Muon Pair Production from Electron Positron Annihilation

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Mathematical Methods: Monte Carlo Integration

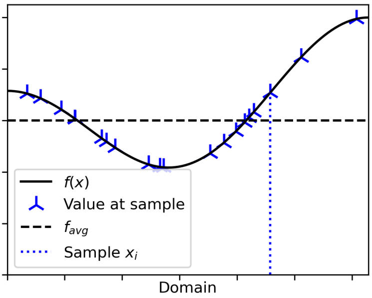
- Using $\langle f(x) \rangle = \frac{1}{b-a} \int_a^b f(x)dx$

Monte Carlo Integration

$$\int_a^b f(x)dx = (b-a)\langle f(x) \rangle$$

Estimated Error

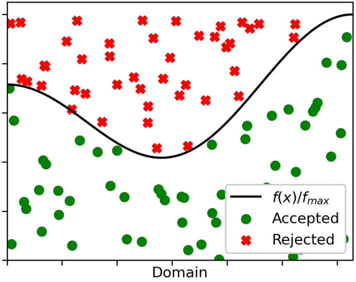
- N uniform random points
- $\sigma_{error} \propto 1/\sqrt{N}$



Rejection Sampling

Reproduce the distribution $f(x)$ on the domain $[a, b]$

1. Sample the domain $[a, b] \times [0, 1]$ labeling points (x_i, v_i)
2. Remove any sample with $v_i > f(x_i)/f_{max}$



Muon Pair Production: $e^+ + e^- \to \mu^+ + \mu^-$

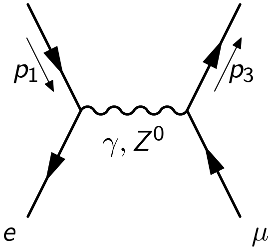
Expected number of events

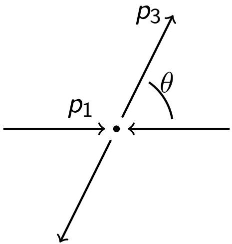
$$N_{exp} = L_{int} \cdot \sigma_{tot}$$

4-Momenta

Using the relativistic limit in the center of momentum frame

- $p_1^\mu = (E, 0, 0, E)^\mu$
- $p_3^\mu = (E, E\sin(\theta), 0, E\cos(\theta))^\mu$





Differential Cross Section

Diagram amplitudes: A_γ, A_Z

Z^0 Vertex factor

- $\frac{-ig_z}{2}\gamma^\mu(c_V^f - c_A^f\gamma^5)$
- For electrons and muons: $c_V^f = -\frac{1}{2} + 2\sin^2\theta_w$, $c_A^f = -\frac{1}{2}$
- $g_z = \frac{g_e}{\sin\theta_w \cos\theta_w}$

Angular dependence of $\langle|A|^2\rangle$

$$\langle|A|^2\rangle = \langle|A_\gamma|^2\rangle + \langle|A_Z|^2\rangle + \langle A_\gamma A_Z^* + A_Z A_\gamma^*\rangle$$

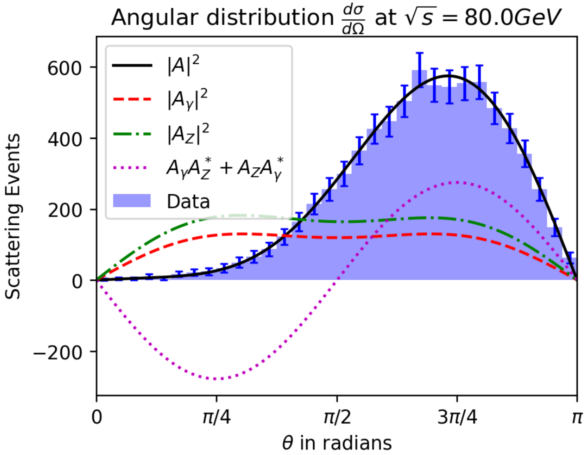
- $\langle|A_\gamma|^2\rangle \propto 1 + \cos^2(\theta)$
- $\langle|A_Z|^2\rangle \propto 1 + \cos^2(\theta) + a\cos(\theta)$
- $\langle A_\gamma A_Z^* + A_Z A_\gamma^*\rangle \propto 1 + \cos^2(\theta) + b\cos(\theta)$

Monte Carlo Simulation

Procedure

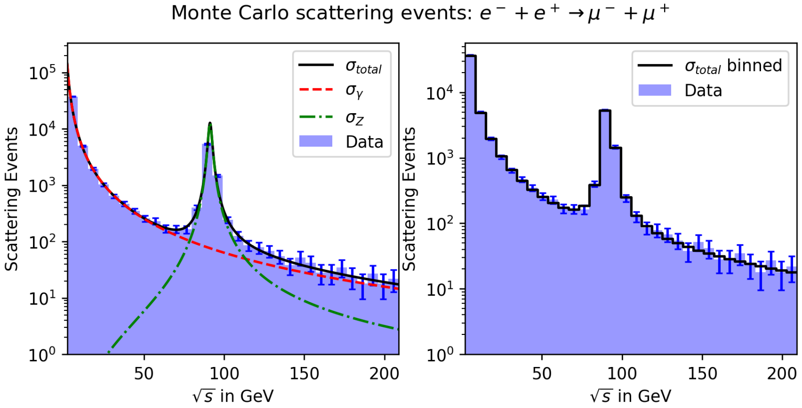
1. Estimate σ_{tot}
2. Expected number of events N_{avg}
3. Sample one value, $N \sim Poisson(N_{avg})$
4. Generate N samples, $\theta \sim \frac{d\sigma}{d\Omega} d\Omega$
5. Calculate p_3^μ

Angular Distribution



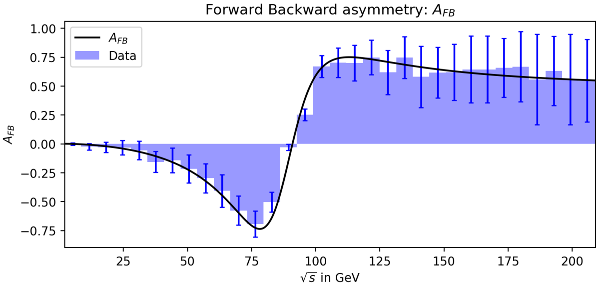
- Integrated luminosity at LEP (year 2000): 2.33×10^{-11} $1/mb$

Cross Section vs Energy



Beam energy range: $[10m_\mu, 104.5GeV]$

Forward Backward Asymmetry



Asymmetry

- $A_{FB} = \frac{N_F - N_B}{N_F + N_B}$
- N_F, number of events with $0 < \theta < \pi/2$
- N_B, number of events with $\pi/2 < \theta < \pi$