Stat 344 – PS 2

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```
mrc <- read.csv('http://sldr.netlify.app/data/mrc.csv')
```

Problem 5.6

X ~ Multinom(π)

$n = \sum x_i$

$l(\pi; x) = logL(\pi; x) = \sum_i log(dmultinnom(x_i, \pi))$

$\infty x_1 log(\pi_1) + ... + x_k log(\pi_k) = \tilde{l}(\pi; x)$ half infinity, not full infinity

Given $g(\pi) = \sum_{i=1}^{k} \pi_i$ when $g(\pi) = 1$

$\Delta\tilde{l}(x; \pi) = \lambda \Delta g(\pi)$

$g(\pi) = 1$

$\frac{x_1}{\pi_1} = \lambda_1$ -> $\pi_1 = \frac{x_1}{\lambda}$

$\frac{x_2}{\pi_2} = \lambda_1$ -> $\pi_2 = \frac{x_2}{\lambda}$

$\frac{x_1}{\pi_3} = \lambda_3$ -> $\pi_3 = \frac{x_3}{\lambda}$

Substituting in, $\pi_1 + \pi_2 + \pi_3 + ... + \pi_k = 1$

$\frac{x_1 + x_2 + x_3 + ... + x_k}{\lambda} = 1$

$\frac{n}{\lambda} = 1$ -> $\lambda = n$

So then $\pi_1 = \frac{x_1}{n}, \pi_2 = \frac{x_2}{n}, ..., \pi_k = \frac{x_k}{n}$ so $\pi_i = \frac{x_i}{n}$

Problem 5.10

Repeat Example 5.1.4 Using numerical methods

Find the Maximum likelihood estimator for

```
#$L(\theta ; x) = 0$

#if $\theta < 8.7$ ,

#$(\frac{1}{\theta})^n $

#if $ \theta \ge 8.7$
```

$log(L(\theta; x)) = log(\Pi_{i=1}^{6}[(\frac{1}{\theta})^n])$

$= log(\frac{1}{\theta})^6$

$= \frac{d}{d\theta}(6log(\frac{1}{\theta})) = -\frac{6}{\theta}$

$-\frac{6}{\theta} = 0$

$\hat{\theta} = \frac{1}{6}$

```
data <- c(1.6, 2.8, 6.2, 8.2, 8.5, 8.7)
```

```
LL <- function(theta, x){
  b <- theta[1]
  dunif(x, min = 0, max = b, log = TRUE)
}
```

```
maxLik(LL, start = c(b = 10), x = data)
```

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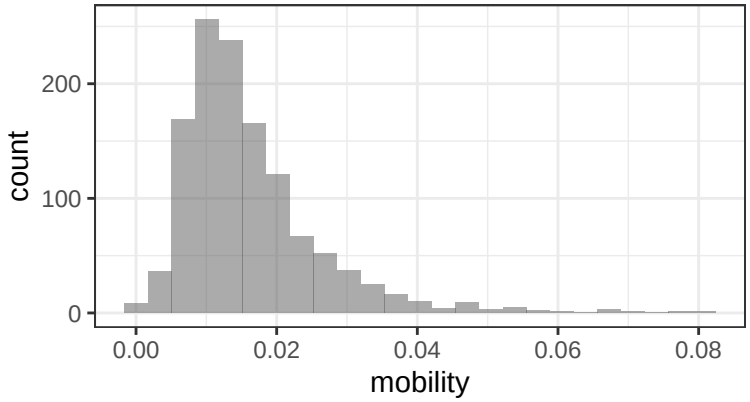
## Maximum Likelihood estimation
## Newton-Raphson maximisation, 6 iterations
## Return code 3: Last step could not find a value above the current.
```

```
## Boundary of parameter space?
## Consider switching to a more robust optimisation method temporarily.
## Log-Likelihood: -12.97997 (1 free parameter(s))
## Estimate(s): 8.700042
```

Problem 5.12

```
mrc <- read.csv('http://sldr.netlify.app/data/mrc.csv') |>
  filter(mobility > 0)
```

```
gf_histogram(~mobility, data = mrc)
```

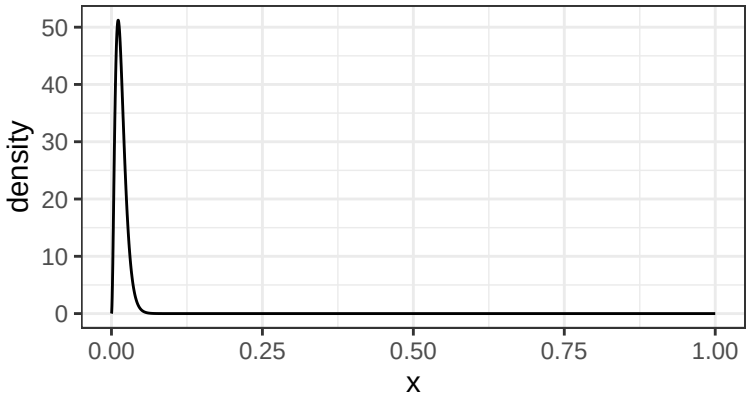



```
LL2 <- function(theta, x){
  alpha <- theta[1]
  beta <- theta[2]
  if (alpha < 0) return(NA)
  if (beta < 0) return(NA)
  dbeta(x, shape1 = alpha, shape2 = beta, log = TRUE)
}
```

```
maxLik(LL2, start = c(alpha = 1, beta = 150), x = mrc$mobility)
```

```
## Maximum Likelihood estimation
## Newton-Raphson maximisation, 10 iterations
## Return code 8: successive function values within relative tolerance limit (reltol)
## Log-Likelihood: 4185.799 (2 free parameter(s))
## Estimate(s): 3.155288 192.6126
```

```
gf_dist("beta", shape1 = 3.155, shape2= 192.61)
```



The model seems pretty good for the model given how well the beta distribution with the given parameters from the MLE function. When overlayed a histogram of the original data, the graphs appear to line up pretty well in distribution.