COMP2261 ARTIFICIAL INTELLIGENCE / MACHINE LEARNING

Cost Functions
For Regression Models
-- Huber-M

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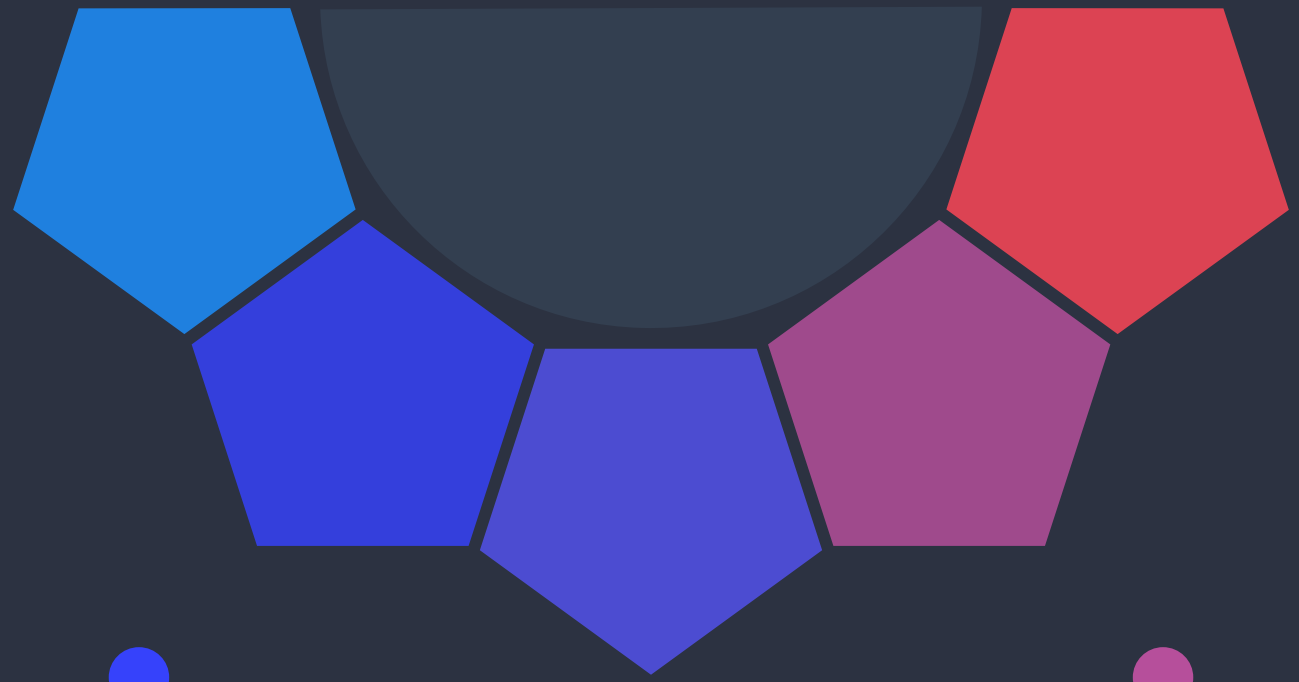
Learning Objectives

- Understand what is Huber-M
- Understand how Huber-M cost function works
- Understand the hyperparameter δ



MSE

Quantile

MAE

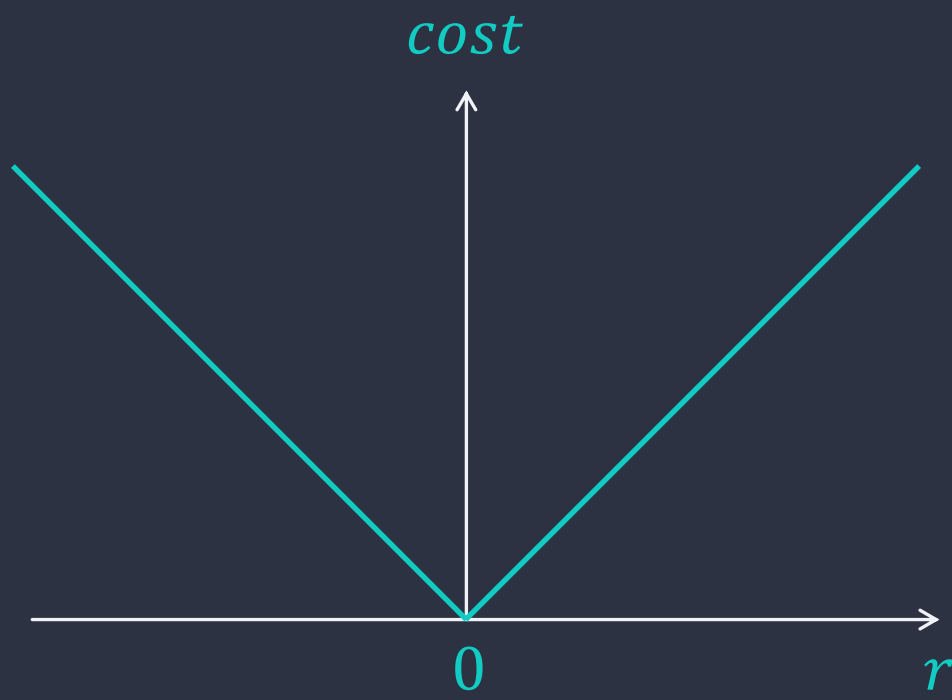
Log-Cosh

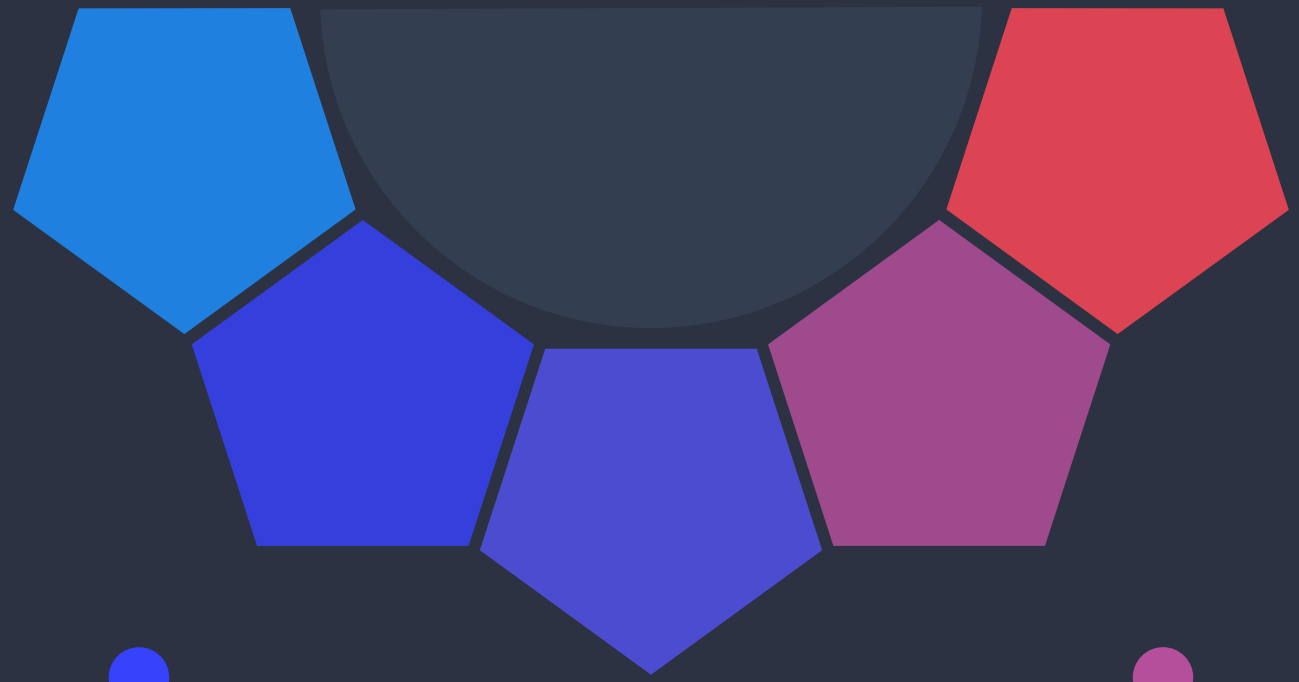
Huber-M

Mean Squared Error (MSE) vs Mean Absolute Error (MAE)

$$J = \frac{1}{m}\sum_{i=1}^{m}(y^{(i)} - \hat{y}^{(i)})^2$$

$$J = \frac{1}{m}\sum_{i=1}^{m}|y^{(i)} - \hat{y}^{(i)}|$$





MSE

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Huber-M

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$$J = \frac{1}{m}\sum_{i=1}^{m}\begin{cases}\frac{1}{2}(y^{(i)} - \hat{y}^{(i)})^2, & \left|y^{(i)} - \hat{y}^{(i)}\right| \le \delta \\ \delta\left(\left|y^{(i)} - \hat{y}^{(i)}\right| - \frac{1}{2}\delta\right), & otherwise\end{cases}$$

Huber-M

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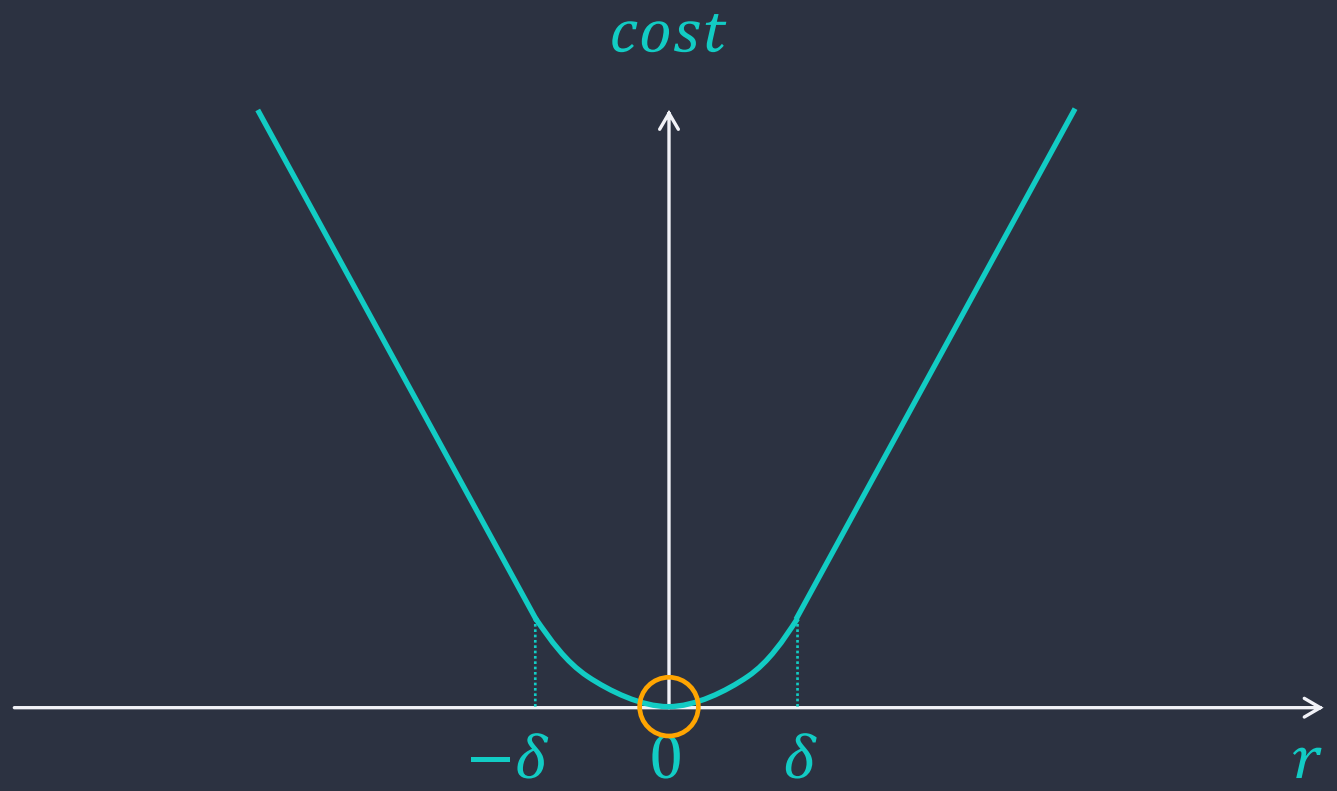

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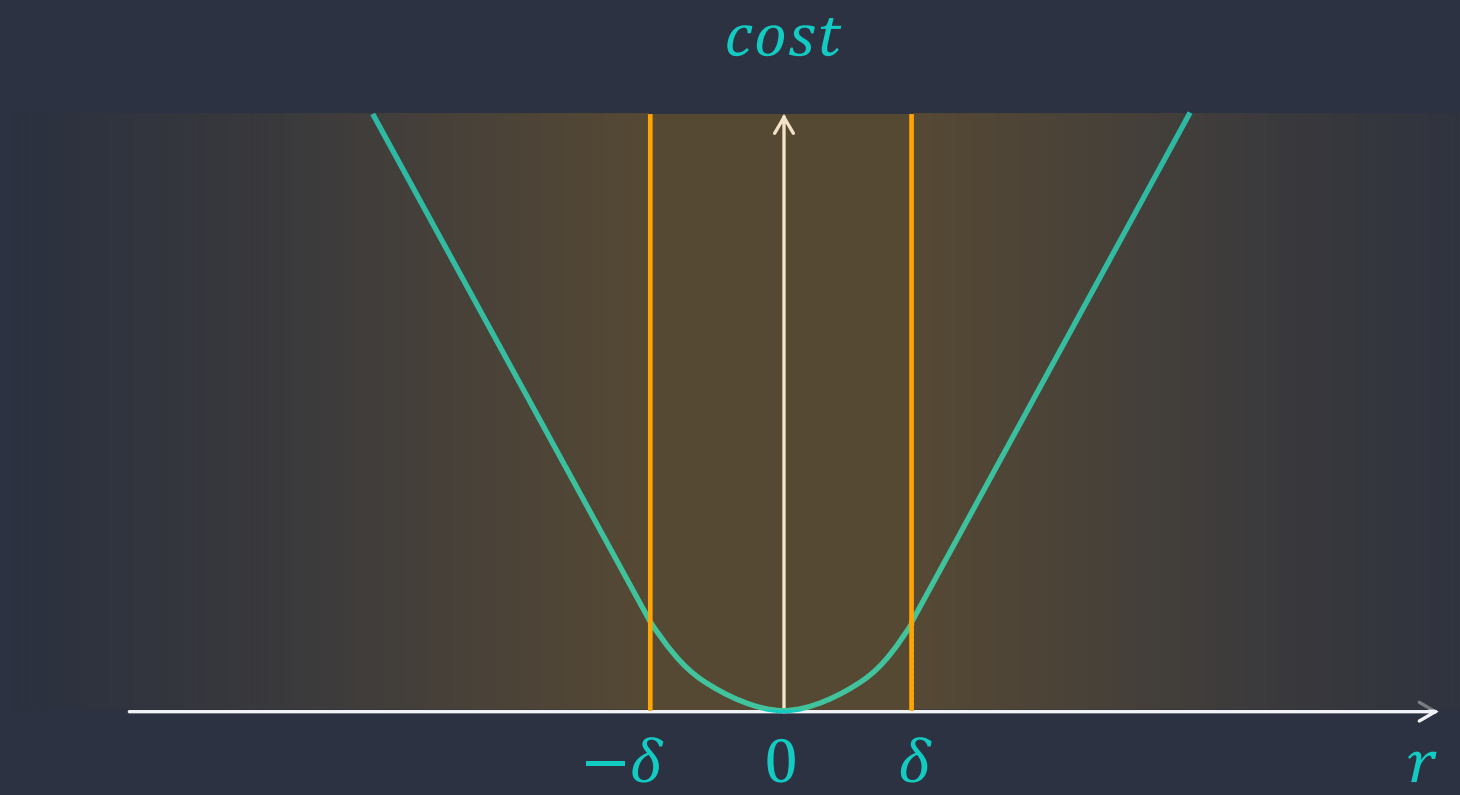

Huber-M

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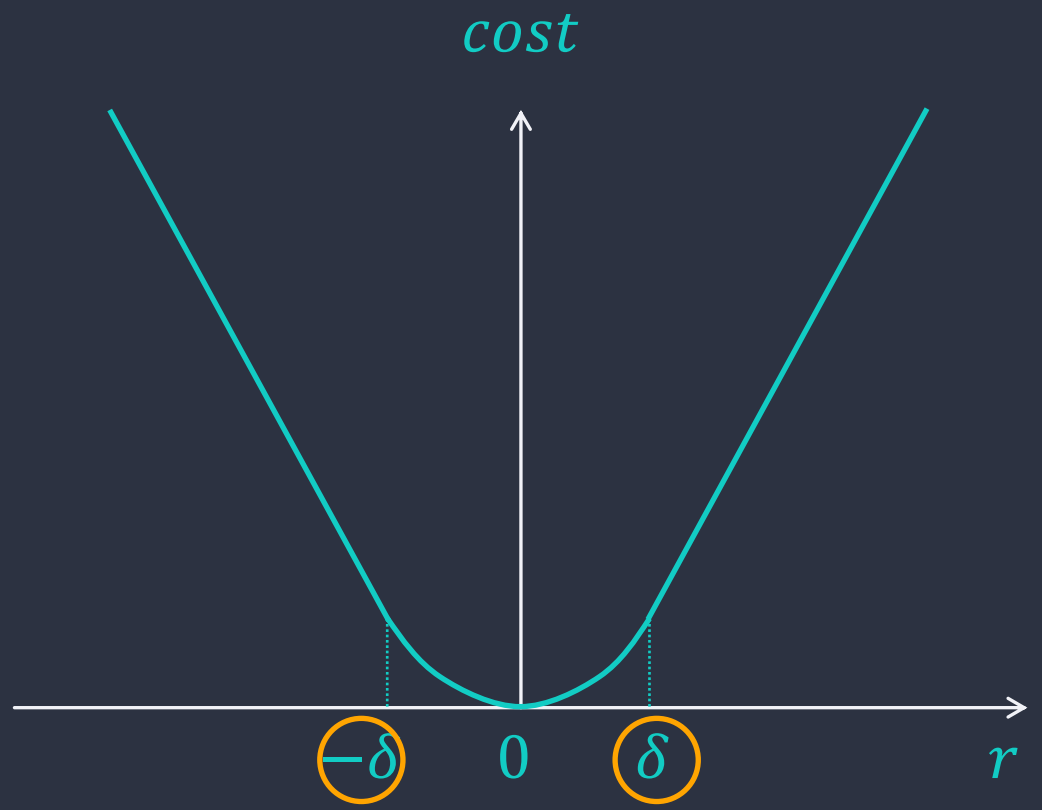

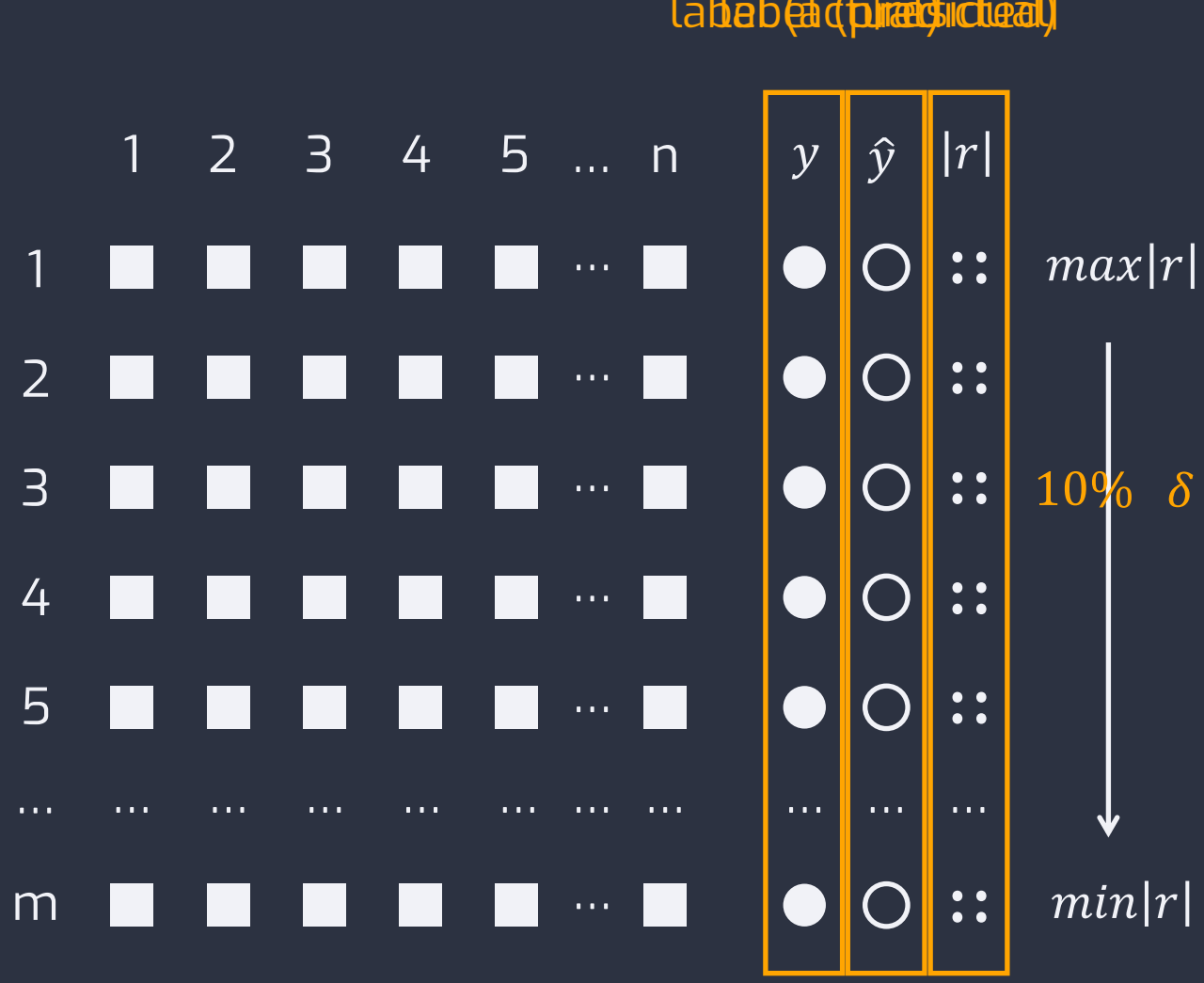

Huber-M

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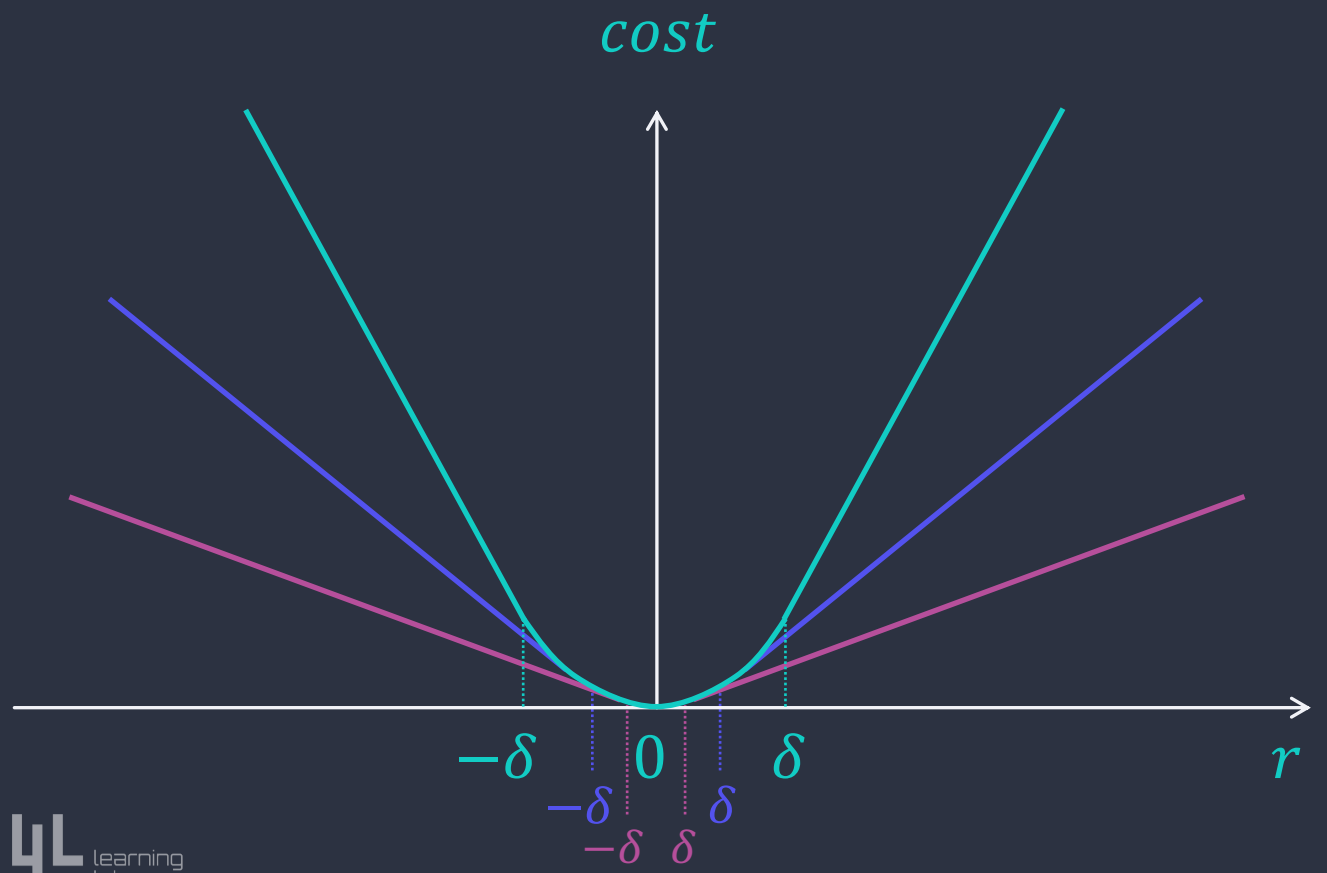

Choice of δ is very important, as it determines what are considered as outliers and how they are delt with.

Takeaway Points

- Huber-M combines good properties from MSE and MAE.
- Curving around minima – reducing gradient during training.
- More robust with outliers.
- Important to choose hyperparameter δ as it determines what are considered as outliers and how to deal with them.