Stat 344 – PS06

Luca Seazzu

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Problem 5.46

a. P(X|) = (e^(-) * ^Σx) / (x ! * x ! * … * x !)

gamma pdf

f(| ,) = (1 / (^ * Γ( ))) * ^(-1) * e^(- /)

Apply Bayes

P(|X) P(X|) * P( )

Log both sides

log(P(|X)) log(P(X|)) + log(P( ))

Then substitute the right side of the equation

log(P(|X)) Σx * log( ) - - log(x ! * x ! * … * x !) + (- 1) * log( ) - / - log(^ * Γ( ))

Thus,

P(|X) ~ gamma(+ Σx , n + 1/)

b.

E( ) = * , so given P(|X) ~ gamma(+ Σx , n + 1/),

E(|X) = (+ Σx) * (n + 1/)

c.

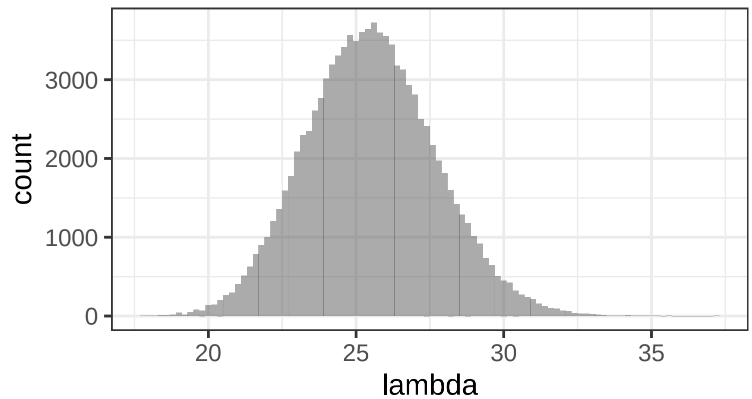
As n approaches infinity, so does the posterior distribution

d.

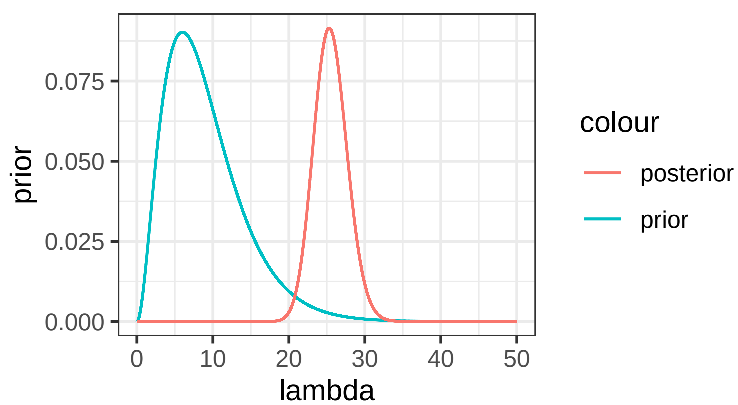
```r
classes <- c(9, 39, 22, 35, 28)

NormalGrid <-
  expand.grid(
    lambda = seq(0, 50, length.out = 100000)
  ) |>
  mutate(
    prior = dgamma(lambda, shape = 3, rate = 1/3), ## prior choices
    likelihood = mapply(function(l) {prod(dpois(classes, lambda = l))}, l = lambda),
    posterior = prior * likelihood /  sum(prior * likelihood) * 1000
  )

NormalGrid |> arrange(-posterior) |> head(3)
```

```
##     lambda       prior   likelihood  posterior
## 1 25.31225 0.002570064 1.818233e-11 0.09150571
## 2 25.31275 0.002569737 1.818464e-11 0.09150570
## 3 25.31175 0.002570391 1.818002e-11 0.09150570
```

```
PosteriorSample <-
  sample(NormalGrid, size = 1e5, replace = TRUE, prob = NormalGrid$posterior)

gf_histogram(~lambda, data = PosteriorSample, binwidth = 0.2)
```




```
gf_line(prior ~ lambda, color = ~ "prior", data = NormalGrid) |>
  gf_line(posterior ~ lambda, color = ~ "posterior", data = NormalGrid)
```




```
cdata(~ lambda, data = PosteriorSample, p = 0.95)
```

```
##        lower   upper central.p
## 2.5% 21.3862 29.9858      0.95
```

Problem 5.47

```
Positions <- rgeo(25)
leaflet_map(position = Positions, mark = TRUE)
```

```
## PhantomJS not found. You can install it with webshot::install_phantomjs(). If it is installed, please
```

```
water <- 16
land <- 9
```

```r
qbeta(c(0.05, 0.95), 17 , 10 )
```

```
## [1] 0.4738376 0.7743001
```

0.4738376 0.7743001 90% credible interval with a uniform prior

Problem 5.49

P(X|tau) = (1 / (sqrt(2 /tau))^n) * e$^{(-\Sigma(x}$ 2) * tau/2)

pdf: f(tau|alpha, beta) = (1 / (beta^alpha * Γ(alpha))) * tau^(alpha-1) * e^(-tau * beta)

Bayes: P(tau|X)   P(X|tau) * P(tau)

Log: log(P(tau|X))   log(P(X|tau)) + log(P(tau))

Substitute: log(P(tau|X)) -Σ(x ^2) * tau/2 + (alpha - 1) * log(tau) - tau * beta - log(beta^alpha * Γ(alpha))

log(P(tau|X))   -(Σ(x ^2)/2 - beta) * tau + (alpha - 1) * log(tau) - log(beta^alpha * Γ(alpha))

Thus, P(tau|X) ~ gamma(alpha + n/2, (Σ(x ^2)/2 - beta))