The Monty Hall Problem

Help Center

The Monty Hall problem is a probability problem that is often posed as a brain teaser. Monty Hall was the host of the popular US TV game show, *Let's Make a Deal*, from the 1970's and 80's. Here is a succinct description of the Monty Hall problem from Parade Magazine:

"Suppose you're on a game show, and you're given the choice of three doors: Behind one door is a car; behind the others, goats. You pick a door, say No. 1, and the host, who knows what's behind the doors, opens another door, say No. 3, which has a goat. He then says to you, "Do you want to pick door No. 2?" Is it to your advantage to switch your choice?"

Mathematical analysis

In deciding whether it is advantageous to switch, many people follow a line of reasoning similar to this. Monty has done me a favor and reduced the number of choices from three doors to two doors. Since there are two remaining doors, the chance that I originally chose the door that hides the car is now 50-50. So, it doesn't matter whether I switch doors or not. Both choices have probability $\frac{1}{2}$ of being correct. Unfortunately, this line of reasoning is false. Monty Hall was a sly game show host and he's not trying to do you any obvious favors when he opens a door that hides a goat. In fact, the phrase "who knows what's behind the doors" is a tip-off.

The correct analysis for the problem is as follows: you have a $\frac{1}{3}$ chance that you chose the door hiding the car on your initial guess. Monty (who knows what's behind each door) always has a choice of at least one door that he can open which hides a goat. So, his action of opening a door that hides a goat didn't improve the probability that your original choice was correct. The probability that you originally chose the door hiding the car remains $\frac{1}{3}$.

However, if you know basic probability, you turn the tables on Monty. Since the probability of the original door being correct is still $\frac{1}{3}$, the probability the remaining closed door hides the car is $\frac{2}{3}$ since the probabilities must sum to one. Thus, the advantageous choice is to always switch.

Simulating the Monty Hall problem

If you aren't convinced by this analysis, you are not alone. The famous mathematician Paul Erdős was unswayed by this argument and remained convinced that the probability was 50-50. To convince Paul or other skeptics of this analysis, we have a powerful tool at our disposal for analyzing this problem: computation. This program implements a bare-bones simulation of the Monty Hall problem.

The canvas displays three white panels (representing the original three doors). Clicking on a panel turns the selected panel green (signifying that the door remains closed and in play). At the same time, the programs turns one of the remaining panels gray (signifying that the door has been opened to reveal a goat) and the remaining panel green (signifying the door remains closed and in play).

At this point, you may click on either of the two green panels to select your final choice for the door. The simulation then opens both remaining doors with a gray panel signifying a goat and a gold panel signifying a car. To aid in the analysis, the simulation keeps tracks of the wins (finding a car) and losses (finding a goat) as you play. You may start a new game by clicking on the canvas.

As you work with the simulation, the simplest action is to just repeatedly click on one of the doors.

This choice simulates the strategy of always staying with your original choice. You will notice that the ratio of wins to losses trends toward one to two. This ratio corresponds to the probability $\frac{1}{3}$ for the strategy of staying with your original choice.

You can also simulate the strategy of always switching by clicking on the other green door after your initial selection. In this case, you will notice that the ratio of wins to losses tends towards two to one. This ratio corresponds to the probability of $\frac{2}{3}$ for the strategy of always switching.

For the stubborn skeptic

If you are still not convinced, the simulation has one extra feature with which you can experiment. You may add extra doors to the simulation. If you have n doors, Monty will always reveal $n-2$ doors that hide goats (i.e; $n-2$ panels turn gray). You are then left with two choices. We suggest that you experiment with $8-10$ doors and see what probabilities result from applying the stay and switch strategies.

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