 0.764572 172 188.90 CH ( 0.76163 0.23837 )
##        12) ListPriceDiff < 0.235 70 95.61 CH ( 0.57143 0.42857 ) *
##        13) ListPriceDiff > 0.235 102 69.76 CH ( 0.89216 0.10784 ) *
##      7) LoyalCH > 0.764572 278 86.14 CH ( 0.96403 0.03597 ) *
```

Let's look at terminal node (8). The predictor region defined by this terminal node encompasses 57 of the original 800 training observations, which is 7.125%. This region is defined by **LoyalCH** < 0.0356415 (the conditions for branches leading to this terminal node all happened to be checking for **LoyalCH** less than a certain value as well). The model predicts any observations meeting that criteria to have a response value of **MM** (indicating a purchase of Minute Maid orange juice). 98.246% of the training observations in this region had a response value of **MM**.

d. Create a plot of the tree and interpret the results.

```
plot(tree.model)
text(tree.model, pretty = 0)
```



Again, we see that we have 8 terminal nodes, 3 of which correspond to a prediction of **MM**. **LoyalCH** is considered to be the most important predictor in determining **Purchase**.

e. Predict the response on the test data and produce a confusion matrix comparing the test labels to the predicted test labels. What is the test error rate?

```
tree.prob = predict(tree.model, newdata = OJ[-train,])
tree.predict = rep('MM', nrow(tree.prob))
tree.predict[tree.prob[, 'CH'] >= .5] = 'CH'
table(tree.predict, OJ[-train, 'Purchase'])
```

```
##
## tree.predict  CH  MM
##              CH 147 49
##              MM  12 62
```

```
tree.testError = (12 + 62) / length(tree.predict)
tree.testError
```

```
## [1] 0.2740741
```

The full grown tree yields a test error rate of 27.4%.

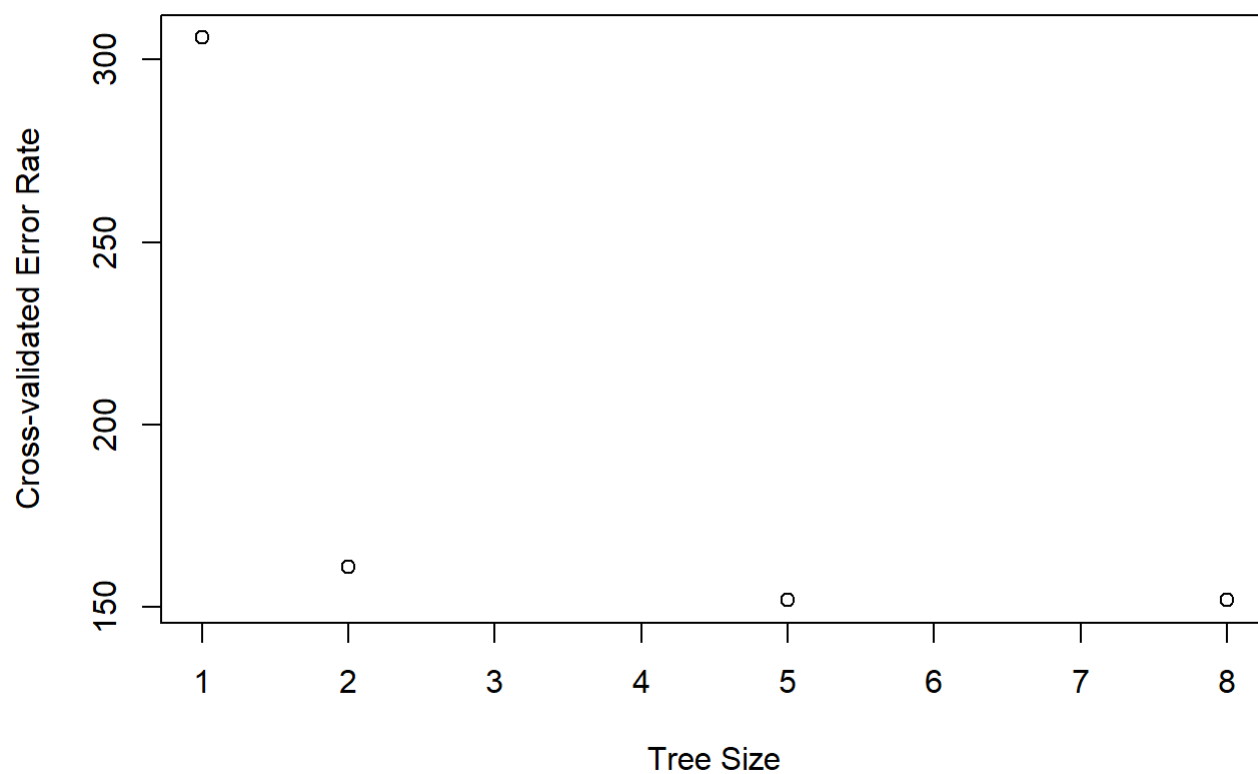
f. Apply the **cv.tree()** function to the training set in order to determine the optimal tree size.

```
set.seed(1)
cv.model = cv.tree(tree.model, FUN = prune.misclass)
cv.model
```

```
## $size
## [1] 8 5 2 1
##
## $dev
## [1] 152 152 161 306
##
## $k
## [1]      -Inf    0.000000    4.666667 160.000000
##
## $method
## [1] "misclass"
##
## attr(,"class")
## [1] "prune"          "tree.sequence"
```

g. Produce a plot with tree size on the x-axis and cross-validated classification error rate on the y-axis.

```
plot(cv.model$size, cv.model$dev, xlab = "Tree Size", ylab = "Cross-validated Error Rate")
```

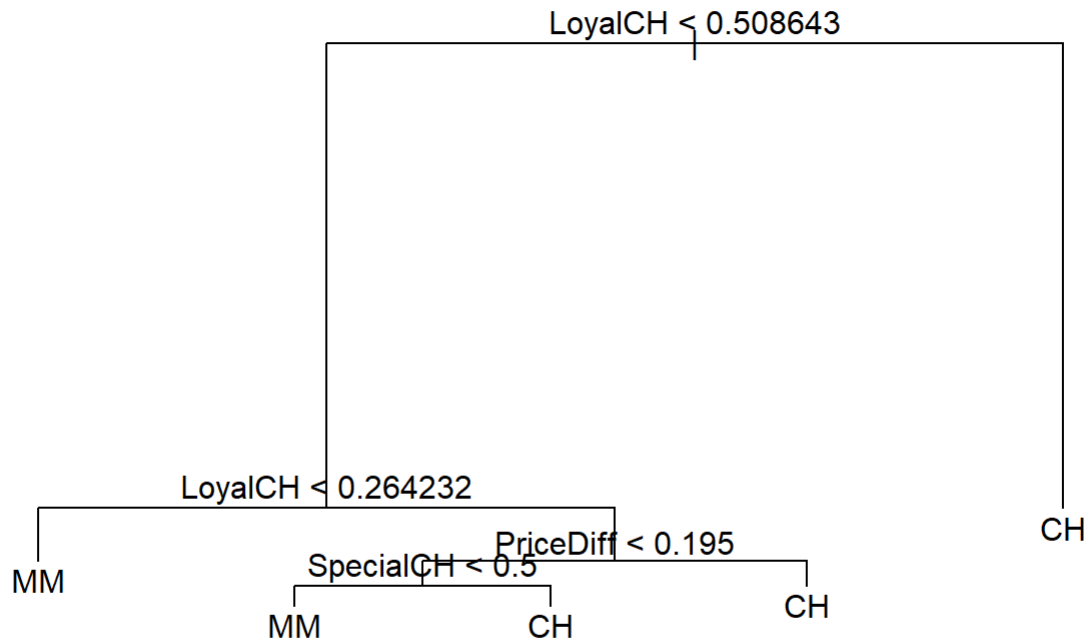


h. Which tree size corresponds to the lowest cross-validated classification error rate?

Both 5 and 8 terminal nodes yield the lowest CV error rate. We'll look at pruning the tree to 5 terminal nodes, since we already examined the full tree with 8 terminal nodes.

i. Produce a pruned tree corresponding to the optimal tree size obtained using cross-validation. If cross-validation does not lead to selection of a pruned tree, then create a pruned tree with five terminal nodes.

```
prune.model = prune.misclass(tree.model, best = 5)
plot(prune.model)
text(prune.model, pretty = 0)
```



j. Compare the training error rates between the pruned and unpruned trees. Which is higher?

```
summary(prune.model)
```

```
##
## Classification tree:
## snip.tree(tree = tree.model, nodes = 3:4)
## Variables actually used in tree construction:
## [1] "LoyalCH" "PriceDiff" "SpecialCH"
## Number of terminal nodes: 5
## Residual mean deviance: 0.8256 = 656.4 / 795
## Misclassification error rate: 0.165 = 132 / 800
```

The training error rate for the pruned tree is 16.5%, which is exactly the same as the training error rate observed with the full tree.

k. Compare the test error rates between the pruned and unpruned trees. Which is higher?

```
prune.probab = predict(prune.model, newdata = OJ[-train,])
prune.predict = rep('MM', nrow(prune.probab))
prune.predict[prune.probab[, 'CH'] >= .5] = 'CH'
prune.testError = mean(prune.predict != OJ[-train, "Purchase"])

tree.testError
```



```
## [1] 0.2740741
```

```
prune.testError
```

```
## [1] 0.2259259
```

The pruned tree has a test error rate of 22.6%, which is a bit lower than the full tree test error rate.

Exercise 10

We now use boosting to predict **Salary** in the **Hitters** data set.

- Remove the observations for which the salary information is unknown and then log-transform the salaries.

```
Hitters = Hitters[complete.cases(Hitters$Salary),]  
Hitters$Salary.log = log(Hitters$Salary)
```

- Create a training set consisting of the first 200 observations and a test set consisting of the remaining observations.

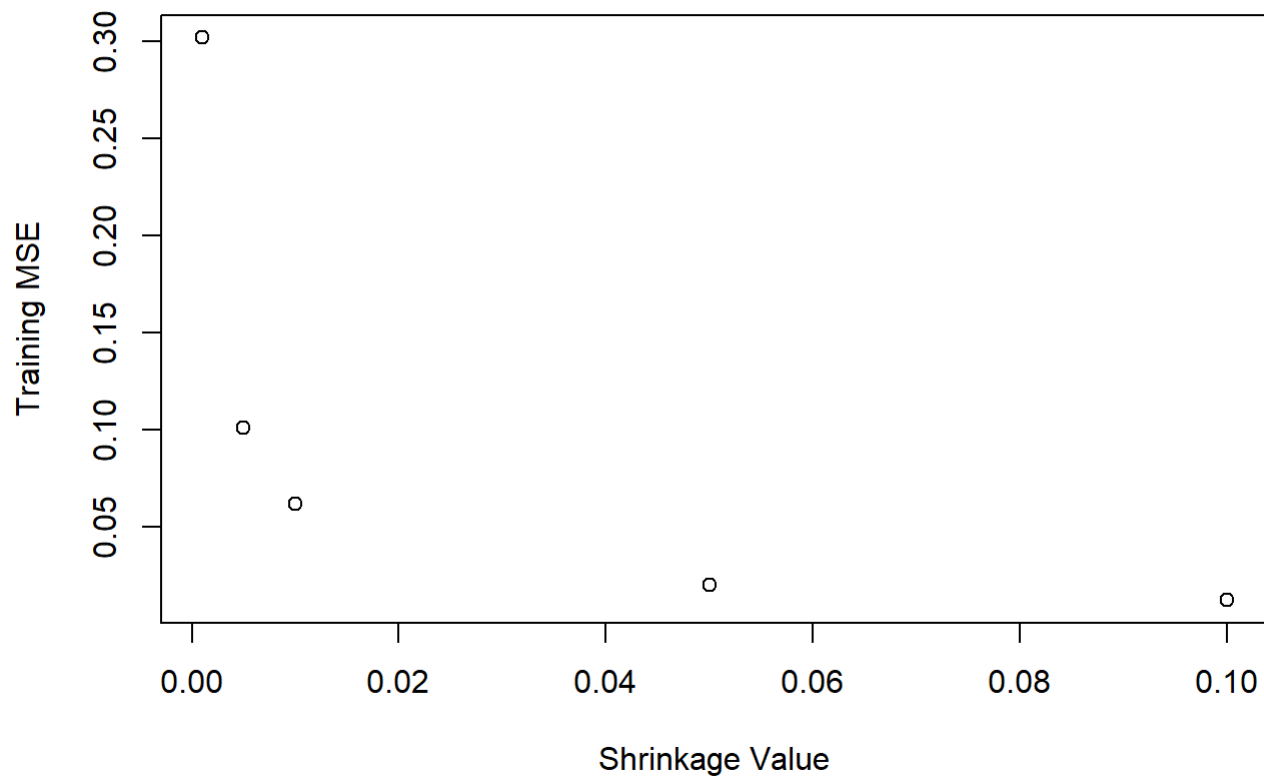
```
train = 1:200
```

- Perform boosting on the training set with 1,000 trees for a range of values of the shrinkage parameter λ . Produce a plot with different shrinkage values on the x-axis and the corresponding training set MSE on the y-axis.

```
# need to gbm library to perform boosting  
library(gbm)  
  
# According to the book, typical values are 0.01 or 0.001, so we'll use that as a starting point  
shrinkage.values = c(0.01, 0.001, 0.1, 0.005, 0.05)  
test.mse = rep(NA, length(shrinkage.values))  
training.mse = rep(NA, length(shrinkage.values))  
  
for (index in 1:length(shrinkage.values)){  
  boost.model = gbm(Salary.log ~ . - Salary, data = Hitters[train,], distribution = "gaussian",  
    n.trees = 1000, shrinkage = shrinkage.values[index])  
  boost.predict = predict(boost.model, newdata = Hitters[-train,], n.trees = 1000)  
  test.mse[index] = mean((Hitters[-train, "Salary.log"] - boost.predict)^2)  
  training.mse[index] = mean(boost.model$train.error^2)  
}
```

```
plot(shrinkage.values, training.mse, xlab = 'Shrinkage Value', ylab = 'Training MSE', main = 'Using Boosting to Predict Salary')
```

Using Boosting to Predict Salary

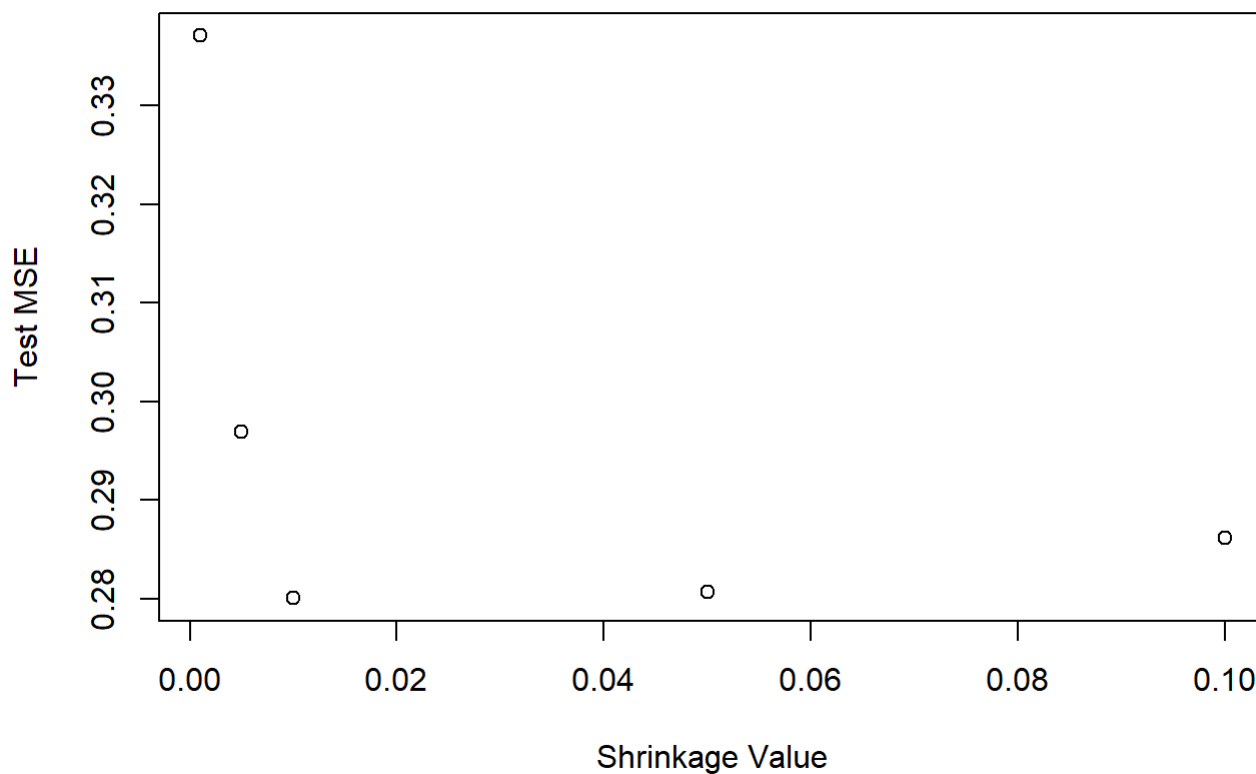


And of course, the largest shrinkage value produces the smallest training MSE due to the boosting algorithm, which focusing on minimizing residuals.

- d. Produce a plot with different shrinkage values on the x-axis and the corresponding test set MSE on the y-axis.

```
plot(shrinkage.values, test.mse, xlab = 'Shrinkage Value', ylab = 'Test MSE', main = 'Using Boosting to Predict Salary')
```

Using Boosting to Predict Salary



The boosting model using $\lambda = 0.05$ was by far the best in terms of test MSE.

- e. Compare the test MSE of boosting to the test MSE that results from applying two of the regression approaches seen in Chapters 3 and 6.

```
best.index = which.min(test.mse)
best.shrinkage = shrinkage.values[best.index]
best.mse = test.mse[best.index]
best.shrinkage
```

```
## [1] 0.01
```

```
best.mse
```

```
## [1] 0.2800611
```

Boosting yields a test MSE of 0.259.

```
# In Chapter 3, we looked at linear regression, so we'll just do a multiple linear regression on the transformed Salary data
lm.fit = lm(Salary.log ~ . - Salary, data = Hitters, subset = train)
lm.predict = predict(lm.fit, newdata = Hitters[-train,])
lm.mse = mean((Hitters[-train, "Salary.log"] - lm.predict)^2)
lm.mse
```

```
## [1] 0.4917959
```

The multiple linear regression yields a test MSE of 0.492, which is almost double that of the boosting model.

```
# In Chapter 6, we looked at a number of methods, including Best Subset Selection, Ridge
Regression, Lasso, PCR, and PLS
# For this exercise, we'll utilize Best Subset Selection, which requires the leaps libra
ry
library(leaps)

# Number of predictors; -2 for Salary and Salary.log columns
p = ncol(Hitters) - 2

# We need a matrix to get response predictions from the best subset method, because ther
e is no predict method for regsubsets
test.mat = model.matrix(Salary.log ~ . - Salary, data = Hitters[-train,])

# Find which subset yields the lowest test MSE and roll with that one
regfit.best = regsubsets(Salary.log ~ . - Salary, data = Hitters[train,], nvmax = p)
best.mse = Inf
for (i in 1:p){
  coefi = coef(regfit.best, id = i)
  pred = test.mat[,names(coefi)]%*%coefi
  this.mse = mean((Hitters[-train, "Salary.log"] - pred)^2)
  if (this.mse < best.mse){
    best.mse = this.mse
    best.coef = coefi
  }
}

length(best.coef)
```

```
## [1] 9
```

```
best.coef
```

```
## (Intercept)      AtBat      Hits      Walks      Years
## 4.4722395349 -0.0032378215 0.0149552471 0.0105182255 0.0647798866
##          CRuns      CWalks  DivisionW      PutOuts
## 0.0012175332 -0.0010136188 -0.1482230049 0.0005050225
```

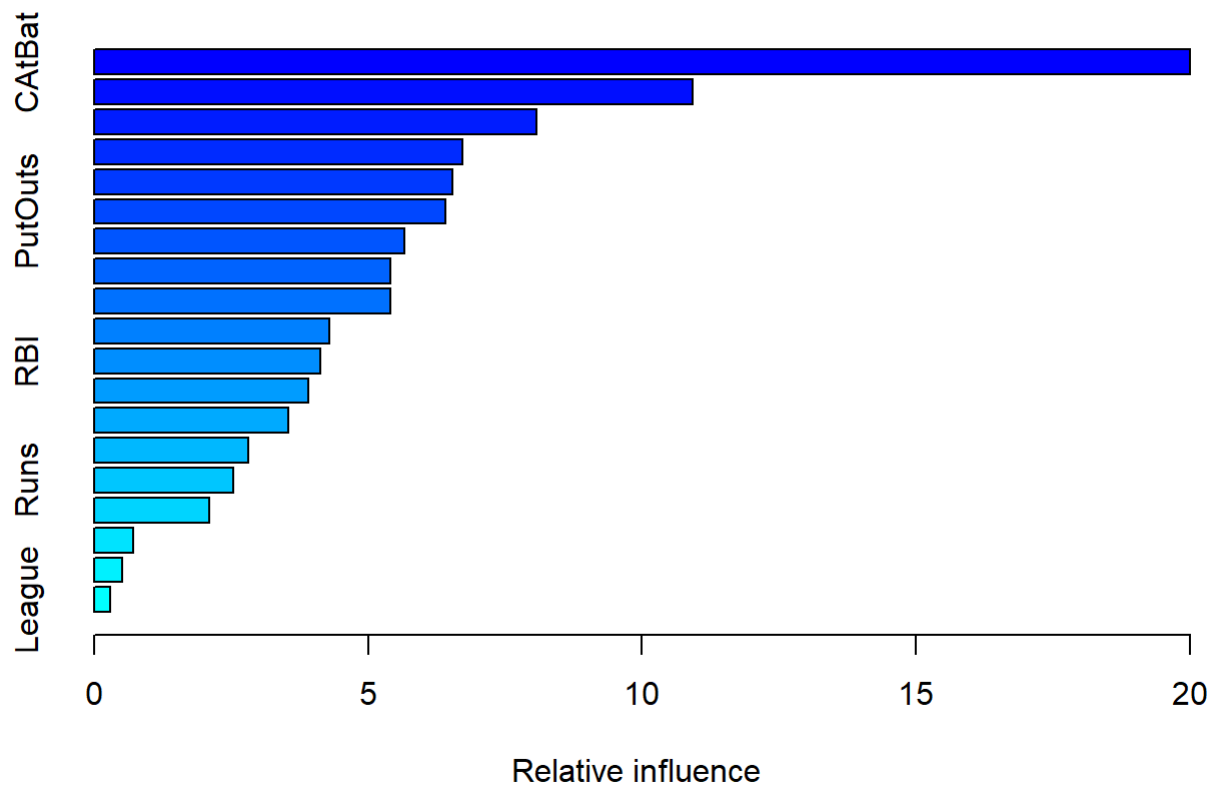
```
best.mse
```

```
## [1] 0.468457
```

The best subset selection yields an 8 predictor model (not counting the intercept) with a test MSE of 0.468, which is only slightly better than the full multiple linear regression. It is still much worse than the test MSE from boosting.

f. Which variables appear to be the most important predictors in the boosted model?

```
summary(boost.model)
```



```
##          var    rel.inf
## CAtBat    CAtBat 20.0016313
## CRuns     CRuns 10.9257398
## CWalks    CWalks 8.0815671
## Years     Years 6.7184333
## CRBI      CRBI 6.5456552
## PutOuts   PutOuts 6.4155962
## CHmRun    CHmRun 5.6683505
## Walks     Walks 5.4062511
## CHits     CHits 5.4028331
## Hits      Hits 4.2915263
## RBI       RBI 4.1287601
## Assists   Assists 3.9069660
## AtBat     AtBat 3.5494994
## HmRun     HmRun 2.8195913
## Runs      Runs 2.5350925
## Errors    Errors 2.0937102
## Division  Division 0.7106661
## NewLeague NewLeague 0.5115559
## League    League 0.2865745
```

In the boosted model, the number of times at bat during a career (**CAtBat**) had far and away the greatest relative influence on a player's salary. The second most influential predictor, the number of runs during a career (**CRuns**), had only half as much relative influence on a player's salary.

g. Now apply bagging to the training set. What is the test set MSE for this approach?

```
set.seed(1)
bag.model = randomForest(Salary.log ~ . - Salary, data = Hitters, subset = train, mtry =
p, importance = T)
bag.predict = predict(bag.model, newdata = Hitters[-train,])
bag.mse = mean((Hitters[-train, "Salary.log"] - bag.predict)^2)
bag.mse
```

```
## [1] 0.2301184
```

The bagging approach yields a test MSE of 0.230, which is a bit less than the boost test MSE. Out of all the techniques we applied to this data set, the bagging has yielded the lowest test MSE.

Exercise 11

This question uses the **Caravan** data set.

a. Create a training set consisting of the first 1,000 observations and a test set consisting of the remaining observations.

```
train = 1:1000

# Convert the Purchase response to a factor
Caravan$Purchase.factor = ifelse(Caravan$Purchase == "Yes", 1, 0)
```

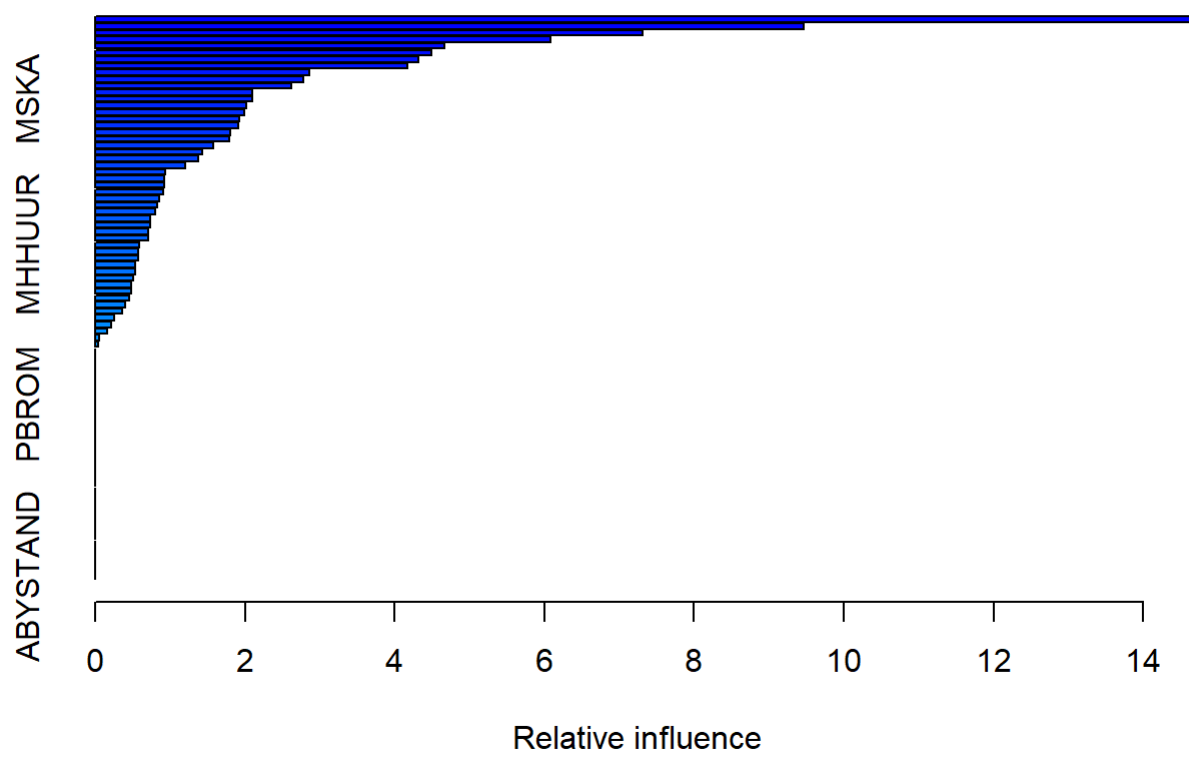
b. Fit a boosting model to the training set with **Purchase** as the response and the other variables as predictors. Use 1,000 trees and a shrinkage value of 0.01. Which predictors appear to be the most important?

```
set.seed(1)
boost.model = gbm(Purchase.factor ~ . - Purchase, data = Caravan[train,], distribution =
"bernoulli", n.trees = 1000, shrinkage = 0.01)
```

```
## Warning in gbm.fit(x, y, offset = offset, distribution = distribution, w =
## w, : variable 50: PVRAAUT has no variation.
```

```
## Warning in gbm.fit(x, y, offset = offset, distribution = distribution, w =
## w, : variable 71: AVRAAUT has no variation.
```

```
summary(boost.model)
```



##		var	rel.inf
##	PPERSAUT	PPERSAUT	14.63504779
##	MK00PKLA	MK00PKLA	9.47091649
##	MOPLH00G	MOPLH00G	7.31457416
##	MBERMIDD	MBERMIDD	6.08651965
##	PBRAND	PBRAND	4.66766122
##	MGODGE	MGODGE	4.49463264
##	ABRAND	ABRAND	4.32427755
##	MINK3045	MINK3045	4.17590619
##	MOSTYPE	MOSTYPE	2.86402583
##	PWAPART	PWAPART	2.78191075
##	MAUT1	MAUT1	2.61929152
##	MBERARBG	MBERARBG	2.10480508
##	MSKA	MSKA	2.10185152
##	MAUT2	MAUT2	2.02172510
##	MSKC	MSKC	1.98684345
##	MINKGEM	MINKGEM	1.92122708
##	MGODPR	MGODPR	1.91777542
##	MBERH00G	MBERH00G	1.80710618
##	MGODOV	MGODOV	1.78693913
##	PBYSTAND	PBYSTAND	1.57279593
##	MSKB1	MSKB1	1.43551401
##	MFWEKIND	MFWEKIND	1.37264255
##	MRELGE	MRELGE	1.20805179
##	MOPLMIDD	MOPLMIDD	0.93791970
##	MINK7512	MINK7512	0.92590720
##	MINK4575	MINK4575	0.91745993
##	MGODRK	MGODRK	0.90765539
##	MFGEKIND	MFGEKIND	0.85745374
##	MZPART	MZPART	0.82531066
##	MRELOV	MRELOV	0.80731252
##	MINKM30	MINKM30	0.74126812
##	MHK00P	MHK00P	0.73690793
##	MZFONDS	MZFONDS	0.71638323
##	MAUT0	MAUT0	0.71388052
##	MHHUUR	MHHUUR	0.59287247
##	APERSAUT	APERSAUT	0.58056986
##	MOSH00FD	MOSH00FD	0.58029563
##	MSKB2	MSKB2	0.53885275
##	PLEVEN	PLEVEN	0.53052444
##	MINK123M	MINK123M	0.50660603
##	MBERARBO	MBERARBO	0.48596479
##	MGEMOMV	MGEMOMV	0.47614792
##	PMOTSCO	PMOTSCO	0.46163590
##	MSKD	MSKD	0.39735297
##	MBERBOER	MBERBOER	0.36417546
##	MGEMLEEF	MGEMLEEF	0.26166240
##	MFALLEEN	MFALLEEN	0.21448118
##	MBERZELF	MBERZELF	0.15906143
##	MOPLLAAG	MOPLLAAG	0.05263665
##	MAANTHUI	MAANTHUI	0.03766014
##	MRELSA	MRELSA	0.00000000
##	PWABEDR	PWABEDR	0.00000000


```
## PWALAND    PWALAND    0.00000000
## PBESAUT    PBESAUT    0.00000000
## PVRAAUT    PVRAAUT    0.00000000
## PAANHANG   PAANHANG   0.00000000
## PTRACTOR   PTRACTOR   0.00000000
## PWERKT     PWERKT     0.00000000
## PBROM      PBROM      0.00000000
## PPERSONG   PPERSONG   0.00000000
## PGEZONG    PGEZONG    0.00000000
## PWAOREG    PWAOREG    0.00000000
## PZEILPL    PZEILPL    0.00000000
## PPLEZIER   PPLEZIER   0.00000000
## PFIETS     PFIETS     0.00000000
## PINBOED    PINBOED    0.00000000
## AWAPART    AWAPART    0.00000000
## AWABEDR    AWABEDR    0.00000000
## AWALAND    AWALAND    0.00000000
## ABESAUT    ABESAUT    0.00000000
## AMOTSCO    AMOTSCO    0.00000000
## AVRAAUT    AVRAAUT    0.00000000
## AAANHANG   AAANHANG   0.00000000
## ATRACTOR   ATRACTOR   0.00000000
## AWERKT     AWERKT     0.00000000
## ABROM      ABROM      0.00000000
## ALEVEN     ALEVEN     0.00000000
## APERSONG   APERSONG   0.00000000
## AGEZONG    AGEZONG    0.00000000
## AWAOREG    AWAOREG    0.00000000
## AZEILPL    AZEILPL    0.00000000
## APLEZIER   APLEZIER   0.00000000
## AFIETS     AFIETS     0.00000000
## AINBOED    AINBOED    0.00000000
## ABYSTAND   ABYSTAND   0.00000000
```

There are 85 predictors in this data set, but the **PPERSAUT** is deemed to be much more important than the others. The next most important predictors are **MKOOKPLA** and **MOPLHOOG**.

- c. Use the boosting model to predict the response on the test data. Predict that a person will make a purchase if the estimated probability of purchase is greater than 20%. Form a confusion matrix. What fraction of the people predicted to make a purchase do in fact make one? How does this compare with the results obtained from applying KNN or logistic regression to this data set?

```
boost.prob = predict(boost.model, newdata = Caravan[-train,], n.trees = 1000, type = "response")
boost.predict = ifelse(boost.prob > .20, "Yes", "No")
table(boost.predict, Caravan[-train, "Purchase"])
```

```
##
## boost.predict    No    Yes
##                No  4410  256
##                Yes   123   33
```

```
33 / (123 + 33)
```

```
## [1] 0.2115385
```

21.15% of people predicted to make a purchase actually do make one.

```
# use KNN, which is part of the class library, to predict purchase
library(class)

# Need to set a seed for consistency because knn() uses a random choice to settle ties
set.seed(1)

# the knn function requires the input to be a matrix, which should be standardized
standardized.mat = scale(subset(Caravan, select = -c(Purchase, Purchase.factor)))
train.mat = standardized.mat[train,]
test.mat = standardized.mat[-train,]

# We'll consider a range of K values, from 1 to the sqrt(n), with n being the number of
# observations in the training set
best.rate = 0
for (k in 1:floor(sqrt(length(train)))){
  knn.pred = knn(train.mat, test.mat, Caravan[train, "Purchase"], k = k)
  this.table = table(knn.pred, Caravan[-train, "Purchase"])
  this.rate = this.table["Yes", "Yes"] / sum(this.table["Yes",])
  if (!is.na(this.rate) & (this.rate > best.rate)){
    best.rate = this.rate
    best.table = this.table
    best.k = k
  }
}

best.table
```

```
##
## knn.pred   No  Yes
##      No 4507 279
##      Yes  26  10
```

```
best.k
```

```
## [1] 5
```

```
best.rate
```

```
## [1] 0.2777778
```

KNN was able correct about 27.78% of the test observations predicted to make a purchase. This percentage was yielded by a KNN with $k = 5$ and is significantly higher than the boosting model, but there are also much fewer test observations that are predicted to make a purchase. Note that we are not using a 20% probability threshold here; I don't believe that makes as much sense using KNN in R. You could make a prediction of purchase if more than 20% of the K nearest neighbors are predicted to make a purchase, but I do not believe that is convenient to implement in R, so I won't.

```
# use logistic regression to predict purchase
glm.model = glm(Purchase.factor ~ . - Purchase, data = Caravan, subset = train, family =
binomial)
```

```
## Warning: glm.fit: fitted probabilities numerically 0 or 1 occurred
```

```
glm.prob = predict(glm.model, newdata = Caravan[-train,], type = "response")
```

```
## Warning in predict.lm(object, newdata, se.fit, scale = 1, type =
## ifelse(type == : prediction from a rank-deficient fit may be misleading
```

```
glm.pred = ifelse(glm.prob > .20, "Yes", "No")
table(glm.pred, Caravan[-train, "Purchase"])
```

```
##
## glm.pred    No  Yes
##          No 4183 231
##          Yes 350  58
```

```
58 / (350 + 58)
```

```
## [1] 0.1421569
```

Only 14.22% of the test observations predicted to make a purchase actually did. This percentage is much lower than boosting. It did actually predict much more of the test observations to make a purchase, and we were able to easily implement the 20% threshold in the logistic regression.

Exercise 12

Apply boosting, bagging, and random forests to a data set of your choice. Be sure to fit the models on a training set and to evaluate their performance on a test set. How accurate are the results compared to simple methods like linear or logistic regression? Which of these approaches yields the best performance?

```
# We'll work with the airquality data set from the datasets package
```

```
library(datasets)
```

```
# drop any NaNs
```

```
airquality = airquality[complete.cases(airquality),]  
names(airquality)
```

```
## [1] "Ozone" "Solar.R" "Wind" "Temp" "Month" "Day"
```

```
dim(airquality)
```

```
## [1] 111 6
```

```
# We'll create models to predict the maximum daily temperature in degrees Fahrenheit (Temp)
```

```
# Number of observations
```

```
n = nrow(airquality)
```

```
# Number of predictors
```

```
p = ncol(airquality) - 1
```

```
# First, split the data into a training set and a test set
```

```
set.seed(1)
```

```
train = sample(n, n/2)
```

```
# Create a data frame to store test MSE results for each model
```

```
df = data.frame(model.name = rep(NA, 5), test.mse = rep(NA, 5))
```

```
# Create the boosting model
```

```
df[1, "model.name"] = "boosting"
```

```
boost.model = gbm(Temp ~ ., data = airquality[train,], distribution = "gaussian", n.trees = 1000)
```

```
boost.predict = predict(boost.model, newdata = airquality[-train,], n.trees = 1000)
```

```
boost.mse = mean((airquality[-train, "Temp"] - boost.predict)^2)
```

```
df[1, "test.mse"] = boost.mse
```

```
# Create the bagging model
```

```
set.seed(1)
```

```
df[2, "model.name"] = "bagging"
```

```
bag.model = randomForest(Temp ~ ., data = airquality, subset = train, mtry = p, importance = T)
```

```
bag.predict = predict(bag.model, newdata = airquality[-train,])
```

```
bag.mse = mean((airquality[-train, "Temp"] - bag.predict)^2)
```

```
df[2, "test.mse"] = bag.mse
```

```
# Create the random forests model
set.seed(1)
df[3, "model.name"] = "random forests"
rf.model = randomForest(Temp ~ ., data = airquality, subset = train, mtry = p/2, importance = T)
rf.predict = predict(rf.model, newdata = airquality[-train,])
rf.mse = mean((airquality[-train, "Temp"] - rf.predict)^2)
df[3, "test.mse"] = rf.mse
```

```
# Create the multiple linear regression model
df[4, "model.name"] = "linear regression"
lm.model = lm(Temp ~ ., data = airquality, subset = train)
lm.predict = predict(lm.model, newdata = airquality[-train,])
lm.mse = mean((airquality[-train, "Temp"] - lm.predict)^2)
df[4, "test.mse"] = lm.mse
```

```
# Logistic regression is specific to qualitative response variables, but we can perform
  a log transformation on Temp and fit a linear regression on that
df[5, "model.name"] = "log transform regression"
log.model = lm(log(Temp) ~ ., data = airquality, subset = train)
log.predict = exp(predict(log.model, newdata = airquality[-train,]))
log.mse = mean((airquality[-train, "Temp"] - log.predict)^2)
df[5, "test.mse"] = log.mse
```

```
# Show the results
df
```

```
##           model.name test.mse
## 1           boosting 64.10107
## 2           bagging 40.37541
## 3      random forests 36.05172
## 4    linear regression 51.59265
## 5 log transform regression 55.33704
```

We can see that the random forests model performed better than any of the other models. It had a much lower test MSE than the linear regression and the log transform regression models. The bagging model had a test MSE which was very close to that of the random forests model. The boosting model performed very poorly on this data set compared to the other models.