Stat 344 – PS 03

Luca Seazzu

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Problem 5.24

a. See attached pdf.

b.

```
x <- c(.64, .92, .73, .96, .98, .33, .8, .96, .81, .76, .98, .75, .87, .82, .44, .96, .61, .32, .67, .98
```

```
W <- function(x, theta0 = 0){
  x_sum <- sum(x)
  n <- length(x)
  theta.hat <- (n / -sum(log(x)) - 1)
  -2 * log(theta0 + 1) + 2 * log(theta.hat + 1) + (log(x_sum) * (2 * theta.hat - 2 * theta0))
}
```

```
W(x)
```

```
## [1] 18.53735
```

c.

```
p_value <- 1 - pchisq(W(x), df = 1)
p_value
```

```
## [1] 1.666069e-05
```

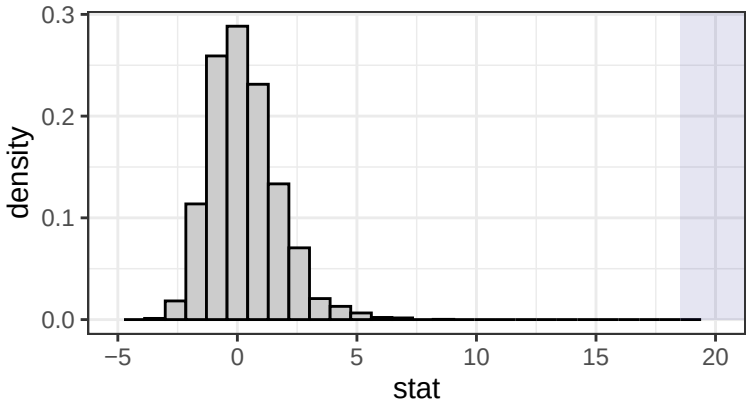
There doesn't seem to be any reason to be concerned about this p-value since it is so small to reject the null hypothesis that theta = 0 when the data doesn't seem to suggest that.

d.

```
rdata <- do(5000) * runif(30, 0,1)

statTally(x, rdata, W)
```

```
##
## Null distribution appears to be asymmetric. (p = 7.41e-06)
##
## Test statistic applied to sample data =  18.54
##
## Quantiles of test statistic applied to random data:
##      50%      90%      95%      99%
## 0.109609 2.146103 2.852096 4.644143
##
## Of the  5001  samples (1 original +  5000  random),
##   1 ( 0.02 % ) had test stats = 18.54
##   1 ( 0.02 % ) had test stats >= 18.54
```



Simulated data reveals a p-value of p = 0.000287 which is consistent with the real data.

e. The two p-values are slightly different but both are really small and consistent with each other. Both reject the null hypothesis that theta = 0 because the data shows that theta isn't really even close to 0.

Problem 5.29

See attached hand-written pdf

Problem 5.30

a.

theta = p(1)^n1 * p(2)^n2 * p(3)^n3 * p(4)^n4 = .25^4(2theta - 3theta^2 + theta$^{4)}$n

```
n <- c(1997, 906, 904, 32)

LL <- function(theta, x){
  x[1] * log(.25 * (2 + theta)) +  x[2] * log(.25 * (1-theta)) + x[3] * log(.25 * (1-theta)) + x[4] * l
}

maxLik(LL, start = .5 , x = n)
```

```
## Maximum Likelihood estimation
## Newton-Raphson maximisation, 3 iterations
## Return code 8: successive function values within relative tolerance limit (reltol)
## Log-Likelihood: -4074.879 (1 free parameter(s))
## Estimate(s): 0.0357123
```

MLE = 0.0357123

b.

```
W2 <- 2 * (LL(0.0357123, n) - LL(0.05, n))

p_value2 <- 1 - pchisq(W2, df = 1)
p_value2
```

```
## [1] 0.03260412
```

P-value = 0.03 therefore reject the hypothesis that theta = 0.05

c.

```
W3 <- 2 * (LL(0.0357123, n) - LL(0.03, n))
```

```r
p_value3 <- 1 - pchisq(W3, df = 1)
p_value3
```

```
## [1] 0.3180208
```

P-value = 0.318 therefore fail to reject the hypothesis that theta = 0.03

d.

```r
W4 <- 2 * (LL(0.0357123, n) - LL(0.07, n))

p_value4 <- 1 - pchisq(W4, df = 1)
p_value4
```

```
## [1] 3.969742e-06
```

P-value = 0.0000039 therefore reject the hypothesis that theta = 0.07