MA3H5 Manifolds

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1   Differential Forms in $\mathbb{R}^n$

1.1   Exterior Algebra

If $f : U \to V$ is a linear map then we define its pullback $f^* : \Lambda^r V^* \to \Lambda^r U^*$ by

$$f^* \beta(u_1, \ldots, u_r) = \beta(f(u_1), \ldots, f(u_r))$$

for every $\beta \in \Lambda^r U^*$ and every $u_1, \ldots, u_r \in U$.

Prop. 1.
$$f^*(\alpha \wedge \beta) = f^*(\alpha) \wedge f^*(\beta)$$

If $f : U \to V$ and $g : V \to W$ are linear maps then

$$(g \circ f)^* = f^* \circ g^*.$$

If $f : U \to U$ is a linear map and $\beta \in \Lambda^n U^*$ where $n = \dim U$ then

$$f^* \beta = (\det f)\beta.$$