Statistical Methods for Research Workers

R.A. Fisher
Rothamsted Experimental Station

4. Scope of this Book

The prime object of this book is to put into the hands of research workers, and especially of biologists, the means of applying statistical tests accurately to numerical data accumulated in their own laboratories or available in the literature.

Ex. 41. Analysis of Variation in Experimental Field Trials

The table on the following page gives the yield in 1b. per plant in an experiment with potatoes (Rothamsted data). A plot of land, the whole of which had received a dressing of dung, was divided into 36 patches, on which 12 varieties were grown, each variety having 3 patches scattered over the area. Each patch was divided into three lines, one of which received, in addition to dung, a basal dressing only, containing no potash, while the other two received additional dressings of sulphate and chloride of potash respectively.

From data of this sort a variety of information may be derived. The total yields of the 36 patches give us 35 degrees of freedom, of which 11 represent differences among the 12 varieties, and 24 represent the differences between different patches growing the same variety. By comparing the variance in these two classes we may test the significance of the varietal differences in yield for the soil and climate of the experiment. The 72 additional degrees of freedom given by the yields of the separate rows consist of 2 due to manurial treatment, which we can subdivide into one representing the differences due

Editor's note: This example could be read in conjunction with Fisher's Paper (1926) reproduced in this volume.

S. Kotz et al. (eds.), *Breakthroughs in Statistics*
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Table 46

Variety	Sulphate Row	Sulphate Row	Sulphate Row	Chloride Row	Chloride Row	Chloride Row	Basal Row	Basal Row	Basal Row
Ajax	3.20	4.00	3.86	2.55	3.04	4.13	2.82	1.75	4.71
Arran Comrade	2.25	2.56	2.58	1.96	2.15	2.10	2.42	2.17	2.17
British Queen	3.21	2.82	3.82	2.71	2.68	4.17	2.75	2.75	3.32
Duke of York	1.11	1.25	2.25	1.57	2.00	1.75	1.61	2.00	2.46
Epicure	2.36	1.64	2.29	2.11	1.93	2.64	1.43	2.25	2.79
Great Scot	3.38	3.07	3.89	2.79	3.54	4.14	3.07	3.25	3.50
Iron Duke	3.43	3.00	3.96	3.33	3.08	3.32	3.50	2.32	3.29
K. of K.	3.71	4.07	4.21	3.39	4.63	4.21	2.89	4.20	4.32
Kerr's Pink	3.04	3.57	3.82	2.96	3.18	4.32	2.00	3.00	3.88
Nithsdale	2.57	2.21	3.58	2.04	2.93	3.71	1.96	2.86	3.56
Tinwald Perfection	3.46	3.11	2.50	2.83	2.96	3.21	2.55	3.39	3.36
Up-to-Date	4.29	2.93	4.25	3.39	3.68	4.07	4.21	3.64	4.11

to a potash dressing as against the basal dressing, and a second representing the manurial difference between the sulphate and the chloride; and 70 more representing the differences observed in manurial response in the different patches. These latter may in turn be divided into 22 representing the difference in manurial response of the different varieties, and 48 representing the differences in manurial response in different patches growing the same variety. To test the significance of the manurial effects, we may compare the variance in each of the two manurial degrees of freedom with that in the remaining 48; to test the significance of the differences in varietal response to manure, we compare the variance in the 22 degrees of freedom with that in the 48; while to test the significance of the difference in yield of the same variety in different patches, we compare the 24 degrees of freedom representing the differences in the yields of different patches growing the same variety with the 48 degrees representing the differences of manurial response on different patches growing the same variety.

For each variety we shall require the total yield for the whole of each patch, the total yield for the 3 patches and the total yield for each manure; we shall also need the total yield for each manure for the aggregate of the 12 varieties; these values are given in Table 47.

Table 47

Variety	Manuring			Total	Plot		
	Sulphate	Chloride	Basal		I	II	III
Ajax	11.06	9.72	9.28	30.06	8.57	8.79	12.70
Arran Comrade	7.39	6.21	6.76	20.36	6.63	6.88	6.85
British Queen	9.85	9.56	8.82	28.23	8.67	8.25	11.31
Duke of York	4.61	5.32	6.07	16.00	4.29	5.25	6.46
Epicure	6.29	6.68	6.47	19.44	5.90	5.82	7.72
Great Scot	10.34	10.47	9.82	30.63	9.24	9.86	11.53
Iron Duke	10.39	9.73	9.11	29.23	10.26	8.40	10.57
K. of K.	11.99	12.23	11.41	35.63	9.99	12.90	12.74
Kerr's Pink	10.43	10.46	8.88	29.77	8.00	9.75	12.02
Nithsdale	8.36	8.68	8.38	25.42	6.57	8.00	10.85
Tinwald Perfection	9.07	9.00	9.30	27.37	8.84	9.46	9.07
Up-to-Date	11.47	11.14	11.96	34.57	11.89	10.25	12.43
Total	111.25	109.20	106.26	326.71	...	...	...

The sum of the squares of the deviations of all the 108 values from their mean is 71.699; divided, according to patches, in 36 classes of 3, the value for the 36 patches is 61.078; dividing this again according to varieties into 12 classes of 3, the value for the 12 varieties is 43.638. We may express the facts so far as follows:

Table 48

Variance	Degrees of Freedom	Sum of Squares	Mean Square	Log (S.D.)
Between varieties	11	43.6384	3.967	.6890
Between patches for same variety	24	17.4401	.727	−.1594
Within patches	72	10.6204	...	...
Total	107	71.6989		

The value of z, found as the difference of the logarithms in the last column, is .8484, the corresponding 1 per cent. value being about .564; the effect of variety is therefore very significant.

Of the variation within the patches the portion ascribable to the two differences of manurial treatment may be derived from the totals for the three manurial treatments. The sum of the squares of the three deviations, divided by 36, is .3495; of this the square of the difference of the totals for the two potash dressings, divided by 72, contributes .0584, while the square of the difference between their mean and the total for the basal dressing, divided by 54, gives the remainder, .2911. It is possible, however, that the whole effect of the dressings may not appear in these figures, for if the different varieties had responded in different ways, or to different extents, to the dressings, the whole effect would not appear in the totals. The 70 remaining degrees of freedom would not be homogeneous. The 36 values, giving the totals for each manuring and for each variety, give us 35 degrees of freedom, of which 11 represent the differences of variety, 2 the differences of manuring, and the remaining 22 show the differences in manurial response of the different varieties. The analysis of this group is shown below:

Table 49

Variance due to	Degrees of Freedom	Sum of Squares	Mean Square
Potash dressing	1	.2911	.2911
Sulphate *v.* chloride	1	.0584	.0584
Differential response of varieties	22	2.1911	.0996
Differential response in patches with same variety	48	8.0798	.1683
Total	72	10.6204	

To test the significance of the variation observed in the yield of patches

bearing the same variety, we may compare the value .727 found above from 24 degrees of freedom, with .1683 just found from 48 degrees. The value of z, half the difference of the logarithms, is .7316, while the 1 per cent. point is about .394. The evidence for unequal fertility of the different patches is therefore unmistakable. As is always found in careful field trials, local irregularities in the nature or depth of the soil materially affect the yields. In this case the soil irregularity was perhaps combined with unequal quality or quantity of the dung supplied.

There is no sign of differential response among the varieties; indeed, the difference between patches with different varieties is less than that found for patches with the same variety. The difference between the values is not significant; $z = .2623$, while the 5 per cent. point is about .33.

Finally, the effect of the manurial dressings tested is small; the difference due to potash is indeed greater than the value for the differential effects, which we may now call random fluctuations, but z is only .3427, and would require to be about .7 to be significant. With no total response, it is of course to be expected, though not as a necessary consequence, that the differential effects should be insignificant. Evidently the plants with the basal dressing had all the potash necessary, and in addition no apparent effect on the yield was produced by the difference between chloride and sulphate ions.