



# A trend factor: Any economic gains from using information over investment horizons?<sup>☆</sup>

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## ABSTRACT

In this paper, we provide a trend factor that captures simultaneously all three stock price trends: the short-, intermediate-, and long-term, by exploiting information in moving average prices of various time lengths whose predictive power is justified by a proposed general equilibrium model. It outperforms substantially the well-known short-term reversal, momentum, and long-term reversal factors, which are based on the three price trends separately, by more than doubling their Sharpe ratios. During the recent financial crisis, the trend factor earns 0.75% per month, while the market loses −2.03% per month, the short-term reversal factor loses −0.82%, the momentum factor loses −3.88%, and the long-term reversal factor barely gains 0.03%. The performance of the trend factor is robust to alternative formations and to a variety of control variables. From an asset pricing perspective, it also performs well in explaining cross-section stock returns.

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## 1. Introduction

Schwert (2003), in his comprehensive study of anomalies, finds that the momentum anomaly is one of the most robust and persistent anomalies. Indeed, there are three major stock price patterns that are difficult to explain by typical factor models, such as the capital asset pricing

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model (CAPM) (Lintner, 1965; Sharpe, 1964), the Fama-French three-factor model (Fama and French, 1993; 1996), and the asset pricing model with a liquidity factor (Pástor and Stambaugh, 2003). The three patterns are investment horizon dependent: the short-term reversals (at daily, weekly, and monthly levels) documented by Lehmann (1990), Lo and MacKinlay (1990b), and Jegadeesh (1990), the momentum effects (6–12 month price continuation) documented by Jegadeesh and Titman (1993), and the long-term reversal effects (3–5 year reversals) documented by DeBondt and Thaler (1985). Since existing studies focus on analyzing each of the three price patterns separately with price information from one investment horizon at a time, a simple and interesting question arises. Are there any economic gains by combining all the price information across the three investment horizons?

In this paper, we provide a trend factor that synthesizes the short-, intermediate-, and long-term price signals of the stock market. Unlike most studies that identify pricing factors by sorting on a few (usually less than three) firm characteristics, we construct our trend factor from cross-section regressions that allow us to incorporate multiple price signals. The signals are based on the moving averages of past prices from three days to as many as 1,000 days (roughly four trading years) to incorporate simultaneously the information of short-, intermediate-, and long-term price on the moving averages to obtain the forecasted returns. Then, as in most studies on constructing pricing factors, we buy those stocks with the highest forecasted expected returns in the top quintile and short those with the lowest. The difference between returns on the two extreme quintiles is the return on our trend factor. Since our methodology utilizes all information from the short- to the long-run, our paper is the first that unifies in a single framework the three major price patterns: the short-term reversal effects, the momentum effects, and the long-term reversal effects.

Why do the moving averages (MAs) of past prices, our signals, have predictive power on stock returns? There are three major reasons. First, theoretically, due to differences in the timing of receiving information or differences in the response to information by heterogeneous investors or behavior biases or feedback trading, Treynor and Ferguson (1985), Brown and Jennings (1989), Cespa and Vives (2012), Hong and Stein (1999), and Edmans, Goldstein, and Jiang (2012), among others, show that past stock prices can predict future prices which implies that the MAs have predictive power. More directly, based on Wang (1993), we also provide a simple general equilibrium model, which appears the first of its kind as existing theoretical studies are rare and impose exogenous prices (Zhu and Zhou, 2009), to justify the use of the MA signals. Second, in the real world, many top traders and successful fund managers do use the MAs to learn about market price trends and make their investment decisions accordingly (see, e.g., Lo and Hasanhodzic, 2009; Narang, 2013; Schwager, 1989). Due to liquidity and uncertainty, the market is likely to trend in taking its time to react fully to big information. An example is the recent financial crisis during which the market trended down for two years to reach its bottom with waves of forced liquidations and hedge trades along

the way. Academic studies, such as Fung and Hsieh (2001), find that trend-following trading is of great importance for explaining hedge fund returns. Burghardt and Walls (2011) show that a simple mechanical trading rule based on the MAs can yield favorable returns in trading futures contracts and the return correlation with the managed futures index exceeds 70%. Third, in the empirical studies on stocks, Brock, Lakonishok, and LeBaron (1992) offer the first major study that provides convincing evidence on the predictive power of the MAs.<sup>3</sup> Lo, Mamaysky, and Wang (2000) further strengthen the evidence with an automated pattern recognition analysis. Recently, Han, Yang, and Zhou (2013) show that the MAs can generate substantial alphas in the cross-section of stock returns, and Neely, Rapach, Tu, and Zhou (2014) find that technical indicators, primarily the MAs, have forecasting power on the stock market matching or exceeding that of macroeconomic variables.

Our empirical results show that there are substantial economic gains from using all the price information across investment horizons. With data from June 1930 through December 2014, the trend factor earns an average return of 1.63% per month, compared with 0.79%, 0.79%, and 0.34% per month of the short-term reversal, momentum, and long-term reversal factors, respectively; and at the same time, the market portfolio earns on average 0.62% per month, the Fama-French size and book-to-market factors earn even less, 0.27% and 0.41% per month, respectively. In terms of the Sharpe ratio, the trend factor has a monthly value of 0.47, more than doubling that of the short-term reversal factor, and more than quadrupling those of the momentum factor, long-term reversal factor, the Fama-French two factors, and the market portfolio. Moreover, the trend factor earns 0.75% per month during the recent financial crisis, while the market loses –2.03% per month, the short-term reversal factor loses –0.82%, the momentum factor loses –3.88%, and the long-term reversal factor barely gains 0.03%.

Since the momentum factor is the most widely studied among the three major price patterns, it is of great interest to compare it with the trend factor further. We find that the long (short) legs of the trend and momentum factors are highly correlated with a correlation of 88% (84%). However, the trend factor performs better, with an average return of 1.93% versus 1.81% on the long leg, and 0.31% versus 1.02% on the short leg. The superior performance of the trend factor is persistent over both recession and expansion periods. These results explain why the trend factor has earned about four times the Sharpe ratio of the momentum factor.

Daniel, Jagannathan, and Kim (2012), Daniel and Moskowitz (2014), and Barroso and Santa-Clara (2015) recently find that the momentum strategy suffers from episodes of crash, an extreme downside risk.<sup>4</sup> For example, the four worst monthly returns of the momentum factor are –89.70%, –83.25%, –59.99%, and –55.74%, making

<sup>3</sup> Cowles (1933) provides perhaps the first academic study on the MA or technical analysis, and Fama and Blume (1966) examine more filter rules. But results of the earlier studies are mixed and inconclusive.

<sup>4</sup> Han, Zhou, and Zhu (2016) show that the crash risk, however, can be reduced substantially with a simple stop-loss strategy.

it unattractive as an investment to most investors. In contrast, the four worst monthly returns of the trend factor are only  $-19.96\%$ ,  $-15.06\%$ ,  $-12.66\%$ , and  $-10.53\%$ . These values are quite favorable compared even with those of the stock market as a whole whose four worst monthly returns are  $-28.97\%$ ,  $-23.70\%$ ,  $-22.64\%$ , and  $-21.91\%$ . In general, the trend factor has far fewer negative outliers and has a large positive skewness of 1.47. But the momentum factor has much more negative outliers and a large negative skewness of  $-4.43$ .

Since the trend factor relies on stock price trends from the short- to the long-run, an important question is whether it can be replicated by a suitable portfolio of the short-term reversal, momentum, and long-term reversal factors. If this were the case, running the cross-section regressions at the stock level to forecast expected returns to form the trend factor would be totally unnecessary. To answer this question, we carry out six different mean-variance spanning tests [see, e.g., Kan and Zhou, 2012 for a complete review], and find that the tests strongly reject the hypothesis that a portfolio of the three factors can yield a factor that is close to the trend factor in terms of the Sharpe ratio. In other words, the cross-section regression based on the MAs is more efficient in capturing the price trends of the short-, intermediate-, and long-term than a combination of the short-term reversal, momentum, and long-term reversal factors.

While the three price trend factors cannot span the trend factor, it is important to know to what extent they can explain the returns on the trend factor. For this purpose, we run (Sharpe, 1988) style regressions and find interestingly that, if a portfolio with positive weights on the three factors is used to capture the performance of the trend factor, the loadings are time-varying and they weigh more on the short-term reversal factor, and especially so during recessions.

The high returns on the trend factor are robust to alternative factor construction, to various control variables such as market size, book-to-market ratio, past returns, and liquidity, and to alternative methodologies (e.g., Fama-MacBeth regressions). For example, regardless of whether or not to impose the price and/or size filters on stocks or to follow Fama and French (1996) to construct the spread portfolio, the resulting trend factors have similar or even larger returns. Furthermore, we show that, while it is extremely difficult to overcome concerns on transaction costs and data-mining, the trend factor appears to perform at least as well as the well-known momentum factor on these two important issues.

We also explore the link between the performance of the trend factor and information uncertainty. When information about stocks is very uncertain, fundamental signals are likely to be imprecise, and hence investors tend to rely more heavily on trend signals. Similar to Han, Yang, and Zhou (2013) who show that the profitability of a simple moving average timing strategy is positively related to information uncertainty, we find that the trend factor performs better when information about stocks is more uncertain as proxied by various variables such as idiosyncratic volatility, analyst coverage, firm age, etc.

From an asset pricing perspective, the trend factor seems to perform better than the widely used momentum factor in explaining the cross-section portfolio returns. For standard stock portfolios sorted by industry or by both size and book-to-market, the aggregate pricing errors of the CAPM with the trend factor are smaller than those of the CAPM with the momentum factor. We also examine the hedge fund portfolios sorted by style and find that the aggregate pricing error is again smaller.

Section 2 discusses the data, methodology, and theory for constructing the trend factor as well as its basic properties and relation to other factors. Section 3 examines the robustness of the trend factor in various dimensions. Section 4 explores the link of the trend factor to information uncertainty. Section 5 investigates its cross-section pricing power as compared with the momentum factor. Section 6 concludes.

## 2. Trend factor

In this section, we discuss the data used in this paper and provide the detailed methodology for constructing the trend factor. Then we outline the economic theories underlying the trend factor and propose a new model. Next, we examine the properties of the trend factor along with other factors, conduct the spanning tests, and run (Sharpe, 1988) style regressions. Finally, we investigate the characteristics of the associated quintile portfolios and the alphas of the trend factor.

### 2.1. Data

We use the *daily* stock prices from January 2, 1926 through December 31, 2014 obtained from the Center for Research in Security Prices (CRSP) to calculate the moving average price signals at the end of each month (where the prices are adjusted for splits and dividends when necessary). Based on the month-end moving average signals, we form our portfolios and factors, and rebalance them at the usual *monthly* frequency. We include all domestic common stocks listed on the NYSE, AMEX, and Nasdaq stock markets, and exclude closed-end funds, real estate investment trusts (REITs), unit trusts, American depository receipts (ADRs), and foreign stocks (or stocks that do not have a CRSP share code of 10 or 11). In addition, at the end of each month, we exclude stocks with prices below \$5 (price filter) and stocks that are in the smallest decile sorted with NYSE breakpoints (size filter). Jegadeesh and Titman (1993) use the same price and size filters when constructing the momentum strategy. A relaxation of either or both of the filters and other alternative procedures will be examined in Section 3.

### 2.2. Methodology

To construct the trend factor, we first calculate the MA prices on the last trading day of each month. The MA on the last trading day of month  $t$  of lag  $L$  is defined as

$$A_{jt,L} = \frac{P_{j,d-L+1}^t + P_{j,d-L+2}^t + \cdots + P_{j,d-1}^t + P_{j,d}^t}{L}, \quad (1)$$

where  $P_{jd}^t$  is the closing price for stock  $j$  on the last trading day  $d$  of month  $t$ , and  $L$  is the lag length. Then, we normal-

ize the moving average prices by the closing price on the last trading day of the month,

$$\tilde{A}_{jt,L} = \frac{A_{jt,L}}{P_{jd}^t}. \quad (2)$$

There are three reasons for this normalization. First, according to our simple model to be presented in the next subsection, it is this normalized average that predicts the future stock returns. Second, econometrically, the normalization makes the MA signals stationary.<sup>5</sup> Third, the normalization can also mitigate the undue impact of high priced stocks.

To predict the monthly expected stock returns cross-sectionally, we use a two-step procedure. In the first step, we run in each month  $t$  a cross-section regression of stock returns on observed normalized MA signals to obtain the time-series of the coefficients on the signals,

$$r_{j,t} = \beta_{0,t} + \sum_i \beta_{i,t} \tilde{A}_{jt-1,L_i} + \epsilon_{j,t}, \quad j = 1, \dots, n, \quad (3)$$

where

- $r_{j,t}$  = rate of return on stock  $j$  in month  $t$ ,
- $\tilde{A}_{jt-1,L_i}$  = trend signal at the end of month  $t - 1$  on stock  $j$  with lag  $L_i$ ,
- $\beta_{i,t}$  = coefficient of the trend signal with lag  $L_i$  in month  $t$ ,
- $\beta_{0,t}$  = intercept in month  $t$ ,

and  $n$  is the number of stocks.<sup>6</sup> It should be noted that only information in month  $t$  or prior is used above to regress returns in month  $t$ .

Following, for example, Brock, Lakonishok, and LeBaron (1992), we consider in the above regressions using MAs of lag lengths 3-, 5-, 10-, 20-, 50-, 100-, and 200-days. In addition, we include 400-, 600-, 800-, and 1,000-days. These MA signals indicate the daily, weekly, monthly, quarterly, 1-year, 2-year, 3-year, and 4-year price trends of the underlying stock. Note that we simply use all the popular MA indicators without any alterations or optimization to mitigate concerns of data-mining, an issue to be examined further in Section 3.

Then, in the second step, as in Haugen and Baker (1996), we estimate the expected return for month  $t + 1$  from

$$E_t[r_{j,t+1}] = \sum_i E_t[\beta_{i,t+1}] \tilde{A}_{jt,L_i}, \quad (4)$$

where  $E_t[r_{j,t+1}]$  is our forecasted expected return on stock  $j$  for month  $t + 1$ , and  $E_t[\beta_{i,t+1}]$  is the estimated expected coefficient of the trend signal with lag  $L_i$ , and is given by

$$E_t[\beta_{i,t+1}] = \frac{1}{12} \sum_{m=1}^{12} \beta_{i,t+1-m}. \quad (5)$$

which is the average of the estimated loadings on the trend signals over the past 12 months. Note that we do not include an intercept above because it is the same for all stocks in the same cross-section regression, and thus it plays no role in ranking the stocks below. Again, it is worth noting that only information in month  $t$  or prior is used to forecast the expected returns in month  $t + 1$ , so our study is an out-of-sample analysis.

Now, we are ready to construct the trend factor. We sort all stocks into five portfolios by their expected returns. The portfolios are equal-weighted and rebalanced every month.<sup>7</sup> The return difference between the quintile portfolio of the highest expected returns and the quintile portfolio of the lowest is defined as the return on the trend factor. Intuitively, the trend factor buys stocks that are forecasted to yield the highest expected returns (Buy High) and shorts stocks that are forecasted to yield the lowest expected returns (Sell Low).

Note that above approach of forming factors based on the spread portfolio between high and low is the standard procedure in cross-section studies, such as Jegadeesh and Titman (1993), Easley, Hvidkjaer, and O'Hara (2002), Fama and French (1993), Ang, Hodrick, Xing, and Zhang (2006), and Han and Lesmond (2011). However, unlike most studies that sort stocks by firm characteristics, we sort here by their expected returns. In characteristics sorting, it is easy to do double-sorting, but difficult to go beyond three characteristics. In contrast, following Haugen and Baker (1996), we use cross-section regressions which can accommodate many characteristics or signals. Interestingly, the popular momentum factor (Jegadeesh and Titman, 1993) can be interpreted as a special case of our procedure. If there is only one trend signal, past year return, and if the beta is one, then the trend factor coincides with the momentum factor. However, rather than using only information in one horizon, the goal of our paper is to combine price trends across investment horizons to see whether there are economic gains of doing so.

### 2.3. Theoretical support

For the trend factor to be valid empirically, there must be price trends in the real world (which is necessary too for the momentum factor and the other two major price patterns). Moreover, the moving average signals must also be effective predictors of future returns. Is there any economic basis for them?

Theoretically, there are various economic forces that can contribute to the price trends in the stock market that make the MA signals useful. Due to differences in the timing of receiving information or in responses to information, Brunnermeier (2001) reviews quite a few models that justify predictable trends in rational equilibria. Recently, Cespa and Vives (2012) show that the presence of liquidity traders and asset payoff uncertainty will generate rational price trends in the market. Moreover, the positive feedback

<sup>5</sup> Keim and Stambaugh (1986) use a similar strategy to make the S&P 500 index stationary.

<sup>6</sup> Jegadeesh (1990) also uses similar cross-sectional regressions to predict individual stock returns, but he uses past returns instead of MA signals.

<sup>7</sup> Following the original momentum strategy of Jegadeesh and Titman (1993), we use equal-weighting. If decile portfolios/value-weighting are used, the results will be similar (stronger/weaker) and qualitatively unchanged.



effects, where an uptrend in stock prices can positively affect firm fundamentals that in turn lead to even higher prices, are the basis for the trading strategies of hedge fund guru (Soros, 2003) and are validated in a theoretical model by Edmans, Goldstein, and Jiang (2012). In addition, from the perspective of behavior finance, investors' underreaction or overreaction can induce price trends because investors are overconfident about their private information and overreact to confirming news (self-attribution bias) or because investors initially underreact and subsequently overreact to information if information diffuses gradually. Barberis, Shleifer, and Vishny (1998) argue that prices can trend slowly when investors underweight new information in making decisions. Finally, any theory that explains momentum can be potentially useful for explaining the trend factor as well since the momentum is a particular trend.

While the above theories justify past prices as signals for predicting future prices, none of them link explicitly the moving averages (MA) to future returns. Based on Wang (1993), we provide a simple model to do exactly this.

Following Wang (1993), we assume that the market is endowed with a certain amount of one risky asset, each unit of which provides a dividend flow given by

$$dD_t = (\pi_t - \alpha_D D_t)dt + \sigma_D dB_{1t}, \quad (6)$$

where  $\pi_t$  is the mean level of dividend flow given by another stochastic process

$$d\pi_t = \alpha_\pi (\bar{\pi} - \pi_t)dt + \sigma_\pi dB_{2t}, \quad (7)$$

where  $B_{1t}$  and  $B_{2t}$  are independent innovations. In addition, we assume that the supply of the risky asset is  $1 + \theta_t$  with

$$d\theta_t = -\alpha_\theta \theta_t dt + \sigma_\theta dB_{3t}, \quad (8)$$

where  $B_{3t}$  is another Brownian motion independent of both  $B_{1t}$  and  $B_{2t}$ . Now, we replace the uninformed traders of Wang (1993) by technical traders who infer information from the historical prices via the MA, defined in the model as an exponentially weighted average of the past prices,

$$A_t \equiv \int_{-\infty}^t \exp[-\alpha(t-s)]P_s ds, \quad (9)$$

with  $\alpha > 0$ . Then, we have

**Theorem 1.** In an economy as defined in Wang (1993) adapted with technical traders, there exists a stationary rational expectations equilibrium. The equilibrium price function has the following linear form:

$$P_t = p_0 + p_1 D_t + p_2 \pi_t + p_3 \theta_t + p_4 A_t, \quad (10)$$

where  $p_0, p_1, p_2, p_3$ , and  $p_4$  are constants determined only by the model parameters.

*Proof.* See Appendix.

The theorem says that the equilibrium price is a linear function of the state variables and the MA signal  $A_t$ . In contrast to a standard model without technical traders,  $A_t$  is the new priced factor in the market. Economically, it is clear that  $p_0, p_1$  and  $p_2$  are all positive, reflecting the positive price impact of the state variables. However,  $p_4$  can be either positive or negative, which depends primarily on the proportion of technical traders in the market.

**Table 1**

The trend factor and other factors: Summary statistics.

This table reports the summary statistics for the trend factor (*Trend*), the short-term reversal factor (*SREV*), the momentum factor (*MOM*), the long-term reversal factor (*LREV*), and the Fama-French three factors including the market portfolio (*Market*), *SMB*, and *HML* factors. For each factor, we report sample mean in percentage, sample standard deviation in percentage, Sharpe ratio, skewness, and excess kurtosis. The *t*-statistics are in parentheses and significance at the 1% level is given by \*\*\*. The sample period is from June 1930 through December 2014.

Factor	Mean (%)	Std dev (%)	Sharpe ratio	Skewness	Excess kurtosis
Trend	1.63*** (15.0)	3.45	0.47	1.47	11.3
SREV	0.79*** (7.21)	3.49	0.23	0.99	8.22
MOM	0.79*** (3.29)	7.69	0.10	-4.43	40.7
LREV	0.34*** (3.09)	3.50	0.10	2.93	24.8
Market	0.62*** (3.69)	5.40	0.12	0.27	8.03
SMB	0.27*** (2.63)	3.24	0.08	2.04	19.9
HML	0.41*** (3.64)	3.58	0.11	2.15	18.9

A major implication of our model is that the stock price can be predicted by the MA signal. Specifically, taking the finite difference in Eq. (10), we obtain

$$\Delta P_t = p_1 \Delta D_t + p_2 \Delta \pi_t + p_3 \Delta \theta_t + p_4 (P_t - \alpha A_t) \Delta t.$$

After dividing by price, we have the following predictive regression,

$$r_{t+1} = a + \beta \frac{A_t}{P_t} + \epsilon_t, \quad (11)$$

where the impact of other variables is summarized in the noise term of the regression. In empirical applications,  $A_t$  can be approximated by the simple moving average

$$A_t = \frac{1}{L} \sum_{i=0}^{L-1} P_{t-i\Delta t}, \quad (12)$$

where  $L$  is lag length or the moving average window. Theoretically, our model states that  $A_t/P_t$  should be a predictor of the asset return, but the slope can have either a positive or negative sign depending on the proportion of technical traders in the market. Since there are traders in the real world who use the MA with various lags, our forecasting Eq. (3) uses all the reasonable lags, covering the short-, intermediate-, and long-term price signals.

#### 2.4. Summary statistics

In this subsection, we provide first the summary statistics of the trend factor and compare them with those of other common factors.

In order to compute the trend signals and estimate their expected coefficients, we have to skip the first 1,000 days and subsequent 12 months. So the effective sample period for our study is from June 1930 with a total of 1,015 observations, during which the trend factor is well defined.

Table 1 reports the summary statistics of the trend factor, short-term reversal factor (*SREV*), momentum factor (*MOM*), long-term reversal factor (*LREV*), as well as

the Fama-French three factors, eg, Market, small minus big (SMB), and high minus low (HML).<sup>8</sup> The average monthly return of the trend factor from June 1930 through December 2014 is 1.63% per month, or 19.56% per annum, more than doubling the average return of any of the other factors including SREV and MOM, whose average returns are the highest among the other factors but are only 0.79% per month. The standard deviation of the trend factor is 3.45%, lower than that of any other factor except SMB. As a result, the Sharpe ratio of the trend factor is much higher than those of the other factors. For example, the trend factor has a Sharpe ratio of 0.47, whereas the next highest Sharpe ratio is only 0.23 generated by SREV. Daniel and Moskowitz (2014) and Barroso and Santa-Clara (2015) show that returns generated from the momentum strategies are negatively skewed with large kurtosis, which implies a very fat left tail. Consistent with these results, Table 1 shows that the momentum factor has a very large negative skewness (−4.43) and very large kurtosis (40.7). In contrast, the trend factor has a large positive skewness (1.47) and large kurtosis (11.3), indicating a fat right tail, and great chances for large positive returns.

It is of interest to see how the factors perform in bad times. Panel A of Table 2 shows that the average return and volatility of the trend factor are both higher in recession periods than in the whole sample period. The average return rises from 1.63% to 2.34%, and the volatility rises from 3.45% to 5.05%. However, the Sharpe ratio is virtually the same, from 0.47 to 0.46. In contrast, the momentum factor and Fama-French factors experience much lower returns and much higher volatilities, and hence much lower Sharpe ratios. Interestingly, though, all of the other factors still have positive average returns, while the market suffers an average loss of −0.67% per month in recessions. Another interesting fact is that the momentum factor experiences the greatest increase, more than 5%, in volatility (from 7.69% to 11.5%) in recessions. Finally, consistent with Nagel (2012) finding that short-term reversal strategies do well in recessions due to “evaporating liquidity”, both the short-term reversal and trend factors perform better during recessions than over the entire sample period. However, the performance of the trend factor is robust to recessions because it is based on the MA signals of not only the short-term, but also the intermediate- and long-term.

Panel B of Table 2 reports the summary statistics for the most recent financial crisis period as defined by the national bureau of economic research (NBER). The average return of the trend factor is about 0.75% per month, and the Sharpe ratio is about 0.15. In contrast, all the other factors except SMB (0.63% per month) and LREV (0.03% per month) experience large losses: the market yields −2.03% per month, SREV yields −0.82% per month, and MOM yields −3.88% per month. In addition, the volatility of the MOM factor increases to 13.4%, an increase of more than 74% compared with the whole sample period.

**Table 2**

The trend factor and other factors: Recession periods.

This table reports the summary statistics for the trend factor (*Trend*), the short-term reversal factor (*SREV*), the momentum factor (*MOM*), the long-term reversal factor (*LREV*), and the Fama-French three factors including the market portfolio (*Market*), *SMB*, and *HML* factors. For each factor, we report sample mean in percentage, sample standard deviation in percentage, Sharpe ratio, skewness, and excess kurtosis for the recession periods in Panel A, and for the most recent financial crisis period identified by the NBER in Panel B. The *t*-statistics are in parentheses and significance at the 1% level is given by \*\*\*. The sample period is from June 1930 through December 2014.

Factor	Mean (%)	Std dev (%)	Sharpe ratio	Skewness	Excess kurtosis
<i>Panel A: Recession periods</i>					
Trend	2.34*** (6.38)	5.05	0.46	1.02	5.73
SREV	1.20*** (3.05)	5.40	0.22	0.85	3.35
MOM	0.20 (0.25)	11.5	0.02	−3.20	17.6
LREV	0.49 (1.59)	4.15	0.12	1.25	6.22
Market	−0.67 (−1.13)	8.24	−0.08	0.50	3.90
SMB	0.02 (0.08)	3.32	0.01	0.54	2.01
HML	0.18 (0.48)	5.11	0.03	2.99	19.9
<i>Panel B: Financial crisis (12/2007 - 06/2009)</i>					
Trend	0.75 (0.65)	5.06	0.15	0.83	0.28
SREV	−0.82 (−0.63)	5.66	−0.14	−0.11	−1.11
MOM	−3.88 (−1.26)	13.4	−0.29	−1.42	1.77
LREV	0.03 (0.03)	3.73	0.01	0.19	−0.12
Market	−2.03 (−1.25)	7.07	−0.29	−0.21	−0.24
SMB	0.63 (1.10)	2.50	0.25	0.25	−0.79
HML	−0.44 (−0.50)	3.83	−0.11	−0.83	0.87

## 2.5. Tail risk

Table 3 compares the maximum drawdown, Calmar ratio, and frequency of big losses of the trend factor with those of the other common factors (Panel A). The maximum drawdown (MDD), a popular metric of actively managed funds in practice, is defined as the largest percentage drop in price from a peak to a bottom. The MDD measures the maximum loss of an investor who invests in the asset at the worst time. From June 1930 through December 2014, the MDD is 20.0% for the trend factor, 33.4% for SREV, 99.3% for MOM, 46.8% for LREV, and 76.5% for the market. Hence, in terms of the MDD, the trend factor performs the best.

The Calmar ratio, another widely used metric in the investment industry, is defined as the annualized rate of return divided by the MDD, which measures return versus downside risk. The higher the ratio, the better the risk-return tradeoff. From June 1930 through December 2014, the trend factor has a Calmar ratio of 97.8%, whereas the other four factors have much lower Calmar ratios. The

<sup>8</sup> Data on the trend factor and the program will be posted online. The other factors are available from Ken French's online data library with the momentum factor from the equal-weighted momentum deciles.

**Table 3**

The trend factor and other factors: Extreme values and correlation matrix.

This table reports the maximum drawdown (*MDD*), Calmar ratio, and number of big losses of the trend factor (*Trend*), short-term reversal factor (*SREV*), momentum factor (*MOM*), long-term reversal factor (*LREV*), and the market portfolio (*Market*) in Panel A and the correlation matrix of the factors in Panel B, respectively. The sample period is from June 1930 through December 2014.

Panel A: Extreme values						
Factor	MDD (%)	Calma r(%)	$n(R < -10\%)$	$n(R < -20\%)$	$n(R < -30\%)$	$n(R < -50\%)$
Trend	20.0	97.8	4	0	0	0
SREV	33.4	28.4	6	0	0	0
MOM	99.3	9.59	49	18	6	4
LREV	46.8	8.75	3	0	0	0
Market	76.5	9.80	30	5	0	0

Panel B: Correlation matrix					
	Trend	SREV	MOM	LREV	Market
Trend	1.00	0.35	0.03	0.14	0.20
SREV		1.00	−0.19	0.04	0.20
MOM			1.00	−0.30	−0.32
LREV				1.00	0.26
Market					1.00

**Table 4**

Comparison of trend and momentum.

This table compares the long and short portfolios of the trend factor and momentum factor, respectively. The summary statistics are reported for each of the long and short portfolios over the whole sample period (Panel A), the recession periods (Panel B), and the expansion periods (Panel C) identified by the NBER. A one-sided test of equal mean between the long (short) portfolios of the trend factor and momentum factor is reported in the table labeled as *Differ*. For the long portfolio, the test is  $H_0 : \mu_{trd}^l = \mu_{mom}^l$ ;  $H_1 : \mu_{trd}^l > \mu_{mom}^l$ ; for the short portfolio, the test is  $H_0 : \mu_{trd}^s = \mu_{mom}^s$ ;  $H_1 : \mu_{trd}^s < \mu_{mom}^s$ , where the subscripts *trd* and *mom* denote the trend and momentum factors, respectively; superscripts *l* and *s* denote the long and short portfolios, respectively. The last column (*Corr*) reports the correlation between the long (short) portfolios of the trend factor and momentum factor. Significance at the 1%, 5%, and 10% levels is given by \*\*\*, \*\*, and \*, respectively. The sample period is from June 1930 through December 2014.

Portfolio	Mean (%)	Std dev (%)	Skewness	Excess Kurtosis	Differ (%)	Corr
Panel A: Whole sample period						
Trend long	1.93	7.52	0.58	10.3	0.12	0.88
Momentum long	1.81	7.37	0.18	8.87		
Trend short	0.31	6.86	0.56	13.8	−0.71***	
Momentum short	1.02	11.2	2.92	25.9		
Panel B: Recession periods						
Trend long	0.75	11.3	0.52	5.78	0.80**	0.87
Momentum long	−0.05	8.53	−0.33	3.87		
Trend short	−1.59	10.2	1.14	11.3	−1.33**	
Momentum short	−0.26	16.6	2.31	15.3		
Panel C: Expansion periods						
Trend long	2.21	6.30	0.81	12.2	−0.04	0.91
Momentum long	2.24	7.01	0.48	11.0		
Trend short	0.74	5.74	0.22	10.3	−0.57***	
Momentum short	1.32	9.52	3.24	29.8		

momentum factor yields a Calmar ratio of only 9.59%, the long-term reversal factor delivers a Calmar ratio of only 8.75%, and the market 9.80%. The SREV is the distant second best, with a Calmar ratio of 28.4%.

## 2.6. Further comparison with momentum

Panel B of Table 3 reports the correlation matrix of the trend factor with the other four factors. The trend factor is correlated with the short-term reversal factor (35%), long-term reversal factor (14%), and the market (20%), but is virtually uncorrelated with the momentum factor.

Why does the trend factor have virtually zero correlation with the momentum factor? This is easily understood once we separate the long and short sides of both factors.

Table 4 reports the summary statistics for their long and short portfolios. Due to capturing overall the same trend, the long (short) portfolios of both factors are indeed positively correlated as expected, and the correlation is in fact as high as 88% (84%) (Panel A). However, the trend factor does a much better job in capturing the trend, so it has a greater average return than the momentum factor, 1.93% versus 1.81% for the long leg, and has a much smaller average return, 0.31% versus 1.02% for the short leg. The differences are statistically significant as shown in the second to the last column of the table. This means that both the long and short portfolios of the trend factor outperform those of the momentum factor, and so the trend factor, as the spread portfolio, must outperform the momentum factor. Moreover, we find that the trend factor earns a much

**Table 5**

Crash months.

This table reports the performance of the trend factor for the crash months, periods when the momentum factor performs poorly, i.e., when the Winners–Losers portfolio experiences losses over –25%. We report the returns for the Losers, Winners, and Winners–Losers for the momentum factor, and similarly the Low, High, and High–Low for the trend factor, respectively. Returns are in percentage. The sample period is from June 1930 through December 2014.

Date	Losers (%)	Winners (%)	Winners–Losers (%)	Low (%)	High (%)	High–Low (%)
<i>Panel A: Momentum</i>				<i>Panel B: Trend</i>		
09/1939	104.78	15.08	–89.70	30.09	26.48	–3.61
08/1932	113.98	30.73	–83.25	63.76	43.79	–19.96
01/2001	64.74	4.75	–59.99	17.82	2.76	–15.06
07/1932	68.76	13.02	–55.74	30.20	47.07	16.87
04/2009	45.91	5.46	–40.45	12.90	25.18	12.28
11/1935	40.98	10.69	–30.29	7.16	11.01	3.85
01/1932	31.32	3.18	–28.14	–0.97	13.71	14.68
01/1975	45.85	18.16	–27.69	18.82	28.30	9.48
11/2002	32.73	5.25	–27.48	11.03	6.70	–4.32
06/1938	36.99	10.37	–26.62	16.66	39.27	22.61

higher average return during recessions while the momentum factor earns a small positive average return only because the short leg is a bit more negative than the long leg (Panel B). Similar conclusions hold for the expansion periods. This is the economic reason why returns on the two factors are not correlated over the entire sample period. A statistical reason is that one can set a pair of random variables *A* and *C* to be highly correlated and another pair *B* and *D* to be highly correlated, so that *A*–*B* and *C*–*D* are not correlated.

At the extreme of the tail risk is the crash risk. Daniel, Jagannathan, and Kim (2012) show that the momentum strategy suffers a loss exceeding –20% per month in 13 out of the 978 months from years 1929–2010. Barroso and Santa-Clara (2015) show that the momentum strategy delivers a –91.59% return in just two months in 1932 and a –73.42% over three months in 2009. This evidence motivates us to examine the worst case scenarios for both the momentum and the trend factors.

An intuitive assessment is simply to examine a few of their worst return months. The four worst monthly returns of the momentum factor are –89.70%, –83.25%, –59.99%, and –55.74%, which occurred in September 1939, August 1932, January 2001, and July 1932 (Table 5). In contrast, the trend factor only suffered, during its four worst months, losses of –19.96%, –15.06%, –12.66%, and –10.53% that occurred in August 1932, January 2001, June 2000, and July 1934. In general, for the extreme losses, out of the 1,015 months, only in four months did the trend factor experience a negative return exceeding –10% and not a single month exceeding –20%. In contrast, the momentum factor (MOM) experiences a negative return exceeding –10% in 49 months, exceeding –20% in 18 months, exceeding –30% in six months, and exceeding –50% in four months (Table 3).

To provide further insights on the crash risk of the momentum strategy, Panel A of Table 5 provides the ten biggest losses of the momentum strategy (10–1 spread portfolio) over the entire sample period. Note that all of the biggest losses are greater than –25%. The extreme downside risks are rare but not rare enough (about 1% of all the months). Panel B provides the returns of the trend factor in the same months. It is remarkable that

there are only four months of negative returns; two of the biggest losses are only –19.96% and –16.87%, and the other two losses are below –5%. In fact, in most of the losing months for the momentum strategy, the trend factor earns large and positive returns. For example, in June 1938 when the momentum strategy lost over 26% (–26.62%), the trend factor gained over 22% (22.61%). The results indicate that the trend factor outperforms the momentum factor not only in terms of average returns, but also in terms of downside risks.

## 2.7. Mean-variance spanning tests

Since our trend factor uses information on the short-term, intermediate-term, and long-term price trends, and since the three trends are traditionally captured by the short-term reversal, momentum, and long-term reversal factors, it is a logical question whether a portfolio of the three factors can mimic the performance of the trend factor. For example, can the trend factor outperform an equal-weighted portfolio of the three factors? In fact, the trend factor outperforms any portfolio of the three factors in terms of the Sharpe ratio.

To see why, it is sufficient to show that the trend factor lies outside the mean-variance frontier of the three factors. Huberman and Kandel (1987) are the first to provide a mean-variance spanning test on the hypothesis of whether *N* assets can be spanned or replicated in the mean-variance space by a set of *K* benchmark assets. De Santis (1993), Bekaert and Urias (1996), De Roan, Nijman, and Werker (2001), Korkie and Turtle (2002), and Kan and Zhou (2012) provide additional tests of the same hypothesis. Statistically, we run a regression of the trend factor on the other three factors,

$$r_{Trend,t} = \alpha + \beta_1 r_{SREV,t} + \beta_2 r_{MOM,t} + \beta_3 r_{LREV,t} + \epsilon_t, \quad (13)$$

where  $r_{Trend,t}$ ,  $r_{SREV,t}$ ,  $r_{MOM,t}$ , and  $r_{LREV,t}$  are the returns on the trend factor, the short-term reversal, momentum, and long-term reversal factors, respectively. The spanning hypothesis is equivalent to the following parametric restrictions on the model,

$$H_0 : \alpha = 0, \quad \beta_1 + \beta_2 + \beta_3 = 1. \quad (14)$$



**Table 6**

Mean-variance spanning tests.

This table reports the results of testing whether the trend factor can be spanned by the short-term reversal, momentum, and long-term reversal factors.  $W$  is the Wald test under conditional homoskedasticity,  $W_e$  is the Wald test under the IID elliptical,  $W_a$  is the Wald test under the conditional heteroskedasticity,  $J_1$  is the Bekaert-Urias test with the Errors-in-Variables (EIV) adjustment,  $J_2$  is the Bekaert-Urias test without the EIV adjustment, and  $J_3$  is the DeSantis test. All six tests have an asymptotic chi-squared distribution with  $2N(N-1)$  degrees of freedom. The  $p$ -values are in brackets. The tests are conducted for the whole sample period, recession periods, and the most recent financial crisis period. The sample period is from June 1930 through December 2014.

Period	$W$	$W_e$	$W_a$	$J_1$	$J_2$	$J_3$
Whole sample period	166.86 [0.00]	106.95 [0.00]	95.45 [0.00]	62.14 [0.00]	67.45 [0.00]	78.63 [0.00]
Recession periods	35.51 [0.00]	22.96 [0.00]	26.68 [0.00]	22.50 [0.00]	25.51 [0.00]	29.30 [0.00]
Financial crisis	28.18 [0.00]	25.42 [0.00]	28.17 [0.00]	17.66 [0.00]	17.65 [0.00]	35.58 [0.00]

Following Kan and Zhou (2012), we carry out six spanning tests: Wald test under conditional homoskedasticity, Wald test under independent and identically distributed (IID) elliptical distribution, Wald test under conditional heteroskedasticity, Bekaert-Urias spanning test with errors-in-variables (EIV) adjustment, Bekaert-Urias spanning test without the EIV adjustment and DeSantis spanning test. All six tests have asymptotic chi-squared distribution with  $2N(N-1)$  degrees of freedom.

Table 6 reports the test results for the whole sample period, recession periods, and financial crisis period. The hypothesis is strongly rejected that the trend factor is inside the mean-variance frontier of the short-term reversal, momentum, and long-term reversal factors for all three periods. Overall, the trend factor is clearly a unique factor that captures the cross-section of stock trends and performs far better than the well-known short-term reversal, momentum, and long-term reversal factors.

### 2.8. Sharpe style regressions

In the previous subsection, we present evidence that the trend factor outperforms any portfolio of the short-term reversal, the momentum, and the long-term reversal factors. In this subsection, we ask a different but related question as to how the performance of the trend factor is related to the three factors. To this end, we conduct (Sharpe, 1988) style analysis on the trend factor.

Sharpe (1988) style regression is widely used in fund performance analysis to identify the contribution of various style portfolios to a given fund. In our case, we regress the trend factor on the three factor portfolios,

$$r_{Trend,t} = \alpha + \beta_1 r_{SREV,t} + \beta_2 r_{MOM,t} + \beta_3 r_{LREV,t} + \epsilon_t, \quad (15)$$

s.t.

$$\beta_1 \geq 0, \quad \beta_2 \geq 0, \quad \beta_3 \geq 0, \quad \beta_1 + \beta_2 + \beta_3 = 1,$$

where  $r_{Trend,t}$ ,  $r_{SREV,t}$ ,  $r_{MOM,t}$ , and  $r_{LREV,t}$  are the returns on the trend factor, the short-term reversal, momentum, and long-term reversal factors, respectively. Unlike mean-spanning tests, the style regression does not impose the

**Table 7**

Sharpe style regressions.

This table reports the Sharpe style regression results regressing the returns of the trend factor on the returns of the short-term reversal factor (SREV), momentum factor (MOM), and long-term reversal factor (LREV). The slope coefficients are restricted to be positive and their sum is equal to 100%. Regression results are reported for the whole sample period, recession periods, and expansion periods. Newey and West (1987) robust  $t$ -statistics are in parentheses and significance at the 1%, 5%, and 10% levels is given by \*\*\*, \*\*, and \*, respectively. The sample period is from June 1930 through December 2014.

	Whole sample	Recession	Expansion
SREV	52.2*** (5.77)	69.9*** (8.18)	40.6*** (4.57)
MOM	13.4*** (9.11)	5.0** (2.06)	19.0*** (18.95)
LREV	34.4*** (4.43)	25.1*** (3.57)	40.4*** (5.05)

zero-intercept constraint, allowing for the possibility that the trend factor can outperform a portfolio of the three factors. However, there are nonnegativity constraints on the portfolio weights. Hence, the style regression examines how the best portfolio of the three factors can mimic or explain the variations of the trend factor.

Table 7 reports the results of style regressions for the whole sample period and for the recession/expansion periods. For the whole sample period, the short-term reversal factor accounts for about 52.2% of the movements of the trend factor, the momentum 13.4%, and long-term reversal 34.4%. Hence, the short-term information plays the greatest role in the performance of the trend factor. This is more evident for the recession periods where the contribution of the short-term reversal increases to 69.9%, while that of the momentum decreases to only 5.0%, and that of the long-term reversal decreases to 25.1%. This is also consistent with the average return patterns of the three factors during recession periods, which are 1.20%, 0.20%, and 0.49% (Table 2), respectively. However, in the expansion periods, the contributions are 40.6%, 19.0%, and 40.4%, respectively, which are also consistent with the average returns of 0.70%, 0.93%, and 0.31% of the three factors. In particular, the substantially increased contribution from the momentum factor during the expansion periods is likely due to much higher returns of momentum during those periods.

Overall, the trend factor, which utilizes price information across all the investment horizons, seems to place more emphasis on short-term price patterns than on the intermediate- and long-term. Its superior performance reveals that the factor is able to pick winners under different market conditions with more weights on the winners.

### 2.9. Trend quintile portfolios

Table 8 reports the average returns and characteristics of the equal-weighted quintile portfolios sorted by the expected returns forecasted using the trend signals (trend quintile portfolios). The average returns increase monotonically from the quintile with the lowest forecasted expected returns (Low) to the quintile with the highest forecasted expected returns (High). More specifically, stocks

**Table 8**

Average returns and other characteristics of the trend quintile portfolios.

This table reports the average return and other characteristics of the five trend quintile portfolios. Market size is in millions of dollars.  $R_{-1}(\%)$ ,  $R_{-6,-2}(\%)$ , and  $R_{-60,-25}(\%)$  are prior month return, past six-month cumulative return skipping the last month, past 60-month cumulative return skipping the last 24 months, respectively. IVol(%) is the idiosyncratic volatility relative to the Fama-French three-factor model estimated from the daily returns of each month. %Zero is the percentage of zero returns in a month. Turnover(%) is the monthly turnover rate of the stocks. E/P and S/P are earnings-price ratio and sales-price ratio, respectively. Newey and West (1987) robust *t*-statistics are in parentheses and significance at the 1%, 5%, and 10% levels is given by \*\*\*, \*\*, and \*, respectively. The sample period is from June 1930 through December 2014.

Rank	Return (%)	Market size	logB/M	$R_{-1}(\%)$	$R_{-6,-2}(\%)$	$R_{-60,-25}(\%)$	IVol (%)	%Zero	Turnover (%)	E/P	S/P
Low	0.31 (1.34)	1,244.8*** (8.31)	−1.11*** (−9.89)	7.50*** (16.0)	6.67*** (5.49)	61.5*** (18.7)	2.17*** (31.9)	15.5*** (25.0)	31.6*** (13.5)	1.66*** (5.15)	77.6*** (11.6)
2	0.87*** (4.20)	1,843.1*** (8.36)	−1.06*** (−8.60)	3.29*** (13.0)	6.89*** (7.20)	53.7*** (18.5)	1.76*** (40.8)	15.8*** (25.3)	24.7*** (13.0)	4.16*** (10.9)	88.9*** (13.5)
3	1.12*** (5.62)	2,057.7*** (8.24)	−1.03*** (−8.40)	1.38*** (7.06)	7.59*** (8.49)	51.1*** (17.8)	1.68*** (41.5)	16.2*** (25.2)	23.0*** (12.7)	4.82*** (11.1)	93.1*** (13.8)
4	1.40*** (6.86)	2,046.2*** (7.98)	−1.04*** (−8.51)	−0.41** (−2.11)	8.47*** (9.32)	51.4*** (17.2)	1.75*** (40.0)	16.0*** (25.0)	23.6*** (12.8)	4.73*** (11.1)	97.9*** (13.5)
High	1.93*** (8.18)	1,498.7*** (7.98)	−1.09*** (−9.35)	−3.45*** (−12.5)	10.5*** (9.50)	57.5*** (17.3)	2.17*** (34.3)	15.8*** (25.2)	29.6*** (12.8)	2.28*** (5.25)	91.8*** (13.3)

forecasted to have the highest expected returns (strongest trend forecasts) yield the highest returns on average in the subsequent month, about 1.93% per month, whereas stocks forecasted to have the lowest expected returns (weakest trend forecasts) yield the lowest returns on average in the subsequent month, only about 0.31% per month.

Also worth noting are the large gaps in average returns between the lowest and the second quintiles and the highest and the fourth quintiles—the average return increases by 181% (0.56% per month) and 38% (0.53% per month), respectively.

The market size displays a hump shape across the quintiles – both quintiles Low and High have smaller market size than the other quintiles, while the book-to-market (B/M) ratio stays roughly constant across the quintiles. Not surprisingly, the prior month returns ( $R_{-1}$ ) decrease monotonically across the quintiles, whereas the past six-month cumulative returns ( $R_{-6,-2}$ ) increase monotonically across the quintiles. Idiosyncratic volatility (IVol) displays a U-shaped pattern across the quintiles—the two extreme quintiles have much higher idiosyncratic volatility. We also report the percentage of zero returns (%Zero) and share turnover rate, both of which measure the liquidity of stocks (Lesmond, Ogden, and Trzcinka, 1999). While the percentage of zero returns stays roughly constant across quintiles, the turnover rate displays a U shape—the turnover rate is higher for both extreme quintiles. The last two columns in Table 8 report price ratios. Both the earnings-to-price ratio (E/P) and the sales-to-price ratio (S/P) display a hump-shaped pattern across the quintiles.

## 2.10. Alphas

Can common risk factors explain the return on the trend factor? Table 9 reports Jensen's alpha and risk loadings for the trend quintile portfolios, the trend factor and momentum factor, with respect to the CAPM and Fama-French three-factor model, respectively. The quintile alphas increase monotonically from the lowest quintile to the highest quintile, from −0.71% to 0.84% with respect to the CAPM, and from −0.84% to 0.71% with respect to the Fama-French three-factor model, respectively. As a result, the trend factor, which is the High-Low spread portfolio,

**Table 9**

CAPM and Fama-French alphas.

This table reports Jensen's alpha and risk loadings with respect to the CAPM and Fama-French three-factor model, respectively, for the five trend quintile portfolios, the trend factor, and the MOM factor. The alphas are reported in percentage. Newey and West (1987) robust *t*-statistics are in parentheses and significance at the 1%, 5%, and 10% levels is given by \*\*\*, \*\*, and \*, respectively. The sample period is from June 1930 through December 2014.

Rank	Panel A: CAPM		Panel B: Fama-French			
	$\alpha(\%)$	$\beta_{mkt}$	$\alpha(\%)$	$\beta_{mkt}$	$\beta_{smb}$	$\beta_{hml}$
Low	−0.71*** (−8.19)	1.17*** (41.2)	−0.84*** (−12.1)	1.02*** (40.8)	0.60*** (11.6)	0.16*** (4.22)
2	−0.08 (−1.31)	1.07*** (55.7)	−0.19*** (−4.65)	0.96*** (69.3)	0.42*** (8.63)	0.17*** (4.30)
3	0.17*** (2.61)	1.06*** (43.9)	0.07 (1.58)	0.96*** (49.6)	0.37*** (10.4)	0.19*** (3.65)
4	0.42*** (6.18)	1.12*** (38.9)	0.31*** (7.04)	1.01*** (42.2)	0.37*** (8.02)	0.19*** (3.50)
High	0.84*** (8.93)	1.30*** (51.7)	0.71*** (10.9)	1.15*** (62.4)	0.60*** (12.9)	0.17*** (4.45)
Trend	1.55*** (13.6)	0.13*** (3.28)	1.54*** (12.9)	0.13*** (3.47)	−0.00 (−0.03)	0.01 (0.16)
MOM	1.07*** (6.04)	−0.45*** (−3.23)	1.37*** (7.59)	−0.23*** (−2.93)	−0.49*** (−3.31)	−0.75*** (−3.68)

has a CAPM alpha of 1.55% per month, and a Fama-French alpha of 1.54% per month, only slightly lower than the unadjusted average return (1.63% per month in Table 1). While the market beta and SMB beta are U-shaped, the HML beta is hump-shaped across the quintiles. Hence, the trend factor has a small loading on the market and insignificant SMB and HML betas in the Fama-French three-factor model.

Schwert (2003) finds that, unlike most anomalies, the momentum is alive and well after its publication. Impressively, it also survives the high hurdle proposed by Harvey, Liu, and Zhu (2016) to detect false discoveries. In a comparison with the momentum, the alpha of the trend factor is about 45% larger, and has a *t*-statistic of 13.6 vs. 6.04. Hence, the trend factor appears more reliable statistically than the momentum factor.

Overall, existing factor models cannot explain the return on the trend factor. More complex asset pricing

**Table 10**

Alternative specifications of the trend factor.

This table reports the summary statistics for the various specifications of the trend factor. *Price filter*: stocks whose prices are less than \$5 at the end of the last month are excluded. *Size filter*: stocks in the smallest decile based on the NYSE size breakpoints at the end of the last month are excluded. *No filter*: No size restriction nor price restriction is imposed. *Fama-French*: the trend factor is constructed following the Fama and French (1993) approach; stocks are first independently sorted into two size groups and then three trend groups using NYSE breakpoints, and then averaged across the two size groups. The summary statistics are sample mean in percentage, sample standard deviation in percentage, Sharpe ratio, skewness, and excess kurtosis. The *t*-statistics are in parentheses and significance at the 1%, 5%, and 10% levels is given by \*\*\*, \*\*, and \*, respectively. The sample period is from June 1930 through December 2014.

Specification	Mean (%)	Std dev (%)	Sharpe ratio	Skewness	Excess kurtosis
No filter	2.89*** (20.3)	4.53	0.64	2.07	16.4
Price filter	1.78*** (16.9)	3.37	0.53	1.46	11.2
Size filter	1.85*** (15.6)	3.78	0.49	1.90	12.1
Fama-French	1.89*** (18.8)	3.19	0.59	2.45	15.1

models, such as the consumption-habitat model of Campbell and Cochrane (1999) and the long-run risks model of Bansal and Yaron (2004), are unlikely capable of explaining the trend factor too, similar to the momentum factor case. A simple intuitive reason is that the information in the MAs are often not incorporated in common macroeconomic variables of those models (see e.g. Neely, Rapach, Tu, and Zhou, 2014).

However, it is important to note that in this paper we do not claim that the trend factor is an anomaly. There are two major reasons for this.<sup>9</sup> First, the high alpha of the trend factor does not necessarily imply that it is an anomaly by itself. Since our cited papers and our proposed model justify the predictability of the MA signals, there must be rational risks associated with the predictability. Though challenging, it may be possible to develop a theoretical model that allows the exact trend factor to be traded, and then the alpha (measured by factor models) of the trend factor may simply be a rational compensation of the risks associated with trading it. These risks may manifest themselves as tail risk or jump risk, which are not captured by the common factor models. Moreover, even if the risk premium cannot explain fully the extra return on the trend factor, it is still unclear whether the trend factor is profitable after transaction costs. This is an issue that will be examined in Section 3.

### 3. Robustness

In this section, we show that the superior performance of the trend forecasts is robust to alternative formations. We also present robustness tests that includes controlling for various firm characteristics such as size, book-to-market ratio, past returns, percentage of zero returns, etc. and using the alternative Fama-MacBeth regression methodology. We then discuss the issue of turnover and transaction costs. Finally, we discuss the issue of data-mining and present evidence to mitigate the concerns.

#### 3.1. Alternative formations

Recall that we have excluded stocks priced below \$5 (price filter) and stocks that are in the smallest decile sorted with NYSE breakpoints (size filter). An interesting question is whether the removal of one or both filters can substantially change the performance of the trend factor. Table 10 provides the results. When no filters are applied, the trend factor will have a much greater average return of 2.89% per month instead of 1.63% per month. Moreover, the volatility is slightly higher (4.53%), and the Sharpe ratio is considerably higher (0.64) compared to the early value of 0.47. The skewness and kurtosis are higher too.

Now if we impose only the price filter, the average return is 1.78%, the volatility is 3.37%, and the Sharpe ratio is 0.53, making this version of the trend factor better than the original one. Similarly, if we impose only the size filter, the average return is 1.85%, and the volatility is 3.78%, with a slightly higher Sharpe ratio of 0.49 than before. Therefore, the filters we impose on the trend factor are not to make it better, just to make it more implementable. The results also suggest that small stocks and low priced stocks are more trending. This may be intuitively true as large stocks have more analyst following and more investors, and hence more information transparency and faster price movements to reflect all available information.

Fama and French (1993) use a double sorting procedure to construct their well-known factors. We investigate how our trend factor will perform if it is constructed by their approach. Following Fama and French (1993), we first sort stocks into two groups by NYSE size breakpoints, and then independently sort the stocks into three groups by the forecasted expected returns from the trend signals. The breakpoints are the 30th and 70th NYSE percentiles. The trend factor is then the average of the two portfolios with the high forecasted expected returns minus the average of the two portfolios with the low forecasted expected returns,

$$\begin{aligned} \text{Trend} = & \frac{1}{2} (\text{SmallHigh} + \text{Big High}) \\ & - \frac{1}{2} (\text{Small Low} + \text{Big Low}). \end{aligned} \quad (16)$$

<sup>9</sup> We are grateful to an anonymous referee for these and other insightful suggestions that helped improve the paper substantially.

The average return of the formed trend factor is higher (1.89% per month), the standard deviation is lower (3.19% per month), and thus the Sharpe ratio is higher than that reported in Table 1 (0.59 versus 0.47). The skewness and kurtosis are higher as well.

Finally, consider some alternative specifications of the cross-sectional forecast, Eqs. (3)–(5). First, there is clearly overlap of the data when constructing the predictors and so there are likely correlations of the predictors. Indeed, the highest correlation is over 90% between 800- and 1,000-day MAs. To reduce the high correlations, one way is to use non-overlapping MA signals, 3-day, 4- to 5-day, 6- to 10-day, and 11- to 20-day, etc. Then the average return of the trend factor is 1.63% with a Sharpe ratio of 0.47, the same as before. Another way is to remove some of the correlated ones. A simple way is to choose the MAs of lags 3, 20, 50, 100, 200, and 400. This is also justified intuitively with fewer lags or supported by a training data of say, 10 to 20 years. Interestingly, the results are remarkably insensitive to the choice. The average return of the trend factor remains at 1.63%, and the Sharpe ratio decreases only slightly to 0.46.

Second, we consider how smoothing the predictive betas may affect the trend factor. So far, following Haugen and Baker (1996), we use a lag of 12 months to smooth the betas in (5). If we use a lag of six months instead, the average return of the trend factor is 1.52% with a Sharpe ratio of 0.42. The results are very close to 1.63% and 0.47 of the original trend factor. However, if we do not smooth the betas at all and simply use the current month  $t$  estimates, the average return and the Sharpe ratio become 0.91% and 0.24, respectively. This says that certain smoothing of the betas is necessary to maintain the out-of-sample performance of the trend factor. Econometrically, the estimation of the betas is similar to estimating the slopes in the Fama and MacBeth (1973) regressions. A time-series average of the slopes is necessary to provide a consistent estimate [see, e.g., Shanken and Zhou, 2007 and Bai and Zhou, 2015 for the associated asymptotic theory]. Finally, to see how the estimated betas, from (5), change over time, Fig. 1 provides a plot of the smoothed betas on some of the major MAs, the 20, 100, and 200 lags. It is seen that the betas do change substantially over time, varying from positive values to negative ones. This is consistent with the theoretical results earlier on the predictability of the MAs. It can be shown that the beta in (11) can be positive or negative depending on the proportion of technical traders in the market. If there are too many trend chasers, the slope can be negative. It is positive otherwise. Clearly, the betas of the trend factor are more volatile in the early periods of greater market volatility and recessions. It will be an interesting topic for future research to explore this relation further.

### 3.2. Control variables

To better understand the trend factor, we also sort the stocks while controlling for a variety of firm characteristics that are known to predict cross-section returns. The sorting procedure has been widely used in the literature to check the robustness of the cross-section pricing power

of predictors, examples of which are Ang, Hodrick, Xing, and Zhang (2006), Avramov, Chordia, Jostova, and Philipov (2009), Yu (2011), and Wahal and Yavuz (2013), to name a few.

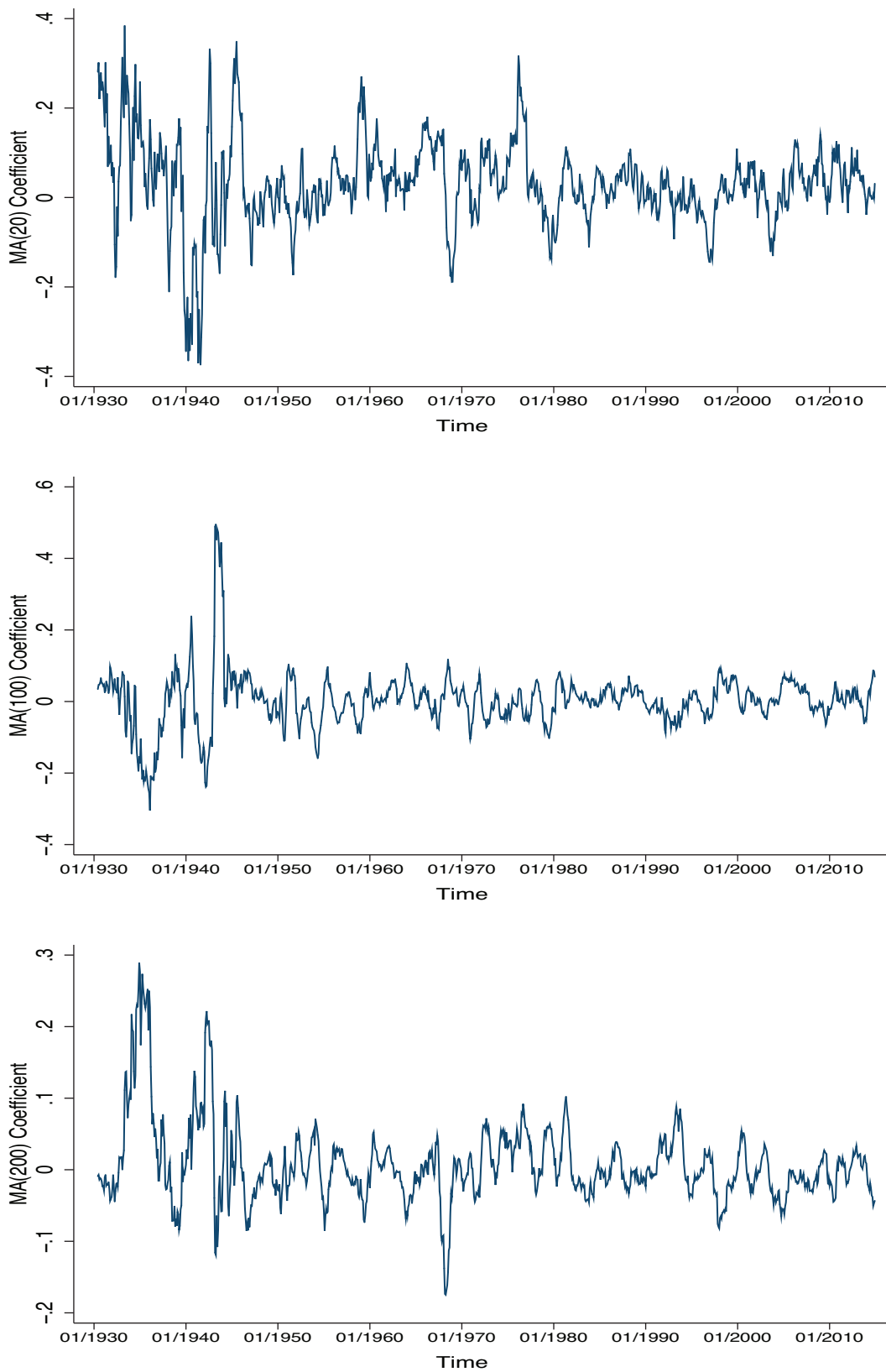
Consider, for example, how to control for market size. We first sort stocks by market size into five quintile groups, and then within each quintile of the market size, we sort stocks further by their trend forecasts to construct five trend quintile portfolios. As a result, a total of  $5 \times 5$  trend quintile portfolios are obtained. Each market size quintile has five trend quintile portfolios ranked from low to high by the trend forecasts, and each trend forecasts quintile has five trend quintile portfolios as well ranked from small to large by the market size. Finally, we average the resulting  $5 \times 5$  trend quintile portfolios across the five quintiles of market size to form five new trend quintile portfolios, all of which should have similar market size to achieve the effect of controlling for market size. We then measure the performance of the trend quintile portfolios by the Fama-French alpha.

Table 11 provides the results of controlling for various firm characteristics. Panel A reports the performance of the  $5 \times 5$  double-sorted trend quintile portfolios using the market size and forecasted expected returns as well as the results of controlling for market size through averaging as described above. It shows that the performance is much stronger for small stocks. For the smallest stocks,<sup>10</sup> the High–Low spread portfolio yields a Fama-French alpha of 2.31% per month. Performance decreases as the market size increases. However, even for the largest stocks, the superior performance of the High–Low spread portfolio is still significant both statistically and economically (0.83% per month). Controlling for market size by averaging across the different market size quintiles still yields a Fama-French alpha of 1.54% per month for the High–Low spread portfolio, identical to the performance reported in Table 9, and consistent with the insignificant size beta reported in Table 9 as well. The robustness to controlling for market size suggests that the performance of the trend factor is not due to small stocks, which may not be too surprising because both the size and price filters are imposed in constructing the trend factor.

Panel B reports the performance of the trend quintile portfolios and the trend factor after controlling for other firm characteristics: the book-to-market ratio (B/M), last month return, past six-month return skipping the first month, past 60-month return skipping the first 24 months, and percentage of zero returns. The superior performance remains largely unchanged. For example, controlling for B/M delivers a Fama-French alpha of 1.37% per month for the High–Low spread portfolio. Controlling for the last month return does reduce the performance to some extent— the High–Low spread portfolio now yields a Fama-French alpha of 1.27% per month. Controlling for liquidity is measured here by the percentage of zero returns, and the performance remains unchanged with the Fama-French alpha of 1.56% per month.

<sup>10</sup> It is worth noting that the smallest decile stocks by NYSE size breakpoints are already excluded.





**Fig. 1.** Times-series of selected MA coefficients. This figure plots the smoothed coefficients of MA(20), MA(100), and MA(200) over the sample period estimated from Eq. (5).

**Table 11**

Performance after controlling for firm characteristics.

This table reports the sort results of controlling for various firm characteristics. Stocks are first sorted by one of the control variables into five quintile groups, and then in each quintile stocks are further sorted to construct five trend quintile portfolios. We then average the resulting  $5 \times 5$  trend quintile portfolios across the five quintiles of the control variable to form five new trend quintile portfolios, all of which should have similar levels of the control variable. In Panel A, we report the performance of the  $5 \times 5$  quintile portfolios and the five new trend quintile portfolios after controlling for the market size. In Panel B, we report the performance of only the new trend quintile portfolios after controlling for one of the firm characteristics. The performance is measured by the Fama-French alpha in percentage. [Newey and West \(1987\)](#) robust *t*-statistics are in parentheses and significance at the 1%, 5%, and 10% levels is given by \*\*\*, \*\*, and \*, respectively. The sample period is from June 1930 through December 2014.

	Trend forecasts					
	Low	2	3	4	High	High–Low
Market size	Panel A: Market size					
Small	–1.28*** (–10.2)	–0.38*** (–4.93)	0.06 (0.99)	0.32*** (4.49)	1.03*** (8.55)	2.31*** (10.3)
2	–0.98*** (–9.85)	–0.23*** (–3.32)	0.13** (1.99)	0.36*** (5.49)	0.84*** (9.77)	1.82*** (11.7)
3	–0.84*** (–10.3)	–0.17*** (–2.93)	0.13** (2.01)	0.35*** (5.58)	0.66*** (8.70)	1.51*** (11.4)
4	–0.62*** (–8.26)	–0.18*** (–3.03)	0.03 (0.50)	0.30*** (4.95)	0.60*** (8.04)	1.23*** (10.1)
Large	–0.47*** (–7.60)	–0.14*** (–3.28)	0.11*** (2.65)	0.19*** (4.52)	0.36*** (6.54)	0.83*** (8.94)
Average over market size	–0.84*** (–12.1)	–0.22*** (–5.37)	0.09** (2.28)	0.30*** (7.27)	0.70*** (10.9)	1.54*** (12.9)
	Panel B: Controlling for firm characteristics					
Average over B/M	–0.73*** (–9.78)	–0.22*** (–4.41)	0.01 (0.11)	0.25*** (4.96)	0.64*** (8.44)	1.37*** (11.2)
Average over $R_{-1}$	–0.69*** (–10.3)	–0.15*** (–3.83)	0.02 (0.53)	0.27*** (7.07)	0.59*** (10.5)	1.27*** (12.2)
Average over $R_{-6,-2}$	–0.82*** (–11.9)	–0.21*** (–5.50)	0.08** (2.15)	0.28*** (7.19)	0.71*** (11.0)	1.53*** (13.0)
Average over $R_{-60,-25}$	–0.82*** (–11.9)	–0.23*** (–5.59)	0.05 (1.07)	0.29*** (6.93)	0.74*** (11.1)	1.56*** (13.1)
Average over %Zeros	–0.84*** (–12.1)	–0.18*** (–4.34)	0.09** (2.11)	0.31*** (7.19)	0.73*** (11.2)	1.56*** (13.4)

### 3.3. Fama-MacBeth regressions

Portfolio sorting, although powerful and capable of capturing nonlinear predictive relations, is often difficult to control for other variables, and it also focuses on extreme portfolios. Fama-MacBeth regression, on the other hand, can control for many variables and focuses on the average (linear) effect. Therefore, we run the Fama-MacBeth regression to further examine the robustness of the results. [Shanken and Zhou \(2007\)](#) argue that weighted least squares (WLS) often generates better results than the ordinary least squares (OLS) used in the first step of the Fama-MacBeth regression. For each stock, we estimate the stock variance using the whole sample period and use the inverse of the variance as the weight.

[Table 12](#) reports the results of regressing the monthly returns on the trend forecasts ( $ER_{trd}^{12}$ ) and various control variables using the weighted Fama-MacBeth cross-sectional regression framework. In the first regression, we examine the predictability of  $ER_{trd}^{12}$  while controlling for market size and book-to-market ratio. As expected,  $ER_{trd}^{12}$  has a significant and positive coefficient indicating that the trend signals can predict future cross-section returns independent of the market size and book-to-market ratio. These results are consistent with the double-sort results in [Table 11](#). In the second regression, we add last-month re-

turn ( $R_{-1}$ , short-term reversal), six-month cumulative return ( $R_{-6,-2}$ , momentum), and last 60-month cumulative return ( $R_{-60,-25}$ , long-term reversal) as additional controls.  $ER_{trd}^{12}$  remains highly significant and the coefficient stays the same. In the third regression, idiosyncratic volatility, percentage of zero returns, and share turnover are included as additional controls, but the results are similar. Similar results are also obtained when three accounting price ratio variables [cash flow-to-price ratio (C/P), E/P, and S/P] are added to the regression.

In the last four regressions in [Table 12](#), we examine the predictability of two alternative trend forecasts,  $ER_{trd}^6$ , the expected return forecasted using the last six-month moving average of coefficients to proxy for the expected coefficients in [Eq. \(5\)](#), and  $ER_{trd}^{60}$ , the expected return forecasted using the last 60-month moving average of coefficients to proxy for the expected coefficients in [Eq. \(5\)](#). The results are surprisingly similar – both alternative trend forecasts are significant and positive.

### 3.4. Transaction costs

In this subsection, we explore the issue of transaction costs. We first compute the turnover rates of the trend factor each month since it is rebalanced monthly. Then, following [Grundy and Martin \(2001\)](#) and [Barroso and Santa-Clara \(2015\)](#), we compute the break-even transaction costs

**Table 12**

Fama-MacBeth regressions.

This table reports the results of regressing monthly returns on the expected returns forecasted by the trend signals and other firm-specific variables. The regression is a modified Fama-MacBeth cross-sectional regression with weighted least squares (WLS) in the first step. The weights are the inverse of the stock variance estimated from the whole sample period. For robustness, the table reports three specifications of the forecasted expected returns,  $ER_{trd}^{12}$ ,  $ER_{trd}^6$ , and  $ER_{trd}^{60}$  using rolling 12-month, 6-month, and 60-month averages, respectively, to estimate the true coefficients. Newey and West (1987) robust t-statistics are in parentheses and significance at the 1% level is given by \*\*\*. The sample period is from June 1930 through December 2014.

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Intercept	−0.17*** (−6.41)	−0.17*** (−3.35)	−0.12*** (−6.46)	−0.18*** (−13.06)	−0.15*** (−6.91)	−0.15*** (−11.76)	−0.18*** (−5.86)	−0.22*** (−15.30)
$ER_{trd}^{12}$	0.67*** (7.84)	0.61*** (4.22)	0.47*** (7.87)	0.58*** (17.15)				
$ER_{trd}^6$					0.54*** (8.85)	0.50*** (15.74)		
$ER_{trd}^{60}$							0.71*** (6.48)	0.69*** (18.78)
Log(size)	−1.00** (−1.98)	−1.00** (−2.19)	−1.21*** (−2.79)	−1.56*** (−3.58)	−1.09** (−2.16)	−1.56*** (−3.59)	−1.02** (−2.01)	−1.59*** (−3.54)
log(B/M)	1.52** (2.06)	1.43** (2.12)	1.39** (2.20)	0.69 (0.80)	1.50** (2.05)	0.68 (0.80)	1.50** (2.03)	0.56 (0.65)
$R_{-1}$		−0.24 (−1.43)	−0.57*** (−7.57)	−0.32*** (−6.20)		−0.37*** (−6.57)		−0.25*** (−4.45)
$R_{-6,-2}$		0.61 (1.03)	0.76** (2.13)	0.27 (0.96)		0.22 (0.77)		0.21 (0.75)
$R_{-60,-25}$		−0.13 (−0.08)	0.16 (0.11)	−1.38** (−2.09)		−1.39** (−2.17)		−1.31** (−1.98)
IVol			−0.15 (−1.39)	−0.10** (−1.98)		−0.11** (−2.03)		−0.12** (−2.26)
Turnover			11.0* (1.76)	9.91** (2.46)		10.1*** (2.61)		10.1** (2.40)
%Zero			−0.39 (−1.14)	−0.55* (−1.73)		−0.56* (−1.79)		−0.55* (−1.66)
C/P				0.40*** (3.84)		0.40*** (3.99)		0.40*** (3.72)
E/P				0.10 (1.37)		0.09 (1.20)		0.12 (1.39)
S/P				−0.35 (−1.64)		−0.32 (−1.56)		−0.41* (−1.67)

**Table 13**

Turnover rates and break-even costs.

This table reports the turnover rate of the trend factor and the corresponding break-even transaction costs (BETCs). Results are also reported for the corresponding (quintile) momentum factor (Panel B). *Zero return*: BETCs that would completely offset the returns or the risk-adjusted returns (Fama-French three-factor alpha); *5% Insignificance*: BETCs that make the returns or the risk-adjusted returns insignificant at the 5% level. Panel C reports the excess turnover rate of the trend factor relative to the momentum factor and the break-even costs to offset the extra returns (risk-adjusted returns) of the trend factor relative to the momentum factor. The sample period is from June 1930 through December 2014.

	Turnover(%)	Break-even costs(%)	
	Mean	Zero return	5% Insignificance
<i>Panel A: Trend factor</i>			
Return	131.2	1.24	1.08
FF Alpha	131.2	1.18	1.00
<i>Panel B: Momentum factor</i>			
Return	75.1	0.68	0.26
FF Alpha	75.1	1.03	0.64
<i>Panel C: Trend – Momentum</i>			
Return	56.1	1.99	1.35
FF alpha	56.1	1.40	0.72

(BETCs). We also do the same for the well-known momentum factor for comparison.

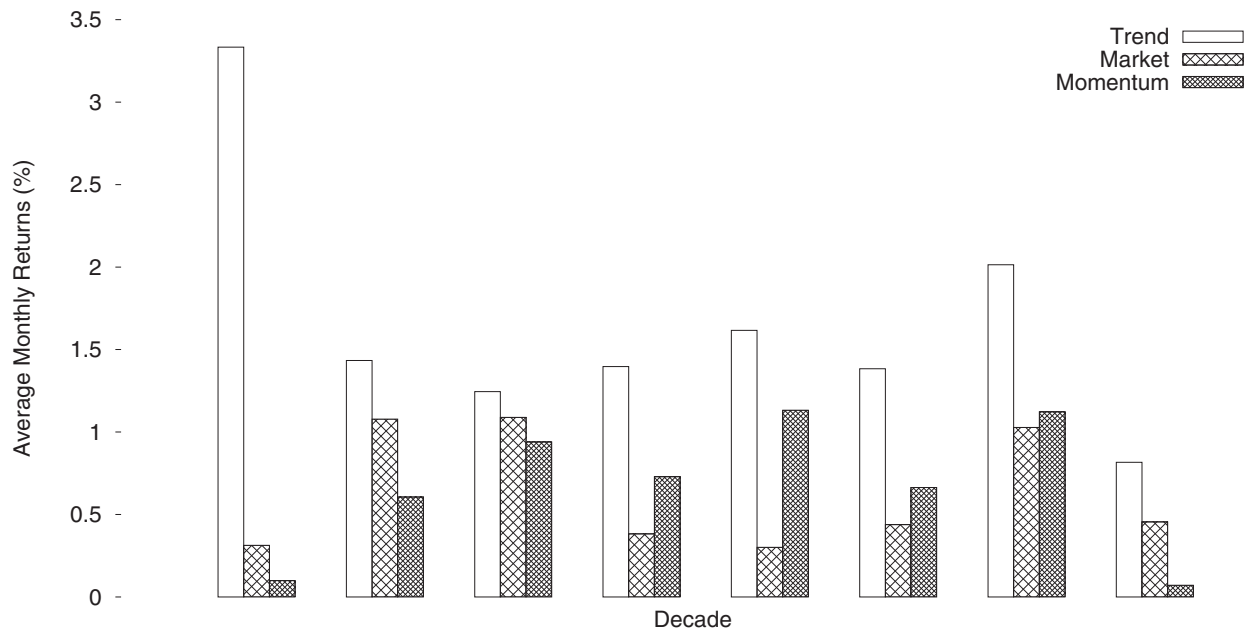
Table 13 reports the results for the trend factor (Panel A) and the momentum factor (Panel B). The turnover rate

of the trend factor is 65.6%. In contrast, the momentum factor has a turnover rate of only 37.6%, which is consistent with both Grundy and Martin (2001) and Barroso and Santa-Clara (2015). Since the trend factor incorporates information across investment horizons, it is not surprising that it has higher turnover rates than the momentum to make use of the information. The question is whether the additional turnover is warranted.

Now we compute the BETCs. Following Grundy and Martin (2001), we compute four types of BETCs. The first is the percentage cost per dollar of trading that one pays so that the trend factor yields a return of exactly zero, and the second is the cost that yields a return statistically insignificant at 5%. The third and the fourth are similar costs that make the Fama and French (1993) alpha of the trend factor to be zero or statistically insignificant.

As shown in Table 13, it takes on average 1.24% of transaction costs to render the trend factor to have zero returns (accounting for the turnover rate). In contrast, it takes only 0.68% to do the same for the momentum factor. In terms of the other three BETCs, the trend factor also outperforms.<sup>11</sup> Since the trend factor has a higher turnover rate than the momentum factor, we also analyze at what level of transaction costs the excess turnover would offset the

<sup>11</sup> The same conclusion holds if the trend factor is value-weighted, and the BETCs will be even higher if it is formed by decile portfolios.



**Fig. 2.** Trend factor performance in subperiods. This figure plots the average monthly returns of the trend factor, the market, and the momentum factor over roughly each of the past eight decades. The first period is from June 1930 to December 1940, the second is from January 1941 to December 1950, and the last is from January 2000 to December 2014.

performance gains of the trend factor relative to the momentum factor. Panel C reports the estimates. For example, it would take about 1.99% per month to eliminate the return difference or about 1.35% to make the difference statistically insignificant at 5%.

Is the trend factor profitable in a practical sense? On the one hand, its BETCs are higher than 50 basis points which is used by some studies as the upper bound for transaction costs (Balduzzi and Lynch, 1999).<sup>12</sup> Also it outperforms the momentum in terms of the BETCs while the momentum is shown profitable by Korajczyk and Sadka (2004). On the other hand, the stocks in the trend factor are not necessarily the same as those of the momentum factor. To obtain the exact real-time transaction costs, we need intraday data as done by Korajczyk and Sadka (2004). In addition, it is not an easy matter to short all the stocks in the short leg of the trend factor and we have to have detailed information on which stocks can be shorted, and then we estimate the costs and their impact on the trend factor. In short, while our evaluation based on the BETCs shows profitability potential, it is limited in scope. Whether the trend factor is truly practically profitable or not requires further research, with a comprehensive analysis such as Korajczyk and Sadka (2004) and Novy-Marx and Velikov (2016), that goes beyond the scope of the current paper.

### 3.5. Data-mining issues

In this subsection, we examine issues associated with data-mining. While it is extremely difficult or impossible

to overcome concerns on data-mining, we discuss four aspects where the trend factor appears to mitigate the concerns.

First, our choice of MAs as predictive signals are economically motivated and the design of the cross-section forecasting procedure is straightforward without any optimization or pre-selecting of the best signals. It is well-known that the MAs are simple technical indicators to capture the price trend in the market. Many studies back in the 1970s find that they are profitable in the commodity futures markets [see, e.g., Park and Irwin, 2007 for a survey of the literature]. Recent studies such as Szakmary, Shen, and Sharma (2010) find that various trend-following strategies continue to be profitable, consistent with Burghardt and Walls (2011) who show that a simple mechanical trading rule based on the MAs can yield over 70% return correlation with the managed futures index. Interestingly, we have shown here that similar trend-following strategies also work in the cross-section of stocks, which has various economic rationales as reviewed in the introduction. Therefore, both theoretically and empirically, there appear to be genuine price trends in the stock market. Otherwise, the trend factor would be pure noise and could not yield significant returns and alphas.

Second, the performance of the trend factor is not driven by a few outliers, but is remarkably stable over time. Fig. 2 plots the average returns of the trend factor over the past eight decades. It is seen that the trend factor does well before and after the 1970s, and actually has more or less similar performance except for the first decade in which it does much better (the overall performance will not change in any significant way if we exclude this decade). In contrast, the performance of the market varies more over the eight decades. It is interesting that

<sup>12</sup> The bound makes more sense after 1975 because fixed commissions on the NYSE were not abolished until then.



the trend factor outperforms the market every decade. In contrast, the momentum factor fails to do so half of the time. Moreover, the trend factor outperforms the momentum every decade too. Its stable performance supports the view that the trend factor is unlikely the result of data-mining.

Third, the trend factor performs well with a false discovery test and with alternative data sets. Clearly, our construction of the trend factor is influenced to a certain degree by previous studies using the same U.S. data, an issue of data-snooping bias (see, e.g., [Lo and MacKinlay, 1990a](#)). Guarding against false discoveries of patterns in such situations, [Harvey, Liu, and Zhu \(2016\)](#) raise the bar of the significant *t*-statistic test from a value of 2 to 3. Our trend factor passes it easily with a value of 13.6 (the momentum has a value of 6.04).

Fourth, [Schwert \(2003\)](#) emphasizes that the use of alternative data sets is one way to mitigate the concern of data-snooping. With this in mind, we consider the trend factor in other G7 countries. Since there are fewer stocks in some of the countries and they are generally more volatile, we use only the less correlated MAs of lags 3, 20, 50, 100, 200, and 400 days and also remove 5% of the outliers (2.5% of stocks that have gone up or down the most in a month) in running the time *t* cross-section regression (3).<sup>13</sup> To make the comparison more on an equal basis, we apply the same winsorization in forming the short-term reversal, momentum, and long-term reversal factors. [Table 14](#) reports the results. The trend factor outperforms the market sharply in all countries, as high as up to 5 times in Japan. The CAPM alphas range from 1–2% per month in all countries except for U.K. (0.83%). However, the Sharpe ratios are mostly less than 0.25 (except for Germany (0.42) and France (0.31)), much lower than the Sharpe ratio in the U.S. (0.47). Given higher transaction costs, the trade factor is likely less profitable overseas. Moreover, the trend factor, while it has higher Sharpe ratios across all the countries, does not outperform the other factors as much as in the U.S. in terms of the average return. The reason is that, in the other G7 countries, at least one of the three factors, the short-term reversal, momentum, and long-term reversal, is not strong. As a result, pooling price information across investment horizons contributes less than the case in the U.S. Nevertheless, the trend factor is clearly valid internationally, yielding the best performance among all the three factors and the local market indexes.

Overall, our discussions above and the analysis in the early part of this section suggest that the trend factor is likely a genuine factor. However, since similar trend-following strategies continue to work in commodity markets after their publications, there are likely risk premia with these strategies and the trend factor too. Theoretical

models are called for to link their superior performance to quantifiable risks.<sup>14</sup>

#### 4. Trends and information uncertainty

In this section, we examine the performance of the trend forecasts for different groups of stocks that are characterized by different degrees of information uncertainty.

When information about stocks is very uncertain, or when the noise-to-signal ratio is very high, fundamental signals, such as earnings and economic outlook, are likely to be imprecise, and hence investors tend to rely more heavily on technical signals. Therefore, trend signals are likely more profitable for high information-uncertain stocks than for low information-uncertain stocks.

We use a number of variables to proxy for information uncertainty, including market size, idiosyncratic volatility, trading turnover rate, analyst coverage (number of analysts following), and firm age. All are proxies used in the previous literature. For example, [Zhang \(2006\)](#) uses market size, firm age, analyst coverage, analyst forecast dispersion, stock volatility, and cash flow volatility as proxies for information uncertainty. [Berkman, Dimitrov, Jain, Koch, and Tice \(2009\)](#) use income volatility, stock volatility, analyst forecast dispersion, and firm age to proxy for information uncertainty.<sup>15</sup>

We use the double-sort procedure described previously to examine the impact of information uncertainty on the performance of the trend forecasts. Briefly, we sort stocks first by the proxy of information uncertainty into three terciles, and then further sort stocks in each tercile into five trend quintile portfolios, thus producing  $3 \times 5$  trend quintile portfolios. We also examine the performance of the trend forecasts after controlling for the information-uncertainty proxy by averaging across all levels of the information-uncertainty proxy as described previously.

The results for using market size as the proxy for information uncertainty are reported earlier in Panel A of [Table 11](#). The Fama-French alpha of the High-Low spread portfolio monotonically increases from 0.83% to 2.31% as the market size (information uncertainty) decreases (increases) from the largest (lowest) to the smallest (highest).

[Table 15](#) reports the performance of the trend forecasts under different levels of information uncertainty as measured by the other four proxies. In Panel A the Fama-French alpha of the High-Low spread portfolio monotonically increases as the idiosyncratic volatility (information uncertainty) increases. The Fama-French alpha of both the weakest and the strongest trend forecast quintiles changes drastically as idiosyncratic volatility (information uncertainty) increases, but in opposite directions. The Fama-French alpha of the weakest trend forecast quintile (Low) decreases as the information uncertainty increases, while the performance of the strongest trend forecast quintile (High) increases at the same time. In other words, stocks

<sup>13</sup> If the issues (which make almost no difference in the U.S. data) are ignored and the original procedure is used, the trend factor continues to outperform the market substantially with high average returns and large CAPM alphas (around or above 1.0% per month). However, in some countries, its average returns can be lower than those of other factors while its Sharpe ratios are higher.

<sup>14</sup> The simple theoretical model developed in this paper can only accommodate one MA signal, not the multiple signals of various trend-following strategies.

<sup>15</sup> We have also examined dispersions in analyst earnings forecasts and quarterly operating income volatility and obtained similar results.

**Table 14**

The trend factor in other G7 countries.

This table reports the summary statistics for the trend factor (*Trend*), the short-term reversal factor (*SREV*), the momentum factor (*MOM*), the long-term reversal factor (*LREV*), and the local market portfolio (*Market*) in the remaining G7 countries. For each factor, we report sample mean in percentage, sample standard deviation in percentage, Sharpe ratio, skewness, minimum, maximum, and CAPM alpha with respect to the respective local market index. The *t*-statistics are in parentheses and significance at the 1%, 5%, and 10% levels is given by \*\*\*, \*\*, and \*, respectively. The sample period is from January 1990 through December 2014.

G7 country	Mean (%)	Std dev (%)	Sharpe ratio	Skewness	Minimum	Maximum	Alpha (%)
France							
Trend	1.52*** (5.11)	4.87	0.31	0.54	−18.6	31.6	1.57*** (6.10)
SREV	1.25*** (3.55)	5.61	0.22	−0.08	−25.8	27.9	1.36*** (4.12)
MOM	0.63* (1.92)	5.28	0.12	0.14	−26.8	30.5	−0.57* (−1.83)
LREV	0.17 (0.53)	4.53	0.04	−0.24	−24.5	14.9	−0.23 (−0.69)
Market	0.48 (1.49)	5.27	0.09	−0.36	−16.0	13.5	
U.K.							
Trend	0.82*** (3.56)	3.76	0.22	0.18	−15.4	20.0	0.83*** (4.26)
SREV	0.57** (2.09)	4.36	0.13	0.13	−23.2	27.8	0.63** (2.30)
MOM	0.18 (0.76)	3.75	0.05	−0.18	−14.8	14.8	−0.13 (−0.52)
LREV	0.22 (1.05)	3.02	0.07	0.64	−6.37	10.8	−0.25 (−0.89)
Market	0.42* (1.66)	4.17	0.10	−0.55	−14.8	10.7	
Germany							
Trend	1.92*** (6.88)	4.57	0.42	0.36	−12.1	20.4	1.96*** (7.41)
SREV	1.65*** (4.46)	5.93	0.28	0.03	−26.2	34.1	1.78*** (5.49)
MOM	0.48 (1.42)	5.46	0.09	1.25	−22.9	34.5	−0.40 (−1.42)
LREV	−0.36 (−1.18)	4.35	−0.08	0.50	−12.8	17.5	0.30 (0.91)
Market	0.63* (1.77)	5.83	0.11	−0.68	−20.9	16.2	
Italy							
Trend	1.25*** (3.48)	5.87	0.21	3.10	−15.7	57.8	1.18*** (3.80)
SREV	0.97*** (2.79)	5.56	0.17	2.28	−21.4	50.0	0.94*** (2.93)
MOM	0.22 (0.80)	4.50	0.05	−0.84	−26.6	13.9	−0.24 (−0.91)
LREV	−0.67** (−2.30)	4.12	−0.16	−0.13	−18.7	13.7	0.66** (2.31)
Market	0.49* (1.74)	4.64	0.11	1.33	−15.1	23.5	
Canada							
Trend	1.05*** (3.41)	5.05	0.21	−0.06	−18.7	17.2	1.02*** (2.95)
SREV	0.71** (2.09)	5.43	0.13	−0.85	−33.5	15.9	0.74** (2.28)
MOM	−0.11 (−0.33)	5.44	−0.02	−0.07	−22.1	22.3	0.23 (0.73)
LREV	0.49 (1.52)	4.57	0.11	0.20	−12.6	15.5	−0.67** (−2.24)
Market	0.56** (2.11)	4.38	0.13	−0.94	−18.1	11.3	

(continued on next page)

Table 14 (continued)

G7 country	Mean (%)	Std dev (%)	Sharpe ratio	Skewness	Minimum	Maximum	Alpha (%)
Japan							
Trend	1.01*** (4.17)	3.97	0.25	0.07	−15.9	17.3	1.00*** (3.50)
SREV	0.11 (0.36)	4.93	0.02	−0.29	−25.7	21.0	0.13 (0.39)
MOM	0.81*** (3.36)	3.91	0.21	1.38	−9.35	26.2	−0.80*** (−3.05)
LREV	0.38* (1.66)	3.22	0.12	−0.17	−10.8	11.1	−0.38 (−1.52)
Market	0.23 (0.67)	5.59	0.04	−0.27	−26.5	13.9	

Table 15

Performance under information uncertainty.

This table reports the performance of the trend quintile portfolios and the trend factor (High–Low) under information uncertainty proxied by idiosyncratic volatility (*IVol*) (Panel A), share turnover rate (Panel B), analyst coverage (Panel C), and firm age (Panel D). Stocks are first sorted by one of the information-uncertainty proxies into three tercile groups, and then in each tercile stocks are further sorted to construct five trend quintile portfolios. For each of the information-uncertainty proxies, the sorted terciles are arranged in the order of increasing information uncertainty. We report the Fama–French alphas for the resulting 3 × 5 trend quintile portfolios and the average across the three terciles of the information-uncertainty proxy. The alphas are reported in percentage. Newey and West (1987) robust *t*-statistics are in parentheses and significance at the 1%, 5%, and 10% levels is given by \*\*\*, \*\*, and \*, respectively. The sample period is from June 1930 through December 2014.

	Trend forecasts					
	Low	2	3	4	High	High–Low
Panel A: Idiosyncratic volatility						
<i>IVol</i>						
Low	−0.24*** (−3.79)	0.09 (1.56)	0.15*** (2.90)	0.33*** (5.95)	0.63*** (9.64)	0.87*** (9.57)
2	−0.53*** (−7.99)	−0.17*** (−3.38)	0.18*** (3.38)	0.40*** (7.33)	0.87*** (12.6)	1.40*** (13.4)
High	−1.45*** (−12.1)	−0.74*** (−12.3)	−0.31*** (−5.60)	0.13** (2.43)	0.80*** (6.57)	2.25*** (10.3)
Average over <i>IVol</i>	−0.74*** (−11.1)	−0.27*** (−7.15)	0.01 (0.20)	0.29*** (7.55)	0.77*** (11.6)	1.51*** (12.7)
Panel B: Turnover rate						
Turnover						
High	−0.80*** (−9.83)	−0.31*** (−4.75)	−0.02 (−0.39)	0.19*** (2.92)	0.38*** (4.33)	1.18*** (10.3)
2	−0.68*** (−10.7)	−0.11** (−2.21)	0.08 (1.46)	0.37*** (7.65)	0.72*** (11.7)	1.40*** (14.9)
Low	−0.95*** (−8.41)	−0.12* (−1.71)	0.06 (0.93)	0.34*** (5.75)	0.95*** (10.4)	1.90*** (10.9)
Average over turnover	−0.81*** (−12.5)	−0.18*** (−4.69)	0.04 (1.00)	0.30*** (7.56)	0.68*** (11.9)	1.49*** (14.3)
Panel C: Analyst coverage						
Analyst coverage						
High	−0.54*** (−5.08)	−0.13 (−1.64)	0.03 (0.47)	0.22*** (2.63)	0.39*** (3.69)	0.93*** (6.06)
2	−0.73*** (−7.94)	−0.16** (−2.28)	0.16** (2.10)	0.39*** (5.10)	0.63*** (7.14)	1.36*** (10.2)
Low	−0.89*** (−11.6)	−0.20*** (−4.21)	0.05 (0.94)	0.30*** (6.09)	0.72*** (10.5)	1.61*** (12.8)
Average over analyst coverage	−0.83*** (−12.1)	−0.18*** (−4.51)	0.07* (1.68)	0.30*** (6.86)	0.70*** (10.8)	1.53*** (12.9)
Panel D: Firm age						
Firm age						
Old	−0.68*** (−10.2)	−0.19*** (−3.36)	0.02 (0.29)	0.21*** (3.60)	0.44*** (6.20)	1.11*** (11.1)
2	−0.75*** (−11.0)	−0.18*** (−4.02)	0.03 (0.60)	0.29*** (6.28)	0.75*** (10.4)	1.50*** (12.9)
Young	−0.91*** (−11.4)	−0.23*** (−4.21)	0.11** (2.12)	0.32*** (6.52)	0.77*** (11.1)	1.68*** (13.5)
Average over age	−0.84*** (−12.2)	−0.19*** (−4.56)	0.07* (1.65)	0.31*** (7.07)	0.74*** (11.3)	1.57*** (13.3)

with poor outlook perform even worse and stocks with strong outlook perform even better when idiosyncratic volatility is high or when information is more uncertain. As a result, the Fama-French alpha of the High–Low spread portfolio increases from 0.87% per month for stocks with the lowest information uncertainty to as high as 2.25% per month for stocks with the highest information uncertainty. In addition, controlling for the information uncertainty (idiosyncratic volatility) by averaging over the three terciles of idiosyncratic volatility yields similar performance to the single sort in Table 9.

In Panel B a similar pattern is observed when trading turnover rate is used to proxy for information uncertainty – low turnover stocks have a high degree of information uncertainty. The performance of the High–Low spread portfolio increases monotonically as the turnover rate (information uncertainty) decreases (increases) – the Fama-French alpha increases from 1.18% per month to 1.90% per month. Similarly, controlling for the trading turnover rate does not reduce the performance.

In Panel C, we use analyst coverage as a proxy for information uncertainty. Stocks that are followed by more analysts should have less information uncertainty. The performance of the High–Low spread portfolio monotonically increases as the number of analysts following (information uncertainty) decreases (increases) across the quintiles – the Fama-French alpha increases from 0.93% per month to 1.61% per month. Controlling for the number of analysts following by averaging across the terciles still yields similar performance.

Finally, we use the age of the firm to proxy for information uncertainty or noise-signal ratio in Panel D. Younger firms are subject to higher information uncertainty.<sup>16</sup> We observe a similar pattern—from the oldest age quintile (Old) to the youngest age quintile (Young), the Fama-French alphas increase monotonically from 1.11% per month to 1.68% per month. Again, controlling for firm age still yields significant Fama-French alphas and the magnitude is similar to what is achieved in the single sort shown in Table 9.

## 5. Cross-section pricing

In this section, we examine how well the trend factor can explain the cross-section stock portfolios sorted by industry, size, and book-to-market, as well as hedge fund portfolios sorted by style.

Consider first the standard ten industry portfolios, which are returns on consumer nondurables, consumer durables, manufacturing, energy, business equipment, telecom, retail, health, utilities, and others (the data are readily available from Ken French's data library). We investigate how well their returns are explained by the CAPM, or by the CAPM with the trend factor, or by the CAPM with the momentum factor, respectively. Panel A of Table 16 reports the CAPM results. All but two industry portfolios have highly significant alphas. The highest alpha is about

0.42% per month (for the energy industry), and the lowest is about 0.07% per month (for the consumer durables industry). To assess further the cross-section overall pricing errors, we also provide a weighted summary of the alphas,

$$\Delta = \alpha' \Sigma^{-1} \alpha, \quad (17)$$

where  $\Sigma$  is the variance-covariance matrix of the residuals across the ten decile portfolios. Shanken (1992) is the first to introduce this measure (different from his paper by a scalar), which is related to the optimal portfolio that exploits the pricing errors. The larger the  $\Delta$ , the greater the difference between investing in the factor portfolio(s) and all the portfolios used on the left-hand side of the regressions. The second to the last column of Panel A reports a value of 2.40% per month. Economically, to a mean-variance investor with a risk aversion of five, this translates into an annual utility gain of 2.88% ( $= 2.40\% \times 12 / (2 \times 5)$ ) over investing in the factor(s) alone. The last column reports the GRS test of Gibbons, Ross, and Shanken (1989), which strongly rejects the hypothesis that all the alphas are zero or that the CAPM can fully explain the ten industry portfolios.

Panel B of Table 16 reports the results when the trend factor is added into the CAPM regressions. The number of significant alphas is reduced from seven to five. In contrast, all industry portfolios have highly significant alphas when the momentum factor is added to the CAPM regressions instead of the trend factor (Panel C). Moreover, the magnitude of the aggregate pricing error ( $\Delta$ ) is 4.30%, which is about 22% greater than 3.20% of the CAPM with the trend factor. However, the CAPM with the addition of either the trend or the momentum factor is still rejected strongly by the GRS test. Nevertheless, the trend factor performs better in explaining the industry returns than the momentum factor.

Consider now six portfolios formed based on an independent double-sort by the market size and book-to-market. Panel A of Table 17 shows that four out of six CAPM alphas are highly significant, with an aggregate pricing error ( $\Delta$ ) of 6.13%. Adding the trend factor reduces the number of significant alphas to two, and the aggregate pricing error is 5.78%, smaller than the one under the CAPM. In contrast, adding the momentum factor increases not only the significance but also the magnitude of the alphas. As a result, the aggregate pricing error is increased to 8.32%, about 43% greater compared to the trend factor. The GRS statistics show that all three models are still rejected, but the CAPM plus the trend factor has the smallest value, suggesting that the trend factor improves performance in terms of explaining the cross-section returns of the six size and book-to-market portfolios.

Finally, we consider portfolios of hedge fund returns sorted by style. We sort the hedge fund returns by 11 popular styles: convertible arbitrage, dedicated short bias, emerging markets, equity market neutral, event driven, fixed income arbitrage, fund of funds, global macro, long/short equity hedge, managed futures, and multi-strategy [the data are from Cao, Rapach, and Zhou, 2014 that span from January, 1994 to December, 2014]. Panel A of Table 18 shows that all style portfolios have significant alphas. In

<sup>16</sup> We exclude firms younger than two years old to allow for enough data.



**Table 16**

Explaining industry portfolios.

This table compares the pricing ability of the trend factor and the momentum factor using ten industry portfolios (*Ind*). Panel A is the CAPM result; Panels B and C include the trend factor and the momentum factor, respectively. The intercept ( $\alpha$ ) is in percentage.  $\beta_{mkt}$ ,  $\beta_{trd}$ , and  $\beta_{mom}$  are the risk loadings on the market portfolio, the trend factor, and the momentum factor, respectively. The  $t$ -statistics are in parentheses and significance at the 1%, 5%, and 10% levels is given by \*\*\*, \*\*, and \*, respectively. The second to the last column is the aggregate pricing error  $\Delta = \alpha' \Sigma^{-1} \alpha$ . The last column is the GRS test statistics with their  $p$ -values in brackets. The sample period is from June 1930 through December 2014.

	Ind1	Ind2	Ind3	Ind4	Ind5	Ind6	Ind7	Ind8	Ind9	Ind10	$\Delta(\%)$	GRS
<i>Panel A: CAPM</i>												
$\alpha(\%)$	0.25** (2.46)	0.07 (0.50)	0.21** (2.07)	0.37** (2.08)	0.23 (1.57)	0.30** (2.18)	0.23* (1.89)	0.42*** (3.14)	0.20 (1.45)	0.23* (1.76)	2.40	2.38*** [0.01]
$\beta_{mkt}$	1.07*** (56.98)	1.43*** (54.68)	1.31*** (68.54)	1.19*** (36.50)	1.43*** (53.78)	1.06*** (42.13)	1.16*** (51.00)	1.07*** (43.90)	0.91*** (36.64)	1.25*** (51.20)		
<i>Panel B: CAPM plus trend factor</i>												
$\alpha(\%)$	0.19* (1.68)	−0.04 (−0.23)	0.14 (1.26)	0.31 (1.58)	0.11 (0.70)	0.40*** (2.65)	0.29** (2.12)	0.28* (1.93)	0.22 (1.48)	0.29** (1.97)	3.20	2.62*** [0.00]
$\beta_{mkt}$	1.06*** (55.58)	1.42*** (53.29)	1.30*** (66.90)	1.18*** (35.60)	1.42*** (52.39)	1.07*** (41.64)	1.17*** (50.16)	1.06*** (42.64)	0.91*** (35.96)	1.25*** (50.35)		
$\beta_{trd}$	0.04 (1.34)	0.07 (1.64)	0.05 (1.50)	0.04 (0.75)	0.08* (1.77)	−0.06 (−1.59)	−0.03 (−0.96)	0.09** (2.26)	−0.01 (−0.37)	−0.04 (−0.91)		
<i>Panel C: CAPM plus momentum factor</i>												
$\alpha(\%)$	0.47*** (5.20)	0.34*** (2.60)	0.44*** (4.73)	0.57*** (3.28)	0.43*** (3.07)	0.45*** (3.38)	0.45*** (3.84)	0.51*** (3.86)	0.30** (2.22)	0.57*** (5.14)	4.30	4.17*** [0.00]
$\beta_{mkt}$	0.98*** (55.85)	1.32*** (52.31)	1.22*** (68.15)	1.10*** (33.17)	1.34*** (50.29)	0.99*** (38.68)	1.07*** (48.05)	1.03*** (40.58)	0.87*** (33.56)	1.10*** (51.59)		
$\beta_{mom}$	−0.21*** (−16.90)	−0.25*** (−14.23)	−0.21*** (−16.72)	−0.19*** (−7.98)	−0.19*** (−9.92)	−0.14*** (−7.98)	−0.20*** (−12.66)	−0.09*** (−4.93)	−0.10*** (−5.29)	−0.32*** (−21.10)		

**Table 17**

Explaining size and book-to-market sorted portfolios.

This table compares the pricing ability of the trend factor and the momentum factor using six size and book-to-market portfolios (*SZBM*). Panel A is the CAPM result; Panels B and C include the trend factor and the momentum factor, respectively. The intercept ( $\alpha$ ) is in percentage.  $\beta_{mkt}$ ,  $\beta_{trd}$ , and  $\beta_{mom}$  are the risk loadings on the market portfolio, the trend factor, and the momentum factor, respectively. The  $t$ -statistics are in parentheses and significance at the 1%, 5%, and 10% levels is given by \*\*\*, \*\*, and \*, respectively. The second to the last column is the aggregate pricing error  $\Delta = \alpha' \Sigma^{-1} \alpha$ . The last column is the GRS test statistics with their  $p$ -values in the brackets. The sample period is from June 1930 through December 2014.

	SZBM1	SZBM2	SZBM3	SZBM4	SZBM5	SZBM6	$\Delta(\%)$	GRS
<i>Panel A: CAPM</i>								
$\alpha(\%)$	−0.09 (−0.66)	0.36*** (3.03)	0.71*** (4.10)	−0.04 (−0.86)	0.15** (2.55)	0.23** (2.21)	6.13	10.2*** [0.00]
$\beta_{mkt}$	1.31*** (51.52)	1.27*** (57.32)	1.38*** (43.26)	1.08*** (123.02)	1.14*** (104.10)	1.31*** (67.71)		
<i>Panel B: CAPM plus trend factor</i>								
$\alpha(\%)$	−0.06 (−0.41)	0.34** (2.55)	0.70*** (3.66)	−0.08 (−1.52)	0.07 (1.05)	0.15 (1.29)	5.78	7.93*** [0.00]
$\beta_{mkt}$	1.31*** (50.56)	1.27*** (56.03)	1.38*** (42.33)	1.08*** (120.31)	1.13*** (101.80)	1.31*** (66.06)		
$\beta_{trd}$	−0.02 (−0.46)	0.02 (0.52)	0.01 (0.18)	0.03* (1.78)	0.05*** (3.10)	0.05* (1.75)		
<i>Panel C: CAPM plus momentum factor</i>								
$\alpha(\%)$	0.17 (1.36)	0.62*** (5.79)	1.16*** (8.00)	−0.02 (−0.50)	0.24*** (4.22)	0.46*** (4.81)	8.32	13.5*** [0.00]
$\beta_{mkt}$	1.20*** (49.05)	1.16*** (56.06)	1.19*** (42.69)	1.07*** (116.28)	1.10*** (100.84)	1.22*** (66.99)		
$\beta_{mom}$	−0.25*** (−14.31)	−0.24*** (−16.59)	−0.42*** (−21.46)	−0.02** (−2.37)	−0.08*** (−10.78)	−0.21*** (−16.29)		

contrast with the passive portfolios sorted by industry and market size and book-to-market, the aggregate pricing error is much higher, indicating the difficulty of using the CAPM alone to explain hedge fund returns. Adding the trend factor slightly reduces the magnitude and significance of the alphas, and the aggregate pricing error comes down from 252.6% to 234.5%. The global macro (HF8) and long/short equity hedge (HF9) are known to be sensitive

to market trends, and the trend factor is able to explain some of their alphas. In contrast, the momentum factor explains less. This is one of the reasons why the aggregate pricing error of the CAPM with the momentum factor is the largest, 255.6%. Overall, while the momentum factor together with the CAPM is far from sufficient in explaining the hedge fund returns, the trend factor performs still slightly better than the momentum factor in that task.

**Table 18**

Explaining hedge fund portfolios.

This table compares the pricing ability of the trend factor and the momentum factor using 11 hedge fund style portfolios (HF). Panel A is the CAPM result; Panels B and C include the trend factor and the momentum factor, respectively. The intercept ( $\alpha$ ) is in percentage.  $\beta_{mkt}$ ,  $\beta_{trd}$ , and  $\beta_{mom}$  are the risk loadings on the market portfolio, the trend factor, and the momentum factor, respectively. The  $t$ -statistics are in parentheses and significance at the 1%, 5%, and 10% levels is given by \*\*\*, \*\*, and \*, respectively. The second to the last column is the aggregate pricing error  $\Delta = \alpha' \Sigma^{-1} \alpha$ . The last column is the GRS test statistics with their  $p$ -values in brackets. The sample period is from January 1994 through December 2014.

	HF1	HF2	HF3	HF4	HF5	HF6	HF7	HF8	HF9	HF10	HF11	$\Delta(\%)$	GRS
<i>Panel A: CAPM</i>													
$\alpha(\%)$	0.23** (2.08)	0.40*** (2.61)	0.37* (1.84)	0.40*** (8.83)	0.42*** (6.00)	0.43*** (6.79)	0.17** (2.33)	0.56*** (5.94)	0.53*** (5.84)	0.66*** (3.72)	0.65*** (13.54)	252.6	227.3*** [0.00]
$\beta_{mkt}$	0.22*** (8.94)	−0.60*** (−17.40)	0.59*** (13.32)	0.08*** (8.13)	0.26*** (16.49)	0.08*** (5.49)	0.19*** (12.12)	0.16*** (7.46)	0.44*** (21.99)	−0.06 (−1.44)	0.14*** (13.62)		
<i>Panel B: CAPM plus trend factor</i>													
$\alpha(\%)$	0.19 (1.62)	0.46*** (2.86)	0.37* (1.78)	0.38*** (7.98)	0.38*** (5.20)	0.44*** (6.64)	0.14* (1.84)	0.51*** (5.13)	0.44*** (4.74)	0.64*** (3.40)	0.60*** (12.19)	234.5	174.6*** [0.00]
$\beta_{mkt}$	0.22*** (8.83)	−0.60*** (−17.29)	0.59*** (13.28)	0.08*** (8.01)	0.25*** (16.39)	0.08*** (5.52)	0.19*** (12.01)	0.15*** (7.33)	0.44*** (22.04)	−0.06 (−1.47)	0.14*** (13.55)		
$\beta_{trd}$	0.03 (1.17)	−0.05 (−1.20)	−0.00 (−0.08)	0.02 (1.48)	0.03* (1.74)	−0.01 (−0.55)	0.02 (1.22)	0.04* (1.82)	0.07*** (2.95)	0.02 (0.47)	0.04*** (2.93)		
<i>Panel C: CAPM plus momentum factor</i>													
$\alpha(\%)$	0.27** (2.54)	0.42*** (2.71)	0.40** (1.99)	0.39*** (8.61)	0.44*** (6.37)	0.44*** (6.88)	0.15** (2.13)	0.56*** (5.86)	0.51*** (5.66)	0.62*** (3.49)	0.65*** (13.47)	255.6	225.0*** [0.00]
$\beta_{mkt}$	0.20*** (8.22)	−0.61*** (−17.26)	0.58*** (12.81)	0.09*** (8.67)	0.25*** (15.82)	0.07*** (5.17)	0.20*** (12.33)	0.16*** (7.41)	0.45*** (21.97)	−0.04 (−0.95)	0.14*** (13.32)		
$\beta_{mom}$	−0.05*** (−3.86)	−0.02 (−0.95)	−0.04 (−1.40)	0.02*** (2.86)	−0.03*** (−2.89)	−0.01 (−1.05)	0.02* (1.89)	0.01 (0.50)	0.02 (1.62)	0.05** (2.31)	−0.00 (−0.06)		

Overall, in comparison with the momentum factor, the trend factor not only has four times greater the Sharpe ratio, but also performs better in explaining the cross-section returns. Hence, the trend factor offers an interesting complement to the popular momentum factor.

## 6. Conclusions

We construct a trend factor with a cross-section regression approach that provides a unified framework incorporating information used in the three major price patterns: the short-term reversal effects, the momentum effects, and the long-term reversal effects. We use moving average prices of various time lengths to capture information which is justified by a proposed general equilibrium model. In our framework, stocks that have high forecasted expected returns from the cross-section regression yield higher future returns on average, and stocks that have low forecasted expected returns tend to yield lower future returns on average. The difference between the highest ranked and lowest ranked quintile portfolios is the trend factor, which earns an average return of 1.63% per month. The return is more than quadruple those of the size and book-to-market factors, and more than twice that of the short-term reversal, momentum, and long-term reversal factors. The superior performance of the trend factor is robust to various firm and market characteristics, such as size, book-to-market ratio, past returns, trading turnover rate, and business cycles.

Future research is called for as to how well similar trend factors perform in other asset classes, such as foreign exchanges, commodities, and bonds. While our methodology here is focused on using price trends across investment horizons, it can also be applied to examine other

economic fundamentals, such as firm earnings, profitability, growth, and investment patterns, over short- and long-terms. There appear to be many potential applications of the trend factors in finance and accounting.

## Appendix

In this Appendix, we outline the major steps for proving Theorem 1, while providing a more detailed proof in an online appendix.

First, we solve the informed investor  $i$ 's optimization problem,

$$\max_{\eta^i, c^i} E \left[ - \int_t^\infty e^{-\rho s - c(s)} ds | \mathcal{F}_t^i \right] \quad \text{s.t.} \quad dW^i = (rW^i - c^i)dt + \eta^i dQ^i, \quad (\text{A.1})$$

where  $W^i$  is the wealth,  $\eta^i$  is the asset allocation, and  $c^i$  is the consumption. Let  $J^i(W^i, D_t, \pi_t, \theta_t, A_t; t)$  be the value function that satisfies the Hamilton-Jacobi-Bellman (HJB) equation,

$$0 = \max_{c^i, \eta^i} \left[ -e^{-\rho t - c^i} + J_W^i(rW^i - c^i + \eta^i e_Q^i \Psi^i) + \frac{1}{2} \sigma_Q^i \sigma_Q^{iT} \eta^{i2} J_{WW}^i + \eta^i \sigma_Q^i \sigma_\Psi^{iT} J_{W\Psi}^i - \rho J^i + (e_\Psi^i \Psi^i)^T J_\Psi^i + \frac{1}{2} \sigma_\Psi^i J_{\Psi\Psi}^i \sigma_\Psi^{iT} \right]. \quad (\text{A.2})$$

The above can be analytically solved, and hence the portfolio solution can be summarized as

*Lemma 1.* The informed investor  $i$ 's optimal demand for the stock is given by

$$\eta^i = f^i \Psi^i = f_0^i + f_1^i D_t + f_2^i \pi_t + f_3^i \theta_t + f_4^i A_t, \quad (\text{A.3})$$

where  $f_0^i, f_1^i, f_2^i, f_3^i$ , and  $f_4^i$  are constants.

Second, we solve the technical trader  $u$ 's optimization problem,

$$\max_{\eta^u, c^u} E \left[ - \int_t^\infty e^{-\rho s - c(s)} ds | \mathcal{F}_t^u \right] \\ \text{s.t. } dW = (rW^u - c^u)dt + \eta^u dQ^u, \quad (\text{A.4})$$

where  $W^u$  is the wealth,  $\eta^u$  is the asset position, and  $c^u$  is the consumption. Let  $J^u(W^u, \Psi^u; t)$  be the value function that satisfies the following HJB equation,

$$0 = \max_{c^u, \eta^u} \left[ -e^{-\rho t - c^u} + J_{W^u}^u (rW^u - c^u + \eta^u e_Q^u \Psi^u) \right. \\ \left. + \frac{1}{2} \sigma_Q^u \sigma_Q^{uT} \eta^{u2} J_{WW}^u + \eta^u \sigma_Q^u \sigma_\Psi^{uT} J_{W\Psi}^u \right. \\ \left. - \rho J^u + (e_\Psi^u \Psi^u)^T J_\Psi^u + \frac{1}{2} \sigma_\Psi^u J_{\Psi\Psi}^u \sigma_\Psi^{uT} \right]. \quad (\text{A.5})$$

This equation can be analytically solved as well, and hence the portfolio solution can be summarized as

**Lemma 2.** The technical trader  $u$ 's optimal demand for the stock is given by

$$\eta^u = f_0^u + f_1^u D_t + f_2^u P_t + f_3^u A_t, \quad (\text{A.6})$$

where  $f_0^u$ ,  $f_1^u$ ,  $f_2^u$ , and  $f_3^u$  are constants.

**Proof of Theorem 1.** Based on Lemmas 1 and 2, and the market clearing condition,

$$\eta^i + \eta^u = 1 + \theta_t,$$

we can solve the equilibrium price as stated by Theorem 1.  $\square$

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