

## 10.009 The Digital World

Term 3. 2019

Problem Set 4 (for Week 4)

Last update: February 22, 2019

- **Problems: Cohort sessions:** Following week: Tuesday 11:59pm.
- **Problems: Homework:** Same as for the cohort session problems.
- **Problems: Exercises:** These are practice problems and will not be graded. You are encouraged to solve these to enhance your programming skills. Being able to solve these problems will likely help you prepare for the midterm examination.

### Objectives

1. Learn nested lists.
2. Learn how to create and use nested lists as tables.
3. Learn how to use looping to process lists.
4. Learn how to traverse sublists.
5. Learn tuples.
6. Learn to use the dictionary data structure.

**Note:** Solve the programming problems listed using your favorite text editor. Make sure you save your programs in files with suitably chosen names, **and try as much as possible to write your code with good style (see the style guide for python code)**. In each problem find out a way to test the correctness of your program. After writing each program, test it, debug it if the program is incorrect, correct it, and repeat this process until you have a fully working program. Show your working program to one of the cohort instructors.

## Problems: Cohort sessions

1. *Lists*: The following problems test your knowledge of lists in Python. There is no need to write a program for these, but you can verify your answers by writing programs. Write 'E' if the code returns any error (or any errors for that line for parts d, e, and f).

- (a) Specify the value of `x[0]` at the end the following code snippet.

```
x=[1,2,3]
x[0]=0
y=x
y[0]=1
```

- (b) Specify the value of `x[0]` after the following code snippet.

```
x=[1,2,3]
def f(1):
    l[0]='a'
f(x)
```

- (c) What is the value of `a[0][0][0][0]` after executing the following code snippet?

Write 'E' if there are any errors.

```
x=[1,2,3]
y=[x]
a=[y,x]
y[0][0] = (1,2)
```

- (d) Specify the values of expressions (a), (b), (c) and (d) in the following code.

```
x=[1,2,3]
y1=[x,0]
y2=y1[: ]
y2[0][0]=0
y2[1]=1
y1[0][0] # (a)
y1[1] # (b)
y2[0][0] # (c)
y2[1] # (d)
```

- (e) Specify the values of expressions (a), (b), (c) and (d) in the following code.

```
import copy
x=[1,2,3]
y1=[x,0]
y2=copy.deepcopy(y1)
y2[0][0]=0
y2[1]=1
y1[0][0] # (a)
y1[1] # (b)
y2[0][0] # (c)
y2[1] # (d)
```

- (f) What is the value of `l` after steps (a), (b), (c) and (d) below?

```
l=[1,2,3]
l[2:3]=4 # (a)
l[1:3]=[0] # (b)
l[1:1]=1 # (c)
l[2:]=[] # (d)
```

2. *Functions: Compound value:* Suppose you deposit \$100 on the first day of each month into a savings account with an annual interest rate of 5%. The bank calculates the interest gained and credits the amount to you at the end of the month. The monthly interest rate is  $0.05/12=0.00417$ . At the end of the first month, the value in the account is

$$100 * (1 + 0.00417) = 100.417$$

At the end of the second month, the value in the account is

$$(100 + 100.417) * (1 + 0.00417) = 201.252$$

At the end of the third month, the value in the account is

$$(100 + 201.252) * (1 + 0.00417) = 302.507$$

and so on.

Write a function named `compound_value_months` that takes in a monthly saving amount, an annual interest rate, and the number of months ( $n$ ), and returns the account value at the end of the  $n^{th}$  month. Round the return value to 2 decimal places. Note: this problem is similar to one of the problems you did in the past. The only different is that the number of months here can be any integer  $n$ , thus requiring you to use loops.

```
>>> ans=compound_value_months(100,0.05,6)
>>> print(ans)
608.81
>>> ans=compound_value_months(100,0.03,7)
>>> print(ans)
707.04
>>> ans=compound_value_months(200,0.05,8)
>>> print(ans)
1630.29
>>> ans=compound_value_months(200,0.03,1)
>>> print(ans)
200.5
```

3. *Loops:* Write a function named `find_average` that takes in a list of lists as an input. Each sublist contains numbers. The function returns a list of the averages of each sublist, and the overall average. If the sublist is empty, take the average to 0.0.

For example, if the input list is `[[3,4],[5,6,7],[-1,2,3]]`, the program returns the list `[3.5,6.0,1.333]`, and the overall average 3.625, calculated by summing all the numbers in all the sublists and dividing this total sum by the total count of all the numbers.

```
>>> ans=find_average([[3,4],[5,6,7],[-1,2,8]])
>>> print(ans)
([3.5, 6.0, 3.0], 4.25)

>>> ans=find_average([[13.13,1.1,1.1],[],[1,1,0.67]])
>>> print(ans)
```

```
([5.11, 0.0, 0.89], 3.0)

>>> ans=find_average([[3.6],[1,2,3],[1,1,1]])
>>> print(ans)
([3.6, 2.0, 1.0], 1.8)

>>> ans=find_average([[2,3,4],[2,6,7],[10,5,15]])
>>> print(ans)
([3.0, 5.0, 10.0], 6.0)
```

4. *Lists and nested loops:* Use of a nested list in Python allows you to implement a data structure as a 2-dimensional matrix along with various matrix operations. Write a function named `transpose_matrix`, which takes a  $n \times m$  integer matrix (i.e. a list with  $n$  items each of which is a list of  $m$  integer items) as an argument, and returns its transposed matrix. For example:

```
>>> a = [[1,2,3], [4,5,6], [7,8,9]]
>>> transpose_matrix(a)
[[1,4,7], [2,5,8], [3,6,9]]
>>>
```

Use a nested for-loop (i.e. a for loop inside a for loop) appropriately to implement this. You are not allowed to use any built-in transpose function from any libraries.

5. *Dictionary:* Write a function named `get_details` that takes in a name, a key search, and a list. The list contains a list of phone book entries, where each entry is a dictionary. For example

```
>>> phonebook=[{'name':'Andrew', 'mobile_phone':9477865, '
office_phone':6612345, 'email':'andrew@sutd.edu.sg'},{'name':'
Bobby','mobile_phone':8123498, 'office_phone':6654321, 'email': '
bobby@sutd.edu.sg'}]
```

The function returns the value of the key search requested for that particular name. It should return `None` if either the name or the key is not found. For example:

```
>>> print(get_details('Andrew', 'mobile_phone', phonebook))
9477865
>>> print(get_details('Andrew', 'email', phonebook))
andrew@sutd.edu.sg
>>> print(get_details('Bobby', 'office_phone', phonebook))
6654321
>>> print(get_details ('Chokey', 'office_phone', phonebook))
None
```

6. *Dictionary:* Write a function named `get_base_counts` that takes a DNA string as an input. The input string consists of letters A, C, G, and T (upper case only). The function returns the count of the number of times each of the four letters A, C, G, and T appear in the input string, in the form of a dictionary. For any input string with letters other than A, C, T, and G, the function will return 'The input DNA string is invalid'. **Test Cases:**

**Test case 1**

Input: 'AACCGT'  
 Output: {'A': 2, 'C': 2, 'G': 1, 'T': 1}

**Test case 2**

Input: 'AACCG'  
 Output: {'A': 2, 'C': 2, 'G': 1, 'T': 0}

**Test case 3**

Input: 'AAB'  
 Output: 'The input DNA string is invalid'

**Test case 4**

Input: 'AaCaGT'  
 Output: 'The input DNA string is invalid'

**Problems: Homework**

1. *Lists and loops*: One may use an approximate formula to quickly converting Fahrenheit (F) to Celsius (C):  $C \approx \tilde{C} = (F - 30)/2$ . Write three functions, one named `f_to_c` that returns the exact value in degree Celsius given a temperature in degree Fahrenheit; the second named `f_to_c_approx` that returns the approximate value in degree Celsius given a temperature in degree Fahrenheit; and the third named `get_conversion_table` to build a table `conversion` (i.e. a nested list). You should use your `f_to_c` and `f_to_c_approx` functions in your `get_conversion_table` function.

- (a) Use the `get_conversion_table()` function you have written to generate a conversion table for values of degree Fahrenheit. You need to convert for each of the following values: F=0, 10, 20, ..., 100. The conversion table should be in the form of `[[F, C, CApprox], [F, C, CApprox], ...]`, where `F` is the temperature in degree Fahrenheit; `C` is the corresponding temperature in degree Celsius to 1 decimal place, and `CApprox` is the approximate temperature in degree Celsius to 1 decimal place.

```
>>> print(get_conversion_table())
[[0, -17.8, -15.0], [10, -12.2, -10.0], [20, -6.7, -5.0],
 [30, -1.1, 0.0], [40, 4.4, 5.0], [50, 10.0, 10.0], [60,
 15.6, 15.0], [70, 21.1, 20.0], [80, 26.7, 25.0], [90,
 32.2, 30.0], [100, 37.8, 35.0]]
```

- (b) Use `get_conversion_table()` function you have written to generate a conversion table for values of degree Fahrenheit. You need to do this for each of the following values: F=0, 10, 20, ..., 100. The conversion table should be in the form of `[[F, F, ...], [C, C, ...], [CApprox, CApprox, ...]]`.

```
>>> print(get_conversion_table())
[[0, 10, 20, 30, 40, 50, 60, 70, 80, 90, 100], [-17.8, -12.2,
 -6.7, -1.1, 4.4, 10.0, 15.6, 21.1, 26.7, 32.2, 37.8],
 [-15.0, -10.0, -5.0, 0.0, 5.0, 10.0, 15.0, 20.0, 25.0,
 30.0, 35.0]]
```

(c) Can you use one of the functions you implemented in one of the cohort exercises to easily convert between the two forms of tables?

2. Write a function named `max_list` which takes a two-level nested list of integers, `inlist`, as an input, and outputs a list, `outlist`, such that `outlist[i]` is the maximum of all numbers in `inlist[i]`. You can assume that `inlist` and all nested lists are never empty.

```
>>> inlist = [[1,2,3],[4,5]]
>>> print(max_list(inlist))
[3, 5]
>>> inlist = [[1,2,3],[4,5],[32,3,4]]
>>> print(max_list(inlist))
[3, 5, 32]
>>> inlist = [[3,4,5,2],[1,7],[8,0,-1],[2]]
>>> print(max_list(inlist))
[5, 7, 8, 2]
>>> inlist = [[100],[1,7],[-8,-2,-1],[2]]
>>> print(max_list(inlist))
[100, 7, -1, 2]
>>> inlist = [[3,4,5,2]]
>>> print(max_list(inlist))
[5]
```

3. Write a Python function named `multiplication_table` that takes a value  $n$  and returns an  $n$  by  $n$  multiplication table. For instance, if  $n$  is seven, your program will return a table as follows.

1	2	3	4	5	6	7
2	4	6	8	10	12	14
3	6	9	12	15	18	21
4	8	12	16	20	24	28
5	10	15	20	25	30	35
6	12	18	24	30	36	42
7	14	21	28	35	42	49

The first element of the nested list should be a list that represents the first row of the table, the second element represents the second row and so on. For  $n < 1$ , your function should return the value `None`.

**Test Cases:****Test case 1**

Input:  $n=7$   
 Output:  $[[1, 2, 3, 4, 5, 6, 7], [2, 4, 6, 8, 10, 12, 14], [3, 6, 9, 12, 15, 18, 21],$   
 $[4, 8, 12, 16, 20, 24, 28], [5, 10, 15, 20, 25, 30, 35],$   
 $[6, 12, 18, 24, 30, 36, 42], [7, 14, 21, 28, 35, 42, 49]]$

**Test case 2**

Input:  $n=1$   
 Output:  $[[1]]$

**Test case 3**

Input:  $n=0$   
 Output:  $\text{None}$

**Test case 4**

Input:  $n=2$   
 Output:  $[[1, 2], [2, 4]]$

**Test case 5**

Input:  $n=-1$   
 Output:  $\text{None}$

4. *Dictionary:* Write a function named **most\_frequent** that takes in a list of integers and returns a list of the ones that have the most occurrences. If more than one number have the most number of occurrences, all of them should be reported. You can assume that the input list is never empty. For example:

```
input=[2,3,40,3,5,4,-3,3,3,2,0]
most_frequent = [3]
```

```
input=[9,30,3,9,3,2,4]
most_frequent = [9, 3]
```

5. *Dictionary:* A polynomial can be represented by a dictionary. Write a function **diff** for differentiating such polynomials. **diff** takes a polynomial as a dictionary argument and returns the dictionary representation of its the derivative. If **p** denotes the polynomial as a dictionary and **dp** a dictionary representing its derivative, we have  $dp[j-1] = j \cdot p[j]$  for  $j$  running over all keys in **p**, except when  $j$  equals 0. Here is an example of the use of **diff**

```
>>> p={0:-3, 3:2, 5:-1}
>>> diff(p)
{2:6, 4:-5}
```

In the above example, the dictionary  
 $p=\{0:-3, 3:2, 5:-1\}$

means that the 0<sup>th</sup> coefficient is -3, the 3<sup>rd</sup> coefficient is 2, and the 5<sup>th</sup> coefficient is -1. This can be written as

$$p(x) = -x^5 + 2x^3 - 3 \quad (1)$$

#### Test Cases:

##### Test case 1

Input:  $p=\{0:-3, 3:2, 5:-1\}$   
Output:  $\{2: 6, 4: -5\}$

##### Test case 2

Input:  $p=\{1:-3, 3:2, 5:-1, 6:2\}$   
Output:  $\{0:-3, 2:6, 4:-5, 5:12\}$

##### Test case 3

Input:  $p=\{0:-3, 3:2, 8:2\}$   
Output:  $\{2:6, 7:16\}$

##### Test case 4

Input:  $p=\{0:-4, 2:12, 3:-2, 4:3, 8:2\}$   
Output:  $\{1:24, 2:-6, 3:12, 7:16\}$

##### Test case 5

Input:  $p=\{0:-3, 1:12, 2:-2, 3:2, 10:2\}$   
Output:  $\{0:12, 1:-4, 2:6, 9:20\}$

#### Problems: Exercises

1. *bool, tuple, None*: The following problems test your knowledge of `bool`, `tuple` and `NoneType` types. There is no need to write a program for these but you can verify your answer by writing programs.

- (a) What are the types of `a` and `b`?

```
a=(1)
b=(1,)
```

- (b) List from the variables below those that will return `False` when converted to `bool`.

```
a='abc'
b=0+0j
c=(1,)
d=''
e='None'
f=None
```

- (c) What will be the values of the expressions at lines indicated by (a), (b) and (c) below?

```
t=(1,2,3)
t+t # (a)
t*2 # (b)
t[1:-1] # (c)
```



2. *Loops*: Two words of equivalent length “interlock” if alternating letters from each word forms a new word, starting with the first word inputted (word1). For example, “shoe” and “cold” interlock to form “schooled”. Write a function named `interlock` which takes (word1, word2, word3) as the input and returns True if and only if word1 and word2 interlocks and generates word3. Return False if word1, word2, or word3 are left blank. Note that words not of equivalent length will never interlock. Similarly, empty strings are not words and thus cannot interlock.

**Test Cases:**

**Test case 1**

Input: word1='shoe', word2='cold', word3='schooled'  
Output: True

**Test case 2**

Input: word1='shoes', word2='cold', word3='schooled'  
Output: False

**Test case 3**

Input: word1='', word2='cold', word3='schooled'  
Output: False

**Test case 4**

Input: word1='shoes', word2='cold', word3=''  
Output: False

**Test case 5**

Input: word1='', word2='', word3=''  
Output: False

**Test case 6**

Input: word1='can', word2='his', word3='chains'  
Output: True

3. Write a function named `throw_dice` to estimate the probability of getting at least one 6 when throwing  $n$  die, where  $n$  is the number of dice thrown. Take `n` and the number of experiments, `nExp` as inputs. When throwing `n` dice, you should break the loop and move to the next experiment as soon as the one of them is a 6. Round the returned value to 2 decimal places. Read up on Python's random library and use `random.randint()` to generate values for the die, to simulate the rolling of die.
4. *Loops, random numbers*: Write a function named `pi_approx_monte_carlo` that approximates the value of  $\pi$  using the Monte Carlo simulation. The function should take an integer argument as the number of random coordinates, located in the circle, to be used to approximate the desired value. Round the returned value to two decimal places. Use `random.random()` or `random.uniform()` to generate the random coordinates. Also, use either a quarter circle in the positive quadrant or a full circle, both centred on (0,0), for this code. Note: your result may differ from the test cases, based on the random seed you use. Furthermore, you should include the boundary of the circle in

approximating pi. For a 5-minute video explanation of the Monte Carlo simulation, see <http://youtu.be/VJTffIq04TU>.

#### Test Cases:

```
>>> pi_approx_monte_carlo(100)
pi = 3.36
>>> pi_approx_monte_carlo(100000)
pi = 3.15
>>> pi_approx_monte_carlo(10000000) (takes approx. 7 seconds)
pi = 3.14
```

5. Somebody suggests the following game: For 1 dollar you are allowed to throw four die. If the sum of all the eyes on the die is less than 9, you win  $r$  dollars. Otherwise, you lose your investment. Should you play this game when  $r = 10$ ? Answer the question by making a function named `game` that simulates this game. Read  $r$  and the number of experiments  $N$  as inputs. The function should return `True`, if the answer is 'Yes' or `False`, if the answer is 'No'. Use `random.randint()` to generate the value reflected on the eye of each dice after rolling it.
6. Previously, we used the Euler's method to obtain a first order approximation of ODE. However, Euler's method is not very accurate. In practice, an improved Euler's method known as the second-order Runge-Kutta method has been found to work very well in many applications. The second-order Runge-Kutta method from  $t_n$  to  $t_{n+1}$  is given by

$$y(t_{n+1}) = y(t_n) + h \left( \frac{1}{2}f(t_n, y(t_n)) + \frac{1}{2}f(t_n + h, y(t_n)) + hf(t_n, y(t_n)) \right)$$

where  $h$  is the step size and  $\frac{dy}{dt} = f(t, y)$ . Now, write a function `approx_ode2` by implementing the Runge-Kutta method, taking in  $h, t_0, y_0$  and  $tn$  as parameters. Use it to find the approximate values of  $y(t)$  for the following initial value problem (IVP):

$$\frac{dy}{dt} = 4 - t + 2y, \quad y(0) = 1$$

from  $t = 0$  to  $t = 5$  at a time interval of 0.1. The function to calculate  $\frac{dy}{dt}$  (or  $f(t, y)$ ) is given on Tutor.

Since the above IVP can be solved exactly by the integrating factor method to obtain  $y(t) = \frac{1}{2}t - \frac{7}{4} + \frac{11}{4}e^{2t}$ , compare your solutions obtained using `approx_ode2` (Runge-Kutta method) with that obtained using `approx_ode` (Euler's method) by finding the approximation error values up to 3 decimal places.

*End of Problem Set 4.*