Machine Learning

Lecture 2 – Linear Regression, Training & Loss

Dr SHI Lei

Last Lecture

- Definition
- Lifecycle
- Types of ML Systems
- Key Terminologies

Last Lecture

Definition

"A computer program is said to learn from **experience E** with respect to some class of **tasks T** and **performance measure P**, if its performance at tasks in T, as measured by P, improves with experience E."

- Tom Mitchell, 1997

It could be used as a design tool to help us think clearly about what data to collect (E), what decisions the software needs to make (T) and how we will evaluate its results (P).

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7 steps in Machine Learning Lifecycle

1. Gathering data
2. Preparing data
3. Choosing model
4. Training
5. Evaluation
6. Hyper-parameter tuning
7. prediction

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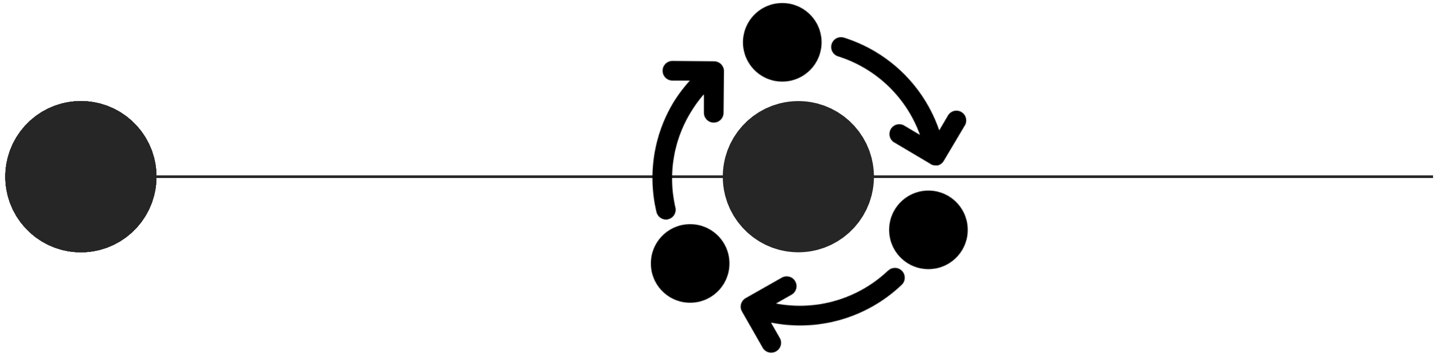
Training / Learning



Using data to answer questions

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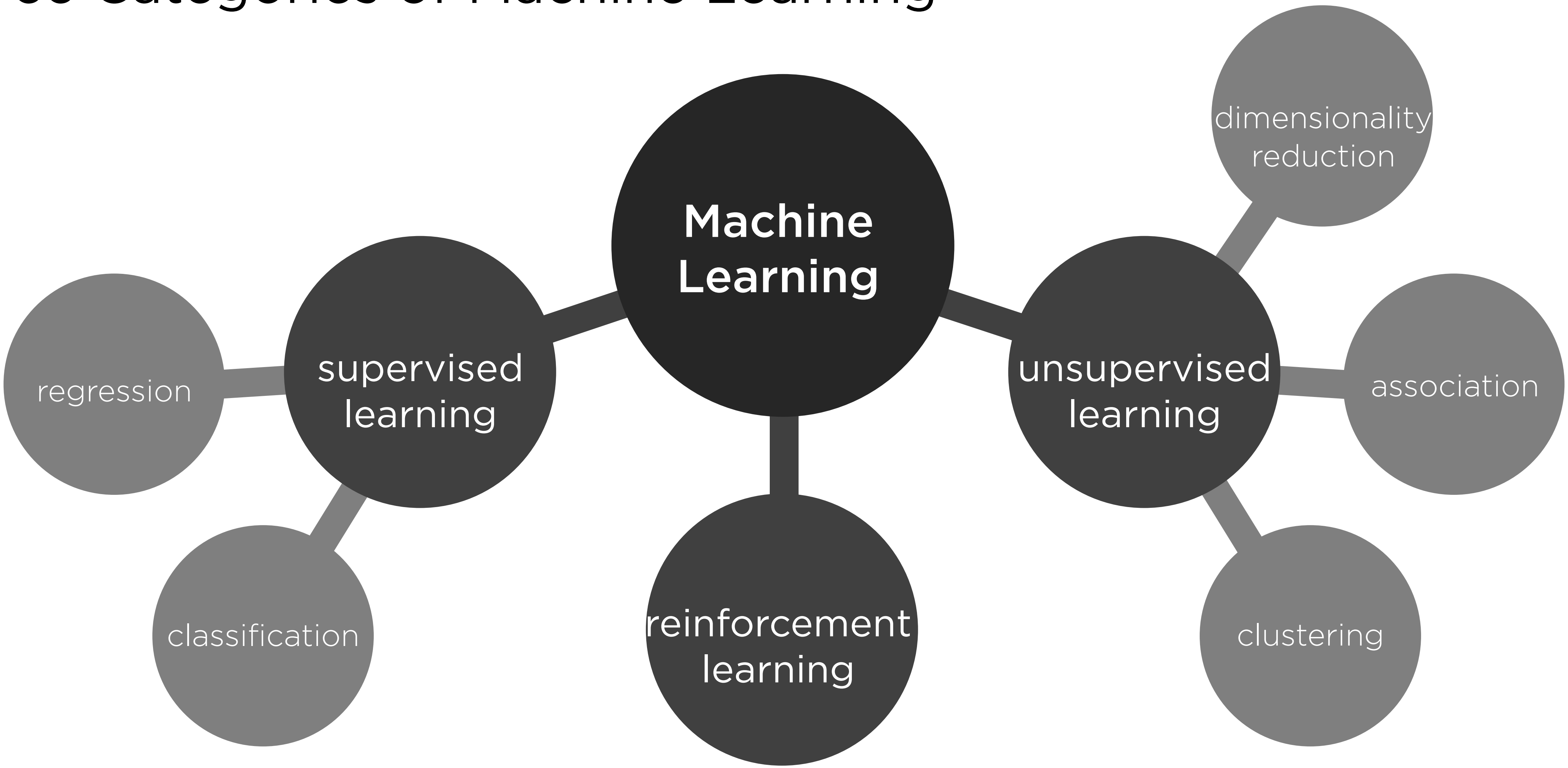
Three Categories of Machine Learning



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Key Machine Learning Terminology

- **Example**: a particular instance of data, $\mathbf{x}$
  - **Label**: the variable to predict ($\mathbf{y}$)
  - **Features**: input variables describing data ($\{\mathbf{x}_1, \mathbf{x}_2, \mathbf{x}_3, \ldots, \mathbf{x}_n\}$ or vector $\mathbf{X}$)
  - **Labelled example** has {features, label}: $(\mathbf{x}, \mathbf{y})$ to train the model
  - **Unlabelled example** has {feature, ?}: $(\mathbf{x}, ?)$ to predict on new data
- **Model** maps *examples* to predict labels: $\hat{y}$

  defined by internal parameters, which are learned
  - **Training** - creating or learning the model.
  - **Inference** - applying the trained model to unlabelled examples.

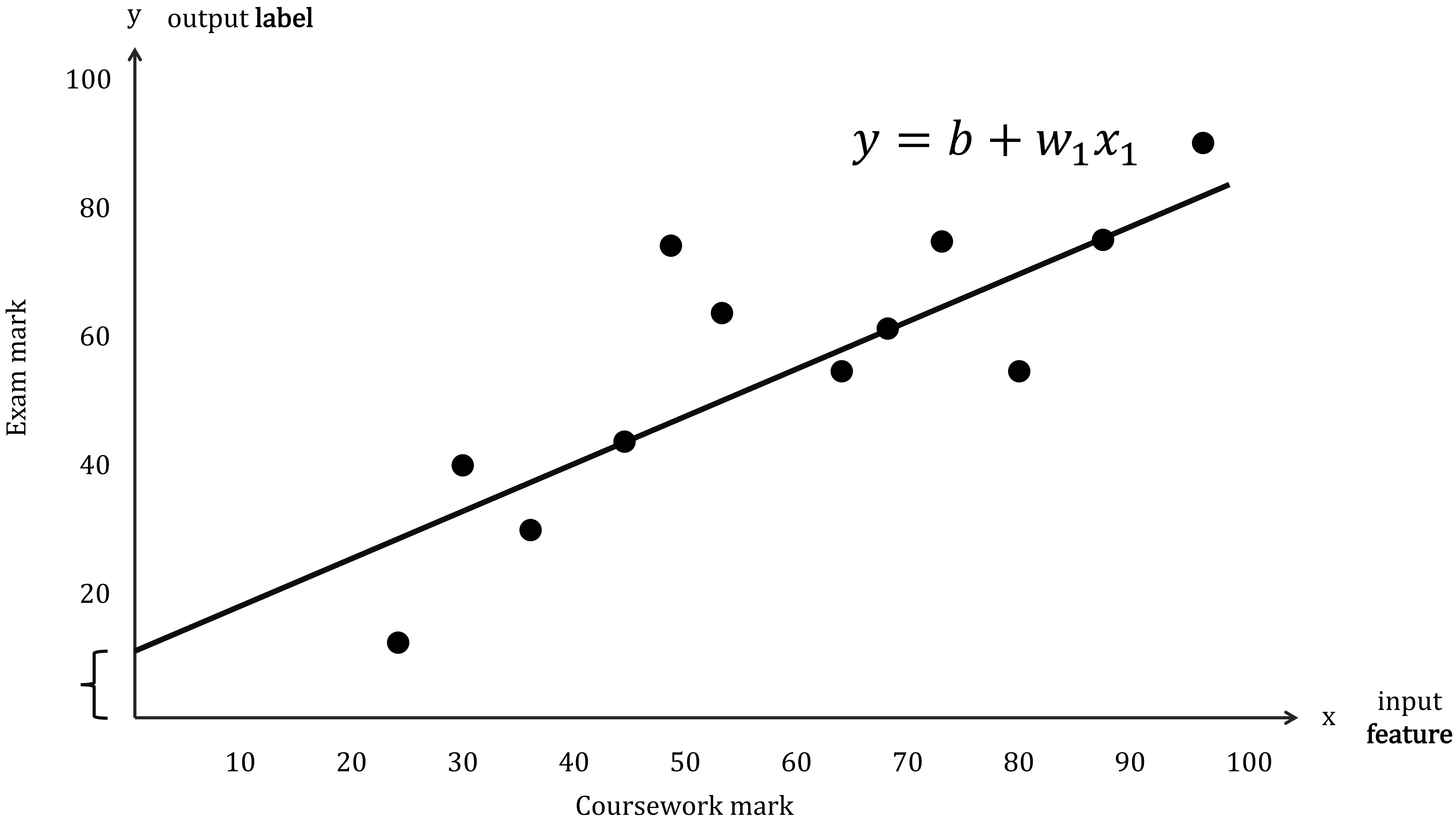
Today

- Linear Regression
- Training & Loss

Linear Regression

Linear Regression

A method for finding the straight line or hyperplane that best fits a set of points.



y
output **label**
100
80
60
40
20
Exam mark
$y = b + w_1 x_1$
x
input
feature
10
20
30
40
50
60
70
80
90
100
Coursework mark

$$y = b + w_1 x_1 \quad \Big| \quad \hat{y} = b + w_1 x_1$$

$\boldsymbol{\hat{y}}$ the predicted label (a desired output).

b the bias (the y-intercept), sometimes referred to as w_0.

$\mathbf{w_1}$ the weight of feature 1 (slope).

$\mathbf{x_1}$ a feature (a known input).

To infer (predict) the exam mark $\hat{y}$ for a new coursework mark value x_1, just substitute the x_1 value into this model.

$$\hat{y} = b + w_1 x_1$$

$$\hat{y} = b + w_1 x_1 + \mathrm{w}_2 \mathrm{x}_2 + \mathrm{w}_3 \mathrm{x}_3 + \ldots$$

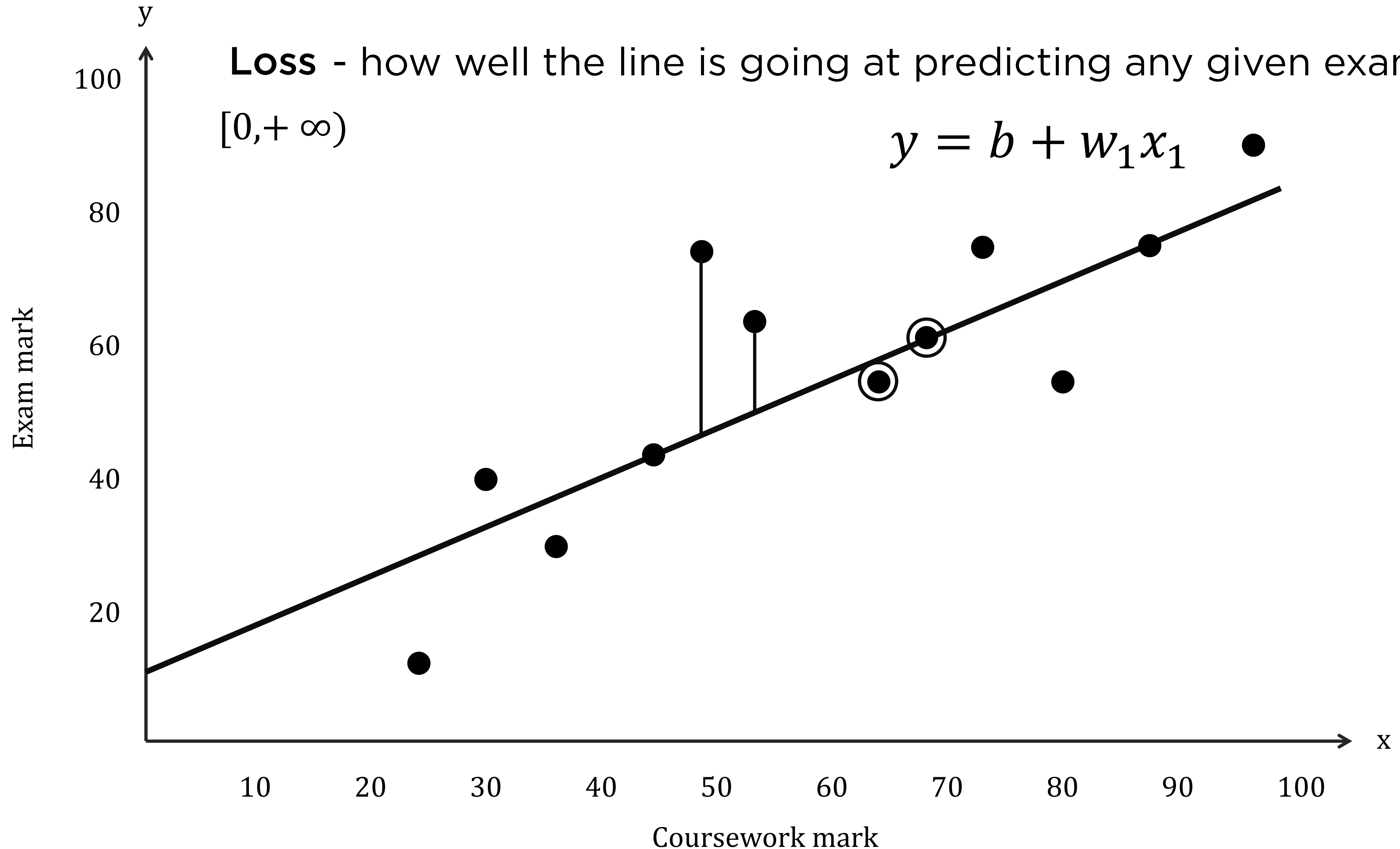
Is this a good line?

Loss

Loss - how well the line is going at predicting any given example.

$[0, +\infty)$

$$y = b + w_1 x_1$$



How do we define Loss?

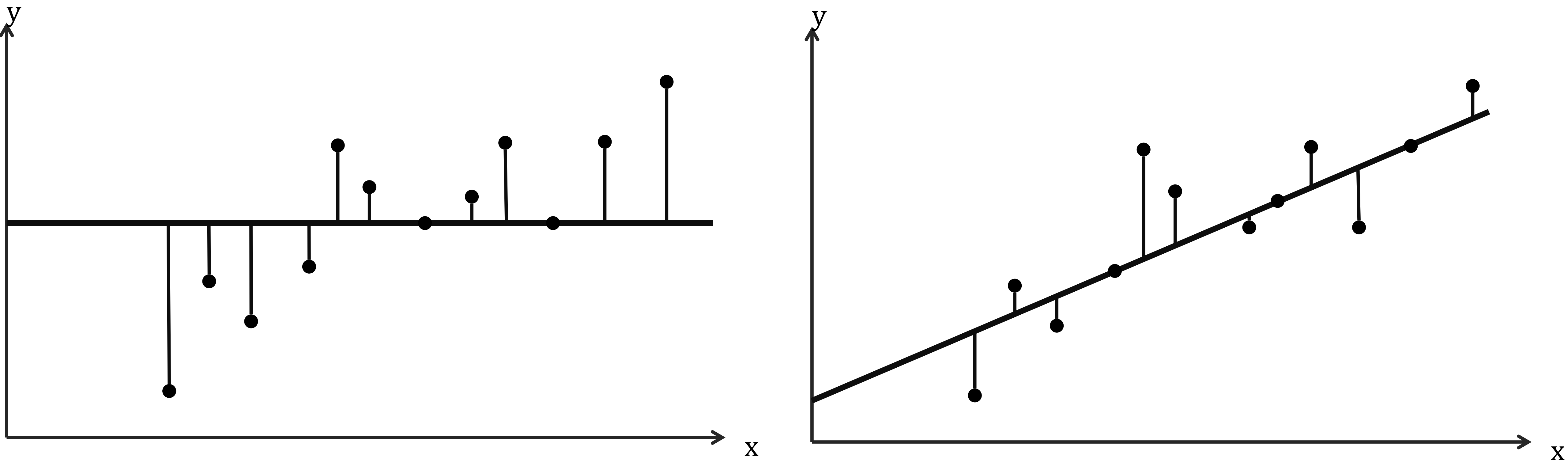
Training & Loss

Training a model: learning (determining) good values for all weights and the bias from labelled examples.

Loss: the penalty for a bad prediction.

Empirical Risk Minimisation: the process of examining many examples and attempting to find a model that minimise loss.

Goal of training: to find a set of weights and biases that have <u>low loss</u>, on average, across all examples.


y
x
y
x

Squared loss (L_2 Loss)

= the square of the difference between the label and the prediction

= $(\text{observation} - \text{prediction}(x))^2$

= $(y - \hat{y})^2$

Mean square error (MSE)

$$MSE = \frac{1}{N} \sum_{(x,y)\in D} (y - prediction(x))^2$$

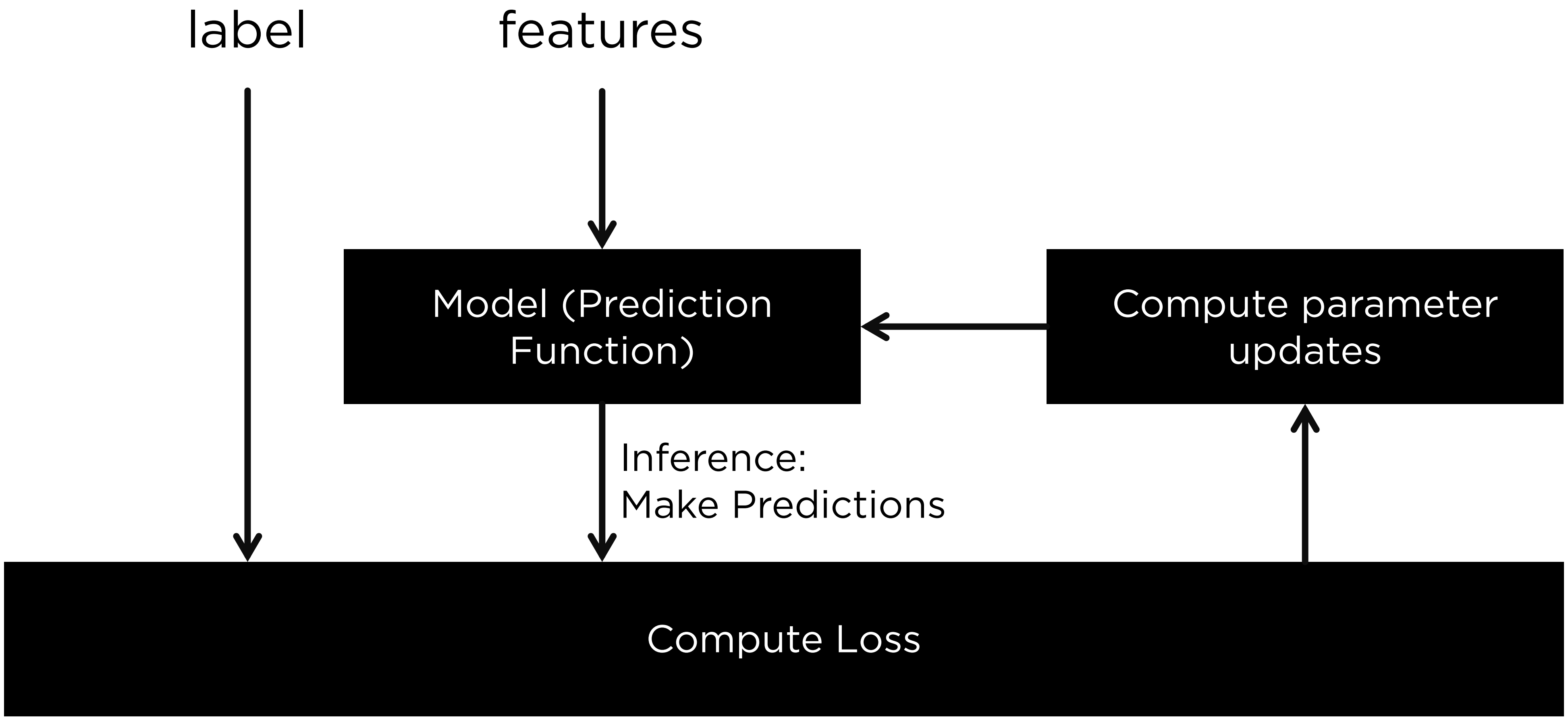
- (x, y) is an example where
  - x is the set of features used by the model to make predictions.
  - y is the example's label
- $\mathrm{prediction(x)}$ is a function of the weights & bias in combination with the set of features x.
- D is a dataset containing many labelled examples - (x, y) pairs.
- N is the number of examples in D.

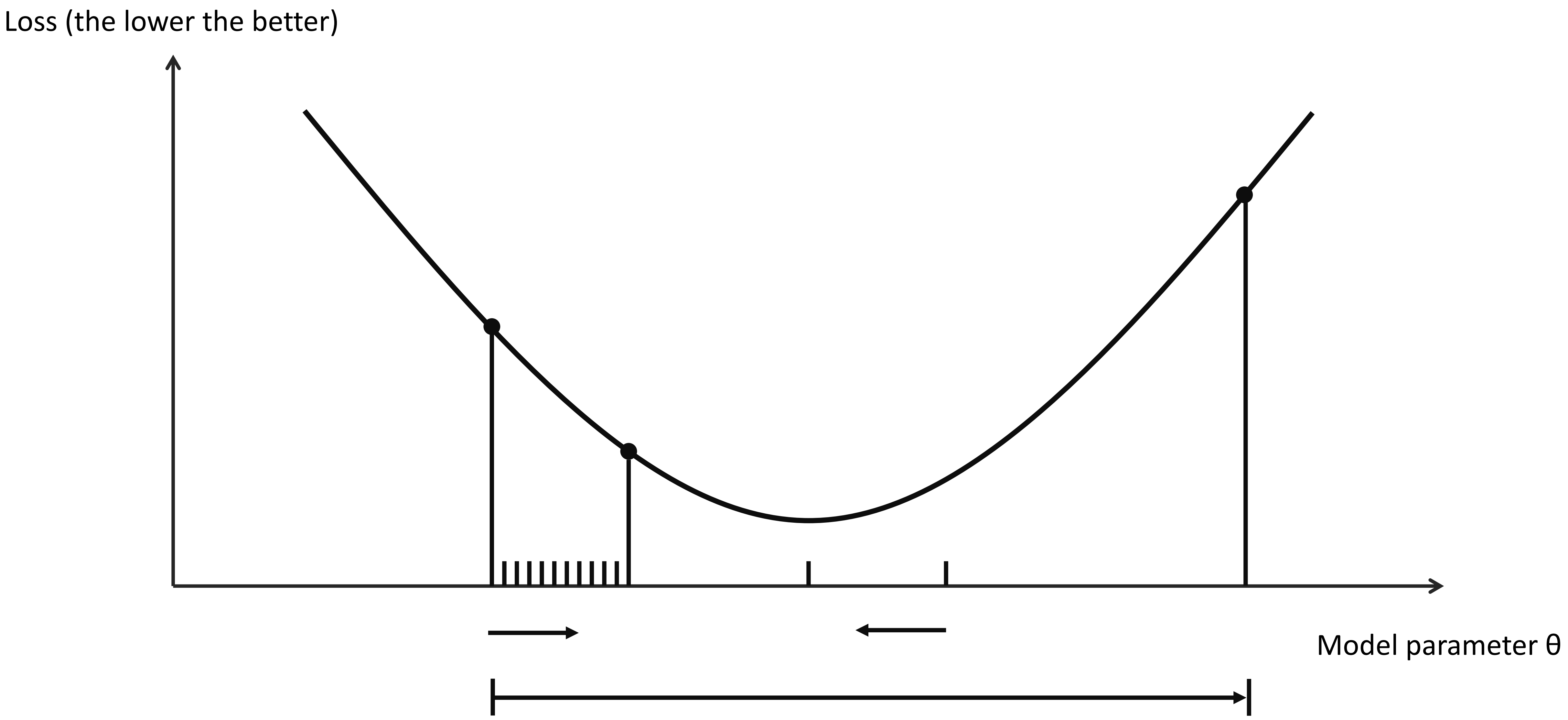
Reducing Loss

Reducing Loss

- Hyperparameters are the configuration settings used to tune how the model is trained.
- Derivative of $(\mathbf{y}\text{-}\,\hat{y})^2$ with respect to the weights and biases tells us how loss changes for a given example
  - Simple to compute and convex
- So we repeatedly take small steps in the direction that minimises loss
  - We call these Gradient Steps (but they are really negative Gradient Steps)
  - This strategy is called **Gradient Descent**

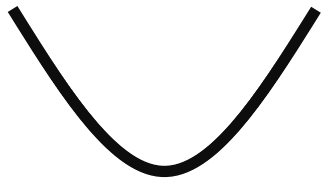
Block Diagram of Gradient Descent: an iterative approach



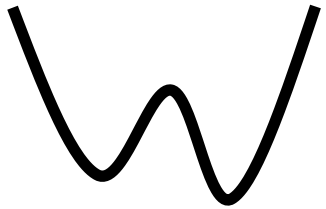

Loss (the lower the better)
Model parameter θ

Weight Initialisation

- For convex problem, weights can start anywhere (say, all 0s)
  - Convex: think of a bowl shape
  - Just one minimum



- Foreshadowing: not true for neural nets
  - Non-convex: think of an egg crate
  - More than one minimum
  - Strong dependency on initial values



Efficiency of Reducing Loss

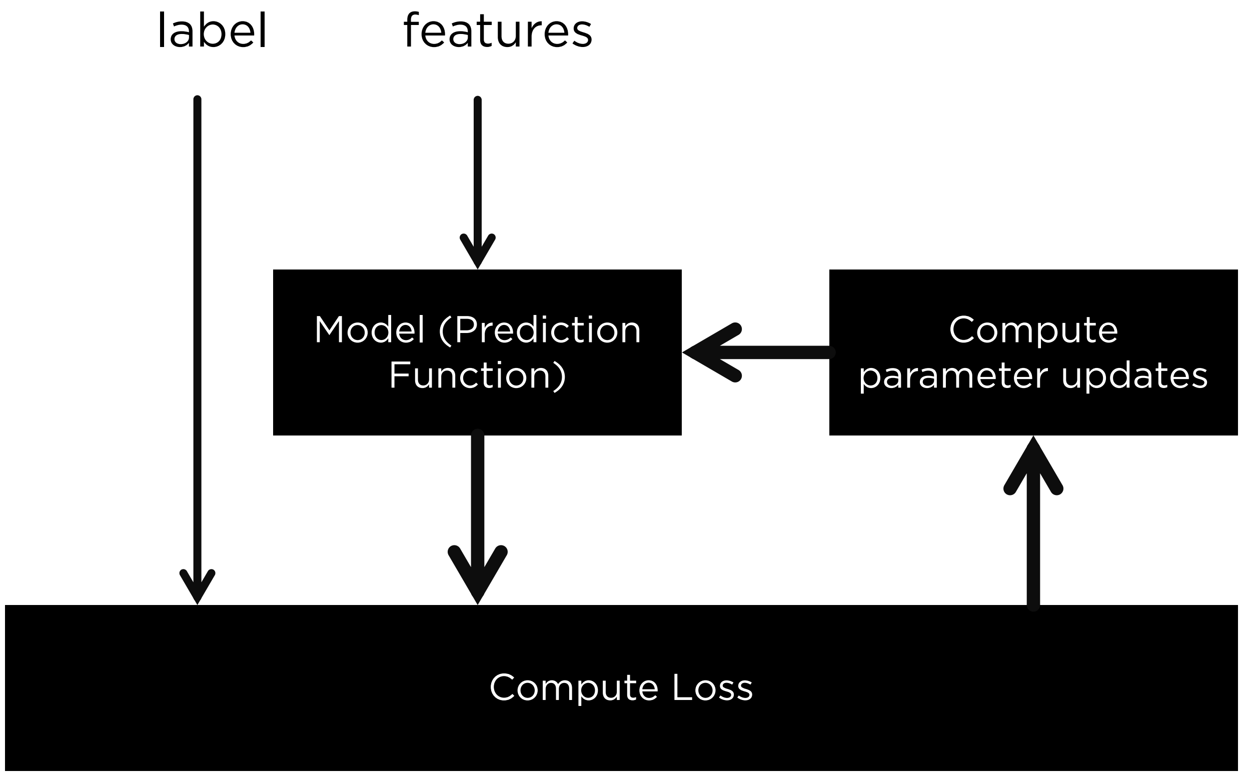
- Could compute gradient over entire dataset on each step, but this turns out to be unnecessary

- Computing gradient on small data examples works well
  - On every step, get a new random sample

- **Stochastic Gradient Descent**: one example at a time

- **Mini-Batch Gradient Descent**: batches of 10 – 1000
  - Loss & gradients are averaged over the batch

An iterative approach to train a model



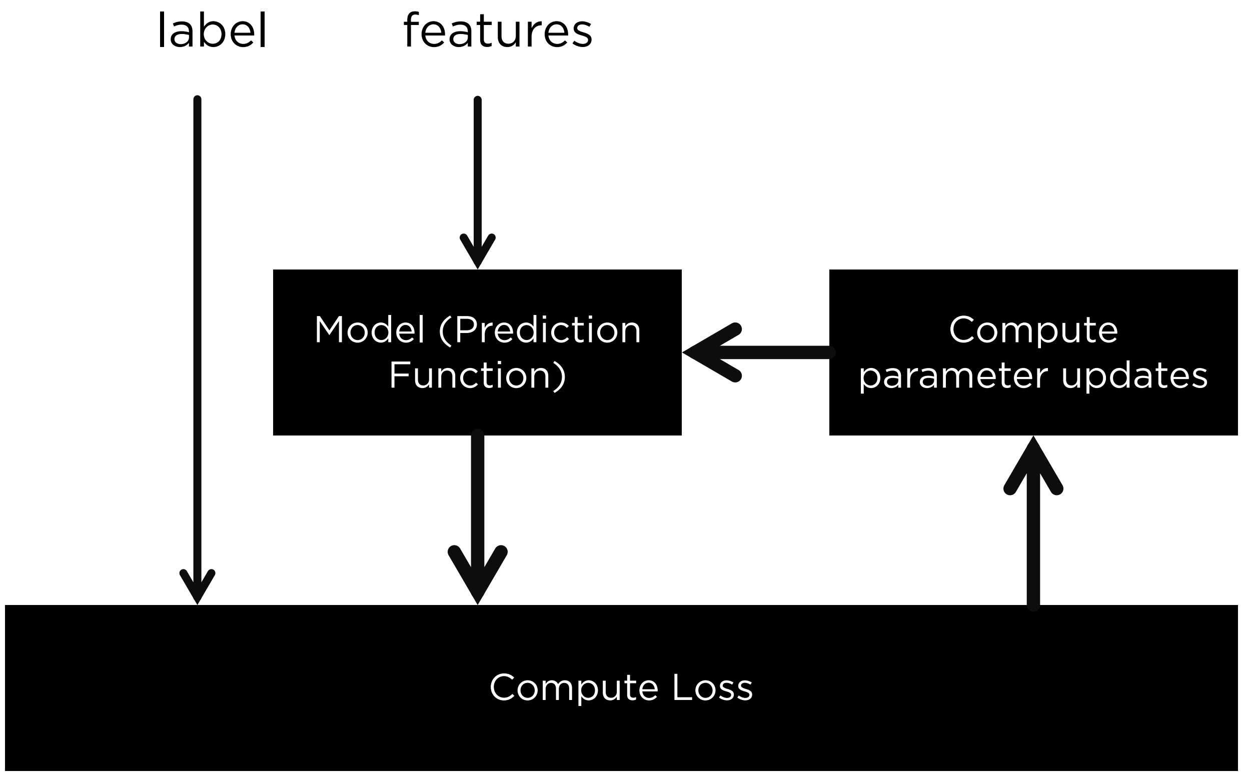
$$\hat{y} = \mathrm{b} + \mathrm{w}_1\mathrm{x}_1$$

What initial values should we set for b and w_1?

- Starting values aren't important.
- Could choose pick random values, but we'll just take the following trivial values instead:

$$\mathrm{b} = 0$$
$$\mathrm{w}_1 = 0$$

An iterative approach to train a model



$$\hat{y} = b + w_1 x_1$$

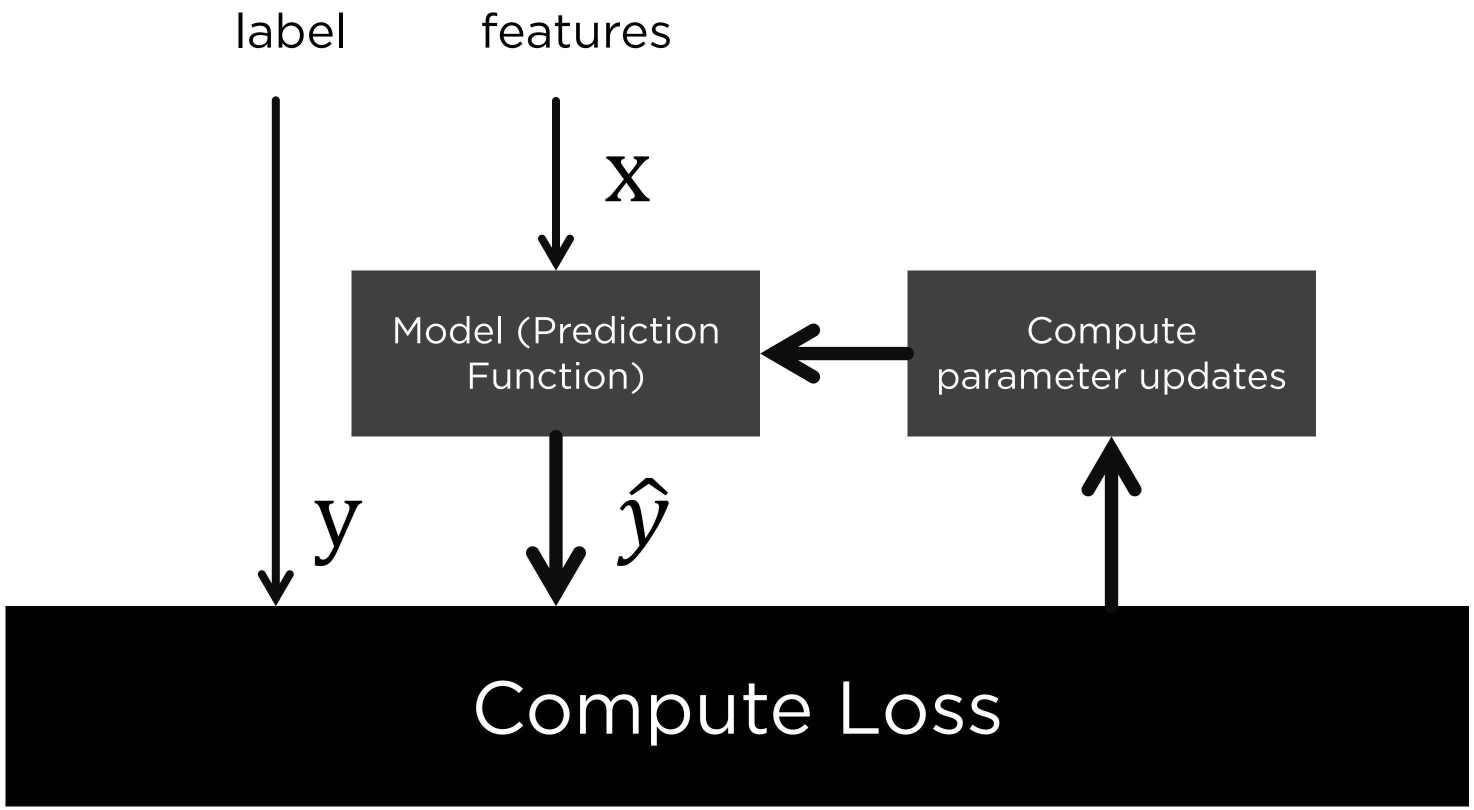
$$b = 0$$
$$w_1 = 0$$

$$\hat{y} = 0 + 0 \times x_1$$

$$x_1 = 10$$

$$\hat{y} = 0 + 0 \times 10$$
$$= 0$$

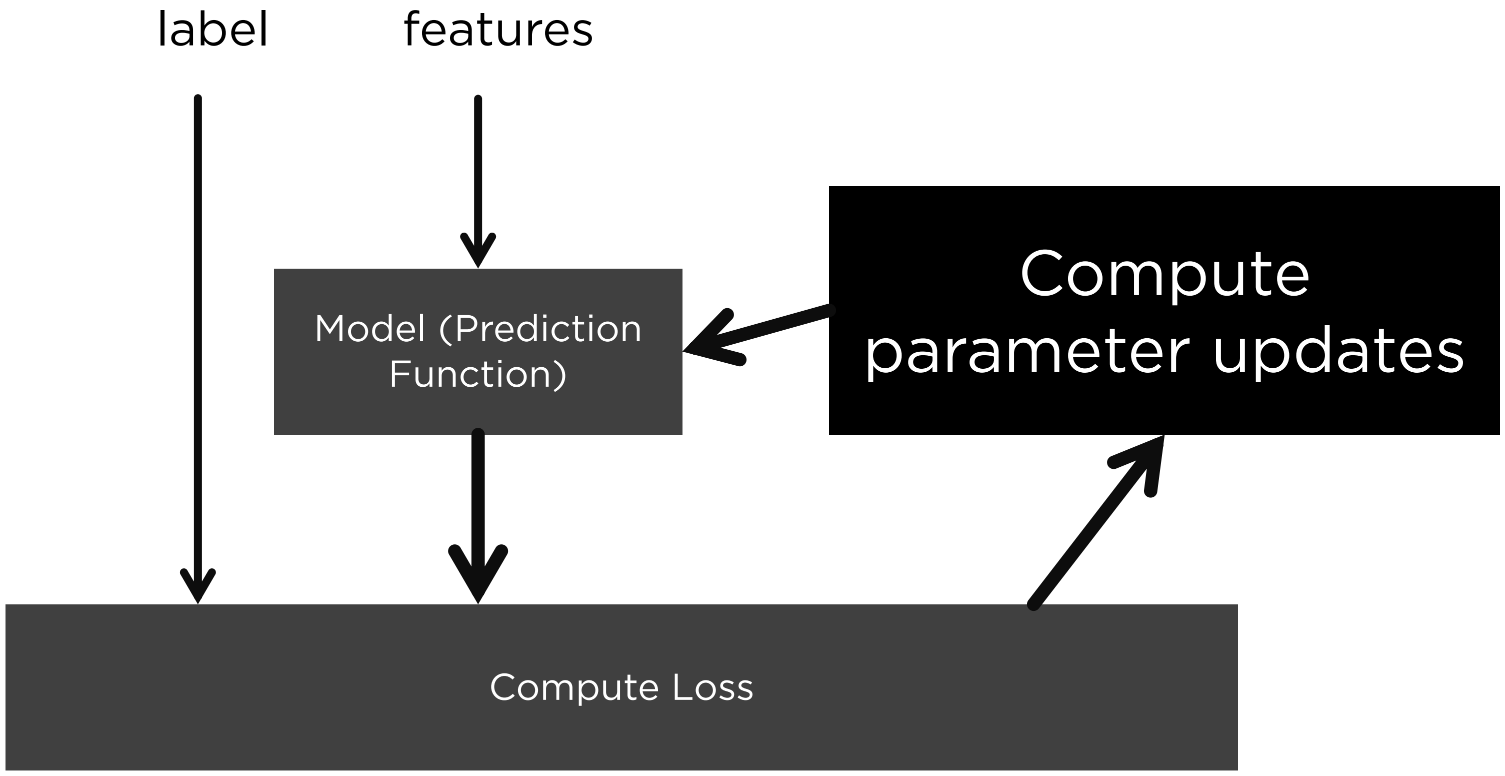
An iterative approach to train a model



Loss Function

- $\hat{\boldsymbol{y}}$ **:** the model's prediction for features x
- **y:** the correct label corresponding to features x.

An iterative approach to train a model



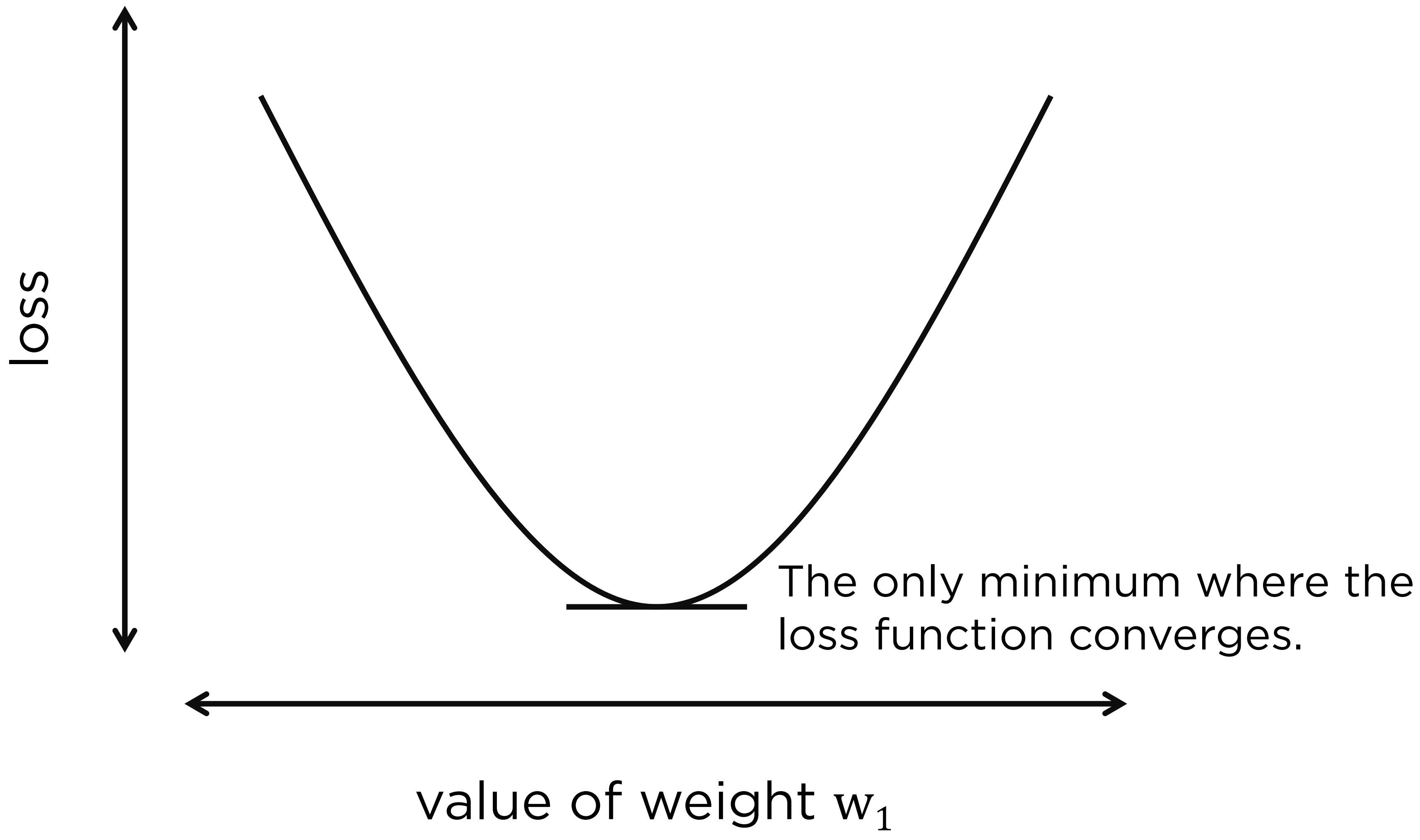
- Examines the value of loss function
- Generates new values for b and w_1

$$\hat{y} = \mathrm{b} + \mathrm{w}_1\mathrm{x}_1$$

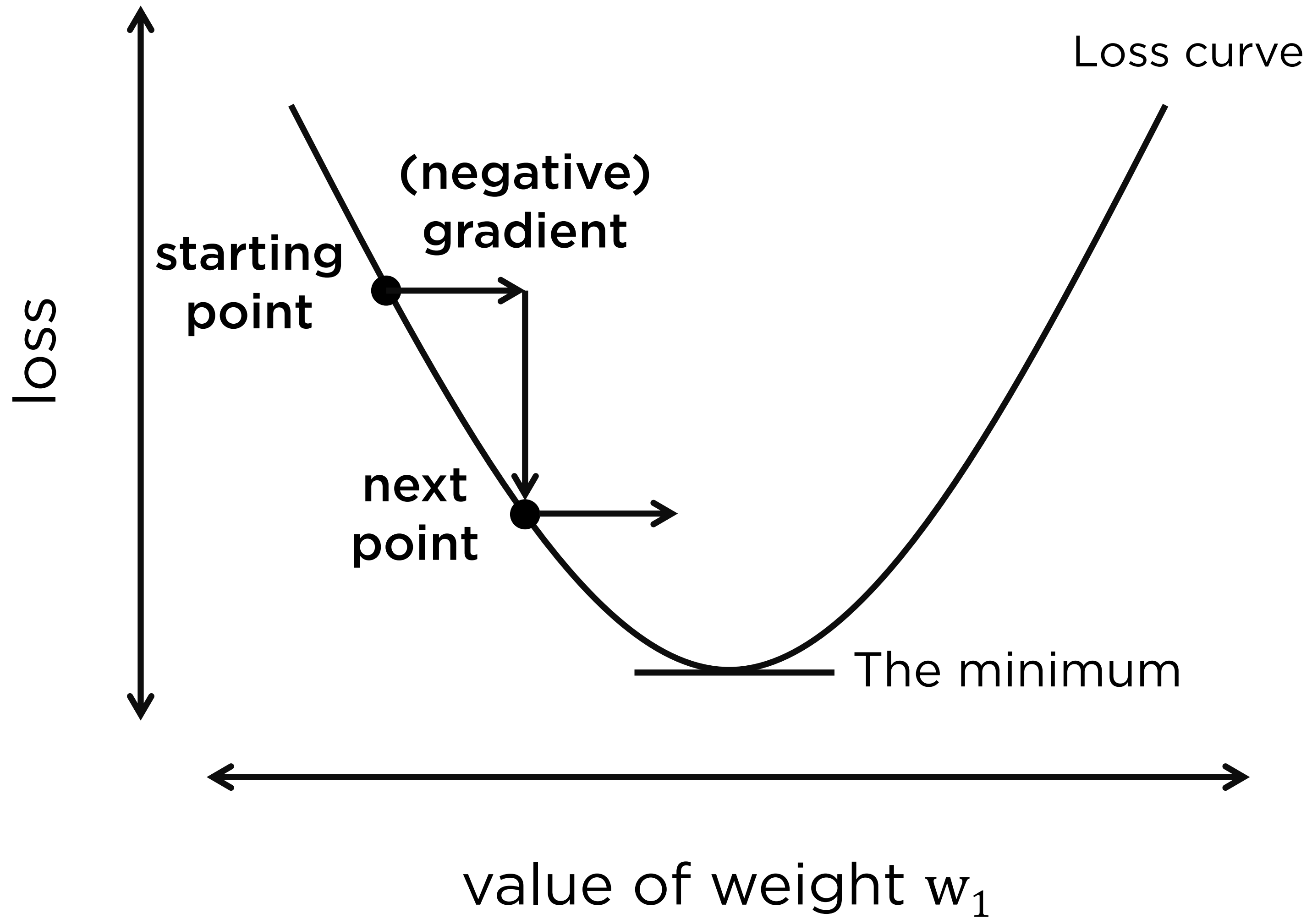
An iterative approach to train a model

Compute
parameter updates

Regression problems yield convex loss vs weight plots

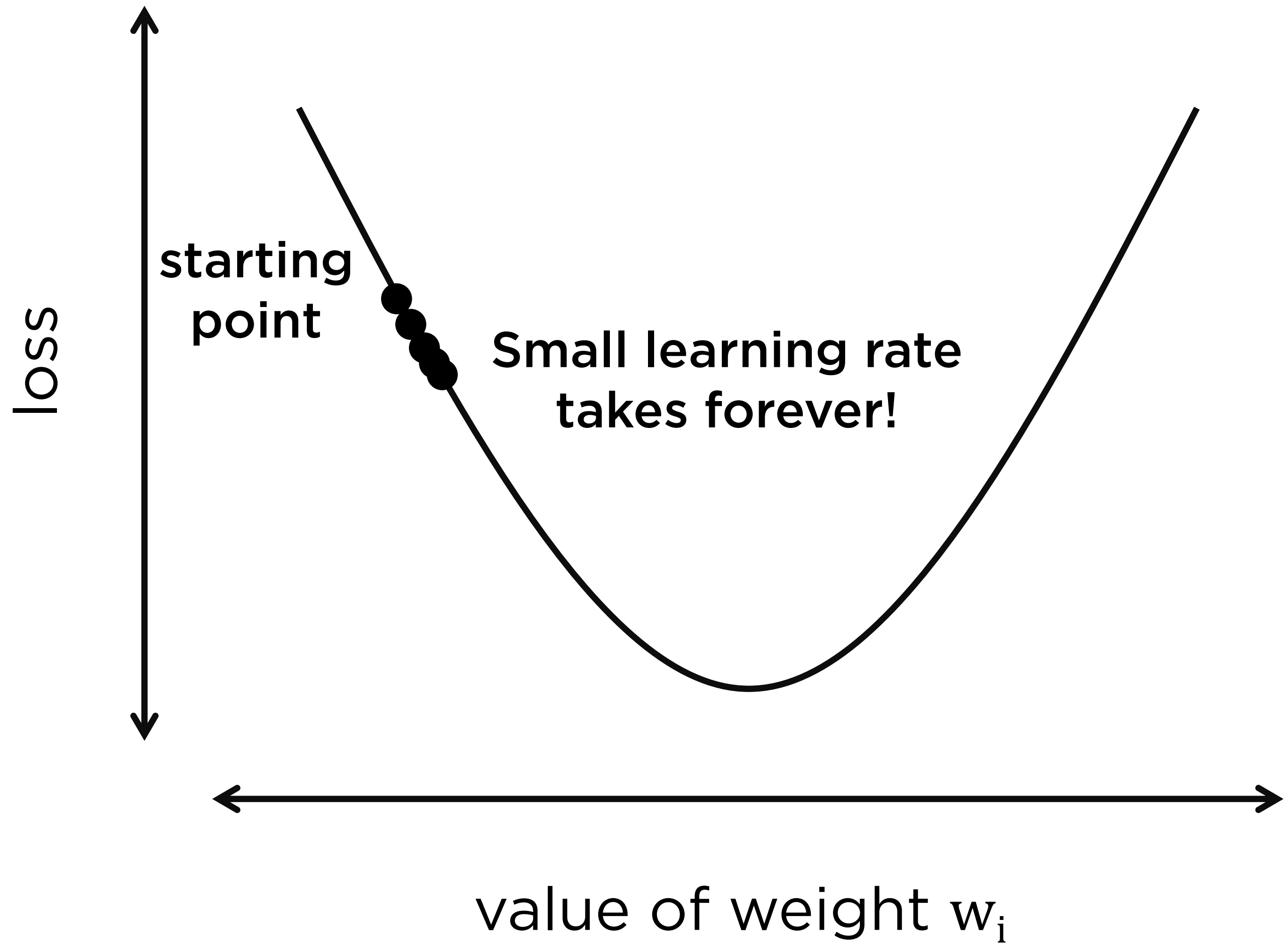


Gradient Descent



Learning Rate

too small

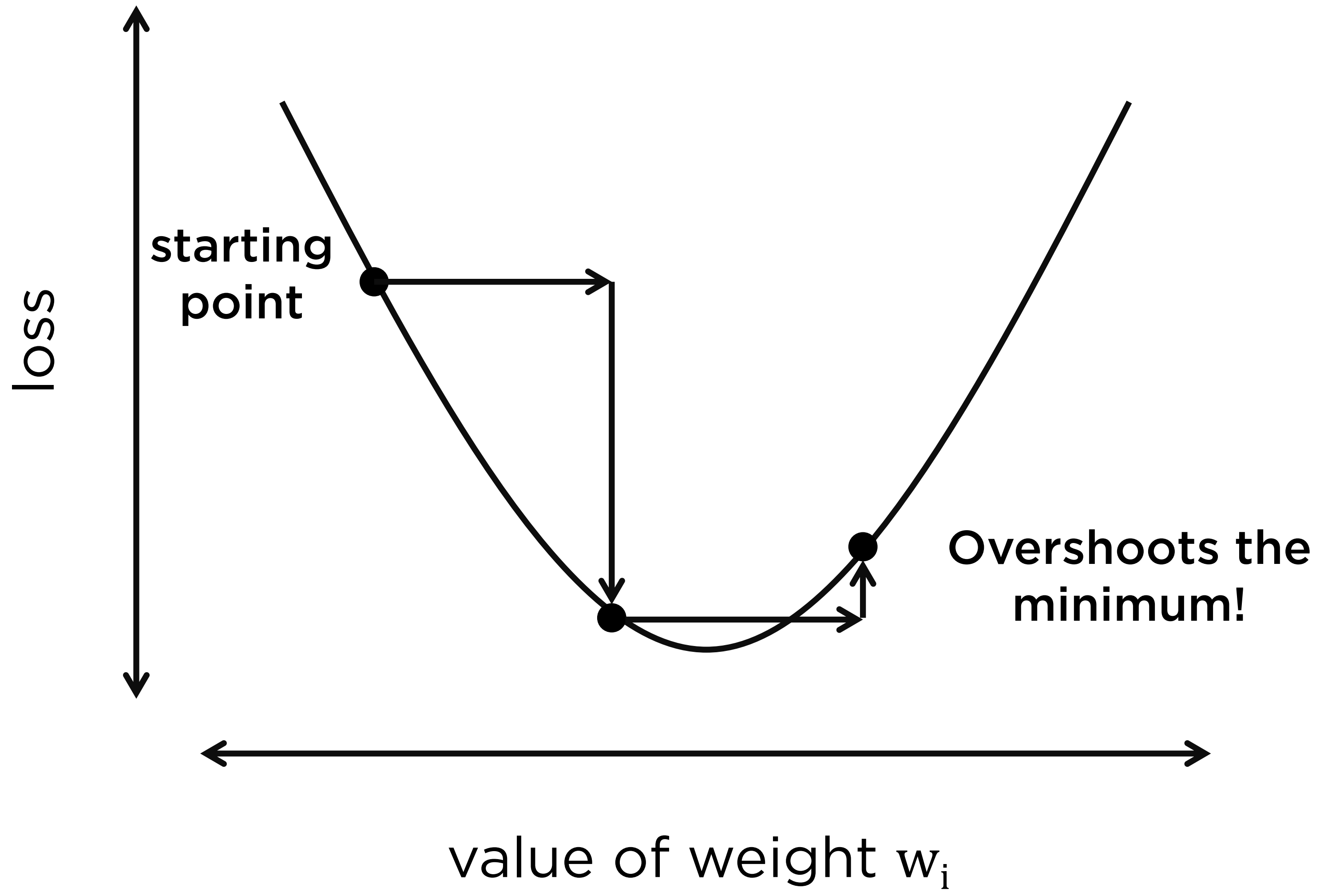


IF
Gradient magnitude = 2.5
Learning rate = 0.01

Then
The next point = 0.025

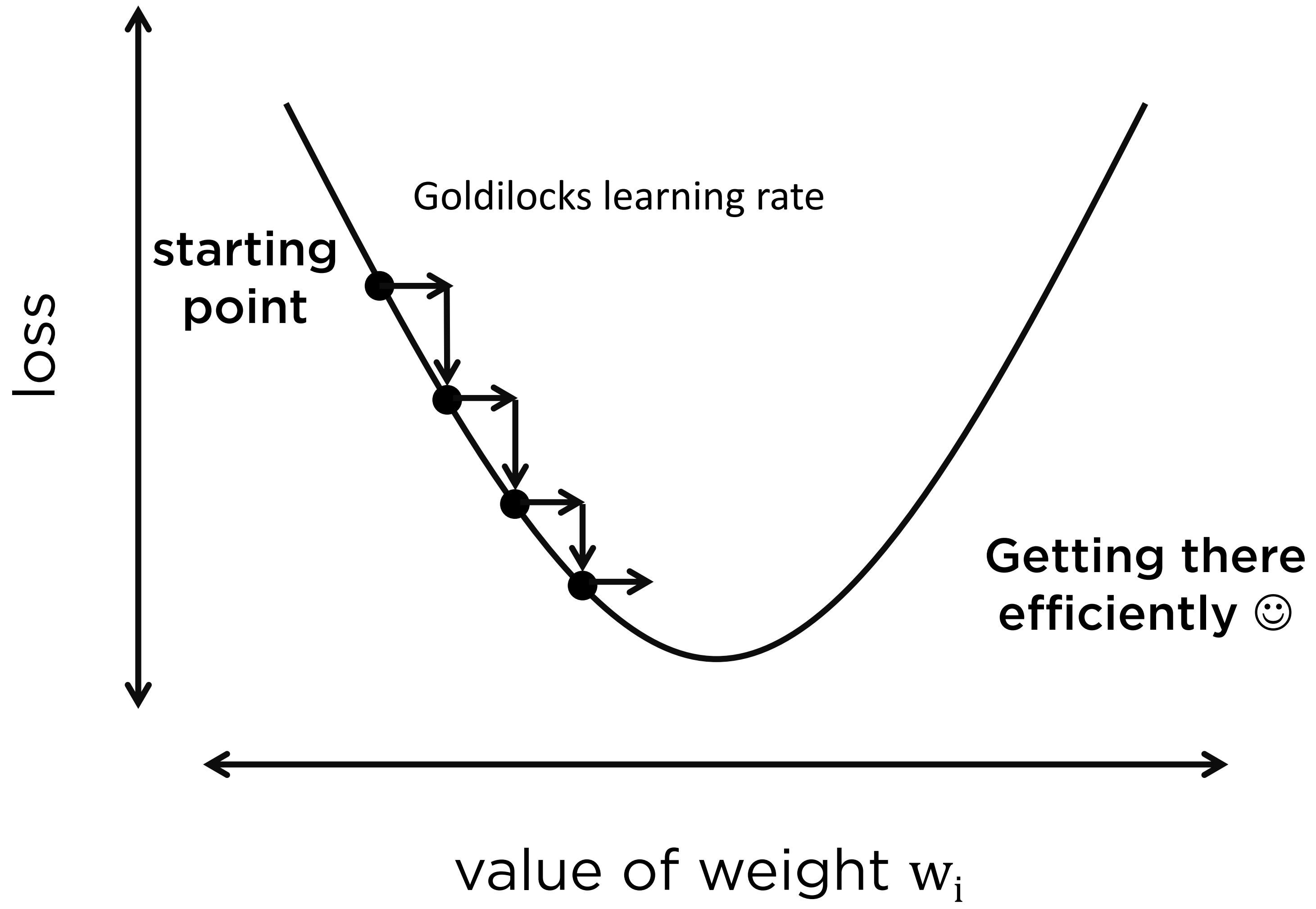
Learning Rate

too large



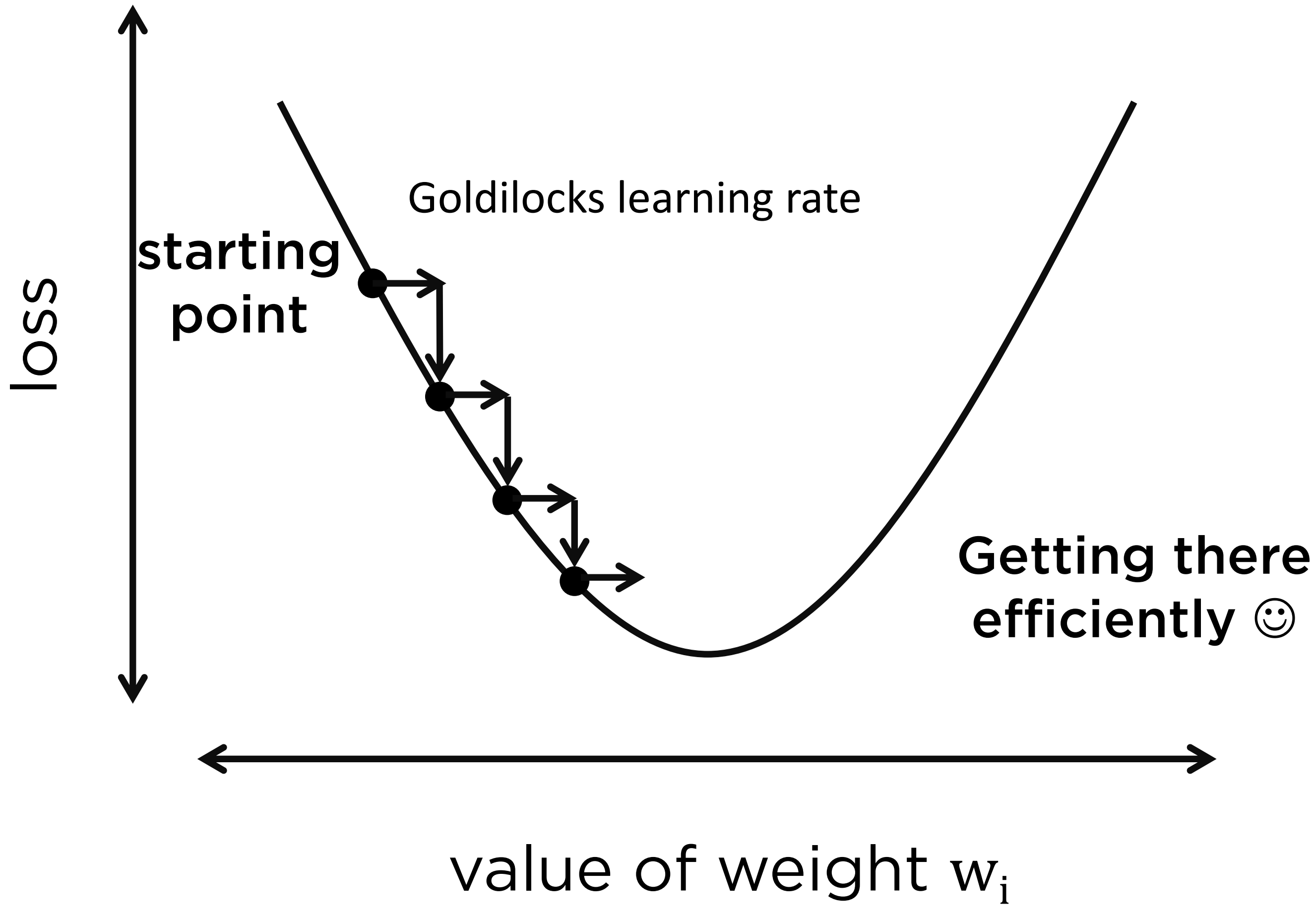
Learning Rate

Just about right (the **Goldilocks** learning rate)



Learning Rate

Just about right (the **Goldilocks** learning rate)



- The ideal learning rate in one-dimension is

$$\frac{1}{f(x)''}$$

(the inverse of the 2^{nd} derivative of f(x) at x).

- The ideal learning rate for 2 or more dimensions is the inverse of the **Hessian** (matrix of second partial derivatives).

Summary

Today

- Linear Regression
- Training & Loss
  - Training (an iterative approach)
  - Reducing Loss
  - Gradient Descent
  - Learning Rate

Homework

- Optimising learning rate (**Goldilocks** and **Hessian**)?
- Chapter 4 – sections 1 & 2 reading & programming exercises

Next Lecture

- Generalisation, Training & Test Set, Representation