Fast and Non-Approximative Language Model Prefixqueries for Word Prediction using Top-k Joining Techniques

Bachelor Thesis

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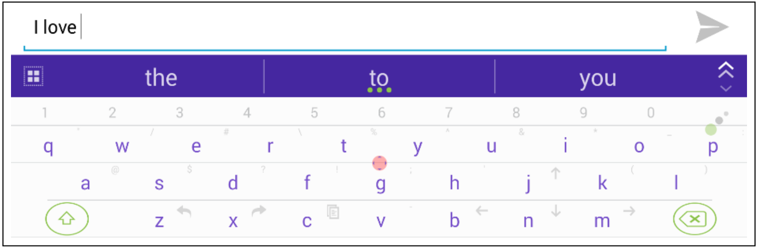
Outline

1. Next Word Prediction
2. Language Models
3. Top-k Joining Techniques

Section 1

Next Word Prediction

What is Next Word Prediction?



Guess the *next intended* word

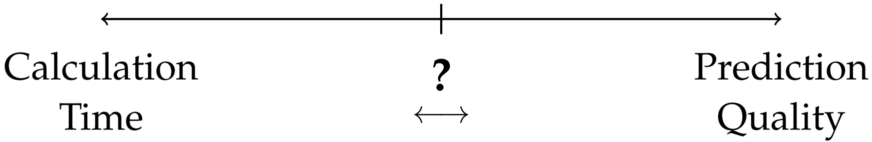
from words already entered

Benefits of good Word Prediction

- Faster typing
- Spelling / Grammar
- ...
- Metric for Language Models?

Thesis Goal

Word prediction usually optimized for speed **or** quality



Thesis Goal

Word prediction usually optimized for speed **or** quality

Calculation Time ⟵ **?** ⟷ ⟶ Prediction Quality

This thesis optimizes for speed while maintaining best quality!

Next Word Prediction

$$\mathrm{NWP}(h) = \underset{w \in \Sigma}{\arg\max}\ \mathrm{P}(w \mid h)$$

Next Word Prediction

history of entered words

$$\mathrm{NWP}(h) = \arg\max_{w \in \Sigma} \mathrm{P}(w \mid h)$$

Next Word Prediction

history of entered words

$$\mathrm{NWP}(h) = \arg\max_{w \in \Sigma} \mathrm{P}(w \mid h)$$

for each word in vocabulary

Section 2

Language Models

How to find probabilities?

$$P(w \mid h)\ ?$$

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$$P(w \mid h) \text{ ?}$$

Solution: Statistical Language Models

- analyze text corpora
- to estimate sequence probabilities

How to find probabilities?

$$P\left(w \mid h\right)\ ?$$

Solution: Statistical Language Models

- analyze text corpora
- to estimate sequence probabilities

We will employ two state-of-the-art Language Models:

- Modified Kneser Ney Smoothing
  (Chen and Goodman 1999)
- Generalized Language Model
  (Pickhardt et al. 2014)

Modified Kneser Ney Smoothing

$$\mathsf{P}_{\mathrm{MKN}}(w_n \mid w_1^{n-1}) = \frac{\mathsf{c}^{\mathsf{d}}(w_1^n) + \gamma(w_1^{n-1})\, \hat{\mathsf{P}}_{\mathrm{MKN}}(w_n \mid w_2^{n-1})}{\mathsf{c}(w_1^{n-1}\,\square\,)}$$

Modified Kneser Ney Smoothing

$$P_{\mathrm{MKN}}(w_n \mid w_1^{n-1}) = \frac{c^d(w_1^n) + \gamma(w_1^{n-1})\,\hat{P}_{\mathrm{MKN}}(w_n \mid w_2^{n-1})}{c(w_1^{n-1}\,\square)}$$

Generalized Language Model

$$P_{\mathrm{GLM}}(w_n \mid w_1^{n-1}) = \frac{c^d(w_1^n) + \frac{\gamma(w_1^{n-1})}{\#_\partial w_1^{n-1}} \sum_{j=1}^{\#_\partial w_1^{n-1}} \hat{P}_{\mathrm{GLM}}(w_n \mid \partial_j w_1^{n-1})}{c(w_1^{n-1} \square)}$$

Generalized Language Model

$$P_{\mathrm{GLM}}(w_n \mid w_1^{n-1}) = \frac{c^d(w_1^n) + \frac{\gamma(w_1^{n-1})}{\#_\partial w_1^{n-1}} \sum_{j=1}^{\#_\partial w_1^{n-1}} \hat{P}_{\mathrm{GLM}}(w_n \mid \partial_j w_1^{n-1})}{c(w_1^{n-1} \square)}$$

Example: Probability of "you" after "I love"

$$\mathsf{P}_{\text{MKN}}(\text{you} \,|\, \text{I love}) = \frac{\mathsf{c}^{\mathsf{d}}(\text{I love you}) + \gamma(\text{I love})\, \hat{\mathsf{P}}_{\text{MKN}}(\text{you} \,|\, \text{love})}{\mathsf{c}(\text{I love} \,\square\,)}$$

$$\hat{\mathsf{P}}_{\text{MKN}}(\text{you} \,|\, \text{love}) = \frac{\mathsf{N}^{\mathsf{d}}_{1+}(\,\bullet\,\text{love you}) + \gamma(\text{love})\, \hat{\mathsf{P}}_{\text{MKN}}(\text{you})}{\mathsf{N}_{1+}(\,\bullet\,\text{love}\,\bullet\,)}$$

$$\hat{\mathsf{P}}_{\text{MKN}}(\text{you}) = \frac{\mathsf{N}_{1+}(\,\bullet\,\text{you})}{\mathsf{N}_{1+}(\,\bullet\,\bullet\,)}$$

Example: Probability of "you" after "I love"

$$P_{\mathrm{MKN}}(\text{you} \mid \text{I love}) = \frac{c^{d}(\text{I love you}) + \gamma(\text{I love})\,\hat{P}_{\mathrm{MKN}}(\text{you} \mid \text{love})}{c(\text{I love} \,\square\,)}$$

$$\hat{P}_{\mathrm{MKN}}(\text{you} \mid \text{love}) = \frac{N^{d}_{1+}(\bullet\,\text{love you}) + \gamma(\text{love})\,\hat{P}_{\mathrm{MKN}}(\text{you})}{N_{1+}(\bullet\,\text{love}\,\bullet)}$$

$$\hat{P}_{\mathrm{MKN}}(\text{you}) = \frac{N_{1+}(\bullet\,\text{you})}{N_{1+}(\bullet\,\bullet)}$$

Idea: express as Weighted Sums

$$P(w \mid h) = \sum_{i=1}^{N} \lambda_i^h \, \alpha_i^h(w)$$

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depends on argument w

$$P\left(w \mid h\right) = \sum_{i=1}^{N} \lambda_i^h \, \alpha_i^h(w)$$

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independent of argument w

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all words in vocabulary

Idea: express as Weighted Sums

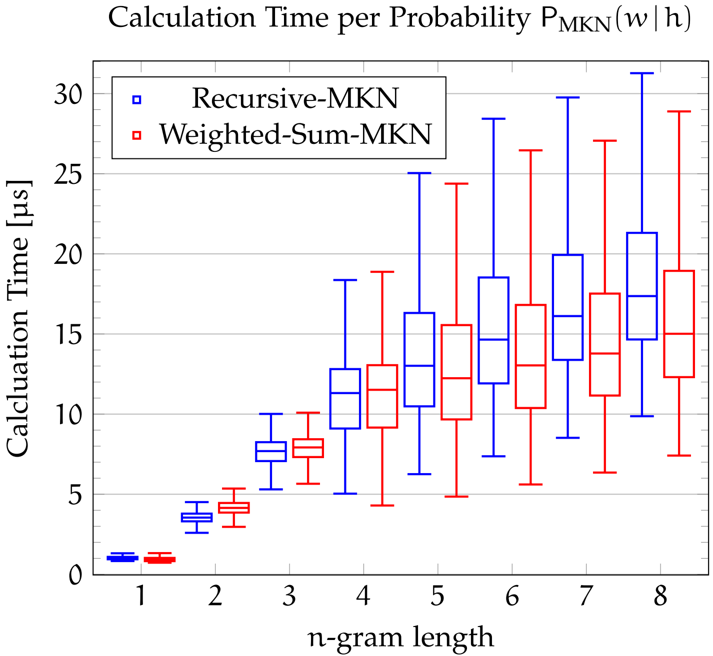
depends on argument w

independent of argument w

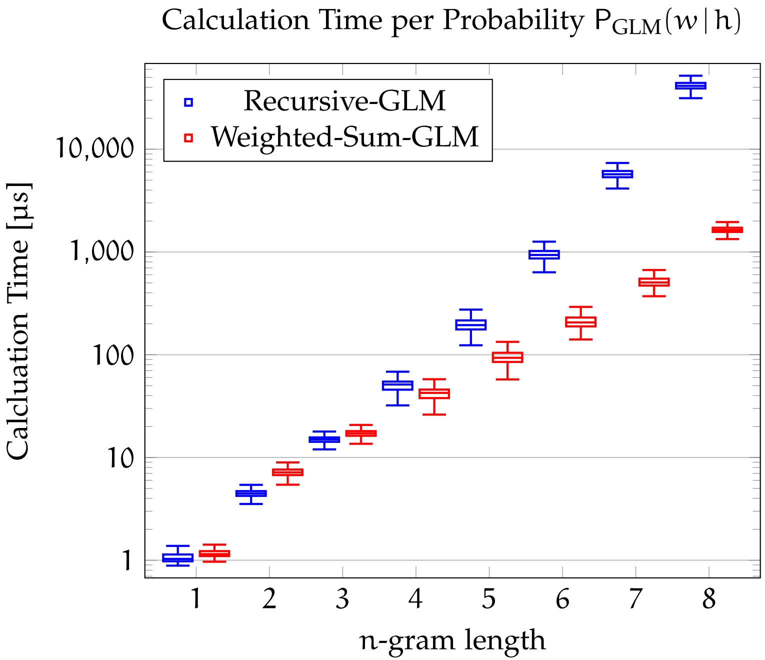
$$P\,(w \,|\, h) = \sum_{i=1}^{N} \lambda_i^h \, \alpha_i^h(w)$$

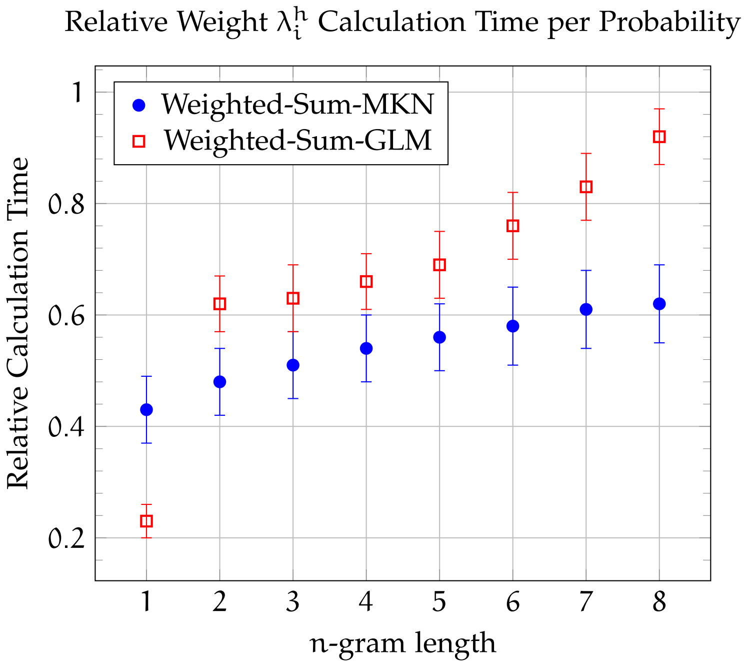
user given, thus fixed

all words in vocabulary



Calculation Time per Probability $P_{MKN}(w \mid h)$
Recursive-MKN
Weighted-Sum-MKN
Calcluation Time [µs]
0
5
10
15
20
25
30
1
2
3
4
5
6
7
8
n-gram length


Calculation Time per Probability $P_{GLM}(w|h)$
Recursive-GLM
Weighted-Sum-GLM
Calcluation Time [μs]
10,000
1,000
100
10
1
1
2
3
4
5
6
7
8
n-gram length



Relative Weight λ_i^h Calculation Time per Probability
Weighted-Sum-MKN
Weighted-Sum-GLM
Relative Calculation Time
n-gram length
1
0.8
0.6
0.4
0.2
1
2
3
4
5
6
7
8

Section 3

Top-k Joining Techniques

What are Top-k Join Queries?

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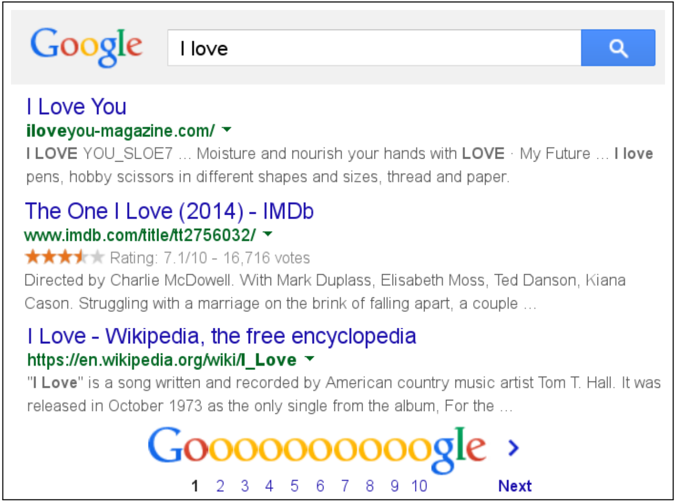
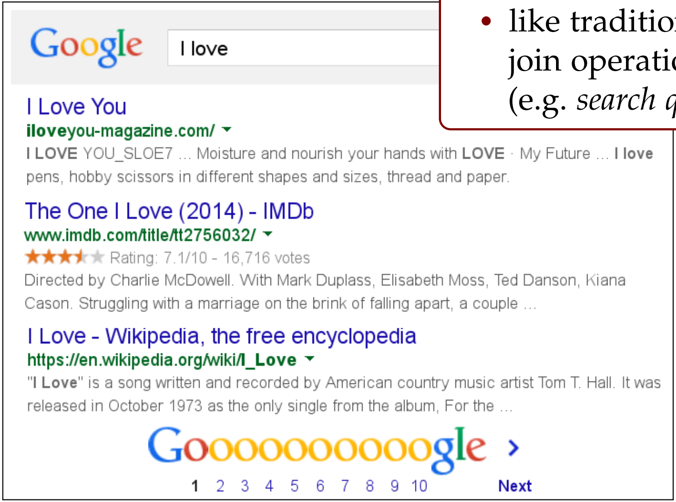


What are Top-k Join Queries?

- like traditional join operations (e.g. *search query*)



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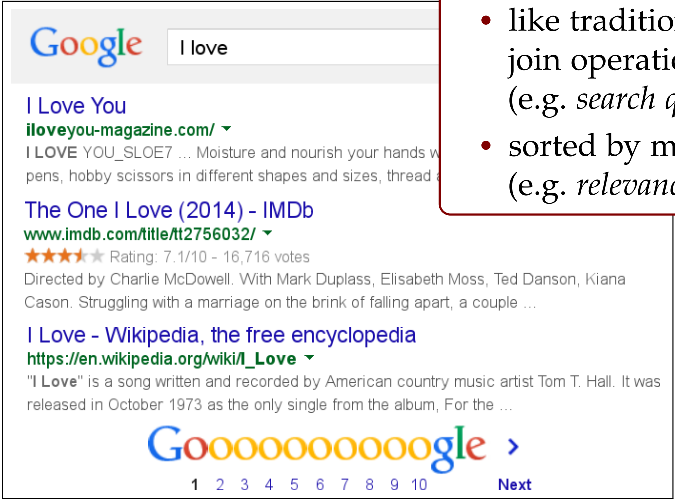
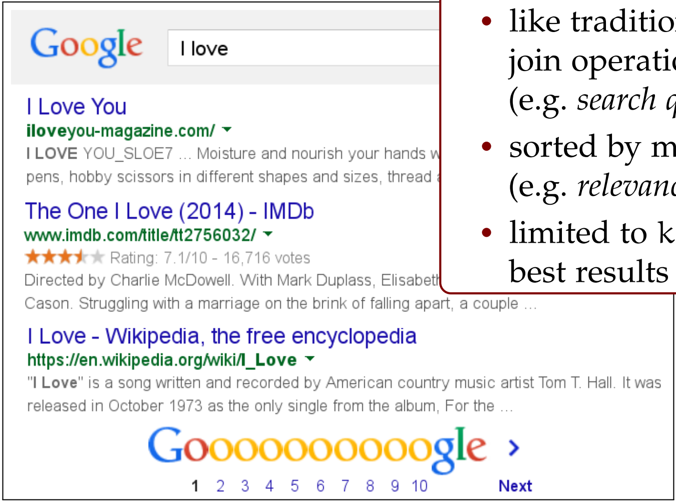


- like traditional join operations (e.g. *search query*)
- sorted by metric (e.g. *relevance*)

What are Top-k Join Queries?



- like traditional join operations (e.g. *search query*)
- sorted by metric (e.g. *relevance*)
- limited to $k = 3$ best results

Calculating probabilities for all words is slow!

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Observation: Words with high probabilities have high occurrence counts (probabilities are monotone)

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Idea: Keep sorted lists of occurrence counts

ϵ		love		I love	
the	50	you	30	you	25
a	40	the	20	a	5
you	35	a	10	the	3

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Observation: Words with high probabilities have high occurrence counts (probabilities are monotone)

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We apply two Top-k Joining Techniques to NWP:

- Threshold Algorithm (Fagin et al. 2001)
- No Random Access Algorithm (Fagin et al. 2001)

Comparison

Threshold Algorithm	No Random Access Algorithm
– Requires Sorted and Random Access	+ Only Sorted Access necessary
+ Faster	
+ Less Memory	

Data structure

Data structure

- Sorted lists are unique for each history h

ϵ		love		I love	
the	50	you	30	you	25
a	40	the	20	a	5
you	35	a	10	the	3

Data structure

- Sorted lists are unique for each history h

ϵ		love		I love	
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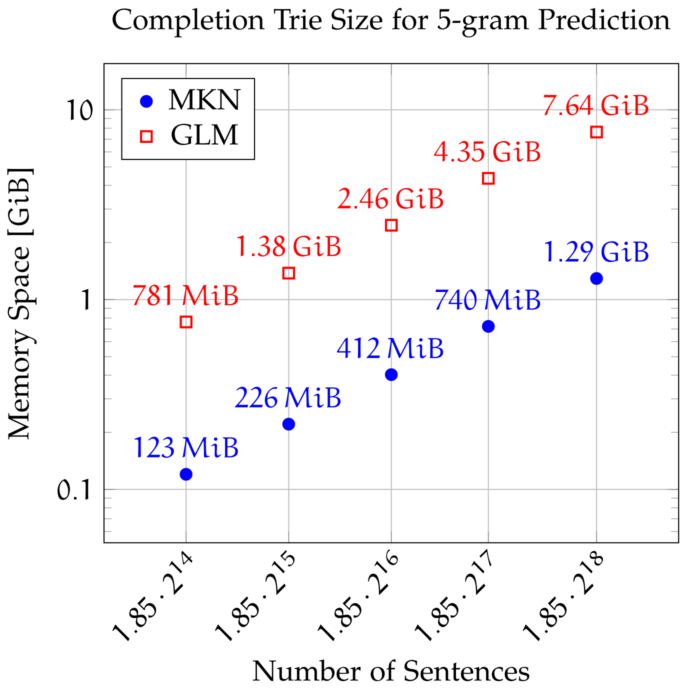
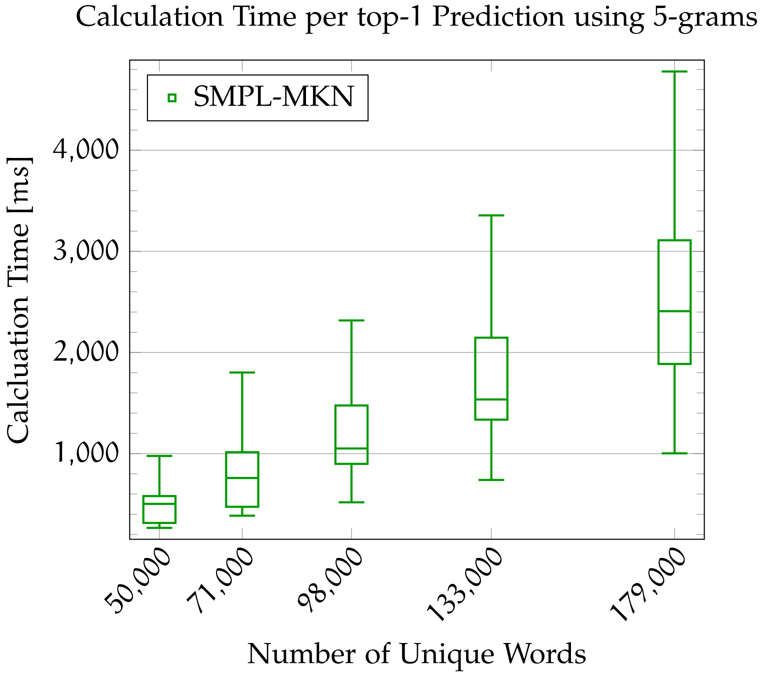
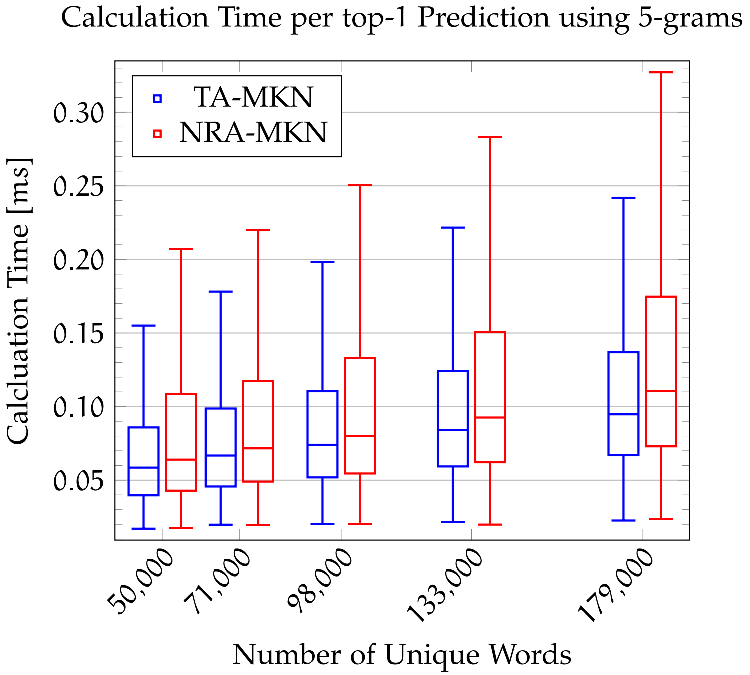
- Optimized data structure: *Completion Trie* (Hsu and Ottaviano 2013)
  - Optimized for prefix-retrieval
  - High data compression

Prediction Quality

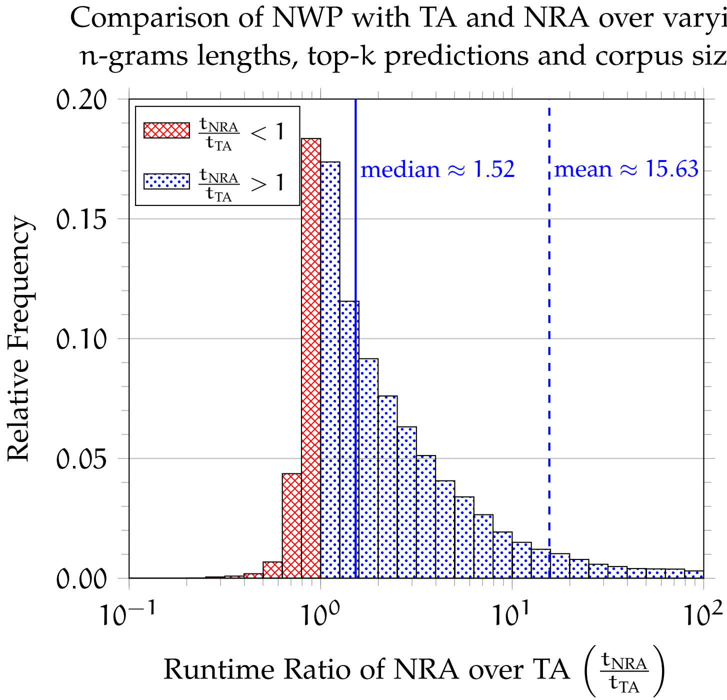
	Keystroke Savings	
n-gram length	MKN	GLM
2	0.44	0.44
3	0.50	0.50
4	0.51	0.51
5	0.51	0.52


Completion Trie Size for 5-gram Prediction
MKN
GLM
Memory Space [GiB]
10
1
0.1
781 MiB
1.38 GiB
2.46 GiB
4.35 GiB
7.64 GiB
123 MiB
226 MiB
412 MiB
740 MiB
1.29 GiB
1.85 · 2^14
1.85 · 2^15
1.85 · 2^16
1.85 · 2^17
1.85 · 2^18
Number of Sentences


Calculation Time per top-1 Prediction using 5-grams
SMPL-MKN
Calcluation Time [ms]
1,000
2,000
3,000
4,000
50,000
71,000
98,000
133,000
179,000
Number of Unique Words



Calculation Time per top-1 Prediction using 5-grams
Calcluation Time [ms]
0.05
0.10
0.15
0.20
0.25
0.30
TA-MKN
NRA-MKN
50,000
71,000
98,000
133,000
179,000
Number of Unique Words


Comparison of NWP with TA and NRA over varying n-grams lengths, top-k predictions and corpus sizes
$\frac{t_{NRA}}{t_{TA}} < 1$
$\frac{t_{NRA}}{t_{TA}} > 1$
median ≈ 1.52
mean ≈ 15.63
Relative Frequency
0.20
0.15
0.10
0.05
0.00
10^{-1}
10^{0}
10^{1}
10^{2}
Runtime Ratio of NRA over TA $\left(\frac{t_{NRA}}{t_{TA}}\right)$

Section 4

Conclusion

Conclusion

- Weighted Sum representation for probabilities
  - Calculation equally fast for MKN, double as fast for GLM
  - Speeds up word prediction by 50% (MKN) to 70% (GLM)
- Top-k Joining
  - Speeds up word prediction by multiple orders of magnitude
  - Threshold Algorithm preferable to No Random Access Algorithm

Thanks for your Attention!

http://141.26.208.6/

Section 5

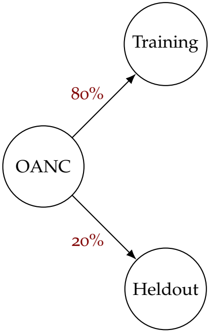
Experimental Setup

Evaluation Corpus

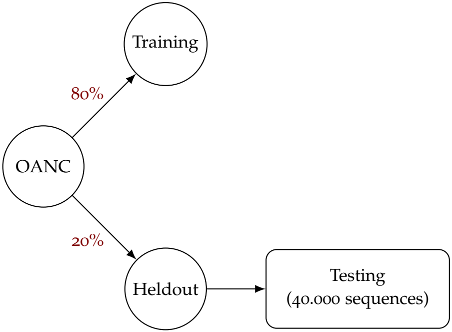
Open American National Corpus (Ide and Suderman 2007)

- open collection of American English
- historical and contemporary
- written text of all genres
- around 600.000 sentences / 14.000.000 words

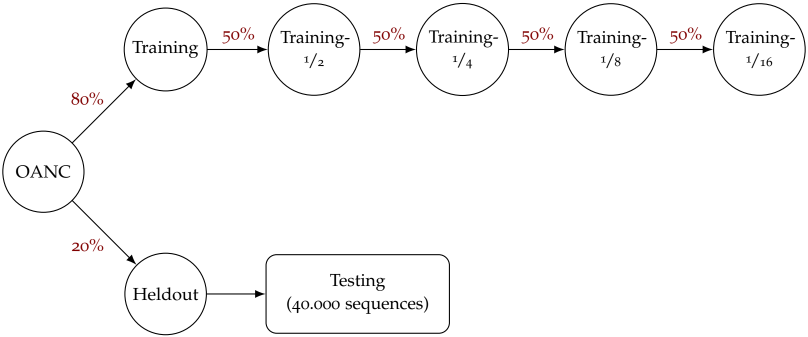
Experimental Setup



Experimental Setup



Experimental Setup



Section 6

Top-k Join Examples

Top-k joins combined with Weighted Sums

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$$P\left(w \,|\, h\right) = \sum_{i=1}^{N} \lambda_i^h \, \alpha_i^h(w)$$

Top-k joins combined with Weighted Sums

$$P(w \mid h) = \sum_{i=1}^{N} \lambda_i^h \, \alpha_i^h(w)$$

- Weights λ_i^h can be precomputed

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$$P\,(w\,|\,h) = \sum_{i=1}^{N} \lambda_i^h \, \alpha_i^h(w)$$

- Weights λ_i^h can be precomputed
- Sorted lists can store terms $\alpha_i^h(w)$

ϵ		love		I love	
the	50	you	30	you	25
a	40	the	20	a	5
you	35	a	10	the	3

Top-k joins combined with Weighted Sums

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- In examples we will assume $\lambda_1^h, \ldots, \lambda_N^h = 1$
  (not possible in practice!)

Top-k joins combined with Weighted Sums

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- In examples we will assume $\lambda_1^h, \ldots, \lambda_N^h = 1$ (not possible in practice!)
- $P(\text{you} \mid \text{I love}) = 35 + 30 + 25 = 95$

Threshold Algorithm

$$\arg\max_{w \in \Sigma} P_{\mathrm{MKN}}(w \mid \mathrm{I\ love})$$

Threshold Algorithm

$$\arg\max_{w \in \Sigma} \mathrm{P}_{\mathrm{MKN}}(w \mid \mathrm{I\ love})$$

ϵ	love	I love

Threshold Algorithm

1. *Sorted Access* to lists in any order (e.g. round robin)

$$\arg\max_{w \in \Sigma} \mathrm{P}_{\mathrm{MKN}}(w \mid \text{I love})$$

ϵ	love	I love

Threshold Algorithm

1. *Sorted Access* to lists in any order (e.g. round robin)
2. For new words, *Random Access* to to all other lists

$$\arg\max_{w \in \Sigma} P_{\mathrm{MKN}}(w \mid \text{I love})$$

ϵ	love	I love

Threshold Algorithm

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3. Compute *threshold* of highest possible unseen probability

$$\arg\max_{w \in \Sigma} \mathrm{P}_{\mathrm{MKN}}(w \mid \text{I love})$$

ϵ	love	I love

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3. Compute *threshold* of highest possible unseen probability
4. Terminate when k probabilities greater threshold have been found

$$\arg\max_{w \in \Sigma} P_{MKN}(w \mid \text{I love})$$

ϵ	love	I love

Threshold Algorithm

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ϵ	love	I love

$$\mathrm{P}_{\mathrm{threshold}} = \infty + \infty + \infty \ = \infty$$

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$$\arg\max_{w \in \Sigma} P_{\mathrm{MKN}}(w \mid \mathrm{I\ love})$$

ϵ	love	I love
$\boxed{\text{the} \quad 50}$		

$$P_{\mathrm{threshold}} = \infty + \infty + \infty \;= \infty$$

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$$\arg\max_{w \in \Sigma} \mathrm{P}_{\mathrm{MKN}}(w \mid \mathrm{I\ love})$$

ϵ	love	I love
$\boxed{\text{the} \quad 50}$		
	the 20	
		the 3

$$\mathrm{P}_{\text{threshold}} = \infty + \infty + \infty = \infty$$

Threshold Algorithm

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$$\arg\max_{w \in \Sigma} \mathsf{P}_{\mathrm{MKN}}(w \mid \mathrm{I\ love})$$

ϵ	love	I love
$\boxed{\text{the} \quad 50}$		
	the 20	
		the 3

$$\mathsf{P}_{\text{threshold}} = \infty + \infty + \infty = \infty$$

$$\mathsf{P}(\text{the} \mid \text{I love}) = 50 + 20 + 3 = 73$$

Threshold Algorithm

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$$\arg\max_{w \in \Sigma} \mathrm{P}_{\mathrm{MKN}}(w \mid \mathrm{I\ love})$$

ϵ	love	I love
$\boxed{\text{the} \quad 50}$		
	the 20	
		the 3

$$\mathrm{P}_{\mathrm{threshold}} = 50 + \infty + \infty = \infty$$

$$\mathrm{P}\left(\mathrm{the} \mid \mathrm{I\ love}\right) = 50 + 20 + 3 = 73$$

Threshold Algorithm

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$$\arg\max_{w \in \Sigma} \mathrm{P}_{\mathrm{MKN}}(w \mid \text{I love})$$

ϵ	love	I love
$\boxed{\text{the} \quad 50}$	$\boxed{\text{you} \quad 30}$	
	the 20	
		the 3

$$\mathrm{P}_{\text{threshold}} = 50 + \infty + \infty = \infty$$
$$\mathrm{P}(\text{the} \mid \text{I love}) = 50 + 20 + 3 = 73$$

Threshold Algorithm

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ϵ		love		I love	
$\boxed{\text{the} \quad 50}$		$\boxed{\text{you} \quad 30}$		you	25
		the	20		
you	35			the	3

$$P_{\text{threshold}} = 50 + \infty + \infty = \infty$$
$$P(\text{the} \mid \text{I love}) = 50 + 20 + 3 = 73$$

Threshold Algorithm

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ϵ		love		I love	
$\boxed{\text{the} \quad 50}$		$\boxed{\text{you} \quad 30}$		you	25
		the	20		
you	35			the	3

$$\mathrm{P}_{\text{threshold}} = 50 + \infty + \infty = \infty$$

$$\mathrm{P}(\text{you} \mid \text{I love}) = 35 + 30 + 25 = 90$$

$$\mathrm{P}(\text{the} \mid \text{I love}) = 50 + 20 + 3 = 73$$

Threshold Algorithm

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$$\arg\max_{w \in \Sigma} P_{\text{MKN}}(w \mid \text{I love})$$

ϵ		love		I love	
$\boxed{\text{the}\quad 50}$		$\boxed{\text{you}\quad 30}$		you	25
		the	20		
you	35			the	3

$$P_{\text{threshold}} = 50 + 30 + \infty = \infty$$
$$P(\text{you} \mid \text{I love}) = 35 + 30 + 25 = 90$$
$$P(\text{the} \mid \text{I love}) = 50 + 20 + 3 = 73$$

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$$\arg\max_{w \in \Sigma} P_{\text{MKN}}(w \mid \text{I love})$$

ϵ		love		I love	
$\boxed{\text{the}}$	$\boxed{50}$	$\boxed{\text{you}}$	$\boxed{30}$	$\boxed{\text{you}}$	$\boxed{25}$
		the	20		
you	35			the	3

$$\begin{aligned} P_{\text{threshold}} &= 50 + 30 + \infty &&= \infty \\ P(\text{you} \mid \text{I love}) &= 35 + 30 + 25 &&= 90 \\ P(\text{the} \mid \text{I love}) &= 50 + 20 + 3 &&= 73 \end{aligned}$$

Threshold Algorithm

1. *Sorted Access* to lists in any order (e.g. round robin)
2. For new words, *Random Access* to to all other lists
3. Compute *threshold* of highest possible unseen probability
4. Terminate when k probabilities greater threshold have been found

$$\arg\max_{w \in \Sigma} \mathrm{P}_{\mathrm{MKN}}(w \mid \mathrm{I\ love})$$

ϵ		love		I love	
$\boxed{\text{the} \quad 50}$		$\boxed{\text{you} \quad 30}$		$\boxed{\text{you} \quad 25}$	
		the	20		
you	35			the	3

$$\mathrm{P}_{\mathrm{threshold}} = 50 + 30 + 25 = 105$$
$$\mathrm{P}(\mathrm{you} \mid \mathrm{I\ love}) = 35 + 30 + 25 = 90$$
$$\mathrm{P}(\mathrm{the} \mid \mathrm{I\ love}) = 50 + 20 + 3 = 73$$

Threshold Algorithm

1. *Sorted Access* to lists in any order (e.g. round robin)
2. For new words, *Random Access* to to all other lists
3. Compute *threshold* of highest possible unseen probability
4. Terminate when k probabilities greater threshold have been found

$$\arg\max_{w\in\Sigma} P_{\mathrm{MKN}}(w \mid \text{I love})$$

ϵ		love		I love	
the	50	$\boxed{\text{you}}$	$\boxed{30}$	$\boxed{\text{you}}$	$\boxed{25}$
$\boxed{\text{a}}$	$\boxed{40}$	the	20		
you	35			the	3

$$\begin{aligned} P_{\text{threshold}} &= 50 + 30 + 25 &&= 105 \\ P(\text{you} \mid \text{I love}) &= 35 + 30 + 25 &&= 90 \\ P(\text{the} \mid \text{I love}) &= 50 + 20 + 3 &&= 73 \end{aligned}$$

Threshold Algorithm

1. *Sorted Access* to lists in any order (e.g. round robin)
2. For new words, *Random Access* to to all other lists
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4. Terminate when k probabilities greater threshold have been found

$$\arg\max_{w \in \Sigma} P_{\mathrm{MKN}}(w \mid \text{I love})$$

ϵ		love		I love	
the	50	$\boxed{\text{you}}$	$\boxed{30}$	$\boxed{\text{you}}$	$\boxed{25}$
$\boxed{\text{a}}$	$\boxed{40}$	the	20	a	5
you	35	a	10	the	3

$$P_{\text{threshold}} = 50 + 30 + 25 = 105$$
$$P(\text{you} \mid \text{I love}) = 35 + 30 + 25 = 90$$
$$P(\text{the} \mid \text{I love}) = 50 + 20 + 3 = 73$$

Threshold Algorithm

1. *Sorted Access* to lists in any order (e.g. round robin)
2. For new words, *Random Access* to to all other lists
3. Compute *threshold* of highest possible unseen probability
4. Terminate when k probabilities greater threshold have been found

$$\arg\max_{w \in \Sigma} P_{\text{MKN}}(w \mid \text{I love})$$

ϵ		love		I love	
the	50	$\boxed{\text{you}}$	$\boxed{30}$	$\boxed{\text{you}}$	$\boxed{25}$
$\boxed{\text{a}}$	$\boxed{40}$	the	20	a	5
you	35	a	10	the	3

$$
\begin{aligned}
P_{\text{threshold}} &= 50 + 30 + 25 &&= 105 \\
P(\text{you} \mid \text{I love}) &= 35 + 30 + 25 &&= 90 \\
P(\text{the} \mid \text{I love}) &= 50 + 20 + 3 &&= 73 \\
P(\text{a} \mid \text{I love}) &= 40 + 10 + 5 &&= 55
\end{aligned}
$$

Threshold Algorithm

1. *Sorted Access* to lists in any order (e.g. round robin)
2. For new words, *Random Access* to to all other lists
3. Compute *threshold* of highest possible unseen probability
4. Terminate when k probabilities greater threshold have been found

$$\arg\max_{w \in \Sigma} P_{\mathrm{MKN}}(w \mid \mathrm{I\ love})$$

ϵ		love		I love	
the	50	$\boxed{\text{you}}$	$\boxed{30}$	$\boxed{\text{you}}$	$\boxed{25}$
$\boxed{\text{a}}$	$\boxed{40}$	the	20	a	5
you	35	a	10	the	3

$$\begin{aligned} P_{\text{threshold}} &= 40 + 30 + 25 &&= 95 \\ P(\text{you} \mid \text{I love}) &= 35 + 30 + 25 &&= 90 \\ P(\text{the} \mid \text{I love}) &= 50 + 20 + 3 &&= 73 \\ P(\text{a} \mid \text{I love}) &= 40 + 10 + 5 &&= 55 \end{aligned}$$

Threshold Algorithm

1. *Sorted Access* to lists in any order (e.g. round robin)
2. For new words, *Random Access* to to all other lists
3. Compute *threshold* of highest possible unseen probability
4. Terminate when k probabilities greater threshold have been found

$$\arg\max_{w\in\Sigma} P_{\text{MKN}}(w \mid \text{I love})$$

ϵ		love		I love	
the	50	you	30	$\boxed{\text{you}}$	$\boxed{25}$
$\boxed{\text{a}}$	$\boxed{40}$	$\boxed{\text{the}}$	$\boxed{20}$	a	5
you	35	a	10	the	3

$$\begin{aligned} P_{\text{threshold}} &= 40 + 30 + 25 &= 95 \\ P(\text{you} \mid \text{I love}) &= 35 + 30 + 25 &= 90 \\ P(\text{the} \mid \text{I love}) &= 50 + 20 + 3 &= 73 \\ P(\text{a} \mid \text{I love}) &= 40 + 10 + 5 &= 55 \end{aligned}$$

Threshold Algorithm

1. *Sorted Access* to lists in any order (e.g. round robin)
2. For new words, *Random Access* to to all other lists
3. Compute *threshold* of highest possible unseen probability
4. Terminate when k probabilities greater threshold have been found

$$\arg\max_{w \in \Sigma} P_{\text{MKN}}(w \mid \text{I love})$$

ϵ		love		I love	
the	50	you	30	$\boxed{\text{you} \quad 25}$	
$\boxed{\text{a} \quad 40}$		$\boxed{\text{the} \quad 20}$		a	5
you	35	a	10	the	3

$$\begin{aligned} P(\text{you} \mid \text{I love}) &= 35 + 30 + 25 &&= 90 \\ P_{\text{threshold}} &= 40 + 20 + 25 &&= 85 \\ P(\text{the} \mid \text{I love}) &= 50 + 20 + 3 &&= 73 \\ P(\text{a} \mid \text{I love}) &= 40 + 10 + 5 &&= 55 \end{aligned}$$

Threshold Algorithm

1. *Sorted Access* to lists in any order (e.g. round robin)
2. For new words, *Random Access* to to all other lists
3. Compute *threshold* of highest possible unseen probability
4. Terminate when k probabilities greater threshold have been found

$$\arg\max_{w \in \Sigma} P_{\text{MKN}}(w \mid \text{I love})$$

ϵ		love		I love	
the	50	you	30	$\boxed{\text{you}}$	$\boxed{25}$
$\boxed{\text{a}}$	$\boxed{40}$	$\boxed{\text{the}}$	$\boxed{20}$	a	5
you	35	a	10	the	3

$$\begin{aligned} P(\text{you} \mid \text{I love}) &= 35 + 30 + 25 &&= 90 \\ P_{\text{threshold}} &= 40 + 20 + 25 &&= 85 \\ P(\text{the} \mid \text{I love}) &= 50 + 20 + 3 &&= 73 \\ P(\text{a} \mid \text{I love}) &= 40 + 10 + 5 &&= 55 \end{aligned}$$

No Random Access Algorithm

No Random Access Algorithm

1. *Sorted Access* to lists in any order

No Random Access Algorithm

1. *Sorted Access* to lists in any order
2. Keep track of all seen counts

No Random Access Algorithm

1. *Sorted Access* to lists in any order
2. Keep track of all seen counts
3. Compute *upper* and *lower bounds* for each probability

No Random Access Algorithm

1. *Sorted Access* to lists in any order
2. Keep track of all seen counts
3. Compute *upper* and *lower bounds* for each probability
4. Terminate when k lower bounds greater than all other upper bounds have been found

Section 7

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