

# Three Implementations of the Categorical Imperative

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April 21, 2021

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`imports Main`

## 1 Introduction

As artificial reasoners become increasingly powerful, computers become increasingly able to perform complex ethical reasoning. The field of machine ethics [25] is attractive for two reasons. First, the proliferation of artificially autonomous agents is creating and will continue to create a demand for ethical autonomous agents. These agents must be able to model and reason about complex ethical theories that withstand philosophical scrutiny. Second, just as automated mathematical reasoning gives mathematicians new powers and proof-finding ability, automated ethical reasoning is a tool that philosophers can use when reasoning about ethics. Many contradictions or paradoxes with an ethical system may not be immediately obvious to the human eye, but can be easily tested using an automated theorem prover. Modelling ethics without sacrificing the intricacies and complexities of an ethical theory is a challenging computational and philosophical problem. Simple and intuitive computational approaches, such as encoding ethical rules as constraints in a constraint satisfaction problem, fail to capture the complexity of most philosophically plausible systems. On the other hand, it is not immediately clear how to formalize many complex moral theories, like virtue ethics.

Computational ethics also requires a sophisticated ethical theory to model. Constraint satisfaction systems often default to some version of utilitarianism, the principle of doing the most good for the most people. Alternatively, they model basic moral principles such as “do not kill,” without modelling the theory that these principles originated from. Modelling a more complex ethical theory will not only enable smarter philosophical machines, it will also empower philosophers to study more complex ethical issues with the computer’s help. The entire field of philosophy is devoted to developing and testing robust ethical theories. Plausible machine ethics must draw on plausible moral philosophy.

The ideal candidate ethical theory will be both philosophically interesting and easy to formalize. Kantian ethics, often described as “formal,” has been often floated as such a candidate [21] [1]. The categorical imperative, Kant’s universal law of morality, is a moral rule that can be used to guide action.

This project’s objective is to automate reasoning about Kantian ethics. I present two different formalizations of Kant’s categorical imperative implemented and tested in the Isabelle/HOL [18] theorem prover. Each formalization is modelled as an extension of Carmo and Jones’ Dyadic Deontic Logic (DDL). The corresponding DDL formalization is then embedded in higher-order logic (HOL) and implemented in Isabelle. Section 3.1 implements and tests the naive formalization, a control group that is equivalent to DDL itself. Section 3.2 examines a more sophisticated formalization inspired by Moshe Kroy’s partial formalization of the categorical imperative.

I contribute implementations of two different interpretations of the categorical imperative, examples of how each implementation can be used to model and solve ethical scenarios, and tests that examine ethical and logical properties of the system, including logical consistency, consistency of obligation, and possibility of permissibility. The implementations themselves are usable models of ethical principles and the tests represent the kind of philosophical work that formalized ethics can contribute.

## 2 System Overview

The goal of this project is to automate sophisticated ethical reasoning. This requires three components. First, the choice of an ethical theory that is both intuitively attractive and lends itself to formalization. Second, the choice of formal logic to model the theory in. Third, the choice of automation engine to implement the formal model in. Section 2.1 introduces Kantian ethics, Section 2.2 explains Carmo and Jones’s Dyadic Deontic Logic [6] as a base logic, and Section 2.3 presents the Isabelle/HOL implementation of the logic.

### 2.1 Kantian Ethics

Kantian ethics is an attractive choice of ethical theory to automate for two reasons. The first is that Kant’s writings have been a major inspiration for much of Western analytic philosophy. In addition to the rich neo-Kantian program, almost all major philosophical traditions after Kant have engaged with his work. Much of Western libertarian political thought is inspired by Kant’s deontology, and his ethics have bled into household ethical thought. Deontology is seen as one of the three major schools of Western normative philosophy.

Understanding the ethic’s potential for formalization requires understanding Kant’s system. Kant argues that if morality exists, it must take the form of an inviolable rule, which he calls the “categorical imperative.” He presents three formulations of the categorical imperative, as well as a robust defense of them in his seminal work on ethics [9]. He argues that all three formulations are equivalent.

The first formulation of the categorical imperative is the “Formula of Universal Law.”

**Definition** (*Formula of Universal Law*)

*I ought never to act except in such a way that I could also will that my maxim should become a universal law [9]*

A “maxim” is roughly a moral rule like “I can murder someone to take their job”. “Willing” a maxim is equivalent to acting on that rule. The FUL creates a thought experiment called the universalizability test: to decide if a maxim is permissible, imagine what would happen if everyone willed that maxim. If your imagined world

yields a contradiction, the maxim is prohibited. Intuitively, the FUL asks the question, "What would happen if everyone did that?" [13].

The universalizability test makes the "formal" nature of Kant's ethics immediately clear. Formal logic has the tools to universalize a maxim (apply a universal quantifier) and to test for contradictions (test for inconsistency).

Kant also presents two additional formulations of the categorical imperative.

**Definition** (*Formula of Humanity*)

*The Formula of Humanity (FUH) is to act in such a way "that you use humanity, whether in your own person or in the person of any other, always at the same time as an end, never merely as a means."*[9]

**Definition** (*Kingdom of Ends*)

*The third formulation of the categorical imperative states that "we should so act that we may think of ourselves as legislating universal laws through our maxims."*[13]

The last two formulations are not as obviously formal as the FUL, but they can still be modelled in logic. Because Kantian ethics presents a series of rules, a logical system can encode the theory by modelling each rule as an axiom.

The above outline is a brief and incomplete picture of a rich philosophical tradition. Kantian scholars debate the meaning of each formulation of the categorical imperative and develop views far more nuanced than those above. For the purposes of this paper, I will adopt Kant's three original formulations presented above. Note additionally that Kantian ethics is widely disputed. I do not present a defense of Kant's ethic in this paper. My approach to formalizing ethics can be applied to other theories as well.

This paper aims to formalize Kant's ethic as faithfully as possible. This is an important choice. While it is tempting to modify or simplify an ethical theory in seemingly insignificant way, these choices often have ramifications. The entire field of neo-Kantian thought has been exploring versions of Kant's ethical theory for centuries. I will not attempt to present a radically new conception of Kant's ethic, but will instead draw on philosophical expertise regarding the content and justification of the categorical imperative.

## 2.2 Dyadic Deontic Logic

### 2.2.1 Deontic Logic

Traditional modal logics include  $\Box$  operators to represent necessity. In simple modal logics like S5 [7] using the Kripke semantics,  $\Box p$  is true at a world  $w$  if  $p$  is true at all of  $w$ 's neighbors. These logics also usually contain the  $\Diamond$  operator, representing possibility, where  $\Diamond p \iff \sim \Box \sim p$ . Additionally, modal logics support operators of propositional logic like  $\sim, \wedge, \vee, \rightarrow$ .

A deontic logic is a special kind of modal logic designed to reason about obligation.

Standard deontic logic [7] [16] essentially replaces  $\Box$  with the obligation operator  $O$ , and  $\Diamond$  with the permissibility operator  $P$ . Using the Kripke semantics for  $O$ ,  $Op$  is true at  $w$  if  $p$  is true at all ideal deontic alternatives to  $w$ . The  $O$  operator in SDL takes a single argument (the formula that is obligatory), and is thus called a monadic deontic operator.

While SDL is appreciable for its simplicity, it suffers a variety of well-documented paradoxes, including contrary-to-duty paradoxes [22]. In situations where duty is violated, the logic breaks down and produces paradoxical results. Thus, I use an improved deontic logic instead of SDL for this work.

### 2.2.2 Dyadic Deontic Logic

I use as my base logic Carmo and Jones’s dyadic deontic logic, or DDL, which improves on SDL [6]. It introduces a dyadic obligation operator  $O\{A|B\}$  to represent the sentence “A is obligated in the context B”. This gracefully handles contrary-to-duty conditionals. The obligation operator uses a neighborhood semantics [23][17], instead of the Kripke semantics. Carmo and Jones define a function  $ob$  that maps from worlds to sets of sets of worlds. Intuitively, each world is mapped to the set of propositions obligated at that world, where a proposition  $p$  is defined as the worlds at which the  $p$  is true.

DDL also includes many other modal operators. In addition to  $\Box$  and  $\Diamond$ , DDL also has a notion of actual obligation and possible obligation, represented by operators  $O_a$  and  $O_p$  respectively. These notions are accompanied by the corresponding modal operators  $\Box_a, \Diamond_a, \Box_p, \Diamond_p$ . These operators use a Kripke semantics, with the functions  $av$  and  $pv$  mapping a world  $w$  to the set of corresponding actual or possible versions of  $w$ .

For more of fine-grained properties of DDL see [6] or this project’s source code. DDL is quite a heavy logic and contains many modal operators that aren’t necessary for my analysis. While this expressivity is powerful, it may also cause performance impacts. In particular, DDL has a large set of axioms involving quantification over complex higher-order logical expressions. Proofs involving these axioms will be computationally expensive. Benzmueller and Parent warned me that this may become a problem if Isabelle’s automated proof tools begin to time out.

## 2.3 Isabelle/HOL Implementation

Isabelle/HOL is an interactive proof assistant [18] built on Haskell and Scala. It allows the user to define types, functions, definitions, and axiom systems. It has built-in support for both automatic and interactive/manual theorem proving.

I started my project by reimplementing Benzmueller, Farjami, and Parent’s [2] [3] implementation of DDL in Isabelle/HOL. This helped me learn how to use Isabelle/HOL, and the implementation showcased in the next few sections demonstrates the power of Isabelle.

BFP use a shallow semantic embedding. This kind of embedding models the semantics of DDL as constants in HOL and axioms as constraints on DDL models. This document will contain a subset of my implementation that is particularly interesting and relevant to understanding the rest of the project. For the complete implementation, see the source code in `paper22.thy`.

### 2.3.1 System Definition

The first step in embedding a logic in Isabelle is defining the relevant terms and types.

**typedec1**  $i$  —  $i$  is the type for a set of worlds.

**type-synonym**  $t = (i \Rightarrow bool)$  —  $t$  represents a set of DDL formulas.

— A set of formulae is defined by its truth value at a set of worlds. For example, the set “True” would be true at any set of worlds.

The main accessibility relation that I will use is the *ob* relation:

**consts**  $ob::t \Rightarrow (t \Rightarrow bool)$  — set of propositions obligatory in this “context”

—  $ob(context)(term)$  is True if  $t$  is obligatory in this context

### 2.3.2 Axiomatization

For a semantic embedding, axioms are modelled as restrictions on models of the system. In this case, a model is specified by the relevant accessibility relations, so it suffices to place conditions on the accessibility relations. These axioms can be quite unweildy, so luckily I was able to lift BFP’s [2] implementation of Carmo and Jones’s original axioms directly. Here’s an example of an axiom:

**and** *ax-5d*:  $\forall X Y Z. ((\forall w. Y(w) \longrightarrow X(w)) \wedge ob(X)(Y) \wedge (\forall w. X(w) \longrightarrow Z(w)))$   
 $\longrightarrow ob(Z)(\lambda w. (Z(w) \wedge \neg X(w)) \vee Y(w))$

— If some subset  $Y$  of  $X$  is obligatory in the context  $X$ , then in a larger context  $Z$ , any obligatory proposition must either be in  $Y$  or in  $Z-X$ . Intuitively, expanding the context can’t cause something unobligatory to become obligatory, so the obligation operator is monotonically increasing with respect to changing contexts.

### 2.3.3 Syntax

The syntax that I will work with is defined as abbreviations. Each DDL operator is represented as a HOL formula. A formula defined with the `abbreviation` command is unfolded in every instance. While the shallow embedding is generally performant (because it uses Isabelle’s original syntax tree), abbreviations may

possibly hurt performance. In some complicated proofs, we want controlled unfolding. Benzmueller and Parent told me that eventually, the performance cost of abbreviations can be mitigated using a definition instead.

Modal operators will be useful for my purposes, but the implementation is pretty standard.

**abbreviation**  $ddlbox::t \Rightarrow t \ (\Box)$   
**where**  $\Box A \equiv \lambda w. \forall y. A(y)$   
**abbreviation**  $ddldiamond::t \Rightarrow t \ (\Diamond)$   
**where**  $\Diamond A \equiv \neg(\Box(\neg A))$

The most important operator for our purposes is the obligation operator.

**abbreviation**  $ddlob::t \Rightarrow t \Rightarrow t \ (O\{-|\cdot\})$   
**where**  $O\{B|A\} \equiv \lambda w. ob(A)(B)$   
—  $O\{B|A\}$  can be read as “B is obligatory in the context A”

While DDL is powerful because of its support for a dyadic obligation operator, in many cases we need a monadic obligation operator. Below is some syntactic sugar for a monadic obligation operator.

**abbreviation**  $ddltrue::t \ (\top)$   
**where**  $\top \equiv \lambda w. True$   
**abbreviation**  $ddlob-normal::t \Rightarrow t \ (O\{-\})$   
**where**  $(O\{A\}) \equiv (O\{A|\top\})$   
— Intuitively, the context `True` is the widest context possible because `True` holds at all worlds.

Validity will be useful when discussing metalogical/ethical properties.

**abbreviation**  $ddlvalid::t \Rightarrow bool \ (\models)$   
**where**  $\models A \equiv \forall w. A \ w$

### 2.3.4 Syntactic Properties

One way to show that a semantic embedding is complete is to show that the syntactic specification of the theory (axioms) are valid for this semantics - so to show that every axiom holds at every world. BFP [2] provide a complete treatment of the completeness of their embedding, but I will include selected axioms that are particularly interesting here. This section also demonstrates many of the relevant features of Isabelle/HOL for my project.

#### Consistency

**lemma** `True nitpick`  $[satisfy, user-axioms, format=2]$  **by** `simp`  
— Isabelle has built-in support for Nitpick, a model checker. Nitpick successfully found a model satisfying these axioms so the system is consistent.[8]  
— [Nitpick found a model for card i = 1:](#)

### Empty assignment

**Nitpick** [5] can generate models or countermodels, so it's useful to falsify potential theorems, as well as to show consistency. **by simp** indicates the proof method. In this case, **simp** indicates the Simplification proof method, which involves unfolding definitions and applying theorems directly. HOL has *True* as a theorem, which is why this theorem was so easy to prove.

### Modus Ponens

**lemma** *modus-ponens*: **assumes**  $\models A$  **assumes**  $\models (A \rightarrow B)$

**shows**  $\models B$

**using** *assms(1) assms(2) by blast*

— Because I have not defined a “derivable” operator, inference rules are written using assumptions.

— The rule **blast** is a classical reasoning method that comes with Isabelle out of the box. [18]

— This is an example of a metalogical proof in this system using the validity operator.

Another relevant operator for our purposes is  $\Box$ , the modal necessity operator. In this system,  $\Box$  behaves as an S5 modal necessity operator.

**lemma** *K*:

**shows**  $\models ((\Box(A \rightarrow B)) \rightarrow ((\Box A) \rightarrow (\Box B)))$  **by blast**

**lemma** *T*:

**shows**  $\models ((\Box A) \rightarrow A)$  **by blast**

**lemma** *5*:

**shows**  $\models ((\Diamond A) \rightarrow (\Box(\Diamond A)))$  **by blast**

As mentioned earlier, the obligation operator is most interesting for my purposes. Here are some of its properties.

**lemma** *O-diamond*:

**shows**  $\models (O\{A|B\} \rightarrow (\Diamond(B \wedge A)))$

**using** *ax-5b ax-5a*

**by** *metis*

— A is only obligatory in a context if it can possibly be true in that context. This is meant to prevent impossible obligations in a context.

**lemma** *O-nec*:

**shows**  $\models (O\{B|A\} \rightarrow (\Box O\{B|A\}))$

**by** *simp*

— Obligations are necessarily obligated. This axiom is faithful to Kant's interpretation of ethics and is evidence of Deontic Logic's power in representing Kant's theory. Kant claimed that the categorical imperative was not contingent on any facts about the world, but instead a property of the concept of morality itself [9]. Under this view, obligation should not be world-specific.



Below is an example of a more involved proof in Isabelle. This proof was almost completely automatically generated. The property itself here is not very interesting for my purposes because I will rarely mix the dyadic and monadic obligation operators.

**lemma** *O-to-O*:

**shows**  $\models (O\{B|A\} \rightarrow O\{(A \rightarrow B)|\top\})$

**proof**—

**have**  $\forall X Y Z. (ob X Y \wedge (\forall w. X w \longrightarrow Z w)) \longrightarrow ob Z (\lambda w. (Z w \wedge \neg X w) \vee Y w)$

— I had to manually specify this subgoal, but once I did Isabelle was able to prove it automatically.

**by** (*smt ax-5d ax-5b ax-5b''*)

— Isabelle’s proof-finding tool, Sledgehammer [20], comes with out-of-the-box support for smt solving [4].

**thus** *?thesis*

**proof** —

**have** *fl*:  $\forall p pa pb. ((\neg (ob p pa)) \vee (\exists i. (p \wedge (\neg pb)) i)) \vee (ob pb ((pb \wedge (\neg p)) \vee pa))$

**using**  $\forall X Y Z. ob X Y \wedge (\models (X \rightarrow Z)) \longrightarrow ob Z ((Z \wedge (\neg X)) \vee Y)$  **by** *force*

**obtain** *ii* ::  $(i \Rightarrow bool) \Rightarrow (i \Rightarrow bool) \Rightarrow i$  **where**

$\forall x0 x2. (\exists v3. (x2 \wedge (\neg x0)) v3) = (x2 \wedge (\neg x0)) (ii x0 x2)$

**by** *moura*

**then have**  $\forall p pa pb. ((\neg ob p pa) \vee (p \wedge (\neg pb)) (ii pb p)) \vee ob pb ((pb \wedge (\neg p)) \vee pa)$

**using** *fl* **by** *presburger*

**then show** *?thesis*

**by** *fastforce*

**qed**

— This entire Isar style proof was automatically generated using Sledgehammer.

**qed**

The implementation of DDL showcases some of the useful features of Isabelle. Abbreviations allow us to embed the syntax of DDL into HOL without defining an entire abstract syntax tree. Automated support for proof-finding using Sledgehammer makes proving lemmas trivial, and proving more complex theorems far easier. Nitpick’s model finding ability is useful to check for consistency and create countermodels.

### 3 The Categorical Imperative

In this section, I will present three formulations of the categorical imperative. In Section 3.1, I will consider a simple, naive formulation of the formula of universal law. This formulation is, as I will show, clearly not a good ethical rule. The purpose of this section is to explore the kinds of ethical tests that Isabelle can carry out. In Section 3.2, I will explore Moshe Kroy’s [15] partial formalization of the first two formulations of the categorical imperative. I will also explore drawbacks of this attempt, particularly in the lack of the machinery to handle the important Kantian concept of a maxim: an action performed for a particular end. In Section 3.3, I will present my own improved formalization of the categorical imperative.

### 3.1 Naive Formulation of the Formula of Universal Law

This section presents a simple and intuitive formalization of the formula of universal law, which is to will only those maxims that you would simultaneously will universalized. The universalizability test presents negative obligations: if a maxim passes the universalizability test, it is permissible. Else, it is prohibited. In order to appropriately formalize this, we need some notion of permissibility.

**abbreviation**  $ddlpermissible::t \Rightarrow t \ (P-)$

**where**  $(P A) \equiv (\neg(O \{ \neg A \}))$

— An act  $A$  is permissible if its negation is not obligated. For example, buying a red folder is permissible because I am not required to refrain from buying a red folder.

This naive formalization will require very little additional logical machinery, but more complex formalizations may require additional logic concepts beyond just that of permissibility.

Let's now consider a naive reading of the Formula of Universal Law (FUL): 'act only in accordance with that maxim through which you can at the same time will that it become a universal law' [9]. An immediate translation to DDL is that if  $A$  is not necessary permissible then it is prohibited. In other words, if we cannot universalize  $PA$  (where universalizing is represented by the modal necessity operator), then  $A$  is prohibited. Let's add this as an axiom to our logic.

**axiomatization where**

$FUL-I: \models ((\neg(\Box(PA))) \rightarrow (O\{\neg A\}))$

Why add the categorical imperative as an axiom of this logic? The purpose of this logic is to perform ethical reasoning. Kant's ethical theory is rule based, so it involves applying the categorical imperative to solve ethical dilemmas. In logic, this is equivalent to adopting the categorical imperative as an axiom and then reasoning in the newly formed logic to come to ethical conclusions. Adding the categorical imperative as an axiom makes it impossible to violate it.

Note that in this system, reasoning about violations of obligation is difficult. Any violation of the categorical imperative immediately results in a contradiction. Developing a Kantian account of contrary- to-duty obligations is a much larger philosophical project that is still open [14]. This paper will focus on the classical Kantian notion of an ideal moral world [19].

The immediate test for any formalization is consistency, which we can check with Nitpick.

**lemma True nitpick**  $[satisfy,user-axioms,format=2]$  **oops**

— Nitpick found a model for card i = 1:

Empty assignment

— Nitpick tells us that the FUL is consistent

### 3.1.1 Specifying the Model

One category of tests is specified models. Specifying the model allows us to encode certain facts into the system and then ask questions about obligations. Without specifying the model, we are limited to showing high-level metaethical facts. Let's start with analyzing an obvious example - that murder is wrong.

**consts**  $M::t$

**abbreviation**  $\text{murder-wrong}::\text{bool}$  **where**  $\text{murder-wrong} \equiv \models(O \{ \neg M \})$

**abbreviation**  $\text{poss-murder-wrong}::\text{bool}$  **where**  $\text{poss-murder-wrong} \equiv \models(\Diamond (O \{ \neg M \}))$

— These are very simple properties. *poss-murder-wrong* is an abbreviation for the axiom that there is some world where murder might be prohibited. Even this is quite a strong assumption - ideally we'd want to give the system nonmoral facts about murder (like a definition) and then make moral claims.

**lemma** *wrong-if-possibly-wrong*:

**assumes** *poss-murder-wrong*

**shows** *murder-wrong*

**using** *assms* **by** *blast*

— This lemma gets to the “heart” of this naive interpretation. We really want to say that if something isn't necessarily obligated, it's not obligated anywhere. ”

The above example does exactly what we expect it to: we show that if something is wrong somewhere it's wrong everywhere. That being said, it seems like quite a weak claim. We assumed a very strong, moral fact about murder (that it is wrong somewhere), so of course we were able to reach our desired conclusion.

Let's try a weaker assumption: Not everyone can lie.

**typeddecl** *person*

**consts**  $\text{lie}::\text{person} \Rightarrow t$

**consts**  $\text{me}::\text{person}$

— Notice that this machinery is quite empty. We don't give an axioms about what a person can or can't do.

**abbreviation**  $\text{lying-not-universal}::\text{bool}$  **where**  $\text{lying-not-universal} \equiv \neg (\forall x. \text{lie}(x) \text{ cw}) \wedge (\text{lie}(\text{me}) \text{ cw})$

This is a rough translation of failure of the universalizability test: we will the maxim universally, as represented by the universal quantifier in the first conjunct, and simultaneously [11], as represented by the second conjunct. The FUL tells us that if this sentence is true, then lying should be prohibited. Let's test it.

**lemma** *breaking-promises*:

**assumes** *lying-not-universal*

**shows**  $(O \{ \neg (\text{lie}(\text{me})) \}) \text{ cw}$

**nitpick** [*user-axioms*]

**oops**

— Nitpick found a counterexample for card person = 2 and card i = 2:

Free variable:  $\text{lie} = (\lambda x. \dots)(p_1 := (\lambda x. \dots)(i_1 := \text{True}, i_2 := \text{False}), p_2 := (\lambda x. \dots)(i_1 := \text{False}, i_2 := \text{False}))$

This formula isn't valid. While the FUL should tell us that lying is prohibited, the fact that it doesn't demonstrates the weakness of this naive formulation of the categorical imperative. Kant's version of the FUL universalizes across people, as we did in the definition of *lying-not-universal*  $\equiv \neg (\forall x. \text{lie } x \text{ } cw) \wedge \text{lie } me \text{ } cw$ . The naive formalization universalizes across worlds using the  $\Box$  operator, so it makes sense that it can't handle this example appropriately.

The above implies that the FUL should prescribe consistent obligations across people. If our formalization doesn't, clearly something has gone wrong somewhere. Let's test that!

**lemma** *universalizability*:

**assumes**  $\models O \{(\text{lie}(me))\}$

**shows**  $\forall x. \models (O \{(\text{lie}(x))\})$

**nitpick** *[user-axioms]* **oops**

— Nitpick found a counterexample for card person = 2 and card i = 2:

Free variable:  $\text{lie} = (\lambda x. \dots)(p_1 := (\lambda x. \dots)(i_1 := \text{False}, i_2 := \text{True}), p_2 := (\lambda x. \dots)(i_1 := \text{False}, i_2 := \text{False}))$  Skolem constant:  $x = p_2$

This lemma demonstrates the problem with the naive interpretation. The FUL universalizes across people but the naive formalization universalizes across worlds. Because this interpretation is so naive, it is limited in its power. However, this serves as an example of the kind of reasoning that Isabelle empowers us to do. Even this simple argument has philosophical consequences. It tells us that reading the FUL as a claim about consistency across possible worlds, instead of consistency across agents, leads to counterintuitive conclusions.

### 3.1.2 Metaethical Properties

The above section specified the model to simulate practical ethical reasoning, or the kind of reasoning that is useful when an agent has to decide what to do. Formalizations of the categorical imperative can also be used to do metaethical reasoning, which can evaluate a particular ethical theory as good or bad. In this case, we can analyze properties of the system in the form of theorems. For example, if we can show that, in this system, nothing is ever obligated, that would indicate that we have a bad ethical system. This is not only philosophical work, but is also a useful way to test different ethical reasoning systems.

An initial property that we might be interested in is permissibility itself. More generally, an ethical theory that doesn't allow for permissibility would require that every action is either obligatory or prohibited. In fact, if that is the case, many counterintuitive theorems follow, including that all permissible actions are obligatory.<sup>1</sup>

---

<sup>1</sup>Proof is in the appendix.

**lemma** *permissible*:

**fixes**  $A$

**shows**  $((\neg (O \{A\})) \wedge (\neg (O \{\neg A\}))) w$

**nitpick** [*user-axioms*, *falsify=false*] **oops**

— Nitpick found a model for card i = 1:

Free variable:  $A = (\lambda x. \_) (i_1 := \text{False})$

— This is exactly the desired result above - we have shown that models exist where an action can be neither obligatory nor prohibited.

— Note that it's not clear [10] if Kant actually thought that permissibility was a coherent concept. Either way, in modern ethics, permissibility is a pretty widely accepted phenomenon.

Another initial test is the possibility of arbitrary obligations. If anything can be shown to be obligatory in this theory, then clearly it doesn't track out intuitions.

**lemma** *arbitrary-obligations*:

**fixes**  $A::t$

**shows**  $O \{A\} w$

**nitpick** [*user-axioms=true*] **oops**

— Nitpick found a counterexample for card i = 1:

Free variable:  $A = (\lambda x. \_) (i_1 := \text{False})$

— This is good! Shows us that any arbitrary term isn't obligatory.

## Conflicting Obligations

A more complex metaethical property is the possibility of conflicting obligations. Many deontological ethics are criticized for prescribing conflicting obligations, but it is clear that in Kantian ethics, obligations are never conflicting [24]. In order for morality to be action-guiding, it needs to be free of conflicting obligations. Let's see if we can have contradictory obligations under the naive formalization.

**lemma** *conflicting-obligations*:

**fixes**  $A$

**shows**  $(O \{A\} \wedge O \{\neg A\}) w$

**nitpick** [*user-axioms*, *falsify=false*] **oops**

— Nitpick found a model for card i = 2:

Free variable:  $A = (\lambda x. \_) (i_1 := \text{False}, i_2 := \text{True})$

— Oh no! Nitpick found a model with conflicting obligations - that's bad!

This is a property of DDL itself, not necessarily of my formalization specifically. A future, more robust formalization should add an axiom that disallows this. Let's see if that causes any obvious problems.

**lemma** *removing-conflicting-obligations*:

**assumes**  $\forall A. \models (\neg (O \{A\} \wedge O \{\neg A\}))$

**shows** *True*

**nitpick** [*satisfy*, *user-axioms.format=2*] **oops**

— Nitpick found a model for card i = 1:

Empty assignment

— We can disallow conflicting obligations and the system is still consistent - that's good.

The above is a rather weak notion of contradictory obligations. Korsgaard [13] argues that Kantian ethics also has the stronger property that if two maxims imply a contradiction, they must not be willed. Let's see if that fact holds in this formalization.

**lemma** *implied-contradiction*:

**fixes**  $A::t$   
**fixes**  $B::t$   
**assumes**  $\models (\neg (A \wedge B))$   
**shows**  $\models (\neg (O \{A\} \wedge O \{B\}))$   
**nitpick** [*user-axioms*]

**proof** –

**have**  $\models (\neg (\Diamond (A \wedge B)))$   
**by** (*simp add: assms*)  
**then have**  $\models (\neg (O \{A \wedge B\}))$  **by** (*smt O-diamond*)

— Notice that this is **almost** the property we are interested in. In fact, if  $O\{A \wedge B\}$  is equivalent to  $O\{A\} \wedge O\{B\}$ , then the proof is complete.

**thus** *?thesis* **oops**

— Nitpick found a counterexample for card i = 2:

Free variables:  $A = (\lambda x..)(i_1 := \text{True}, i_2 := \text{False})$   $B = (\lambda x..)(i_1 := \text{False}, i_2 := \text{True})$

— Sadly the property we're actually interested in doesn't follow.

The above proof yields an interesting observation.  $O\{A \wedge B\}$  is not equivalent to  $O\{A\} \wedge O\{B\}$ . The rough English translation of  $O\{A \wedge B\}$  is “you are obligated to do both A and B”. The rough English translation of  $O\{A\} \wedge O\{B\}$  is “you are obligated to do A and you are obligated to do B.” We think those English sentences mean the same thing, so they should mean the same thing in our logic as well. Let's test that.

**lemma** *distribute-obligations*:

**assumes**  $\models (O \{A\} \wedge O \{B\})$   
**shows**  $\models O \{A \wedge B\}$   
**nitpick** [*user-axioms*] **oops**

— Nitpick found a counterexample for card i = 2:

Free variables:  $A = (\lambda x..)(i_1 := \text{True}, i_2 := \text{False})$   $B = (\lambda x..)(i_1 := \text{False}, i_2 := \text{True})$

Note that this is a property of DDL itself, not just my formalization. A future formalization might add this property as an axiom.<sup>2</sup>

## Miscellaneous Properties

I named this formalization the naive formulation for a reason. Though it seems to be an immediate translation of the FUL into DDL, it doesn't fully respect the properties of modal logic itself. In particular, the formalization as given is equivalent to the below theorem.

**lemma** *FUL-alternate*:

**shows**  $\models ((\Diamond (O \{\neg A\})) \rightarrow (O \{\neg A\}))$

<sup>2</sup>For discussion of why this property doesn't hold in DDL, see the Appendix.

**by simp**

— This means that if something is possibly prohibited, it is in fact prohibited.

— This is a direct consequence<sup>3</sup> of the naive formalization, but it’s not clear to me that this is actually how we think about ethics. For example, we can imagine an alternate universe where smiling at someone is considered an incredibly rude and disrespectful gesture. In this universe, we are probably prohibited from smiling at people, but this doesn’t mean that in this current universe, smiling is morally wrong.

The “ought implies can” principle is attributed to Kant<sup>4</sup> and is rather intuitive: you can’t be obligated to do the impossible. It is worth noting that deontic logics evolved[7] specifically from this principle, so this should hold in both my modified logic and in DDL.

**lemma** *ought-implies-can*:

**shows**  $\forall A. \models (O \{A\} \rightarrow (\Diamond A))$

**using** *O-diamond* **by** *blast*

—  $\models \lambda w. ob \ ?B \ ?A \longrightarrow \neg \models \neg \ ?B \wedge \ ?A$  is an axiom of DDL itself, so this theorem holds in DDL.

## 3.2 Kroy’s Formalization of the Categorical Imperative

This section contains a formalization of the categorical imperative introduced by Moshe Kroy in 1976 [15]. Kroy used Hintikka’s deontic logic to formalize the Formula of Universal Law and the Formula of Humanity. I will first import the additional logical tools that Hintikka’s logic contains that Kroy relies on, then examine the differences between his logic and DDL, and finally implement and test both of Kroy’s formalizations.

### 3.2.1 Additional Logical Tools

Kroy’s logic relies heavily on some notion of identity or agency. We need some notion of “x does action”, which we can write as “x is the subject of the sentence ‘does action’”. This requires defining a subject.

**typed decl** *s* — *s* is the type for a “subject”, like the subject of a sentence

Kroy also defines a substitution operator<sup>5</sup>.  $P(d/e)$  is read in his logic as “P with e substituted for d.” DDL has no such notion of substitution, so I will define a more generalized notion of an “open sentence.” An open sentence takes as input a subject and returns a complete or “closed” DDL formula. For example, “does action” is an open sentence that can be instantiated with a subject.

**type-synonym** *os* =  $(s \Rightarrow t)$

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<sup>3</sup>For a manual proof, see the Appendix.

<sup>4</sup>The exact philosophical credence of this view is disputed, but the rough idea holds nonetheless. See [12] for more.

<sup>5</sup>see page 196 in [15]

- “P sub (d/e)” can be written as “S(e)”, where  $S(d) = P$
- The terms that we substitute into are actually instantiations of an open sentence, and substitution just requires re-instantiating the open sentence with a different subject.

### New Operators

Because Isabelle is strongly typed, we need to define new operators to handle open sentences. These operators are similar to DDL’s original operators. We could probably do without these abbreviations, but they will simplify the notation and make it look more similar to Kroy’s original paper.

**abbreviation**  $os-neg::os \Rightarrow os \ (\neg-)$   
**where**  $(\neg A) \equiv \lambda x. \neg(A(x))$   
**abbreviation**  $os-and::os \Rightarrow os \Rightarrow os \ (-\wedge-)$   
**where**  $(A \wedge B) \equiv \lambda x. ((A(x)) \wedge (B(x)))$   
**abbreviation**  $os-or::os \Rightarrow os \Rightarrow os \ (-\vee-)$   
**where**  $(A \vee B) \equiv \lambda x. ((A(x)) \vee (B(x)))$   
**abbreviation**  $os-ob::os \Rightarrow os \ (O\{-\})$   
**where**  $O\{A\} \equiv \lambda x. (O \{A(x)\})$

Once again, the notion of permissibility will be useful here.

**abbreviation**  $ddl-permissible::t \Rightarrow t \ (P \{-\})$   
**where**  $P \{A\} \equiv \neg (O \{\neg A\})$   
**abbreviation**  $os-permissible::os \Rightarrow os \ (P \{-\})$   
**where**  $P \{A\} \equiv \lambda x. P \{A(x)\}$

### 3.2.2 Differences Between Kroy’s Logic (Kr) and DDL



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