Chapter 1

Material Modelling

1.1 Winfrith concrete model in LS-DYNA (RATE = 1)

Concrete is modelled using the Winfrith material implemented in LS-DYNA as `*MAT_WINFRITH_CONCRETE` (MAT084/085). In this thesis, strain-rate enhancement is neglected by selecting the no-rate option `RATE = 1`. Under this setting, the tensile cracking parameter `FE` is interpreted as the critical crack width at which the normal tensile traction transmitted across the crack plane vanishes. This critical opening is denoted by w_c [3].

1.1.1 Elastic and strength parameters (Eurocode-based estimates)

The Winfrith model requires elastic constants (ρ, E_c, ν) and strength parameters (f_c, f_t). When direct material testing is unavailable, these quantities can be estimated from the mean 28-day cylinder compressive strength f_{cm} using Eurocode-type relationships [2]. The corresponding LS-DYNA inputs are `RO`, `TM`, `PR`, `UCS`, and `UTS`.

Compressive strength

In Eurocode terminology, f_{cm} is the mean cylinder compressive strength at 28 days, while f_{ck} is the characteristic cylinder strength. A commonly used relationship is

$$f_{ck} = f_{cm} - 8 \text{ MPa}. \tag{1.1}$$

In this thesis, the unconfined compressive strength in LS-DYNA is taken as

$$\texttt{UCS} = f_c \approx f_{cm}, \tag{1.2}$$

unless a test-based unconfined value is available.

Tensile strength

The mean axial tensile strength according to Eurocode is taken as

$$f_{ctm} = 0.30\, f_{ck}^{2/3}, \qquad (f_{ck} \leq 50 \text{ MPa}), \tag{1.3}$$

and for higher-strength concrete,

$$f_{ctm} = 2.12 \ln\left(1 + \frac{f_{cm}}{10}\right), \qquad (f_{ck} > 50 \text{ MPa}). \tag{1.4}$$

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The Winfrith tensile strength is set as

$$\texttt{UTS} = f_t = f_{ctm}. \tag{1.5}$$

Elastic modulus, Poisson's ratio, and density

The secant modulus of elasticity is estimated by Eurocode as

$$E_{cm} = 22{,}000 \left(\frac{f_{cm}}{10} \right)^{0.3} \text{ MPa}, \tag{1.6}$$

and used directly in LS-DYNA:

$$\texttt{TM} = E_c = E_{cm}. \tag{1.7}$$

When not measured, $\nu \approx 0.20$ is adopted for normal-weight concrete, and the density is selected in the range 2300–2500 kg/m^3 depending on mix design:

$$\texttt{PR} = \nu, \qquad \texttt{RO} = \rho. \tag{1.8}$$

1.1.2 Crack width variable w and its geometrical meaning

The Winfrith tensile formulation is expressed in terms of the crack width variable w, which represents the separation of two crack faces in the direction normal to the crack plane. Figure 1.1 provides a geometrical interpretation of w and the crack opening angle 2θ. For the idealised V-shaped crack, the opening angle is related to the crack width and a characteristic length L through
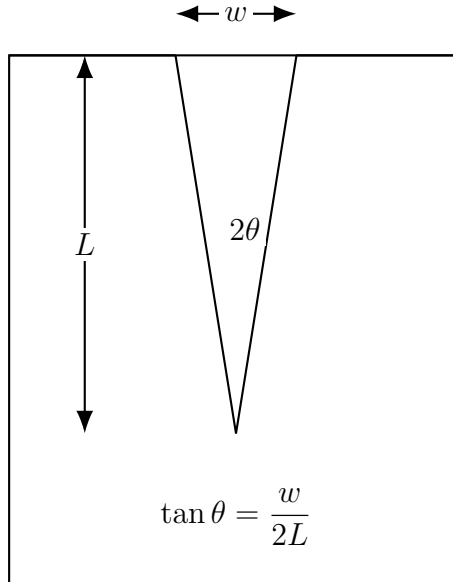
$$\tan \theta = \frac{w}{2L}. \tag{1.9}$$

Figure 1.1: Geometrical interpretation of crack width w and crack opening angle.

1.1.3 Fracture energy and determination of FE (CEB-FIP + RATE=1)

In the Winfrith model, the progressive loss of tensile load transfer is governed by the mode-I fracture energy under uniaxial tension, denoted here as G_{F_t}, and the terminal crack width w_c [3]. When direct fracture tests are unavailable, G_{F_t} can be estimated using CEB-FIP Model Code 1990 relations based on aggregate size and compressive strength [1].

Base fracture energy from aggregate size

The base fracture energy G_{F0} depends on the maximum aggregate size $d_{\max}$:

$$G_{F0} = 0.021 + 5.357 \times 10^{-4}\, d_{\max}, \tag{1.10}$$

where $d_{\max}$ is in mm and G_{F0} is in N/mm. In practice, the Winfrith parameter ASIZE is set consistently with the mix design:

$$\texttt{ASIZE} \approx d_{\max}. \tag{1.11}$$

Strength scaling to obtain G_{F_t}

The fracture energy under uniaxial tension is estimated as

$$G_{F_t} = G_{F0} \left(\frac{f_{cm}}{f_{cm0}} \right)^{0.7}, \qquad \text{for } f_{cm} \leq 50 \text{ MPa}, \tag{1.12}$$

$$G_{F_t} = G_{F0} \ln\left(1 + \frac{f_{cm}}{f_{cm0}} \right), \qquad \text{for } f_{cm} > 50 \text{ MPa}, \tag{1.13}$$

with the reference strength $f_{cm0} = 10$ MPa [1].

Conversion to w_c and FE for RATE=1

For RATE=1, the tensile softening is formulated in the traction–separation space $\sigma_n(w)$ and is commonly taken as linear softening [3]. Enforcing energy equivalence yields

$$w_c = \frac{2 G_{F_t}}{f_t}, \tag{1.14}$$

and the Winfrith input is set as

$$\texttt{FE} = w_c. \tag{1.15}$$

1.1.4 Tensile traction–separation law in the σ_n–w space

For RATE=1, the normal tensile traction is prescribed as a function of crack width [3]:

$$\sigma_n(w) = f_t \left(1 - \frac{w}{w_c} \right), \qquad 0 \leq w \leq w_c, \tag{1.16}$$
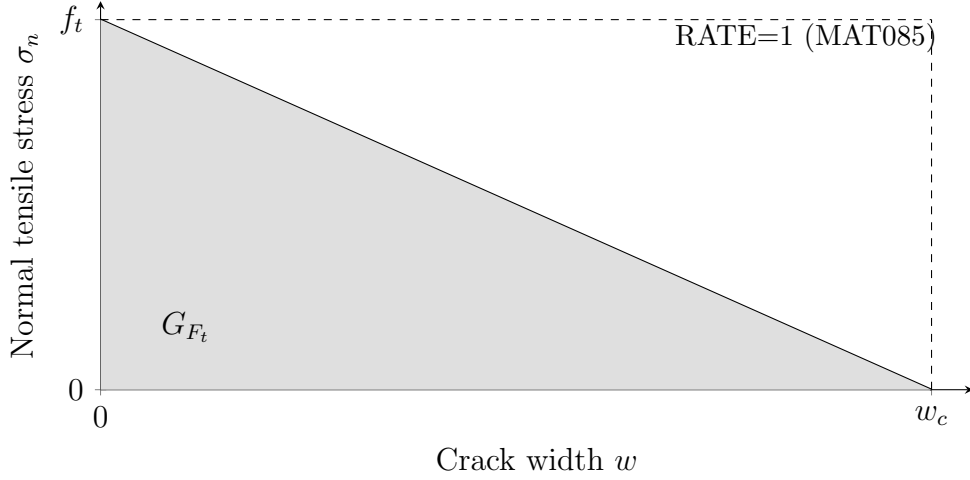
$$\sigma_n(w) = 0, \qquad w \geq w_c. \tag{1.17}$$

Figure 1.2: Linear tensile softening in the σ_n–w space for Winfrith RATE=1. The area under the curve equals G_{F_t}, giving $w_c = 2G_{F_t}/f_t$.

1.1.5 Crack-band regularisation and mesh dependence

Winfrith employs a smeared-crack approach. The crack width is therefore not introduced as an explicit geometric discontinuity. Instead, the crack width is computed from the crack-normal strain and a characteristic element length [3]:

$$w = \varepsilon_n L, \qquad L = \sqrt[3]{V_e}, \tag{1.18}$$
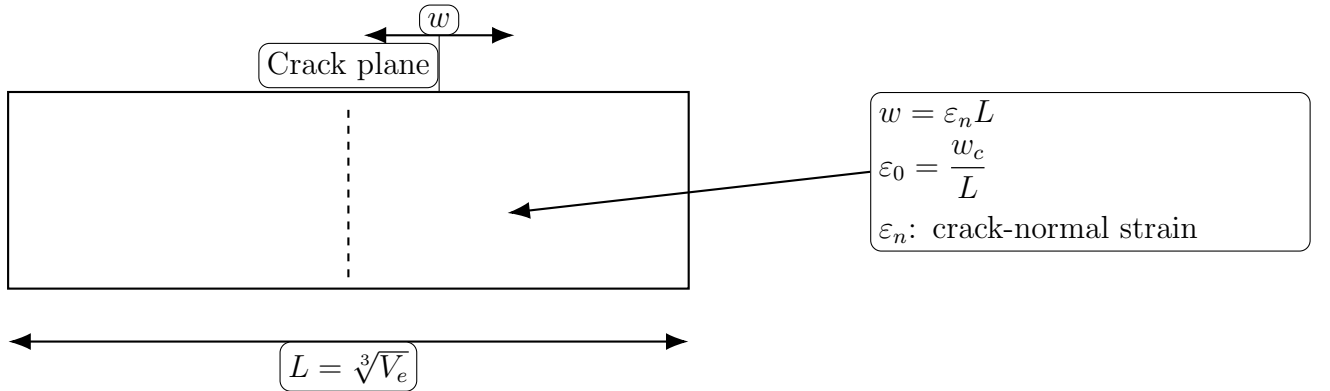
where V_e is the element volume.

Figure 1.3: Crack-band regularisation in Winfrith. The crack opening is computed from the crack-normal strain using a characteristic element length $L = \sqrt[3]{V_e}$, so that $w = \varepsilon_n L$ [3].

The strain at which tensile traction vanishes follows from $w = w_c$:

$$\varepsilon_0 = \frac{w_c}{L}. \tag{1.19}$$

Thus, for a fixed material law in the σ_n–w space (fixed w_c), the apparent softening length in a stress–strain view depends on the element size through L.
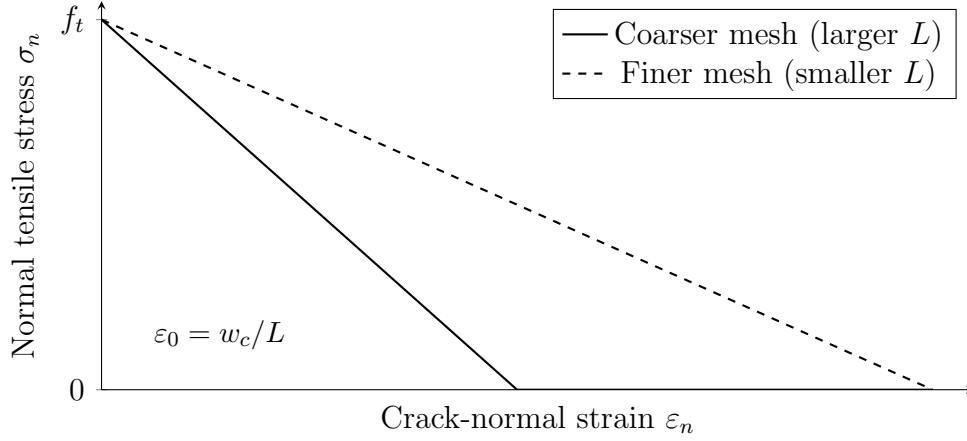
Figure 1.4: Mesh dependence in a stress–strain representation. For fixed w_c (fixed `FE` when RATE=1), the strain at zero traction is $\varepsilon_0 = w_c/L$ and therefore changes with L.

1.2 Summary of Winfrith inputs used in this thesis

Table 1.1: Summary of key Winfrith (LS-DYNA) inputs and recommended estimates (RATE=1).

Quantity	Symbol	LS-DYNA keyword	How to set (this thesis)
Density	ρ	`RO`	Select from mix design or typical normal-weight concrete (2300–2500 kg/m^3).
Young's modulus	E_c	`TM`	Eurocode: $E_c = 22{,}000 \left(\dfrac{f_{cm}}{10} \right)^{0.3}$ MPa.
Poisson's ratio	ν	`PR`	If unknown: $\nu \approx 0.20$.
Unconfined compressive strength	f_c	`UCS`	Set $f_c \approx f_{cm}$ (MPa).
Tensile strength	f_t	`UTS`	Set $f_t = f_{ctm}$ (Eurocode): $f_{ck} = f_{cm} - 8$ MPa; if $f_{ck} \leq 50$, $f_{ctm} = 0.30 f_{ck}^{2/3}$; else $f_{ctm} = 2.12 \ln \left(1 + \dfrac{f_{cm}}{10} \right)$.
Maximum aggregate size	$d_{\max}$	`ASIZE`	Set `ASIZE` $\approx d_{\max}$ (mm) from mix design.
Critical crack width at $\sigma_n = 0$ (RATE=1)	w_c	`FE`	Compute $G_{F0} = 0.021 + 5.357 \times 10^{-4} d_{\max}$ (N/mm). Then obtain G_{F_t} by Eqs. (1.12)–(1.13) with $f_{cm0} = 10$ MPa. Finally set $w_c = 2G_{F_t}/f_t$ and `FE` $= w_c$.

Bibliography

[1] CEB-FIP. *Model Code 1990: Design Code*. Thomas Telford, 1993. Fracture energy formulation and dependence on compressive strength and aggregate size.

[2] CEN. En 1992-1-1: Eurocode 2: Design of concrete structures – part 1-1: General rules and rules for buildings. Technical report, European Committee for Standardization, 2004.

[3] L. E. Schwer. The winfrith concrete model: Beauty or beast? insights into the winfrith concrete model. Technical report, Livermore Software Technology Corporation (LSTC), 2011.