Summary of: “Distributed Coordination Control of Multiagent Systems While Preserving Connectedness”

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Outline

- Problem (graph connectedness)
- Background literature
- Solution
  - In the context of rendezvous problems
    - Simulation results
  - In the context of formation control problems
    - Simulation results
  - In the context of hybrid rendezvous-formation control problems
- Summary
- Future Work

Motivation

- What is the problem being solved?
  - Connectedness issue: maintaining connection **at all times** in a group of mobile agents while they carry out some performance objective with limited sensing and communication ranges
  - Paper focuses on formation control and rendezvous problems
    - Formation control: maintaining a local formation between agents as the body of agents moves
    - Rendezvous problem: agents converge toward one location
    - These problems are considered solved assuming:
      - Connectedness at all times
      - If connectedness is only required at distinct times
        - “Sensing” and “movement” phases (connection is only needed during sensing phase
  - Can we ensure that connection is maintained **at all times**, negating the need for separate sensing and movement phases?

Relevant Background Literature

Formation control:

Leader-follower:

J. Desai, J. Ostrowski, and V. Kumar, “Controlling formations of multiple mobile robots,” in Proc. IEEE Int. Conf. Robot. Autom., Leuven, Belgium, 1998, pp. 2864–2869.
M. Egerstedt, X. Hu, and A. Stotsky, “Control of mobile platforms using a virtual vehicle approach,” IEEE Trans. Autom. Control, vol. 46, no. 46, pp. 1777–1782, Nov. 2001.
M. Egerstedt and X. Hu, “Formation constrained multi-agent control,” IEEE Trans. Robot. Autom., vol. 17, no. 6, pp. 947–951, Dec. 2001.

Leaderless:

T. Balch and R. C. Arkin, “Behavior-based formation control for multirobot teams,” IEEE Trans. Robot. Autom., vol. 14, no. 6, pp. 926–939, Dec. 1998.
J. Lawton, R. Beard, and B. Young, “A decentralized approach to formation maneuvers,” IEEE Trans. Robot. Autom., vol. 19, no. 6, pp. 933–941, Dec. 2003.

Rendezvous or formation control assuming connectedness:

J. A. Fax and R. M. Murray, “Graph laplacian and stabilization of vehicle formations,” in Proc. 15th IFAC, 2002, pp. 283–288.
Z. Lin, M. Broucke, and B. Francis, “Local control strategies for groups of mobile autonomous agents,” IEEE Trans. Autom. Control, vol. 49, no. 4, pp. 622–629, Apr 2004.
H. Tanner, A. Jadbabaie, and G. Pappas, “Stable flocking of mobile agents, part II : Dynamic topology,” in Proc. 42nd IEEE Conf. Decision Control, Maui, HI, Dec.2003, pp. 2016–2021.

Rendezvous or formation control assuming connectedness at discrete times:

H. Ando, Y. Oasa, I. Suzuki, and M. Yamashita, “Distributed memoryless point convergence algorithm for mobile robots with limited visibility,” IEEE Trans. Robot. Autom., vol. 15, no. 5, pp. 818–828, Oct. 1999.
A. Jadbabaie, J. Lin, and A. S. Morse, “Coordination of groups of mobile autonomous agents using nearest neighbor rules,” IEEE Trans. Autom. Control, vol. 48, no. 6, pp. 988–1001, Jun. 2003.
J. Corť es, S. Martı´nez, and F. Bullo, “Robust rendezvous for mobile autonomous agents via proximity graphs in d dimension,” IEEE Trans. Robot. Autom., vol. 51, no. 8, pp. 1289–1298, Aug. 2006.
L. Moreau, “Stability of multiagent systems with time-dependent communication links,” IEEE Trans. Autom. Control, vol. 50, no. 2, pp. 169–182, Feb. 2005.

Relevant Background Literature

Consensus or agreement problems utilizing the graph Laplacian

J. Fax and R. Murray, “Information flow and cooperative control of vehicle formations,” IEEE Trans. Autom. Control, vol. 49, no. 9, pp. 1465–1476, Sep. 2004.

R. Olfati-Saber and R. M. Murray, “Flocking with obstacle avoidance: Cooperation with limited communication in mobile networks,” in Proc. 42nd IEEE Conf. Decision Control, vol. 2, Maui, HI, Dec. 2003, pp. 20222028.

R. Olfati-Saber and R. M. Murray, “Agreement problems in networks with directed graphs and switching toplogy,” in Proc. 42nd IEEE Conf. Decision Control, vol. 4, Maui, HI, Dec. 2003, pp. 41264132.

R. Olfati-Saber and R. M. Murray, “Consensus problems in networks of agents with switching topology and time-delays,” IEEE Trans. Autom. Control, vol. 49, no. 9, pp. 1520–1533, Sep. 2004.

Time-varying network topologies:

M. Mesbahi, “State-dependent graphs,” in Proc. 42nd IEEE Conf. Decision Control, Maui, HI, Dec. 2003, pp. 3058–3063.

M. Mesbahi, “On a dynamic extension of the theory of graphs,” in Proc. 2002 Am. Control Conf., 2002, vol. 2, pp. 1234–1239.

W. Ren and R. Beard, “Consensus of information under dynamically changing interaction topologies,” in Proc. Am. Control Conf., vol. 6, Jun. 30–Jul. 2, 2004, pp. 4939–4944.

R. Olfati-Saber and R. M. Murray, “Consensus problems in networks of agents with switching topology and time-delays,” IEEE Trans. Autom. Control, vol. 49, no. 9, pp. 1520–1533, Sep. 2004.

A. Jadbabaie, N. Motee, and M. Barahona, “On the stability of the Kuramoto model of coupled nonlinear oscillators,” in Proc. 2004 Am. Control Conf., 2004, pp. 4296–4301.

What contribution did these authors make that went beyond any prior research?

- Prior research *assumes* graph is connected rather than *proving* that it's connected
- Method is presented that modifies control law to ensure that the graph stays connected for all times (negates the need for separate "sensing" and "movement" phases)

Model

N agents with positions $x_1, ..., x_N$ ($x_i \in \mathbb{R}^n$):

$$\dot{x_i} = u_i, \quad i = 1, ..., N$$

Δ-disk proximity graph:

$$G = (V, E(t))$$

where edge $(v_i, v_j)(t)$ exists if and only if:

$$|x_i - x_j| \leq \Delta$$

at time t (Note that V is assumed to be static, while E undergoes dynamic changes).

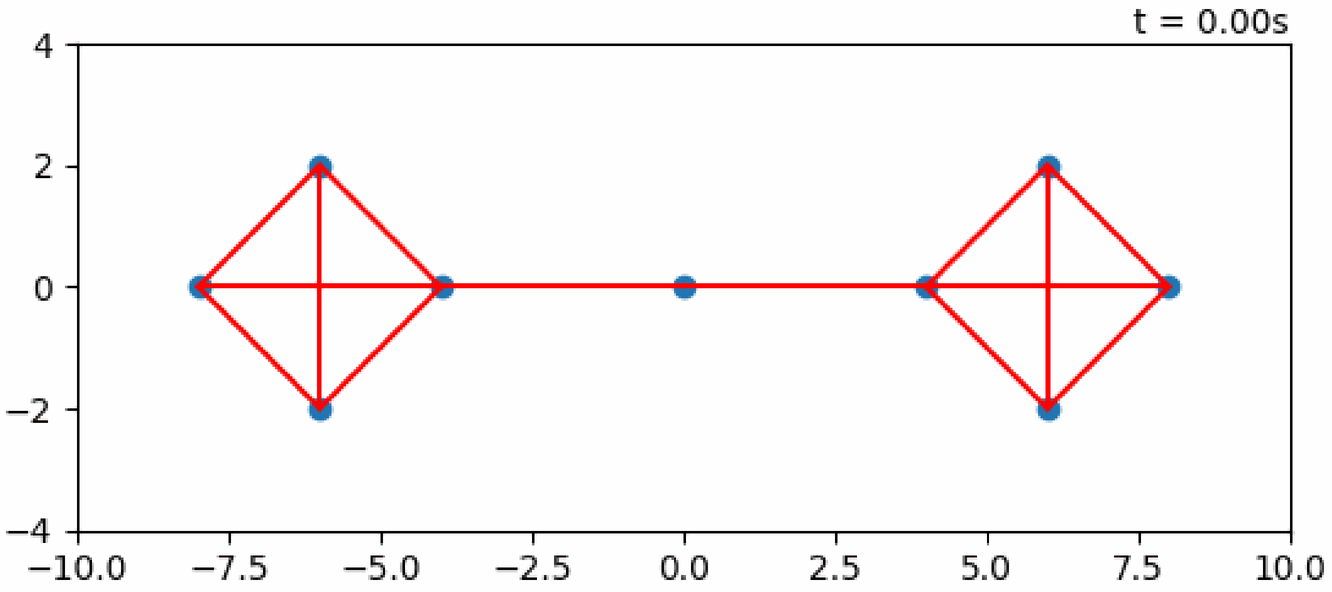
Define a neigbhorhood for agent i:

$$\mathcal{N}_\sigma(i) = \{j|(v_i, v_j) \in E\}$$

Simple Rendezvous

Limited-information time-invariant decentralized control law:

$$u_i = -\sum_{j \in \mathcal{N}_\sigma(i)} (x_i - x_j), \quad i = 1, ..., N$$



New Definitions

Let l_{ij} denote the edge vector between agents i and j:

$$l_{ij}(x) = x_i - x_j$$

Define the ϵ-interior of a δ-constrained realization of graph $\mathcal{G}$ as:

$$\mathcal{D}^{\epsilon}_{\mathcal{G},\delta} = \{x \in \mathbb{R}^{nN} \mid \|l_{ij}\| \leq (\delta - \epsilon)\}$$

Define the edge-tension function $\mathcal{V}_{ij}$ as:

$$\mathcal{V}_{ij}(\delta, x) = \begin{cases} \frac{\|l_{ij}(x)\|^2}{\delta - \|l_{ij}(x)\|}, & \text{if } (v_i, v_j) \in E(\mathcal{G}) \\ 0, & \text{otherwise} \end{cases}$$

with

$$\frac{\partial \mathcal{V}_{ij}(\delta, x)}{\partial x_i} = \begin{cases} \frac{2\delta - \|l_{ij}(x)\|}{(\delta - \|l_{ij}(x)\|)^2}(x_i - x_j), & \text{if } (v_i, v_j) \in E(\mathcal{G}) \\ 0, & \text{otherwise} \end{cases}$$

Lemma 3.1

Lemma 3.1: Given an initial position $x_0 \in \mathcal{D}^{\epsilon}_{\mathcal{G},\delta}$, for a given $\epsilon \in (0, \delta)$. If the SIG $\mathcal{G}$ is connected, then the set $\Omega(\delta, x_0) := \{x | \mathcal{V}(\delta, x) \leq \mathcal{V}(\delta, x_0)\}$ is an invariant set to the system under the control law

$$\dot{x}_i = - \sum_{j \in \mathcal{N}_{\mathcal{G}}(i)} \frac{2\delta - \|\ell_{ij}(x)\|}{(\delta - \|\ell_{ij}(x)\|)^2} (x_i - x_j). \tag{17}$$

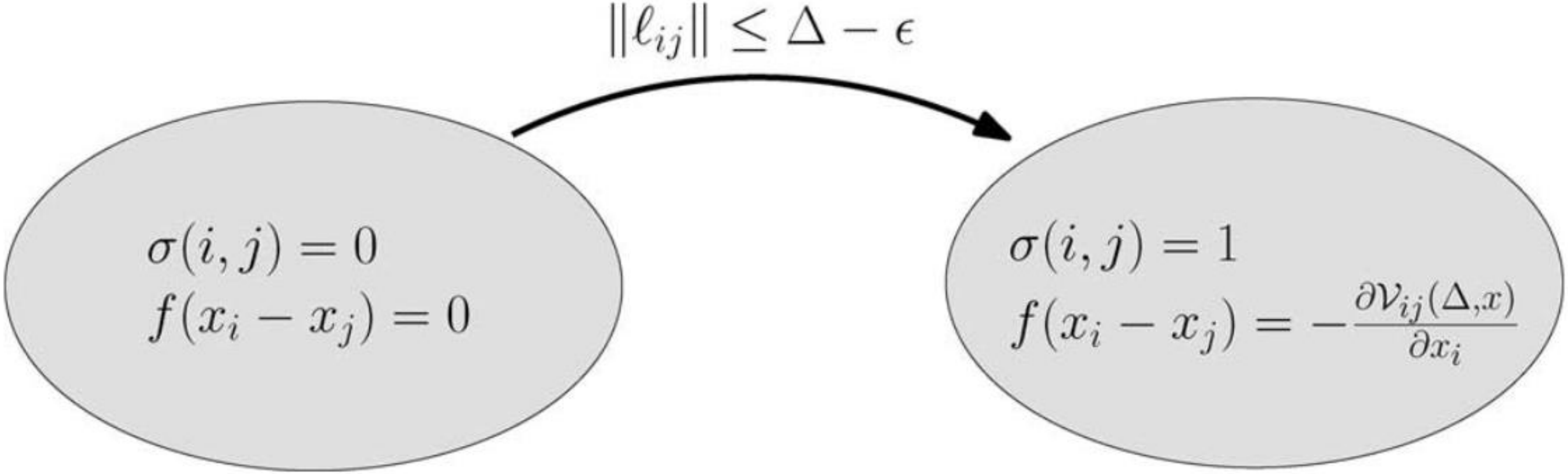
Theorem 3.2

Theorem 3.2: Given a connected SIG $\mathcal{G}$ with initial condition $x_0 \in \mathcal{D}^{\epsilon}_{\mathcal{G},\delta}$, for a given $\epsilon > 0$. Then, the the multiagent system under the control law in (17) asymptotically converges to the static centroid $\bar{x}(x_0)$.

- Proof of this theorem involves LaSalle’s Invariance Principle to show that all solutions with the initial condition defined above converge to the centroid.

New and Improved Control Law

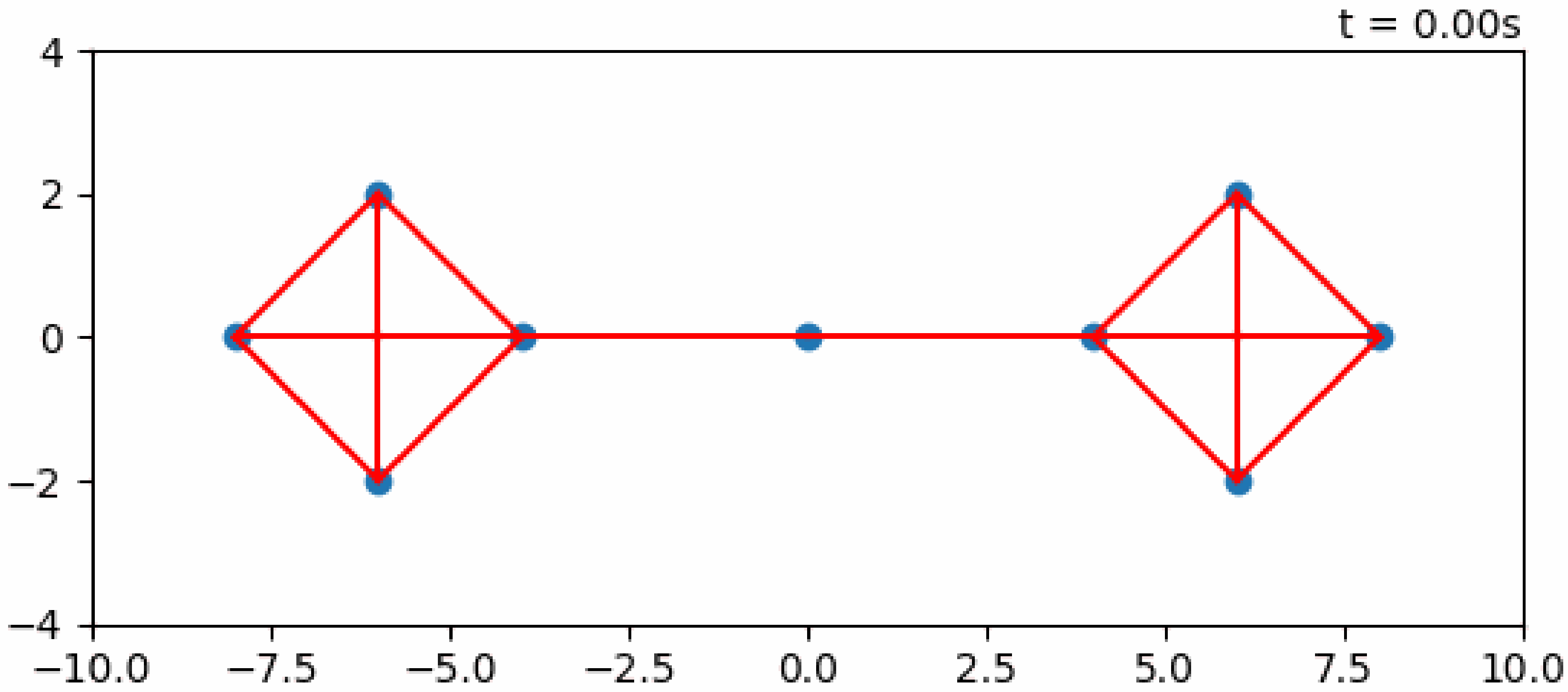
$$u_i = \sum_{j \in \mathcal{N}_\sigma(i)} f(x_i - x_j), \quad i = 1, ..., N$$



Why does this work?

- From Lemma 3.1, we know that no edges will be lost from the starting graph
  - A.K.A. all agents that start within Δ - ϵ of each other will remain within Δ of each other for all future time
- Two possibilities:
  1. No new edges will be added to the graph
  2. New edges will be added to the graph
- From Theorem 3.2, we know that the system will converge to the centroid
  - As they converge, nodes that were not within Δ - ϵ of each other to start will eventually become within Δ - ϵ
  - Therefore, edges will be added until the graph is complete

Simulation results



Recap on Assumptions

- Static nodes, dynamics edges in graph
- Agents are homogeneous (same control laws govern the motion of all agents)
- Edges can be weighted, but nodes are weighted equally
- The graph starts connected (Δ - ϵ)

Formation Control

- "The control objective is to drive the collection of autonomous mobile agents to a specific configuration such that their relative positions satisfy some desired topological and physical constraints"

Define a new desired graph $\mathcal{G}_d$:

$$\mathcal{G}_d = (V, E_d, d)$$

where

$$\|d_{ij}\| \leq \delta$$

Formation Control

- Assumptions:
  - No conflicting constraints (formation is geometrically possible)
  - Target formation is chosen in such a way that rigidity is obtained

Given a desired formation, the goal of the distributed formation control is to find a feedback law such that:

F1 dynamic interaction graph $\mathcal{G}(t)$ converges to a graph that is a supergraph of the desired graph $\mathcal{G}_d$ (without labels) in finite time. In other words, what we want is that $E_d \subset E(t)$ for all $t \geq T$, for some finite $T \geq 0$;

F2 $\|\ell_{ij}(t)\| = \|x_i(t) - x_j(t)\|$ converges asymptotically to $\|d_{ij}\|$ for all i, j such that $(v_i, v_j) \in E_d$; and

F3 feedback law utilizes only local information.

Formation Control

Define a set of arbitrary targets $\tau_i \in R^n$:

$$d_{ij} = \tau_i - \tau_j \quad \forall i, j$$

such that

$$(v_i, v_j) \in E_d$$

Define $y_i(t)$:

$$y_i(t) = x_i(t) - \tau_i$$

New control law:

$$u_i = -\sum_{j \in \mathcal{N}_{\mathcal{G}_d}} \frac{\partial \mathcal{V}_{ij}(\Delta - \|d_{ij}\|, y)}{\partial y_i} = -\sum_{j \in \mathcal{N}_{\mathcal{G}_d}} \frac{2(\Delta - \|d_{ij}\|) - \|l_{ij} - d_{ij}\|}{(\delta - \|d_{ij}\| - \|l_{ij} - d_{ij}\|)^2}(x_i - x_j - d_{ij})$$

A Couple More Assumptions

- The desired graph is a connected spanning graph of the initial graph
  - Meaning that all edges in the desired graph already exist in the initial graph
- The desired graph (which influences the control law) is static
  - Only the “desired” edges are used, and those do not go away
- Agents share the same coordinate system

Corollary 5.1 (similar to Lemma 3.1)

Given $y_0 \in \mathcal{D}^{\epsilon}_{\mathcal{G}_d,\Delta-\|d\|}$, with $\mathcal{G}_d$ being a connected spanning graph, then the set $\Omega(\Delta - \|d\|, y_0) := \{y | \mathcal{V}(\delta - \|d\|, y) \leq \mathcal{V}_0\}$, where $\mathcal{V}_0$ denotes the initial value of the total tension energy function, is an invariant set under the control law in (23), under the assumption that the interaction graph is static.

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Corollary 5.1 (similar to Lemma 3.1)

Given $y_0 \in \mathcal{D}^{\epsilon}_{\mathcal{G}_d, \Delta - \|d\|}$, with $\mathcal{G}_d$ being a connected spanning graph, then the set $\Omega(\Delta - \|d\|, y_0) := \{y | \mathcal{V}(\delta - \|d\|, y) \leq \mathcal{V}_0\}$, where $\mathcal{V}_0$ denotes the initial value of the total tension energy function, is an invariant set under the control law in (23), under the assumption that the interaction graph is static.

Corollaries 5.1 and 5.2

Corollary 5.1 (similar to Lemma 3.1)

Given $y_0 \in \mathcal{D}^{\epsilon}_{\mathcal{G}_d, \Delta - \|d\|}$, with $\mathcal{G}_d$ being a connected spanning graph, then the set $\Omega(\Delta - \|d\|, y_0) := \{y | \mathcal{V}(\delta - \|d\|, y) \leq \mathcal{V}_0\}$, where $\mathcal{V}_0$ denotes the initial value of the total tension energy function, is an invariant set under the control law in (23), under the assumption that the interaction graph is static.

AKA: if the desired graph starts as a spanning graph to the initial proximity *Δ*-disk graph, then it remains a spanning graph for all future time

Corollary 5.2:

Corollary 5.2: Given an initial condition x_0 such that $y_0 = (x_0 - \tau_0) \in \mathcal{D}^{\epsilon}_{\mathcal{G}_d, \Delta - \|d\|}$, with $\mathcal{G}_d$ being a connected spanning graph of $\mathcal{G}(x_0)$, the group of autonomous mobile agents adopting the decentralized control law in (23) can guarantee that $\|x_i(t) - x_j(t)\| = \|\mathcal{L}_{ij}(t)\| < \Delta, \forall t > 0$ and $(v_i, v_j) \in E_d$.

AKA: edges that belong to the desired graph are never lost under the proposed control law

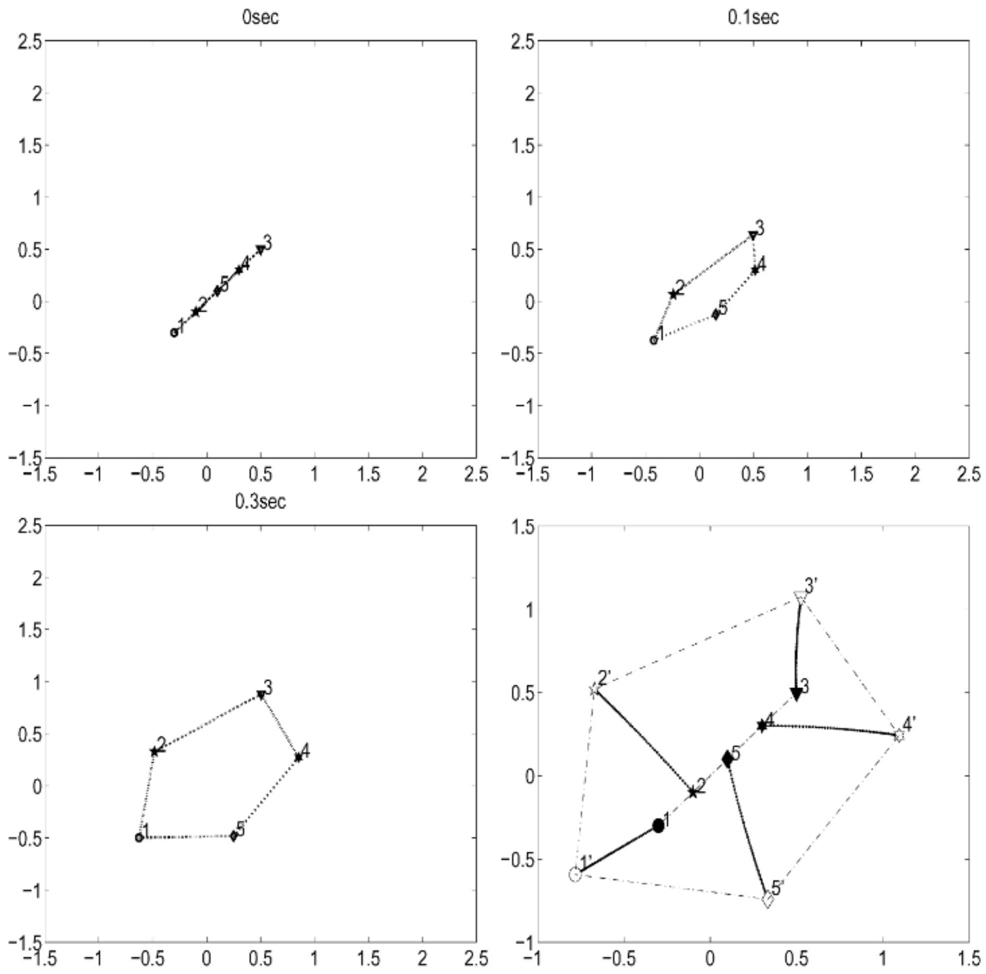
But does this converge to the desired formation?

Theorem 5.3: Under the same assumptions as in Lemma 5.2 $\|\ell_{ij}(t)\| = \|x_i(t)\| - \|x_j(t)\|$ converges asymptotically to $\|d_{ij}\|$ for all i, j such that $(v_i, v_j) \in E_d$.

(Similar proof to Theorem 3.2, which showed asymptotic convergence to the centroid for the rendezvous problem)

Simulation Results

- Could not get simulation to work (very well)

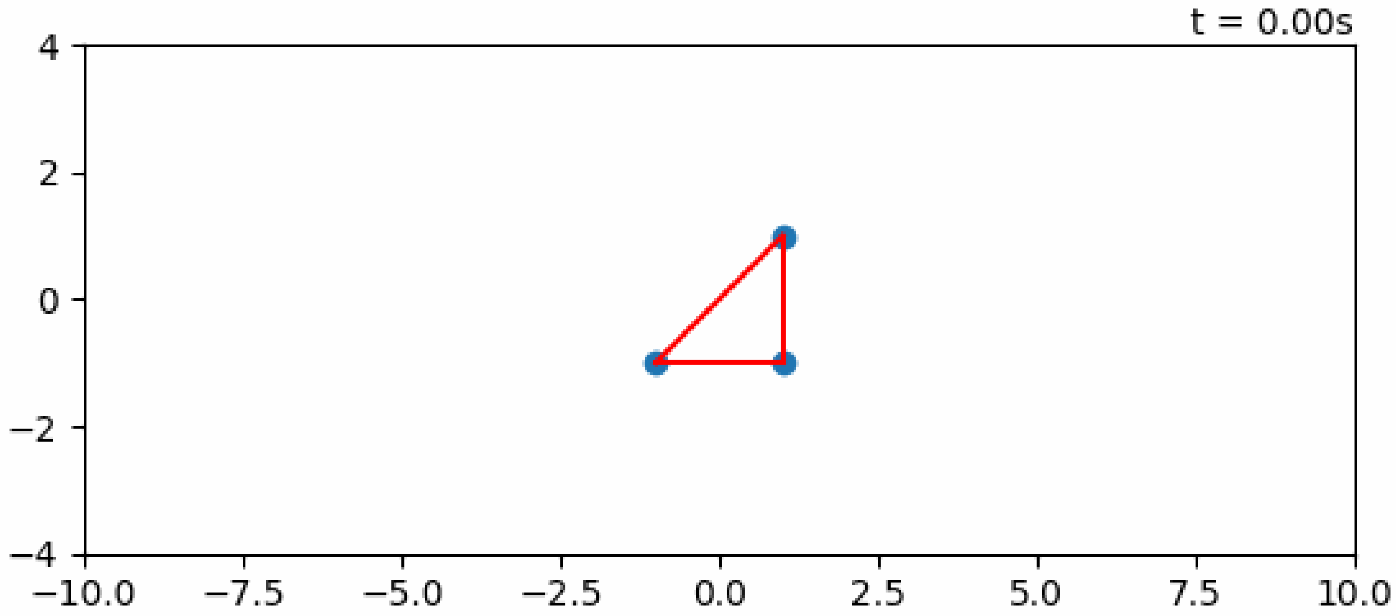


The third simulation highlights the proposed formation control strategy, and is implemented based on the formation control law in (23). In the simulation, five agents starting from a straight line are to form a pentagonal formation, with $\mathcal{G}_d = C_5$ (the cyclic graph with 5 nodes), and the desired interagent distances being $\|d_{ij}\| = 3.2$ for all $(v_i, v_j) \in E_d$. The movement of the group during the first 0.5 s and the trajectories corresponding to the same time period are shown in Fig. 5.

$$u_i = -\sum_{j \in \mathcal{N}_{\mathcal{G}_d}} \frac{2(\Delta - \|d_{ij}\|) - \|l_{ij} - d_{ij}\|}{(\delta - \|d_{ij}\| - \|l_{ij} - d_{ij}\|)^2}(x_i - x_j - d_{ij})$$

$$d_{ij} = \tau_i - \tau_j \quad \forall i, j$$

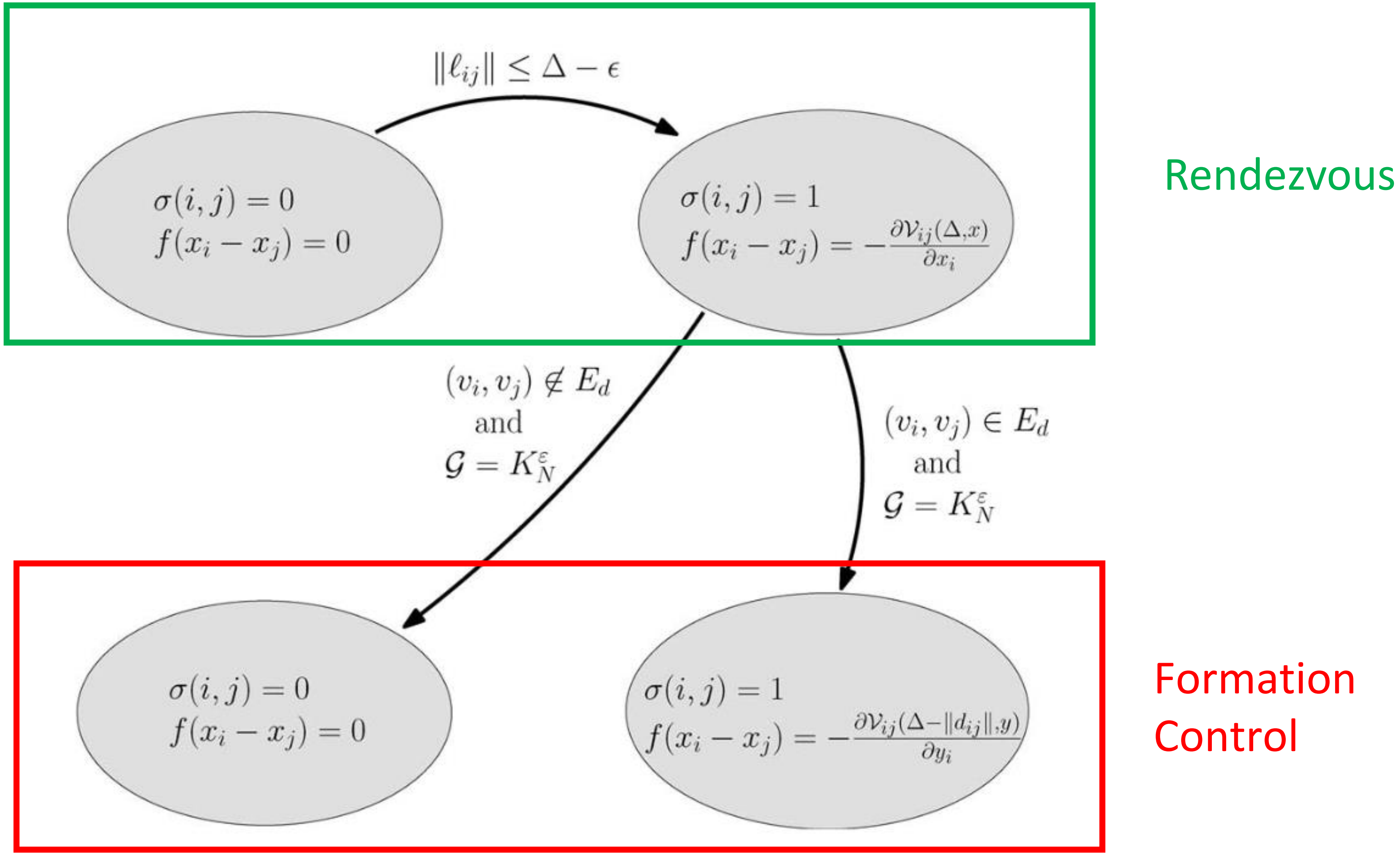
Simulation Implementation



Addressing the Key Assumption

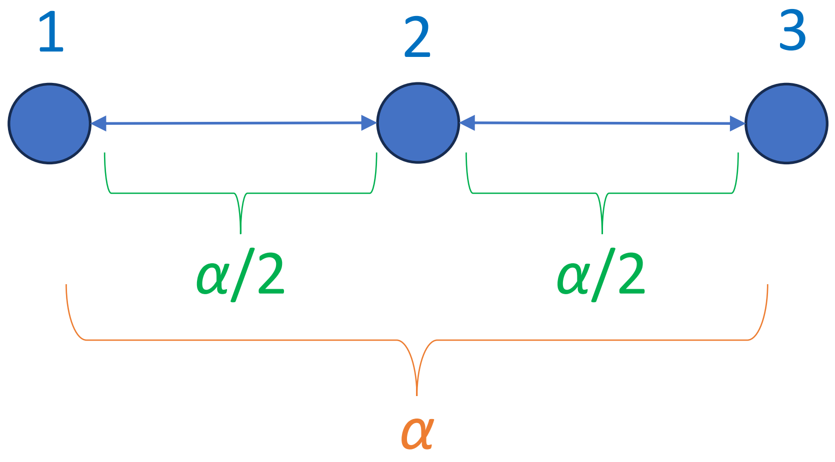
- This formation control solution all hinges on the assumption that the desired graph is a connected spanning graph of the initial graph
- How can we ensure that the graph starts out this way?
  - Solution: hybrid rendezvous-to-formation control
    - Rendezvous gathers the agents into a complete graph
    - State machine switches control from rendezvous to formation control

Hybrid State Machine



De-centralized Switching Mechanism

- Assumption: each node nodes the total number of nodes (*N*)
- When a single node has $N - 1$ neighbors, and the distance between itself and its furthest neighbor is $\alpha/2$, then all agents are within a distance of α of each other.
  - The node that recognizes this then broadcasts the switch to the other $N - 1$ nodes
- Proof:



Note: the graph may become complete before this trigger occurs, but this method provides a de-centralized way to guarantee the graph is complete

Summary

- <u>Problem</u>: Assuming rather than proving graph connectedness
- <u>Solution</u>: Novel control laws for both rendezvous and formation control (and hybrid applications) that guarantee graph connectedness at all times
- <u>Simulation results</u>: Rendezvous results were replicated without much difficulty, but the formation control seems to relay on known waypoint locations (rather than only relative distances)

Future Work

- Formation control that is only dependent on relative information
- Extension of application from 2D to 3D applications
- Explore other de-centralized switching mechanisms (from rendezvous to formation control) that guarantee a complete graph quicker
- Control laws that involve weighted nodes in addition to weighted edges