COMP2261 ARTIFICIAL INTELLIGENCE / MACHINE LEARNING

Multivariate Linear Regression

-- Multiple Variables

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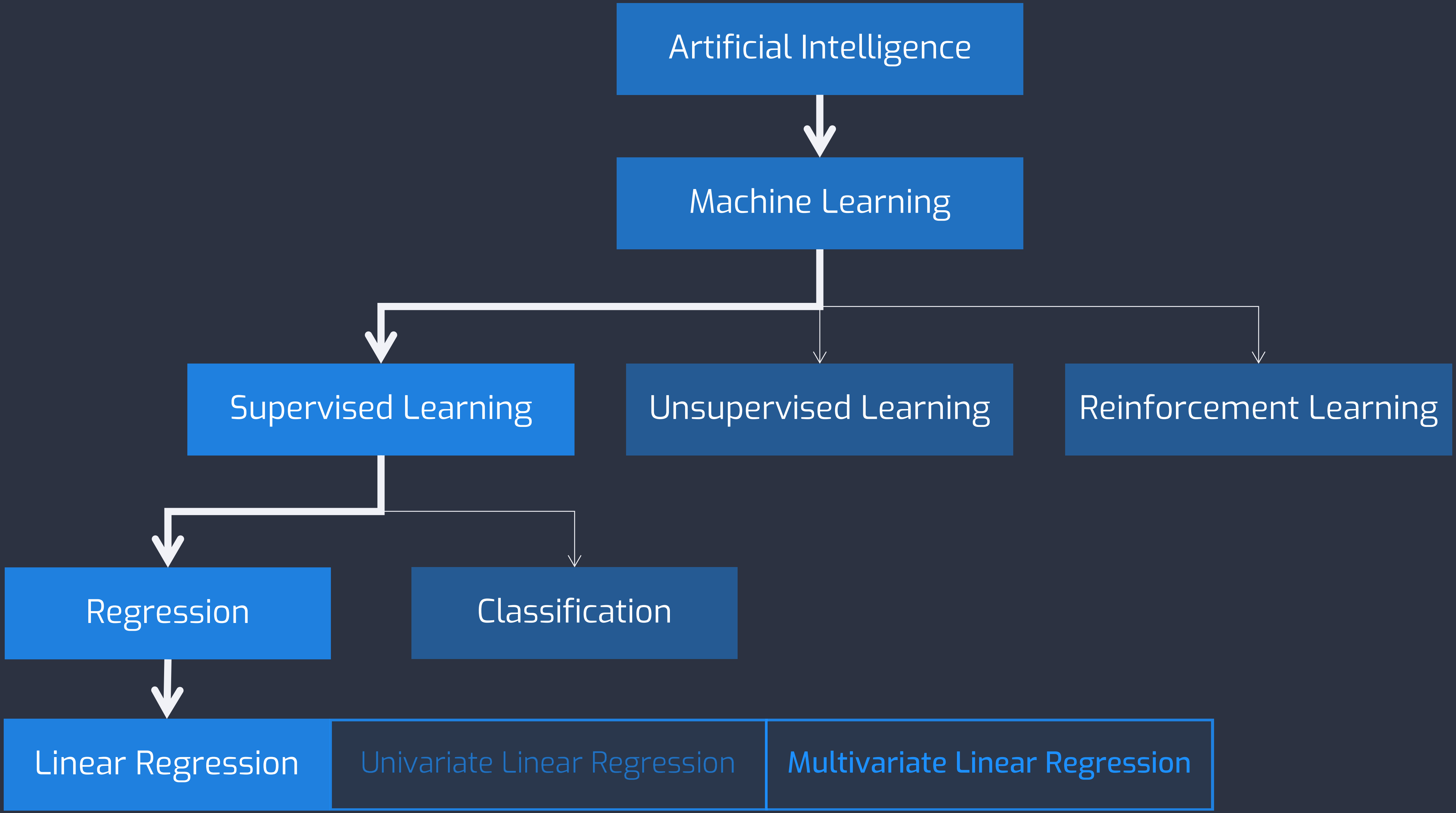
learning lab

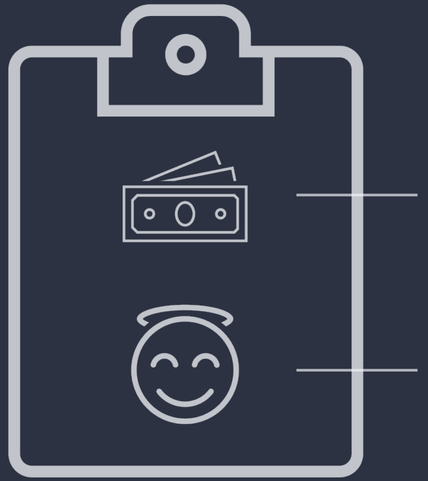

Artificial Intelligence
Machine Learning
Supervised Learning
Unsupervised Learning
Reinforcement Learning
Regression
Classification
Linear Regression
Univariate Linear Regression
Multivariate Linear Regression

Multivariate Linear Regression

- Multiple Variables -

EXAMPLE. annual income to predict happiness

500 people in Durham



survey

annual income — independent variable

£15k -- £75k

happiness — dependent variable

0 -- 10

A supervised learning task

-- we know the “right answer”, the correct label i.e. given annual income, we know exact happiness.

A regression task

-- we want to predict a real-valued output i.e. happiness in a continuous scale from 0 to 10.

A univariate regression task

-- there is a single independent variable, annual income.

EXAMPLE. annual income to predict happiness

	income (k)	happiness
1	38.63	2.314
2	49.79	3.433
3	40.34	4.03
4	21.18	1.45
...	...	...

$$\boxed{h_\theta(x)} = \theta_0 + \theta_1 \boxed{x}$$

EXAMPLE. annual income & age, number of children, cups of tea/week to predict happiness

	income (k)	age	# of children	tea (cups/week)	happiness
1	38.63	46	1	7	2.314
2	49.79	37	0	15	3.433
3	40.34	52	3	20	4.03
4	21.18	25	0	3	1.45
...	...	...	...	...	...

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...	...	...	...	...	...

m number of training instances $m = 350$

n number of independent variables / features $n = 4$

$y^{(i)}$ the label of the i^{th} training instance $y^{(3)} = 4.03$

$x_j^{(i)}$ the j^{th} feature of the i^{th} training instance $x_4^{(2)} = 15$

$\boldsymbol{x}^{(i)}$ feature vector $\boldsymbol{x}^{(1)} = \begin{bmatrix} 38.63 \\ 46 \\ 1 \\ 7 \end{bmatrix}$

$\boldsymbol{x}^{(i)} \in \mathbb{R}^4$

EXAMPLE. annual income & age, number of children, cups of tea/week to predict happiness

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...	...	...	...	...	...

Hypothesis Function $h_{\boldsymbol{\theta}}(\boldsymbol{x}) = \theta_0 + \theta_1 \cdot x_1 + \theta_2 \cdot x_2 + \theta_3 \cdot x_3 + \theta_4 \cdot x_4$

$$h_{\boldsymbol{\theta}}(\boldsymbol{x}) = [\theta_0, \theta_1, \theta_2, \theta_3, \theta_4] \begin{bmatrix} \boxed{1} \\ x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = [\theta_0, \theta_1, \theta_2, \theta_3, \theta_4] \begin{bmatrix} \boxed{x_0} \\ x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}$$

$(\boxed{x_0 = 1})$

constant

Hypothesis Function

$$h_{\boldsymbol{\theta}}(\boldsymbol{x}) = \theta_0 + \theta_1 \cdot x_1 + \theta_2 \cdot x_2 + \theta_3 \cdot x_3 + \theta_4 \cdot x_4 + \cdots + \theta_n \cdot x_n$$

$$h_{\boldsymbol{\theta}}(\boldsymbol{x}) = [\theta_0, \theta_1, \theta_{\mathbf{2}}, \dots, \theta_n] \begin{bmatrix} x_0 \\ x_1 \\ x_2 \\ \cdots \\ x_n \end{bmatrix}$$

$$\boldsymbol{\theta} = \begin{bmatrix} \theta_0 \\ \theta_1 \\ \theta_2 \\ \cdots \\ \theta_n \end{bmatrix} \in \mathbb{R}^{n+1} \qquad \boldsymbol{x} = \begin{bmatrix} x_0 \\ x_1 \\ x_2 \\ \cdots \\ x_n \end{bmatrix} \in \mathbb{R}^{n+1}$$

$$h_{\boldsymbol{\theta}}(\boldsymbol{x}) = \boldsymbol{\theta}^T \boldsymbol{x}$$

Multivariate Linear Regression (with multivariable i.e. multiple variables.)

Takeaway Points

- Multivariate Linear Regression is linear regression with multiple independent variables.
- The hypothesis function is

$$h_{\boldsymbol{\theta}}(\boldsymbol{x}) = \theta_0 + \theta_1 \cdot x_1 + \theta_2 \cdot x_2 + \theta_3 \cdot x_3 + \cdots + \theta_n \cdot x_n$$

- Vectorisation of hypothesis function

$$h_{\boldsymbol{\theta}}(\boldsymbol{x}) = \boldsymbol{\theta}^T \boldsymbol{x}$$