Explaining Adversarial Vulnerability with Differential Geometry

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Introduction

Adversarial vulnerability can be defined as the extreme sensitivity of learning models' predictions to small perturbations of their inputs.
Despite outstanding practical achievements, a consensus is yet to emerge to fully explain this phenomenon.
In this work, we explore how methods borrowed from differential geometry could shed light on adversarial vulnerability.

Framework

Multi-class classification:

$$f : \mathcal{X} \subseteq \mathbb{R}^d \to \Delta^{c-1}, \quad x \mapsto \theta.$$

The probability simplex Δ^{c-1} is the parameter space of the family of categorical distributions.
Fisher information metric on Δ^{c-1}:

$$g_\theta = \sum_{i,j=1}^{c} \frac{1}{\theta^i} \delta_{ij} d\theta^i d\theta^j.$$

Pullback metric $\widetilde{g} = f^* g$. In coordinates:

$$\widetilde{G}_x = J_x^T G_{f(x)} J_x.$$

Adversarial robustness

Let $x \in \mathcal{X}$.
Let $\mathcal{D}$ be the decision boundary in Δ^{c-1}, and $\delta(x) = d(f(x), \mathcal{D})$ be the distance between $f(x)$ and $\mathcal{D}$.
Let $\mathcal{B}(x, \epsilon)$ be the Euclidean ball, and let $\widetilde{\mathcal{B}}(x, \delta(x))$ be the geodesic ball induced by $\widetilde{g}$.
Robustness criteria. If:

$$\mathcal{B}(x, \epsilon) \subseteq \widetilde{\mathcal{B}}(x, \delta(x)),$$

then the model f is adversarially robust at x.

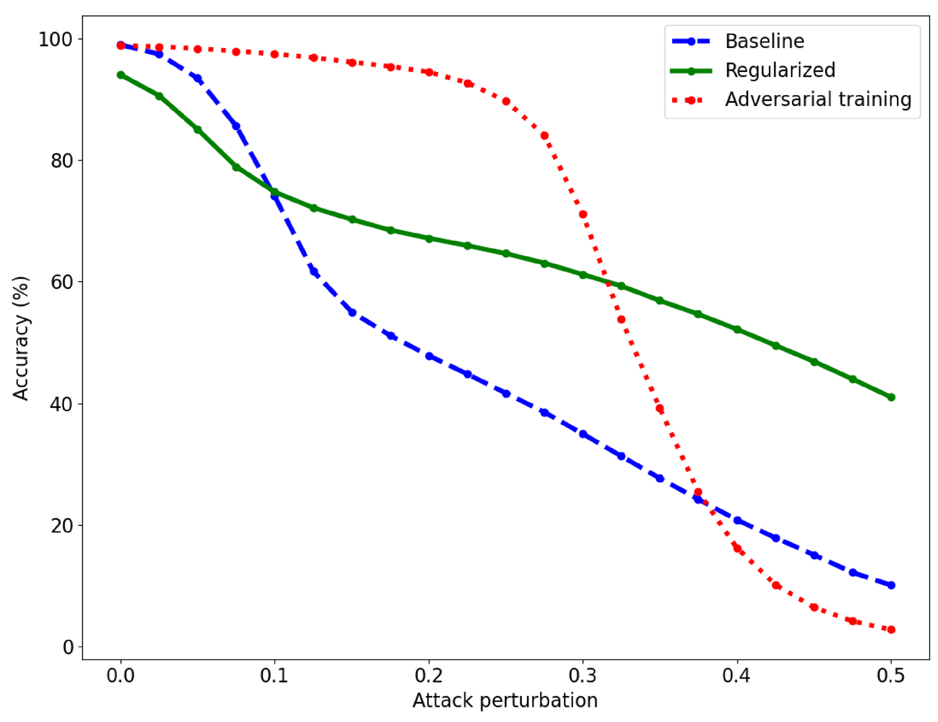
Isometry regularization

Let $P_x = (\ker \widetilde{g}_x)^\perp$, and let $\overline{g}$ be the Euclidean metric. The isometry condition is:

$$\widetilde{g}_x|_{P_x} = \frac{\delta(x)^2}{\epsilon^2} \overline{g}_x|_{P_x}.$$

We define the following regularization:

$$\alpha(x, f) = \left\| \widetilde{J}_x \widetilde{J}_x^T - \frac{\delta(x)^2 \rho(x)^2}{\epsilon^2} I_{c-1} \right\|_F.$$



Accuracy of the baseline (blue), regularized (green), and adversarially trained (red) models for various attack perturbations on the MNIST dataset. The perturbations are obtained with PGD using l_∞ norm.

Jacobian regularization

The robustness condition is, for all $X \in T_x\mathcal{X}$:

$$\widetilde{g}_x(X, X) \leq \frac{\delta(x)^2}{\epsilon^2} \overline{g}_x(X, X).$$

We define the following regularization:

$$\alpha(x, f) = h\left(\|\widetilde{J}_x\|_2^2 - \frac{\delta(x)^2}{\kappa(x)^2 \epsilon^2} \right),$$

where h is a "soft barrier function".

Research directions

1. **Certified defense**
   Derive a certified defense by strongly enforcing the robustness criteria on a chosen proportion of the training examples. Can we prove that the accuracy is maximized under the constraint of a chosen robustness level?
2. **Extensions**
   The proposed regularizations focus on l_2 white-box attacks for multi-class classification. It can be extended to regression tasks (e.g., using the family of multivariate normal distributions) as well as to other attacks (e.g., l_∞ attacks or unrestricted attacks such as spatial attacks).
3. **Other metric**
   Find another metric or another family of distributions such that the robustness criteria is optimal (i.e., $\widetilde{\mathcal{B}}(x, \delta(x))$ is exactly the set of points connected to x with the same class than x).
4. **Exact robustness criteria**
   Derive an exact robustness criteria by taking into account the curvature of $\widetilde{g}$. Is there a formulation of this exact robustness criteria that is computationally tractable?
5. **Data leaf**
   Consider the distribution $P : x \mapsto (\ker \widetilde{g}_x)^\perp$. Under mild assumptions [1], the distribution P is integrable. Moreover, the underlying data distribution may be supported on a unique leaf of the foliation associated to P. The data leaf framework may explain why the generalization and robustness properties are dependent on the training set distribution.

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