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Lista de Exercícios 2 - MAE0228

Exercício 2

a) Sabendo que $E(X) = \int_{-\infty}^{\infty} x \cdot f(x)dx$, pode-se fazer:

$$\begin{aligned} E(aX + b) &= \int_{-\infty}^{\infty} (ax + b) \cdot f(x)dx \\ &= \int_{-\infty}^{\infty} ax \cdot f(x) + b \cdot f(x)dx \\ &= \int_{-\infty}^{\infty} ax \cdot f(x)dx + \int_{-\infty}^{\infty} b \cdot f(x)dx \\ &= a \cdot \int_{-\infty}^{\infty} x \cdot f(x)dx + b \cdot \int_{-\infty}^{\infty} f(x)dx \\ &= a \cdot E(X) + b \end{aligned}$$

b) Usando a propriedade $Var(X) = E(X^2) - E^2(X)$ e o resultado do item **a**:

$$\begin{aligned} Var(aX + b) &= E[(aX + b)^2] - E^2(aX + b) \\ &= \int_{-\infty}^{\infty} (a^2x^2 + 2abx + b^2) \cdot f(x)dx - \left[\left(\int_{-\infty}^{\infty} (ax + b) \cdot f(x)dx \right) \cdot \left(\int_{-\infty}^{\infty} (ax + b) \cdot f(x)dx \right) \right] \\ &= \int_{-\infty}^{\infty} a^2x^2 \cdot f(x)dx + \int_{-\infty}^{\infty} 2abx \cdot f(x)dx + \int_{-\infty}^{\infty} b^2 \cdot f(x)dx - (a \cdot E(X) + b)^2 \\ &= a^2 \cdot \int_{-\infty}^{\infty} x^2 \cdot f(x)dx + 2ab \cdot \int_{-\infty}^{\infty} x \cdot f(x)dx + b^2 \cdot \int_{-\infty}^{\infty} f(x)dx - (a \cdot E(X) + b)^2 \\ &= a^2 \cdot E(X^2) + 2ab \cdot E(X) + b^2 - a^2 \cdot E^2(X) - 2ab \cdot E(X) - b^2 \\ &= a^2 \cdot [E(X^2) - E^2(X)] = a^2 \cdot Var(X) \end{aligned}$$