$$(\mathrm{F}_1, \phi_1) \coloneqq \operatorname*{argmin}_{\mathrm{F},\phi} \mathop{\mathbb{E}}_{x \in \mathcal{D}} \mathcal{L}_1(\mathrm{F} \circ \phi(x), \mathrm{F}^* \circ \phi^*(x))$$

$$(\mathrm{F}_2, \phi_2, \mathrm{K}_2) \coloneqq \operatorname*{argmin}_{\mathrm{F},\phi} \mathop{\mathbb{E}}_{x \in \mathcal{D}} [\mathcal{L}_1(\mathrm{F} \circ \phi(x), \mathrm{F}^* \circ \phi^*(x)) + \alpha \cdot \mathcal{L}_2(\mathrm{K} \circ \phi(x), \mathrm{K}^* \circ \phi^*(x))]$$

$$\mathrm{K}_3 \coloneqq \operatorname*{argmin}_{\mathrm{K}} \mathop{\mathbb{E}}_{x \in \mathcal{D}} \mathcal{L}_2(\mathrm{K} \circ \phi_1(x), \mathrm{K}^* \circ \phi^*(x))$$

$$\begin{aligned}
&\mathop{\mathbb{E}}_{x \in \mathcal{D}} [\mathcal{L}_1(\mathrm{F}_2 \circ \phi_2(x), \mathrm{F}^* \circ \phi^*(x)) + \alpha \cdot \mathcal{L}_2(\mathrm{K}_2 \circ \phi_2(x), \mathrm{K}^* \circ \phi^*(x))] \\
&= \min_{\mathrm{F},\phi,\mathrm{K}} \mathop{\mathbb{E}}_{x \in \mathcal{D}} [\mathcal{L}_1(\mathrm{F} \circ \phi(x), \mathrm{F}^* \circ \phi^*(x)) + \alpha \cdot \mathcal{L}_2(\mathrm{K} \circ \phi(x), \mathrm{K}^* \circ \phi^*(x))] \\
&\leqslant \min_{\mathrm{K},\phi=\phi_1,\mathrm{F}=\mathrm{F}_1} \mathop{\mathbb{E}}_{x \in \mathcal{D}} [\mathcal{L}_1(\mathrm{F} \circ \phi(x), \mathrm{F}^* \circ \phi^*(x)) + \alpha \cdot \mathcal{L}_2(\mathrm{K} \circ \phi(x), \mathrm{K}^* \circ \phi^*(x))] \\
&= \mathop{\mathbb{E}}_{x \in \mathcal{D}} [\mathcal{L}_1(\mathrm{F}_1 \circ \phi_1(x), \mathrm{F}^* \circ \phi^*(x))] + \min_{\mathrm{K}} \mathop{\mathbb{E}}_{x \in \mathcal{D}} \alpha \cdot \mathcal{L}_2(\mathrm{K} \circ \phi_1(x), \mathrm{K}^* \circ \phi^*(x))] \\
&= \mathop{\mathbb{E}}_{x \in \mathcal{D}} [\mathcal{L}_1(\mathrm{F}_1 \circ \phi_1(x), \mathrm{F}^* \circ \phi^*(x)) + \alpha \cdot \mathcal{L}_2(\mathrm{K}_3 \circ \phi_1(x), \mathrm{K}^* \circ \phi^*(x))] \\
&\leqslant \mathop{\mathbb{E}}_{x \in \mathcal{D}} [\mathcal{L}_1(\mathrm{F}_2 \circ \phi_2(x), \mathrm{F}^* \circ \phi^*(x)) + \alpha \cdot \mathcal{L}_2(\mathrm{K}_3 \circ \phi_1(x), \mathrm{K}^* \circ \phi^*(x))]
\end{aligned}$$

The second inequality holds because of the first definition above.