

health studies  
Fitbit

Grades class attendance

Longitudinal Data and Time Series (4/17/2017)

Examples: Climate (CO<sub>2</sub> level, temperature, precip)



Economic data (housing prices, stock/bond prices, interest rates)

**Time Series:** Measuring a single variable over points in time

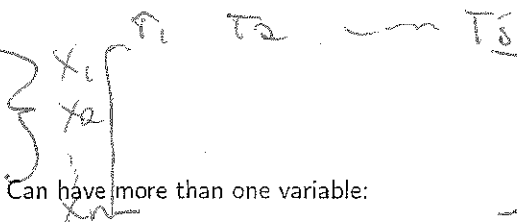
Can be regularly or irregularly spaced time intervals:

Data structure

one person/object,  
J time points

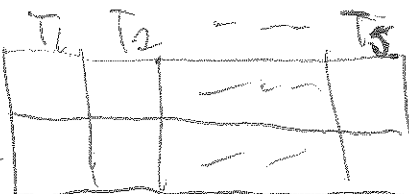


lots of people/objects,  
J time points



Can have more than one variable:

one person/object, v1  
two variables v2



Regularly spaced (h=fixed #)

$T_i, T_{i+h}, T_{i+2h}, T_{i+3h}$

Irregularly spaced

$T_1, T_2, T_3, \dots$

Several <sup>2/5</sup> people/objects

several  $n \times J$  matrices

Analyzing and Comparing Time Series

What are we interested in seeing with time series?

Trends

increasing?  
decreasing?  
no trend?

Cumulative measures  
(random increases or decreases)

Standard deviation  
Range of possible values  
Most common values

**SPIKES**  
(or valleys/dropps)

Periodicity: daily/weekly/yearly trends

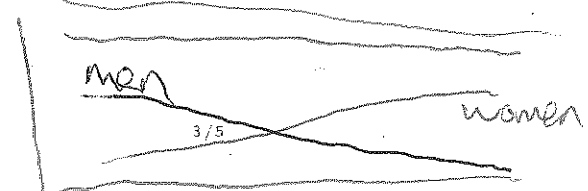
parallel ( $\uparrow\uparrow$  or  $\downarrow\downarrow$ )

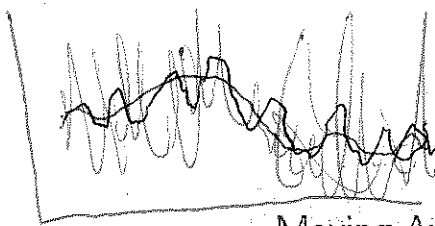
opposite ( $\uparrow\downarrow$  or  $\downarrow\uparrow$ )

What if we have multiple time series? How to compare?

\* Must have measurements at the same time points!!

Check for dependence on a sub group of the data





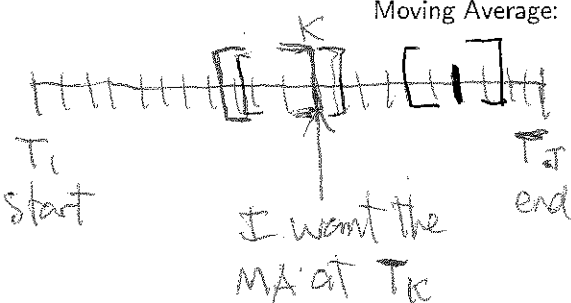
Small  $ww$   
large  $ww$   
original TS

Moving Average Plots

$WW = 1 \Rightarrow$  Not really a moving average — just gives back the original time series

parameter that we get to choose

Moving Average: Want to visualize how the trend changes over time.



$[ ] =$  "window"

$| | =$  "window width"

$$MA_K = \sum_{i=K-ww+1}^K T_i / ww$$

Big  $WW$ :

smoother moving averages, global trends in TS

Small  $WW$ : jagged, high-variation MAs, local trends in TS

Can downweight older observations in your moving window:

"weighted moving average"  $\rightarrow$  often useful in forecasting / prediction

\* How do we choose the weights?  $\rightarrow$  set of parameters that you get to choose

$\hookrightarrow$  cross-validation

$\hookrightarrow$  weights  $\propto 1 / [\text{"how far"}]$

Suppose  $ww = 3$ , you might choose something like weighting by "Importance" Importance =  $[5, 2, 1/2]$

Lags and Autocorrelation

$$\Rightarrow \text{weights} = \left[ \frac{5}{5+2+1/2}, \frac{2}{5+2+1/2}, \frac{1/2}{5+2+1/2} \right]$$

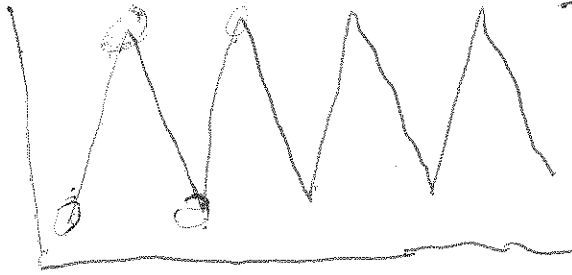
Lags: Does one time point influence future time point(s)?

Lag 1: looking at TS vs. itself one time step  $\rightarrow$  scaled so that weights sum to 1

Lag 2: two steps ago

Lag K: K steps ago

Autocorrelation: Correlation between a time series and a lagged version of itself



$\rightarrow$  high negative AC at lag 1

$\rightarrow$  high positive AC at lag 2

high values at time  $K-2$  correspond to high values at time  $K$

$$R(s, t) = E[(X_s - \mu_s)(X_t - \mu_t)]$$

$\hookrightarrow$  two time points

$T_s \quad T_t$

## Moving Average Plots

Moving Average: Want to visualize how the trend changes over time.

Can downweight older observations in your moving window:

4/5

## Lags and Autocorrelation

Lags: Does one time point influence future time point(s)?

**Autocorrelation:** Correlation between a time series and a lagged version of itself

