| Student ID | | Name | |
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# Computer Vision: Final Exam

3:30 – 4:45 PM, 10 JUN 2017

Write your answer **as compactly as possible** in this question sheet. There are **seven** problems in total.

1. (20 points) Let us prove that the $K$-means algorithm does not always converge to a global minimum. There are 7 scalar values:

   1.0, 2.0, 3.0001, 4.0, 8.0, 9.0, 12.0

   We divide these points into three clusters, i.e. $K = 3$.

   (a) Start with initial cluster centers $\mu_1 = 1.0$, $\mu_2 = 5.0$, and $\mu_3 = 9.0$. After the convergence, what are the cluster centers?

   (b) Find a different set of initial cluster centers that cause the convergence to a different solution from (a).

2. (15 points) For a natural number $n$, $D_n$ denotes the number of different ways to write $n$ as the sum of 1, 3, 4. For example, $D_1 = 1, D_2 = 1, D_3 = 2, D_4 = 4$. Also, $D_5 = 6$ because

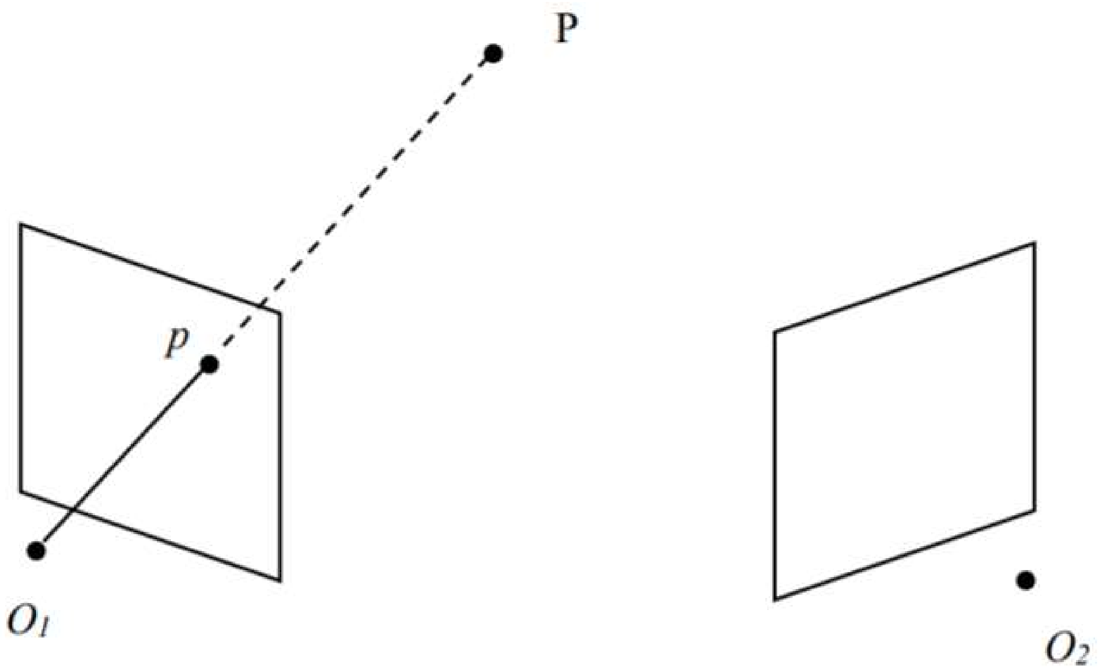
$$\begin{aligned}5 &= 1+1+1+1+1\\ &= 1+1+3\\ &= 1+3+1\\ &= 3+1+1\\ &= 1+4\\ &= 4+1\end{aligned}$$

   (a) What is $D_4$?

   (b) What is $D_6$?

   (c) In general, what is the recurrence equation for solving this problem?

3. (10 points) The figure below shows a stereo camera setup, with cameras $O_1$ and $O_2$. Draw and identify the *baseline*, the *epipoles*, the *epipolar plane*, and the *epipolar line* corresponding to the image point $p$.




4. (15 points) A Gaussian distribution has a known variance $\sigma^2$ and an unknown mean $m$, given by

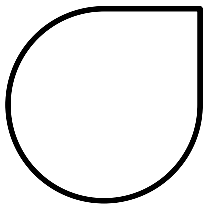
$$f(x|m) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left(-\frac{(x-m)^2}{2\sigma^2}\right)$$

Given $N$ points from the distribution, $x_1, x_2, \cdots, x_N$, the likelihood of the parameter $m$ is given by

$$L(m) = \prod_{i=1}^{N} f(x_i\,|m).$$

What is the maximum likelihood (ML) estimate of the parameter? Explain your answer.

5. (10 points) Suppose that you need to find the following shape of the fixed size in a noisy image. Assume that the image contains only one such shape. Describe the Hough transform algorithm for this problem. Your algorithm should output the position of the sharp corner point of the shape.

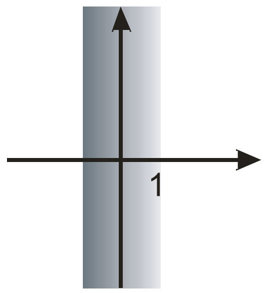


6. (15pts) Find the equation $y = \beta_0 + \beta_1 x$ of the least-squares line that best fits $(0,1),\ (1,1),\ (2,2),\ (3,2)$.

7. (15 points) We have a vertical strip at time $t = 0$, as shown below. Its intensity is given by

$$\psi(x, y, t) = \begin{cases} x - t + 10 & \text{if } |x - t| \leq 1 \\ 0 & \text{otherwise} \end{cases}$$

For example at $t = 0$, it looks like this




(a) Compute the motion vector of the point $(0, 0)$ at time $t = 0$ using the optical flow equation.

(b) Plot the strip at time $t = 2$.

(c) Have you obtained a unique solution in (a)?

(d) If you assume that all $3 \times 3$ points around $(0, 0)$ have the same motion vector as in the Lucas-Kanade algorithm, can you obtain a unique solution in (a)? Explain your answer.