UCLM
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Speech

Ray-Triangle Intersection

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Contents

Bibliography

• Peter Shirley. “Fundamentals of computer graphics” (2005), Chapters 2 & 10.
• Akenine-Möller, Haines, Hoffman. “Real-Time Rendering” (2008), Chapter 16.

Introduction

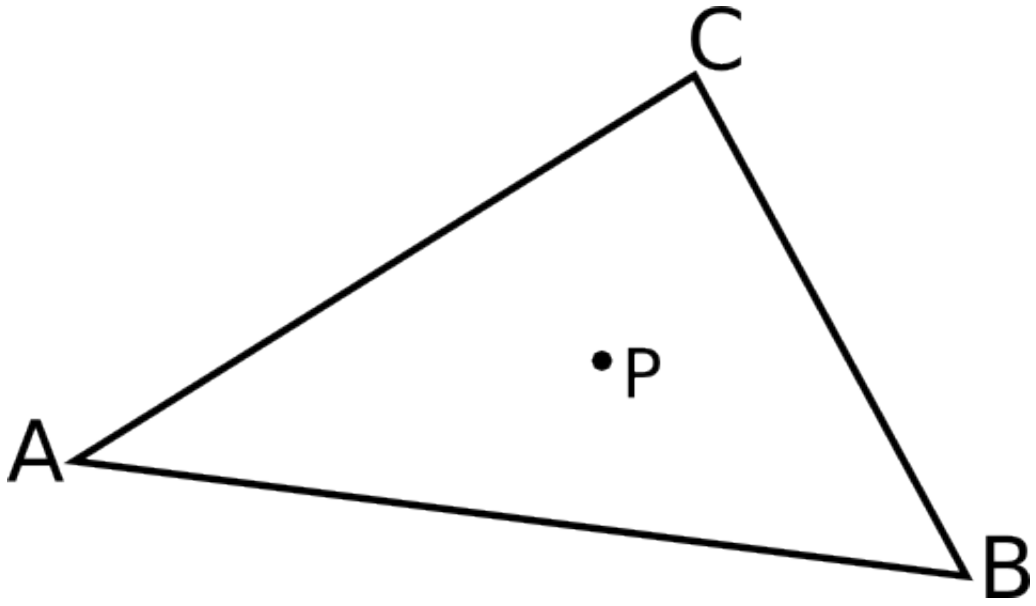
Several alternatives:

- Use Ray-Plane + In Triangle Test
- Use General Ray-Polygon Algorithm
- **Use Barycentric Coordinates**

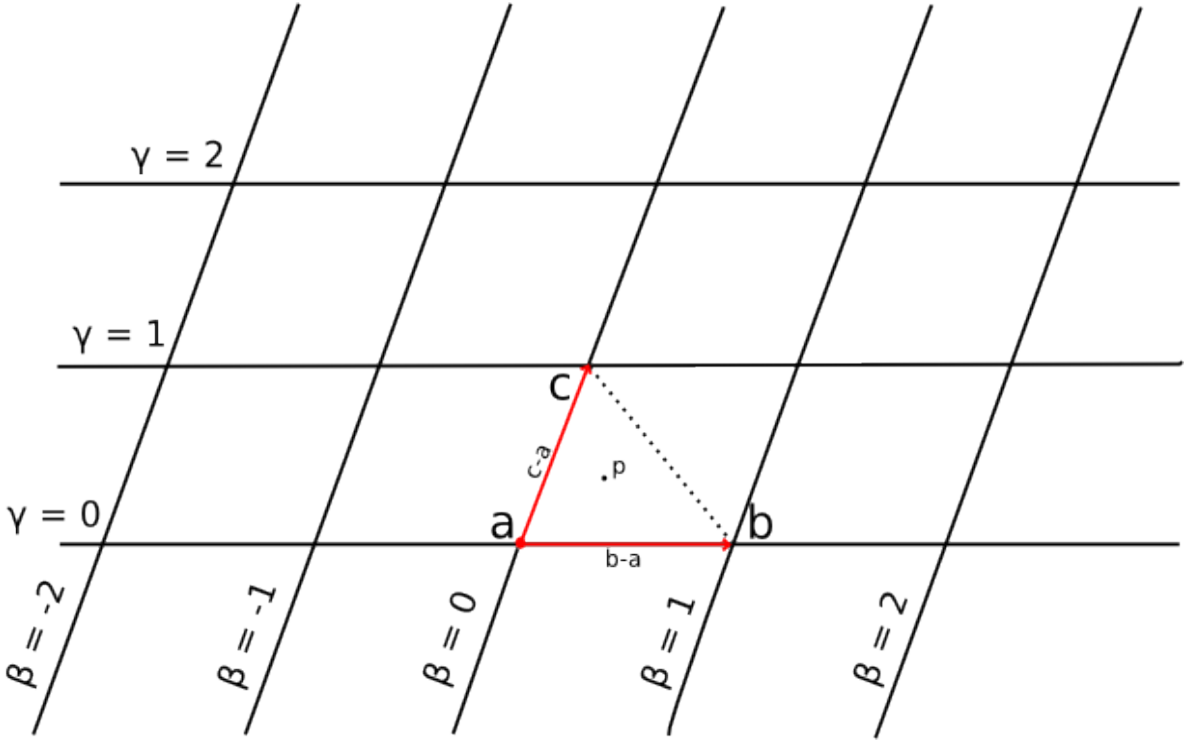
  ⇒ The most efficient

Barycentric Coordinates

Definition: In the context of a triangle, barycentric coordinates are also known as area coordinates, because the coordinates of P with respect to triangle ABC are proportional to the (signed) areas of PBC, PCA and PAB.



Barycentric Coordinates



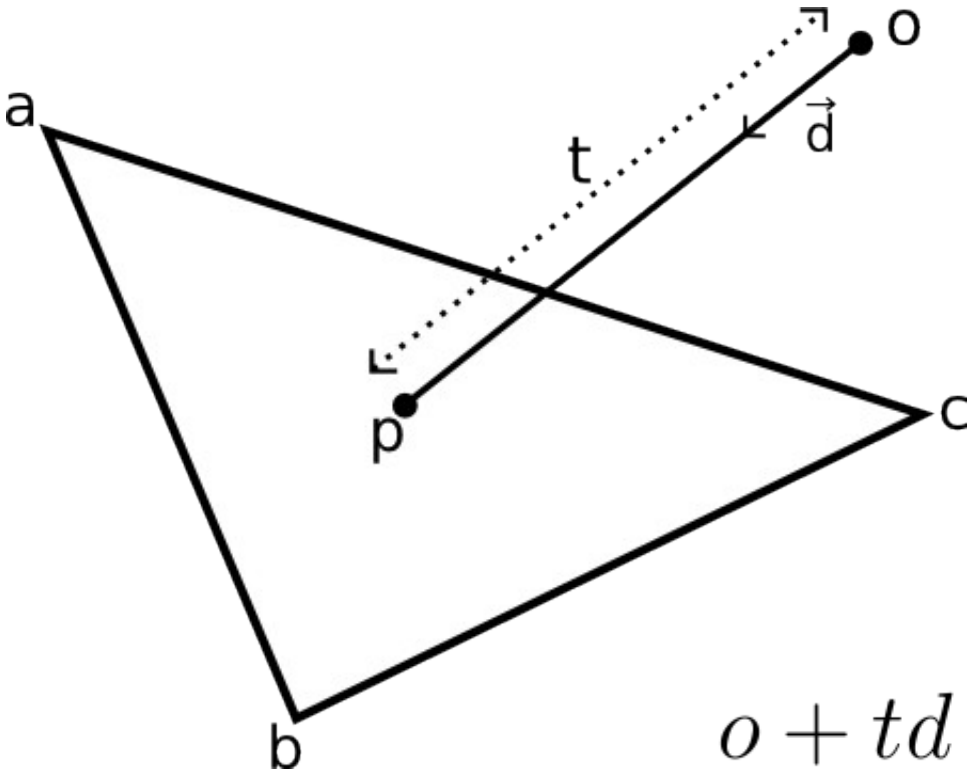
P inside the triangle

$$\Leftrightarrow$$

$$0 < \beta < 1$$
$$0 < \delta < 1$$
$$0 < \beta + \delta < 1$$

$$\mathrm{p}=\mathrm{a}+\beta(b-a)+\gamma(c-a)$$

Ray-Triangle Intersection

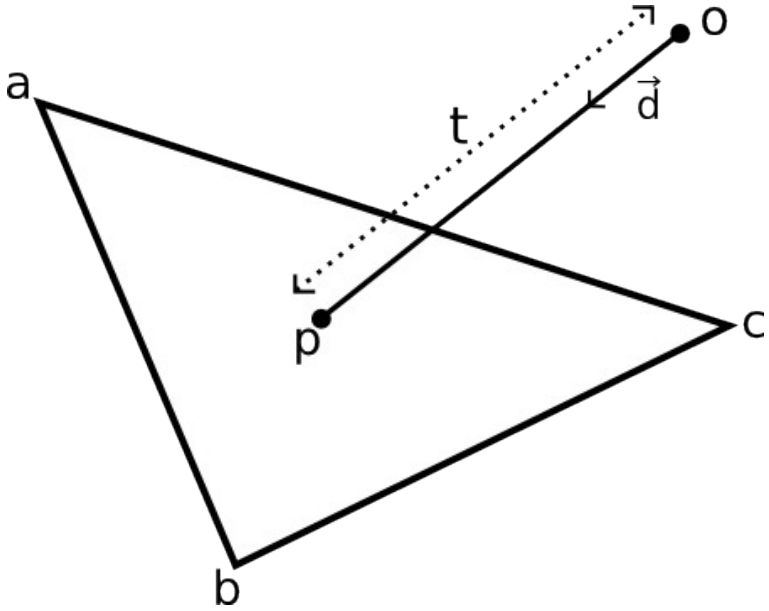


$$o_x + td_x = f(u,v)$$
$$o_y + td_y = g(u,v)$$
$$o_z + td_z = h(u,v)$$

$$o + td = a + \beta(b-a) + \gamma(c-a)$$

$$\beta > 0, \quad \gamma > 0, \quad and \quad \beta + \gamma < 1$$

Ray-Triangle Intersection



$$o_x + td_x = a_x + \beta(b_x - a_x) + \gamma(c_x - a_x)$$
$$o_y + td_y = a_y + \beta(b_y - a_y) + \gamma(c_y - a_y)$$
$$o_z + td_z = a_z + \beta(b_z - a_z) + \gamma(c_z - a_z)$$

Ray-Triangle Intersection

We can express these formulas as a standard linear equation:

$$\begin{bmatrix} a_x - b_x & a_x - c_x & d_x \\ a_y - b_y & a_y - c_y & d_y \\ a_z - b_z & a_z - c_z & d_z \end{bmatrix} \begin{bmatrix} \beta \\ \gamma \\ t \end{bmatrix} = \begin{bmatrix} a_x - o_x \\ a_y - o_y \\ a_z - o_z \end{bmatrix}$$

Ray-Triangle Intersection

Solving this linear equation using *Cramer's Rule:*

$$\beta = \frac{\begin{vmatrix} a_x - o_x & a_x - c_x & d_x \\ a_y - o_y & a_y - c_y & d_y \\ a_z - o_z & a_z - c_z & d_z \end{vmatrix}}{|A|} \qquad \gamma = \frac{\begin{vmatrix} a_x - b_x & a_x - o_x & d_x \\ a_y - b_y & a_y - o_y & d_y \\ a_z - b_z & a_z - o_z & d_z \end{vmatrix}}{|A|}$$

$$t = \frac{\begin{vmatrix} a_x - b_x & a_x - c_x & a_x - o_x \\ a_y - b_y & a_y - c_y & a_y - o_y \\ a_z - b_z & a_z - c_z & a_z - o_z \end{vmatrix}}{|A|}$$

Pseudocode

```
boolean raytri (ray r, vector a, vector b, vector c, interval [t_o,t_1])
    compute t
    if (t < t_o) or (t > t_1) then
        return false
    compute γ
    if (γ < 0) or (γ > 1) then
        return false
    compute β
    if (β < 0) or (β > 1 − γ) then
        return false
    return true
```