Hypothesis Representation

We could approach the classification problem ignoring the fact that y is discrete-valued, and use our old linear regression algorithm to try to predict y given x. However, it is easy to construct examples where this method performs very poorly. Intuitively, it also doesn't make sense for $\mathbf{h}_{\theta}(x)$ to take values larger than 1 or smaller than 0 when we know that

 $y \in \{0, 1\}$. To fix this, let's change the form for our hypotheses $h_{\theta}(x)$ to

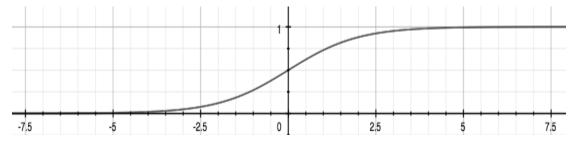
satisfy $0 \le h_{\theta}(x) \le 1$. This is accomplished by plugging $\theta^T x$ into the Logistic Function. Our new form uses the "Sigmoid Function," also called the "Logistic Function":

$$h_{\theta}(x) = g(\theta^{T}x)$$

$$z = \theta^{T}x$$

$$g(z) = \frac{1}{1 + e^{-z}}$$

The following image shows us what the sigmoid function looks like:



The function g(z), shown here, maps any real number to the (0, 1) interval, making it useful for transforming an arbitrary-valued function into a function better suited for classification.



 $\mathbf{h}_{\theta}(x)$ will give us the **probability** that our output is 1. For example, $\mathbf{h}_{\theta}(x)=0.7$ gives us a probability of 70% that our output is 1. Our probability that our prediction is 0 is just the complement of our probability that it is 1 (e.g. if probability that it is 1 is 70%, then the probability that it is 0 is 30%).

$$h_{\theta}(x) = P(y=1|x;\theta) = 1 - P(y=0|x;\theta)$$

 $P(y=0|x;\theta) + P(y=1|x;\theta) = 1$