COMP2261 ARTIFICIAL INTELLIGENCE / MACHINE LEARNING

Polynomial Regression

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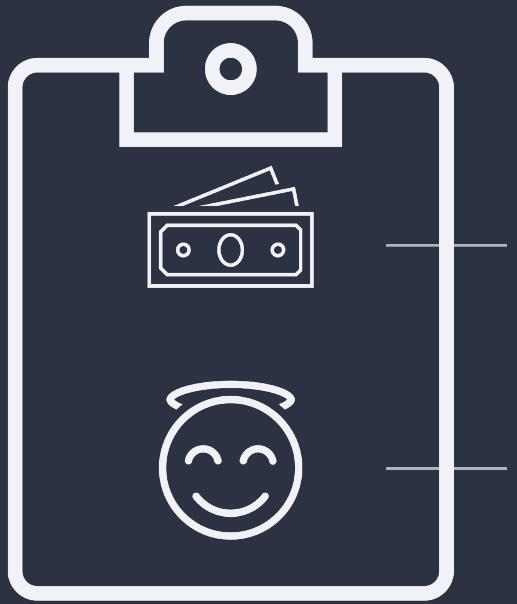
Learning Objectives

- Understand what is a polynomial regression model
- Understand how to transform features
- Understand how to solve parameters

EXAMPLE. annual income to predict happiness

500 people in Durham



survey

annual income

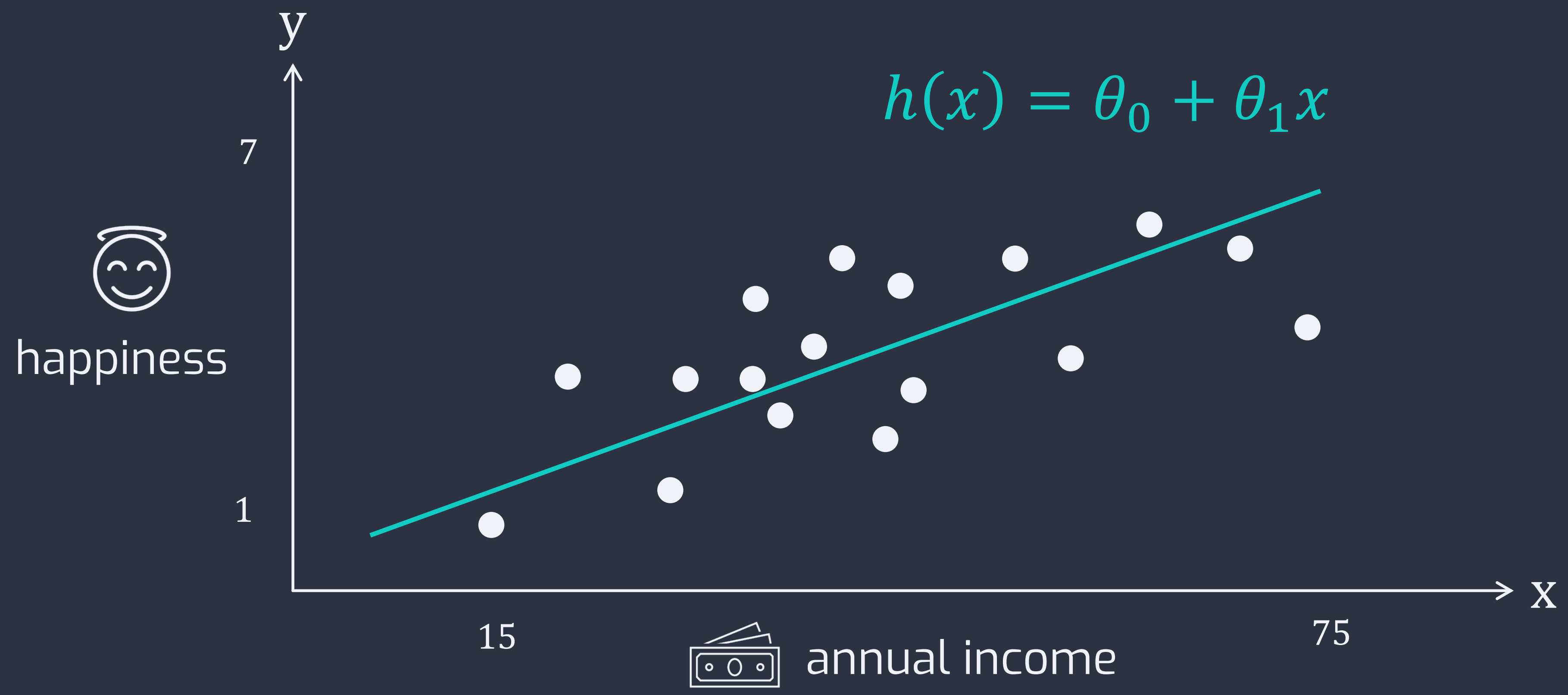
£15k -- £75k

happiness

0 -- 10

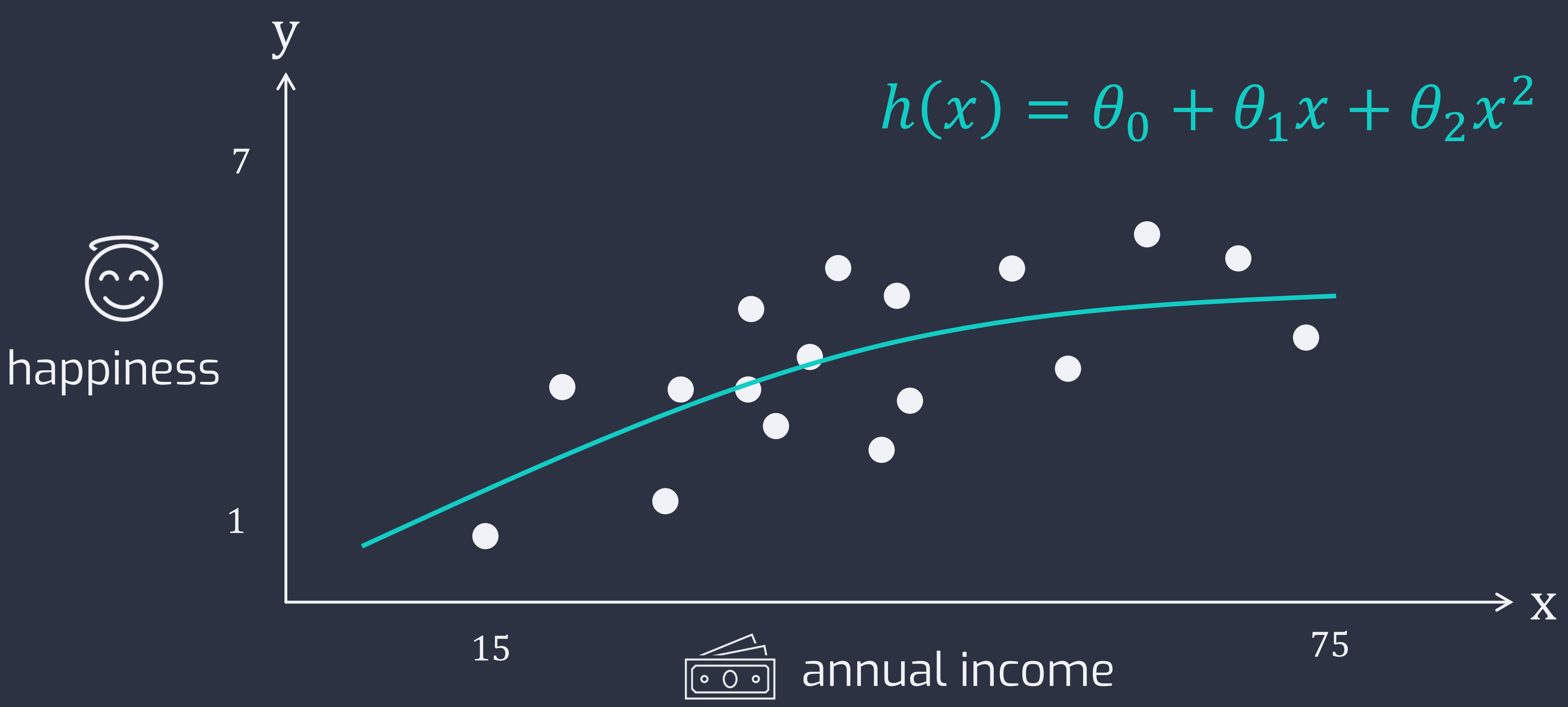
EXAMPLE. annual income to predict happiness

Simple linear regression: a linear regression model with a single explanatory variable (x).



EXAMPLE. annual income to predict happiness

Polynomial linear regression: a linear regression model with higher order.



Polynomial

data x, taken to increasingly high power

$$\theta_0 + \theta_1 x + \theta_2 x^2 + \cdots + \theta_n x^n$$

coefficients, scaling data

Polynomial

$$\boxed{\theta_0} + \theta_1 x + \theta_2 x^2 + \cdots + \theta_n x^n$$

$$\theta_0 x^0 \quad \boxed{2} + 4x + \frac{3}{7}x^2 + 5.6x^3$$

Polynomial

$$\theta_0 + \theta_1 x + \theta_2 x^2 + \cdots + \theta_n x^n$$

$$2 + 4x + \frac{3}{7}x^2 + 5.6x^3$$

$$8 + 9.1x^3 \qquad 8 + \boxed{0}x + \boxed{0}x^2 + 9.1x^3$$

Polynomial

Order: the highest coefficient

$$\theta_0 + \theta_1 x + \theta_2 x^2 + \cdots + \theta_n x^{\boxed{n}}$$

$$2 + 4x + \frac{3}{7}x^2 + 5.6x^3$$

third order polynomial

$$8 + 9.1x^3$$

Polynomial Regression

$h(x) = \theta_0 + \theta_1 x$ first order polynomial

$h(x) = \theta_0 + \theta_1 x + \theta_2 x^2$ second order polynomial

$h(x) = \theta_0 + \theta_1 x + \theta_2 x^2 + \cdots + \theta_n x^n$ n-th order polynomial

Fit a Polynomial Regression with Nonlinearities

$$h(x) = \theta_0 + \theta_1 x + \theta_2 \boxed{x^2} + \cdots + \theta_n \boxed{x^n}$$

nonlinear

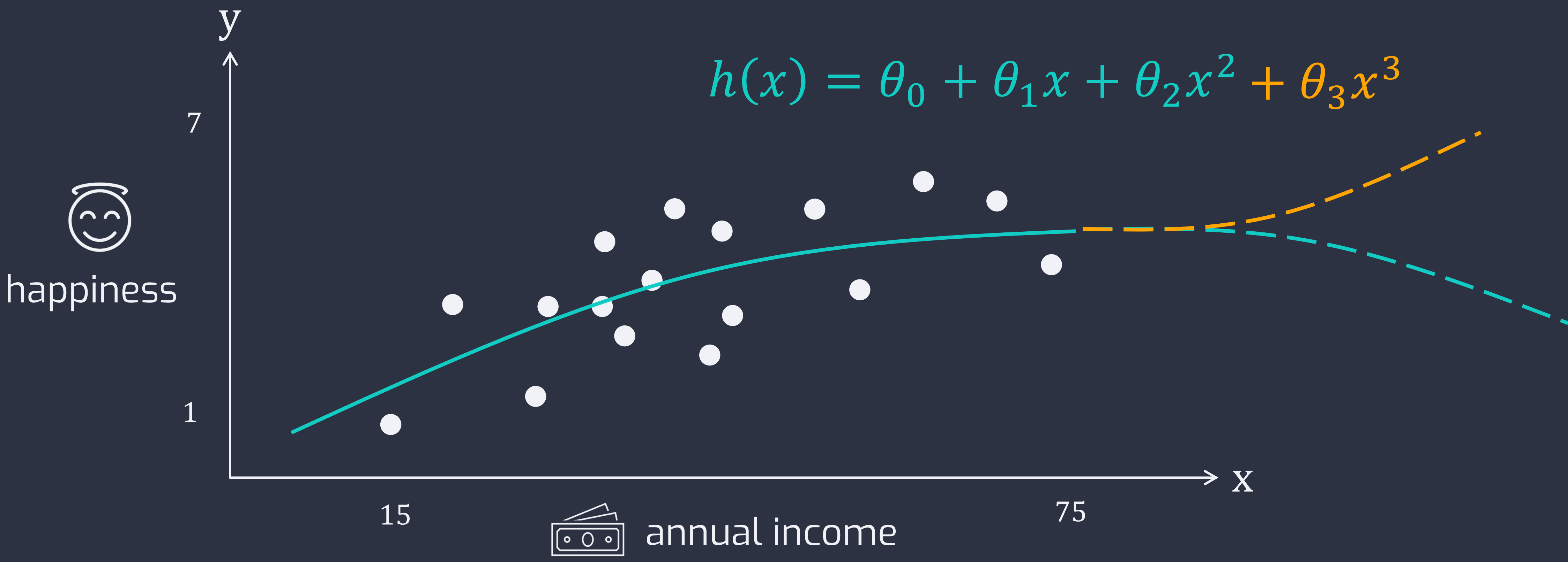
Fit a Polynomial Regression with Nonlinearities

$$h(x) = \boxed{\theta_0} + \boxed{\theta_1}x + \boxed{\theta_2}x^2 + \cdots + \boxed{\theta_n}x^n$$

The coefficients are all linear.

This is a standard linear model!

EXAMPLE. annual income to predict happiness



EXAMPLE. annual income to predict happiness

Transforming polynomial linear regression to multivariate linear regression.

$$h(x) = \theta_0 + \theta_1 x + \theta_2 x^2 + \theta_3 x^3$$

$$h(x) = \theta_0 + \theta_1 x_1 + \theta_2 x_2 + \theta_3 x_3$$

Gradient descent / normal equation to get $\theta_0, \theta_1, \theta_2, \theta_3$

EXAMPLE. annual income to predict happiness

Transforming polynomial linear regression to multivariate linear regression.

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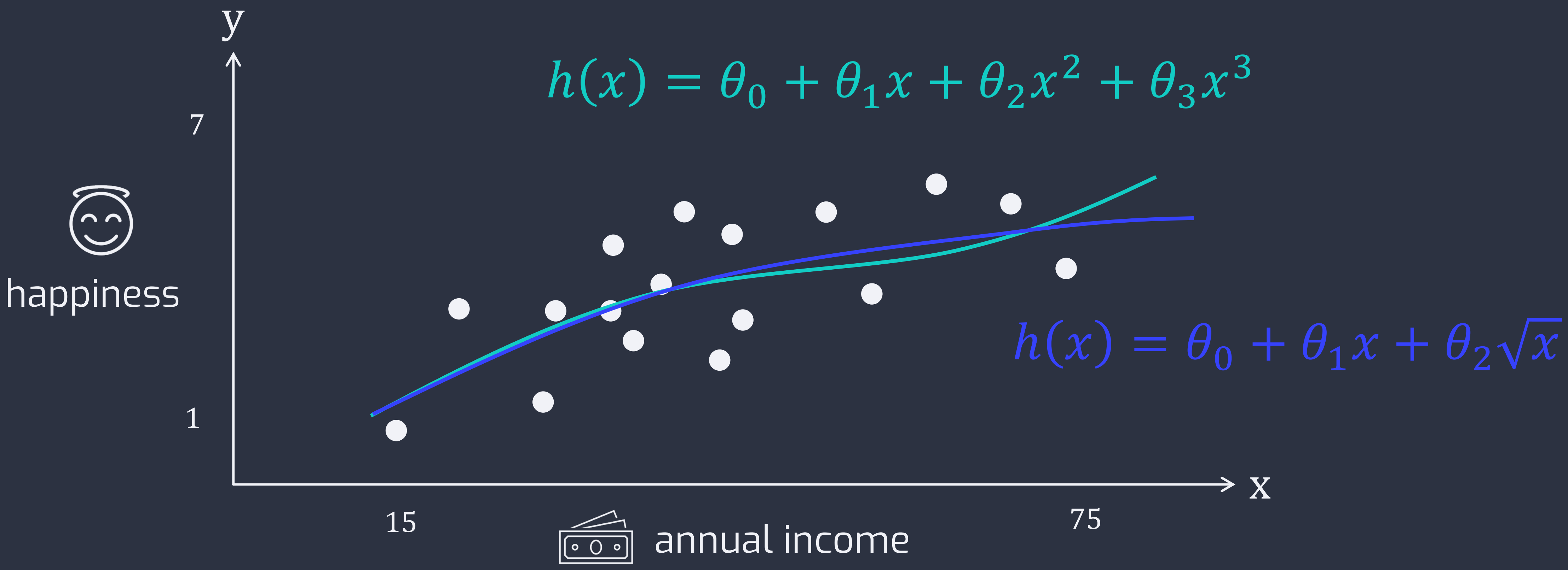
$$h(x) = \theta_0 + \theta_1 x_1 + \theta_2 x_2 + \theta_3 x_3$$

$x_1 =$	x	10…1000
$x_2 =$	x^2	100…1,000,000
$x_3 =$	x^3	1000…1,000,000,000

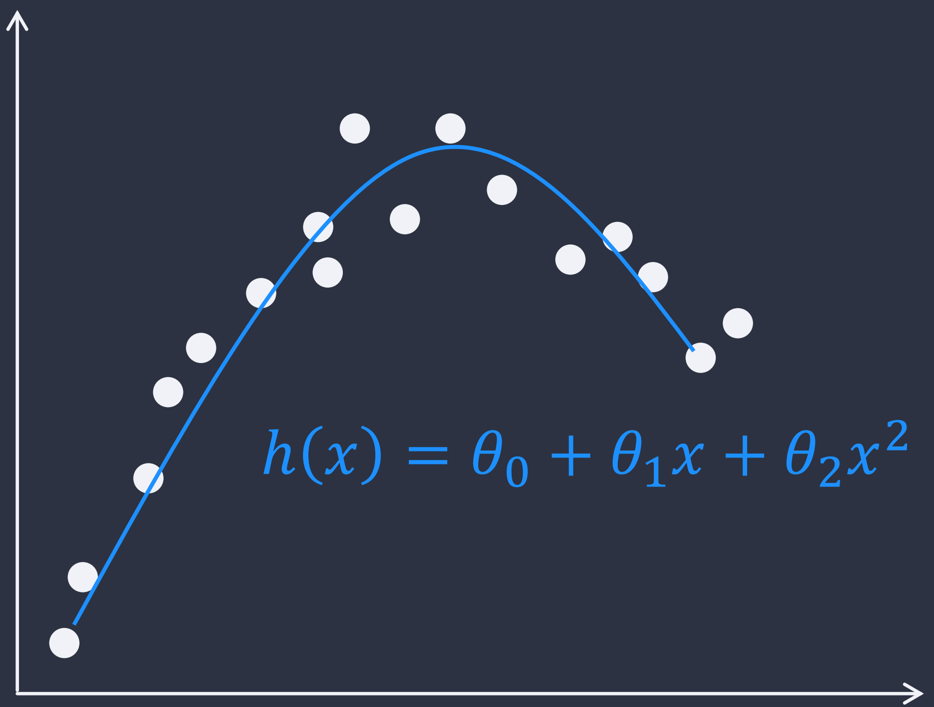
Feature scaling is important!

EXAMPLE. annual income to predict happiness

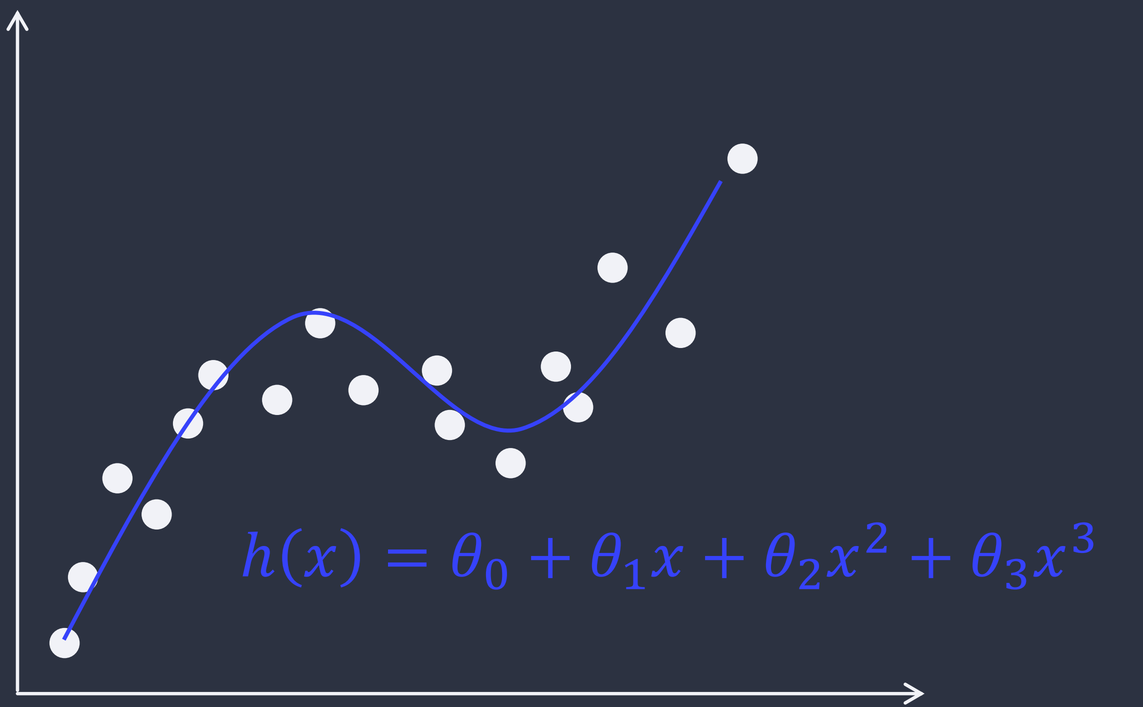


When to use Polynomial

Second-order polynomial



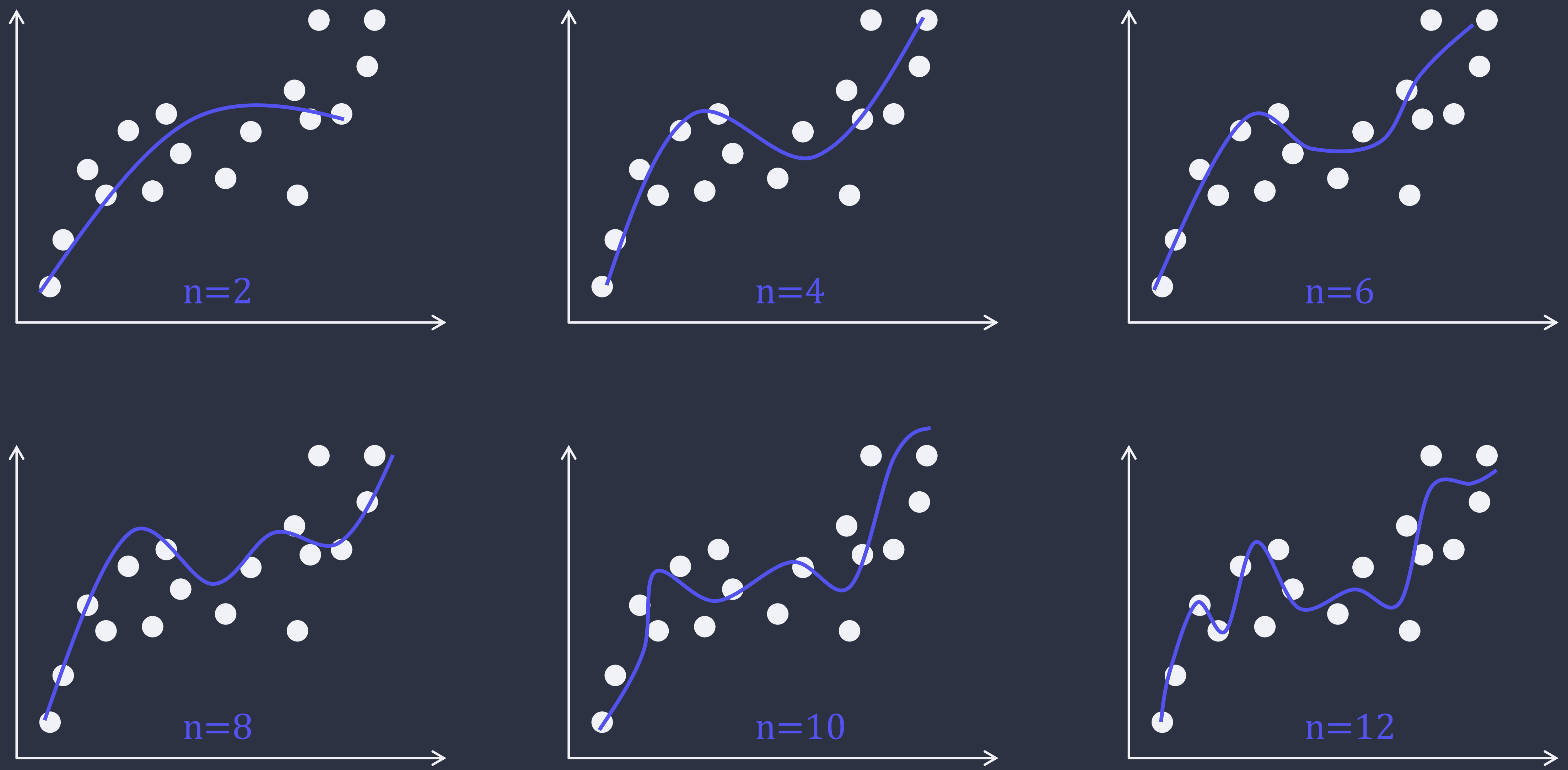
Third-order polynomial



How do we decide the order of a polynomial?

$$h(x) = \theta_0 + \theta_1 x + \theta_2 x^2 + \cdots + \theta_n x^n$$

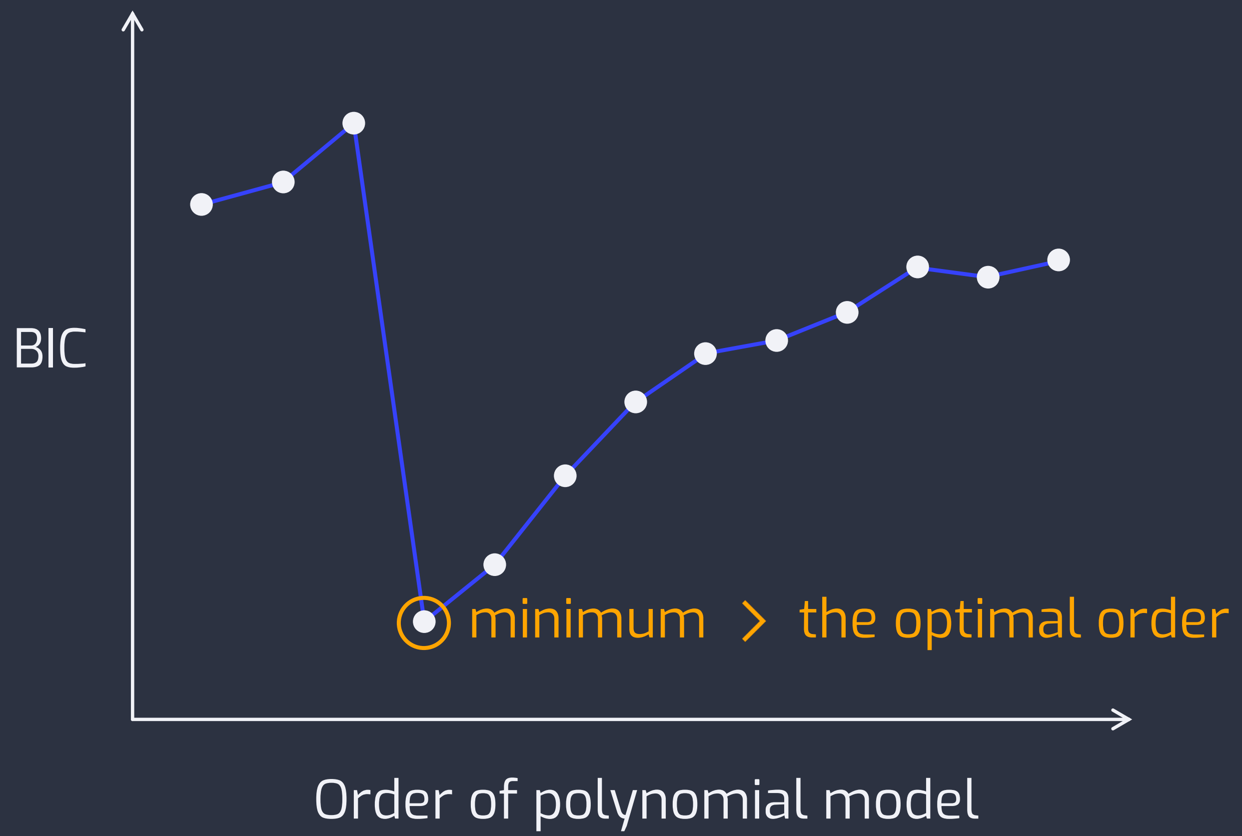
Order of Polynomial



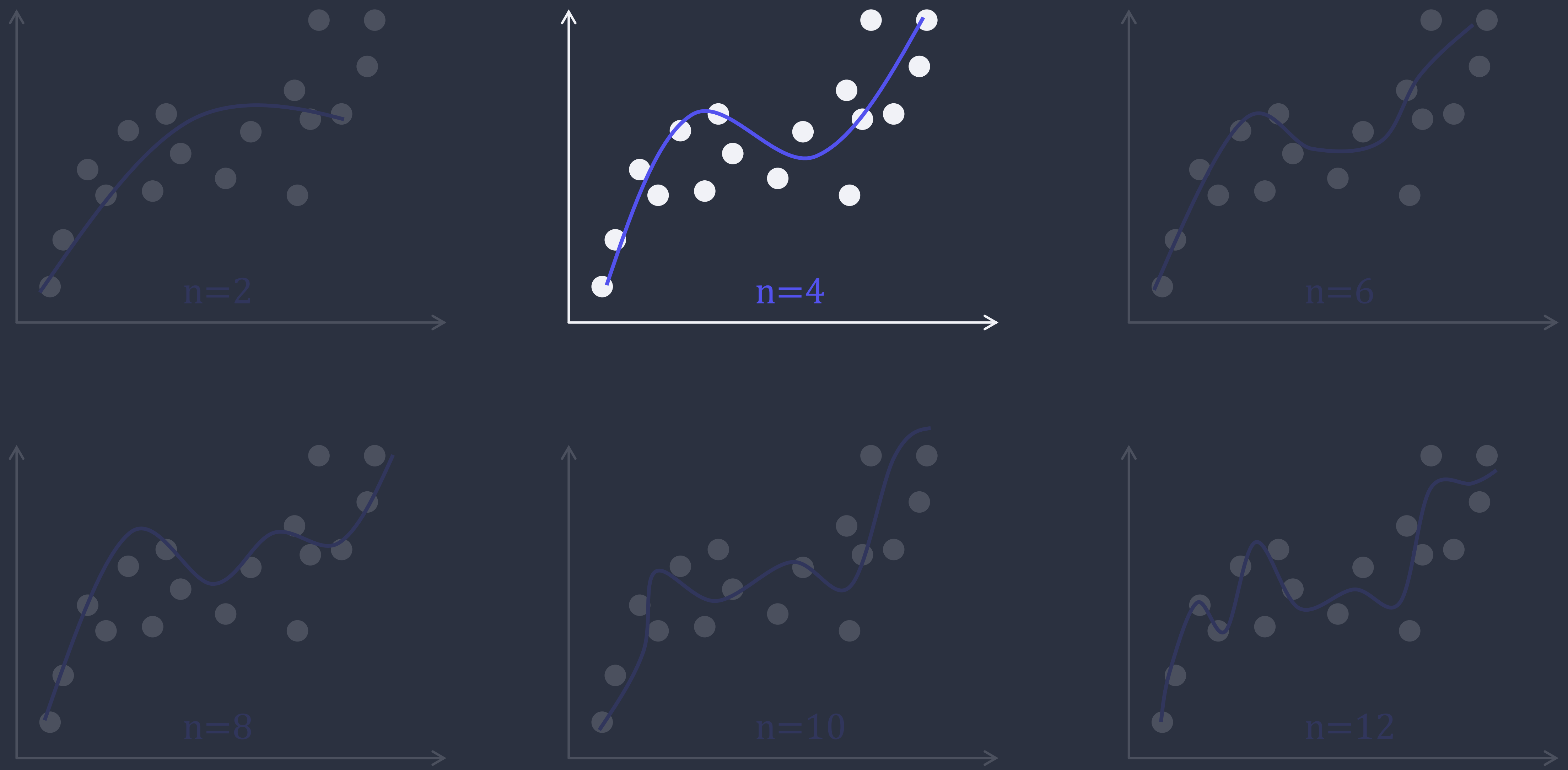
Deciding on Order of Polynomial

Bayesian Information Criterion

$$BIC_n = m \cdot ln(SS_r) + n \cdot ln(m)$$



Deciding on Order of Polynomial



Takeaway Points

- Higher order polynomial linear model may better fit data.
- Multivariate linear regression model method to solve $\theta_0, \theta_1, \theta_2, \ldots$
- Try different features $x^2, x^3, \sqrt{x}, \ldots$
- BIC to decide optimal polynomial order.