Nome: Luís Felipe de Melo Costa Silva
Número USP: 9297961

Lista de Exercícios 2 - MAE0228

Exercício 6

b) Sabendo, do item anterior, que $X + Y \sim Poisson(\lambda + \mu)$ e que X e Y são independentes, teremos:

$$
\begin{aligned}
P(X|X+Y=n) = \frac{P(X=x, X+Y=n)}{P(X+Y=n)} &= \frac{P(X=x, Y=n-x)}{P(X+Y=n)} = \frac{P(X=x)\cdot P(Y=n-x)}{P(X+Y=n)} = \\
&= \left[\frac{e^{-\lambda}\cdot\lambda^x}{x!}\cdot\frac{e^{-\mu}\cdot\mu^{n-x}}{(n-x)!}\right]\cdot\frac{n!}{e^{-(\lambda+\mu)}\cdot(\lambda+\mu)^n} = \\
&= \frac{n!}{x!\cdot(n-x)!}\cdot\frac{e^{-(\lambda+\mu)}}{e^{-(\lambda+\mu)}}\cdot\frac{\lambda^x\cdot\mu^{n-x}}{(\lambda+\mu)^n} = \binom{n}{x}\frac{\lambda^x}{(\lambda+\mu)^x}\cdot\frac{\mu^{n-x}}{(\lambda+\mu)^{n-x}} = \\
&= \binom{n}{x}\left(\frac{\lambda}{\lambda+\mu}\right)^x\cdot\left(\frac{\mu}{\lambda+\mu}\right)^{n-x} \sim Binomial\left(n, \frac{\lambda}{\lambda+\mu}\right)
\end{aligned}
$$

$\square$