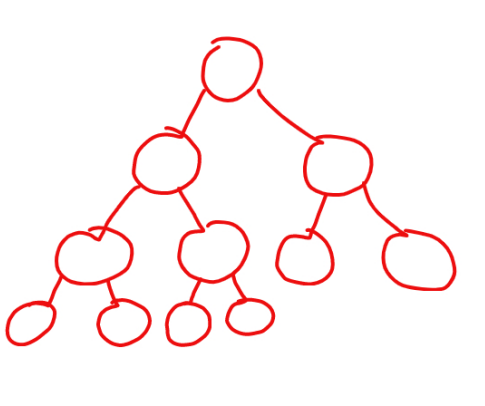
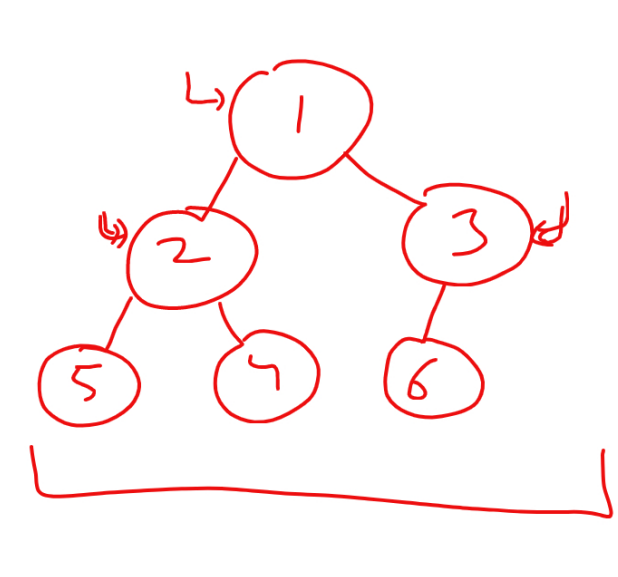


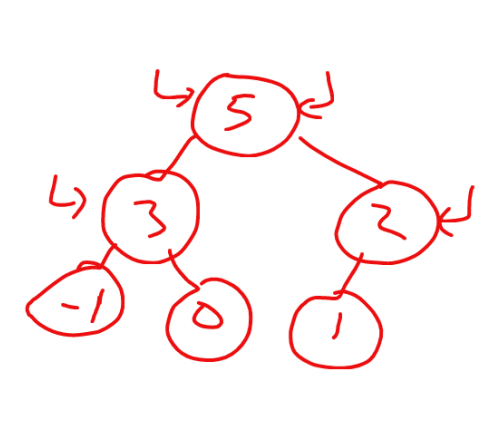
(2, 1, 3, 5, 7)

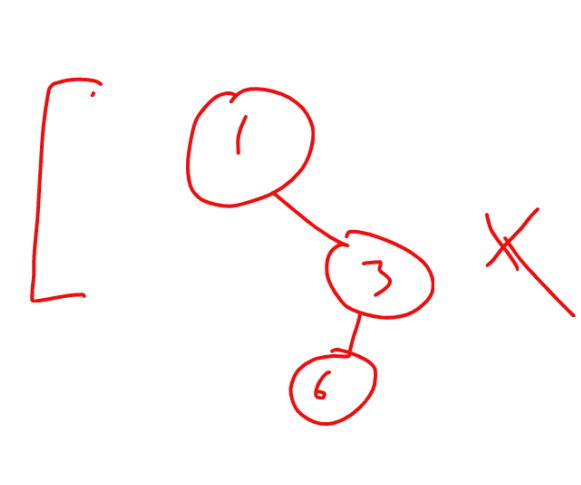
get_min: 1
pop-min: (2, 3, 5, 7)
get_min: 2

min_heap:



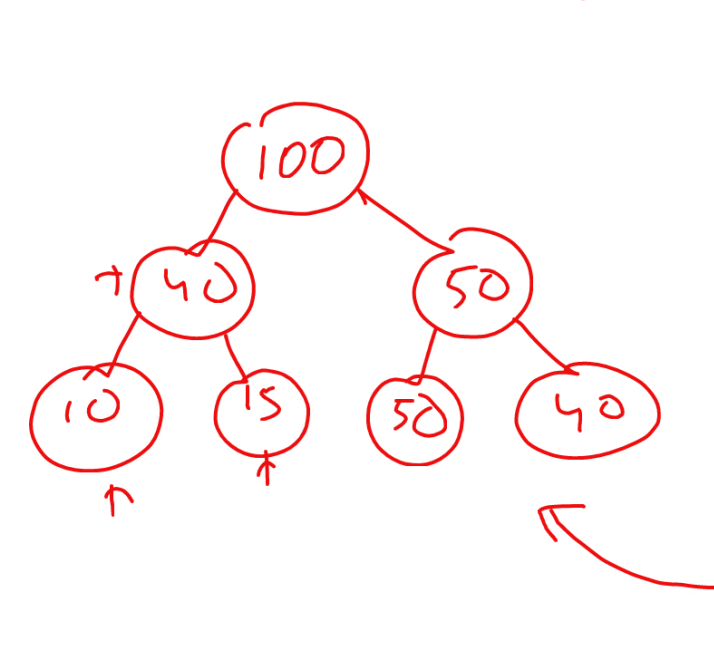
max_heap:





BINARY HEAPS USING ARRAYS

→ We are able to represent heaps using arrays because they are complete binary trees.

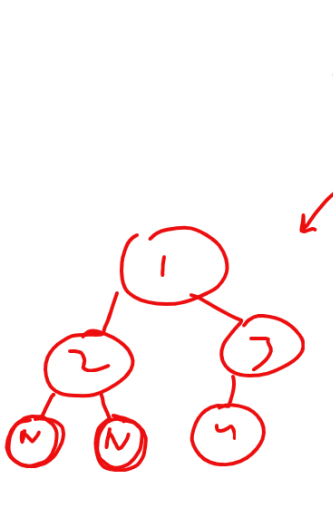

100
40
50
10
15
50
40

```
            0     1    2    3    4    5    6
=> arr[100,  40,  50,  10,  15,  50,  40]
             ↑         ↑         ↑    ↑
           2*1+1
           = 3
           2*1+2
           = 4
                          (3-1)/2 = 1
```

→ Root is at arr[0].

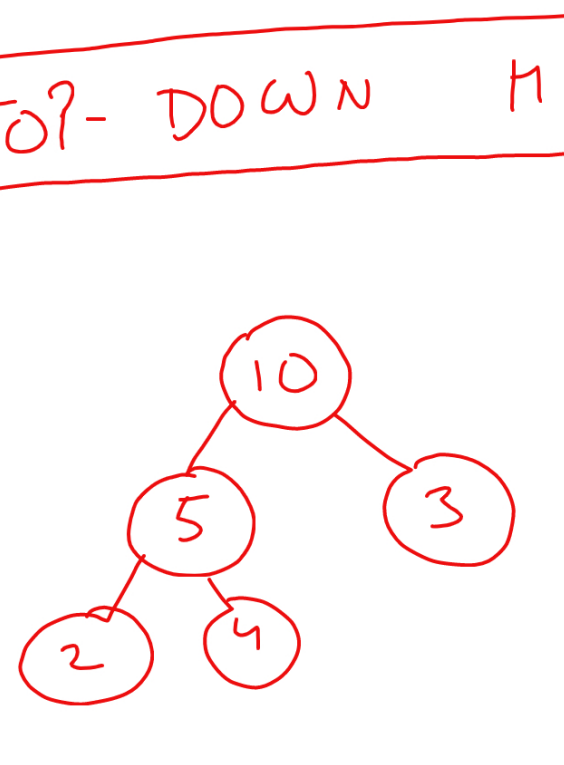
→ For the node at index i:
- → Parent is at index : (i-1)/2
- → Left child " " : 2*i + 1
- → Right child " " : 2*i + 2

"null"



[1, 2, 3, N, N, 4]

TOP-DOWN HEAPIFY: Used for DELETION

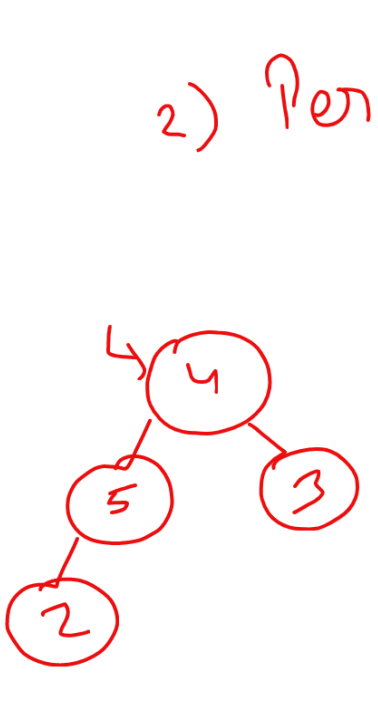


Deletion in a max-heap
⇓
Deleting the maximum value node.

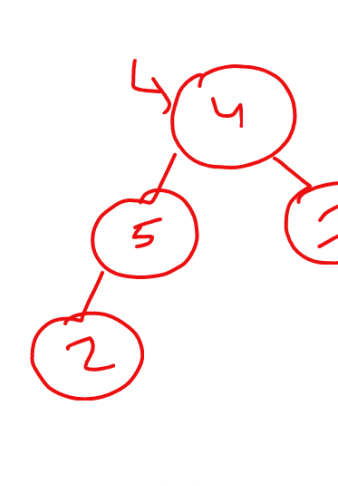
Steps:

1) Replace the root node with the last node and delete the last node.
2) Perform top-down heapify on root.

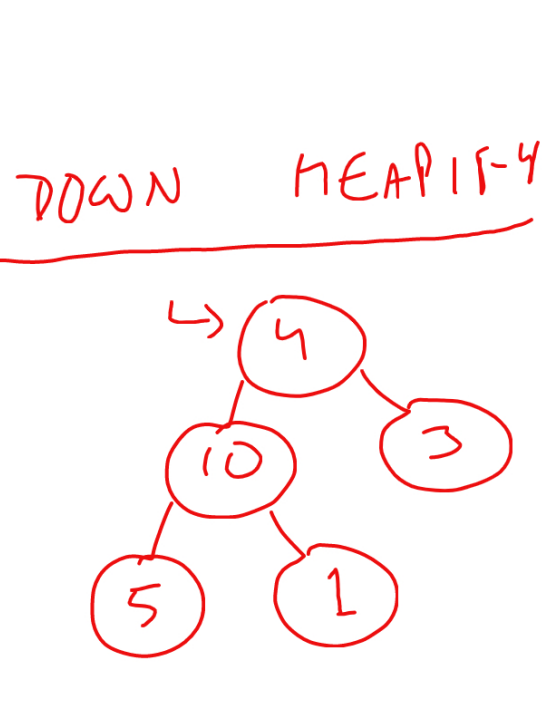
Step-1:



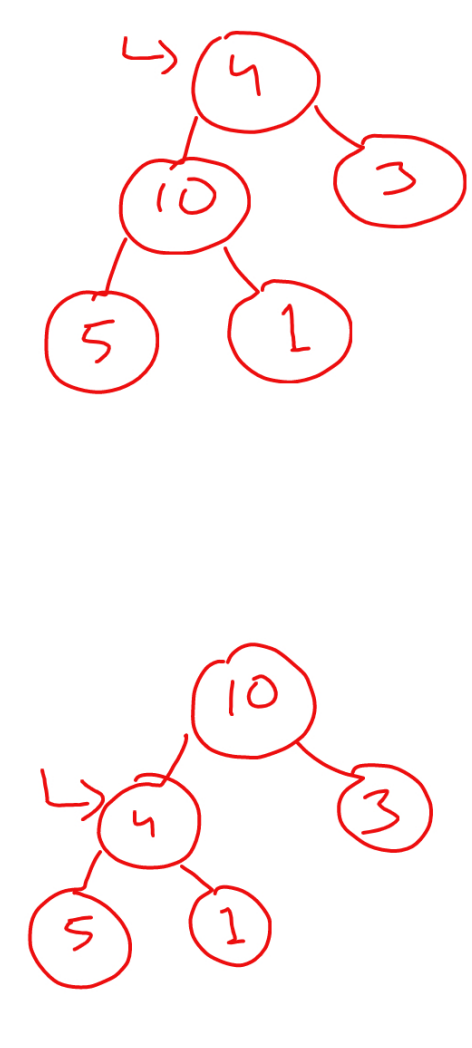
Step-2:



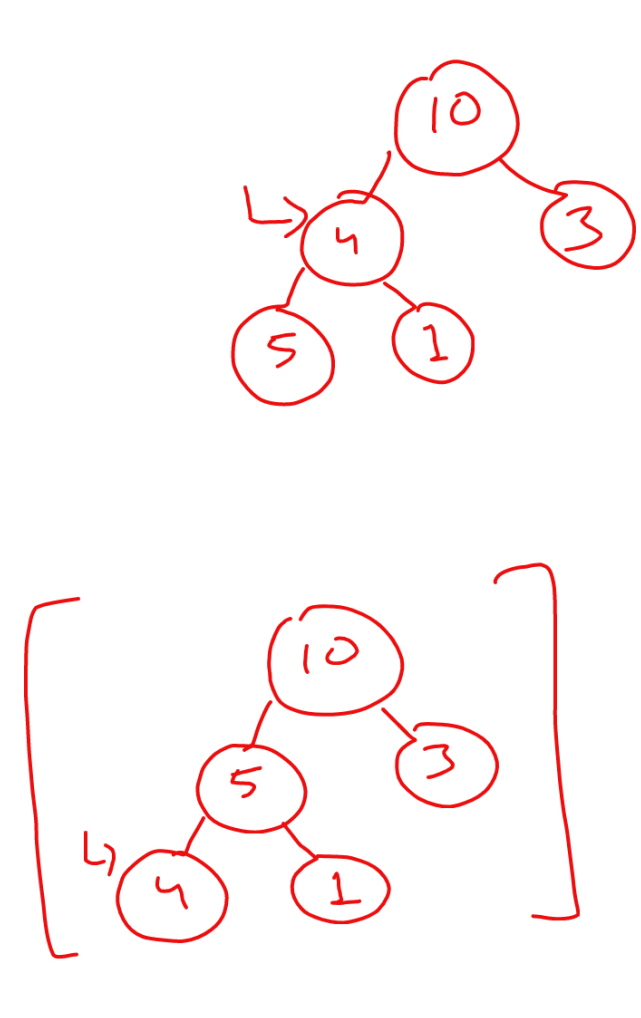
TOP-DOWN HEAPIFY:



```
 ↓
 0    1   2   3   4
[4,  10,  3,  5,  1]
      ‾   ‾
left child: 2*0+1 = 1
right child: 2*0+2 = 2
```

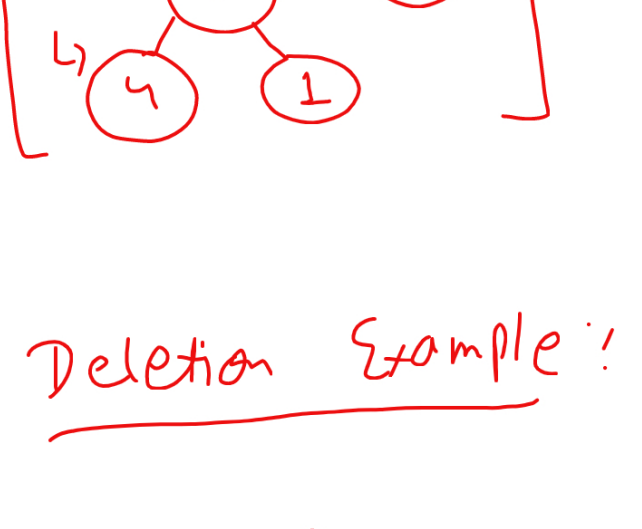


```
      ↓
 0    1   2   3   4
[10,  4,  3,  5,  1]
              ‾   ‾
left child: 2*1+1 = 3
right child: 2*1+2 = 4
```



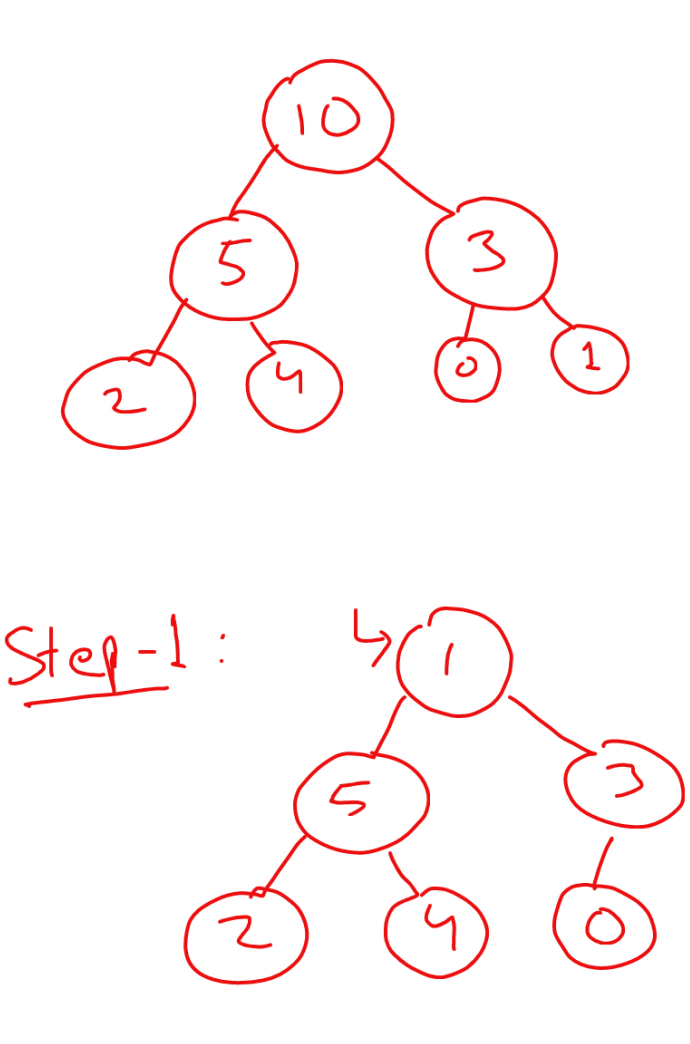
```
              ↓
 0    1   2   3   4
[10,  5,  3,  4,  1]
left child = 2*3+1 = 7 x
right child = 2*3+2 = 8 x
```

Deletion Example:



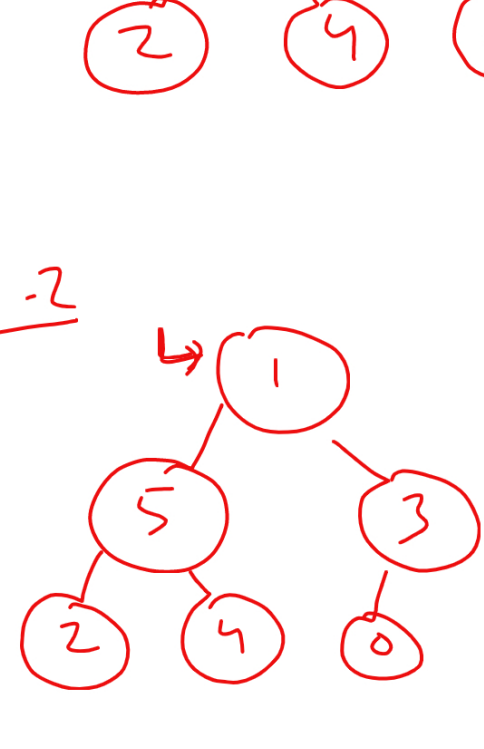
[10, 5, 3, 2, 4, 0, 1]

Step-1:

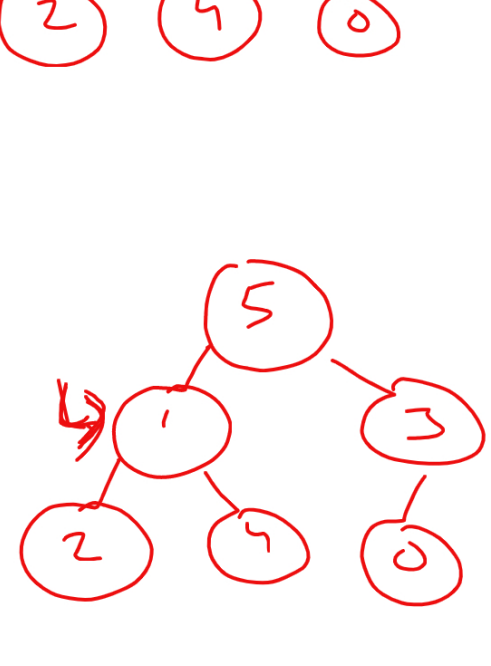


[1, 5, 3, 2, 4, 0]

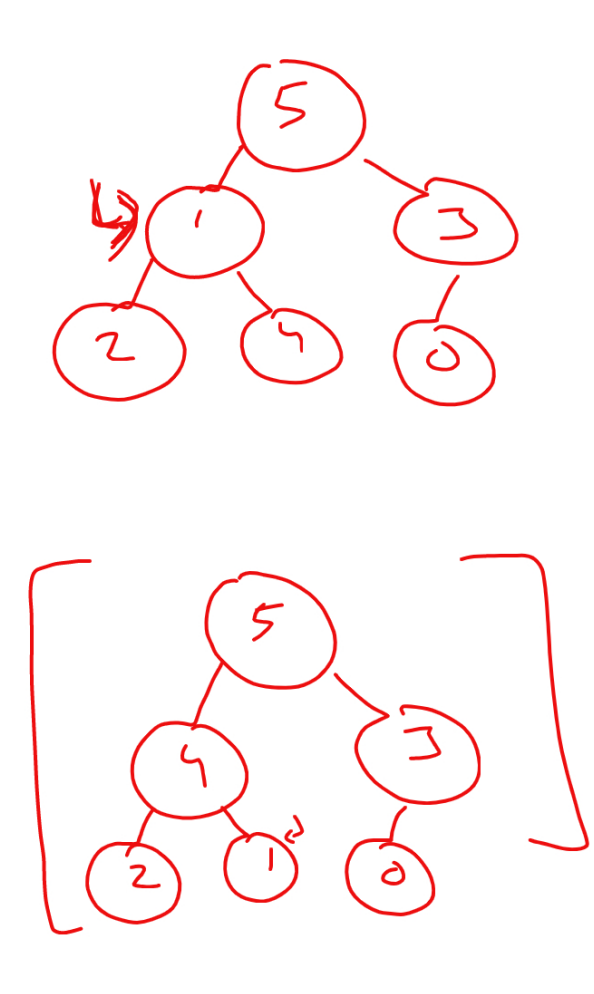
Step-2



```
 ↓
 0   1   2   3   4   5
[1,  5,  3,  2,  4,  0]
     ‾   ‾
                      index = 0
                      lvi = ~~0~~ ①
lc = 2*0+1 = 1
rc = 2*0+2 = 2
```



```
 0   1   2   3   4   5
     ↓
[5,  1,  3,  2,  4,  0]
             ‾   ‾
lc = 2*1+1 = 3
rc = 2*1+2 = 4
```



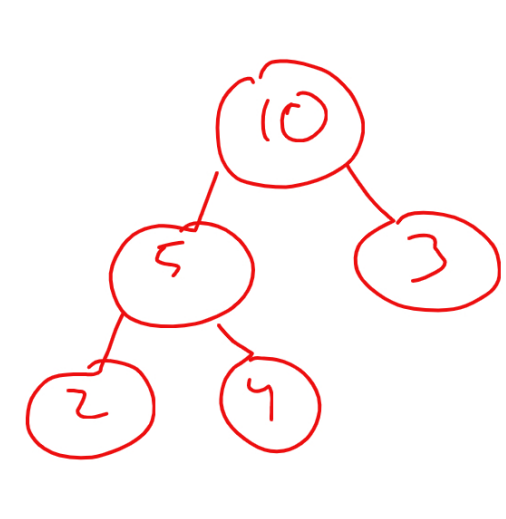
```
 0   1   2   3   4   5
                 ↓
[5,  4,  3,  2,  1,  0]
lc = 2*4+1 = 9 x
rc = 2*4+2 = 10 x
```

TC: O(height)
= O(log(n))

Aux. space = O(log(n))

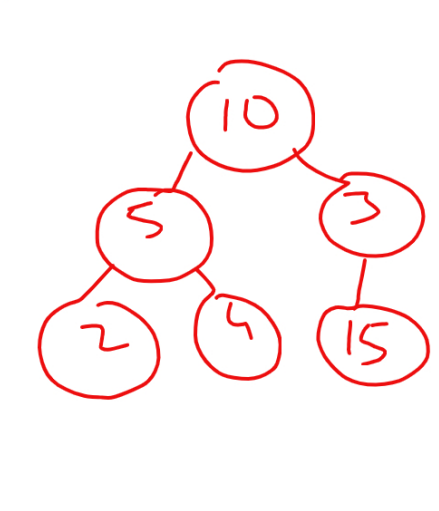
BOTTOM-UP HEAPIFY: USED FOR INSERTION

1) Inc. the heap size by 1
2) Insert the new element at the end
3) Bottom-up heapify on the last index.



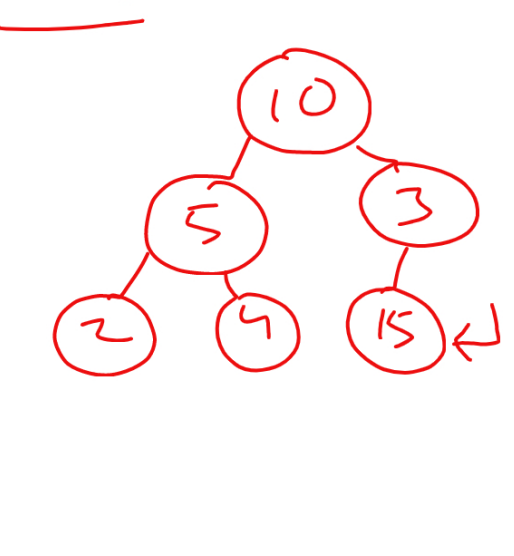
(Insert-15)
[10, 5, 3, 2, 4]

Step-1 & 2:

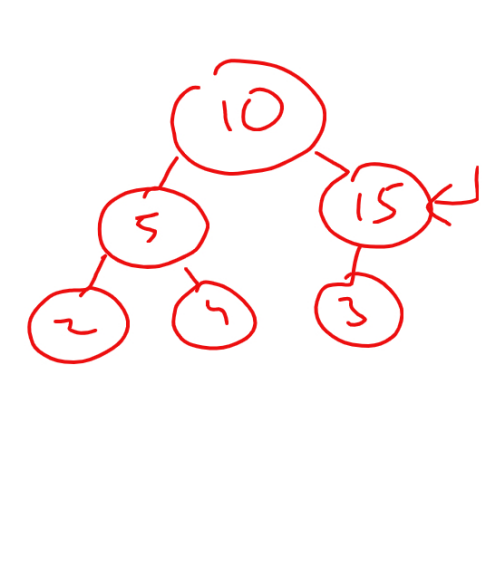


```
 0    1   2   3   4   5
[10,  5,  3,  2,  4,  15]
```

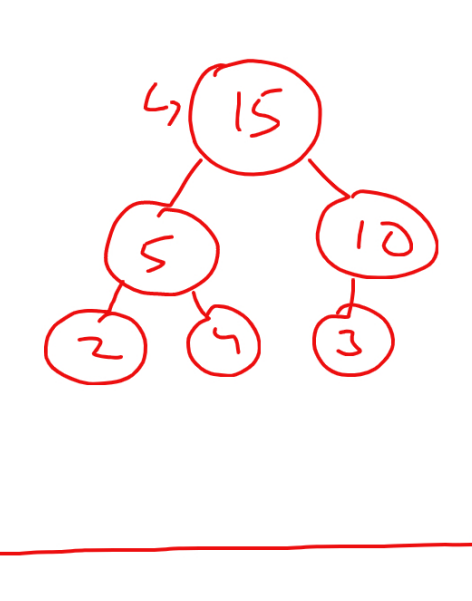
Step-3:



```
                      ↓
 0    1   2   3   4   5
[10,  5,  3,  2,  4,  15]
          ‾
parent = (5-1)/2 = 4/2 = 2
```

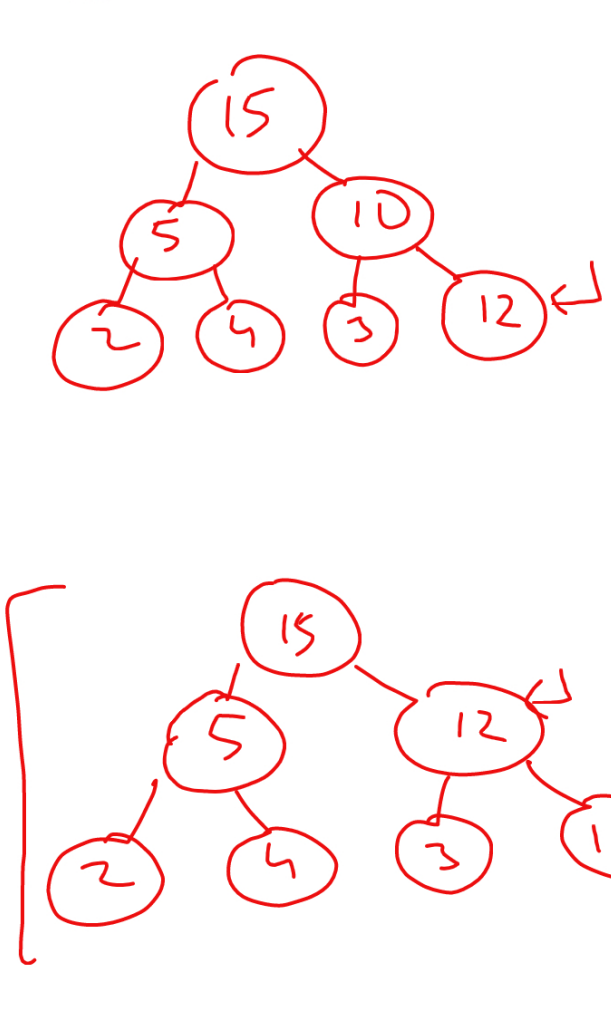


```
          ↓
 0    1   2   3   4   5
[10,  5,  15, 2,  4,  3]
 ‾
parent = (2-1)/2 = 1/2 = 0
```

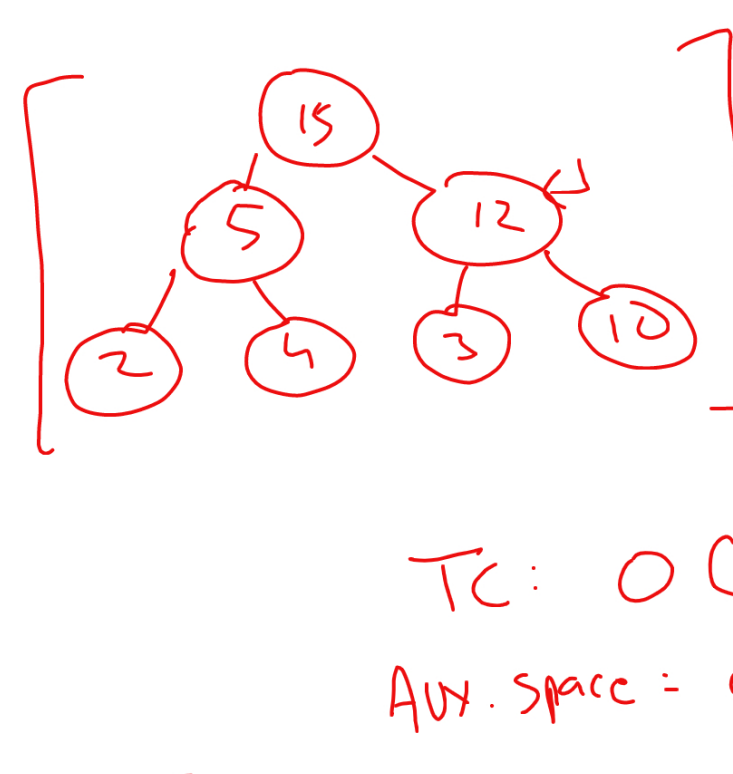


```
 0    1   2   3   4   5
 ↓
[15,  5,  10, 2,  4,  3]
parent: (0-1)/2 = -1/2
```

BOTTOM-UP HEAPIFY:

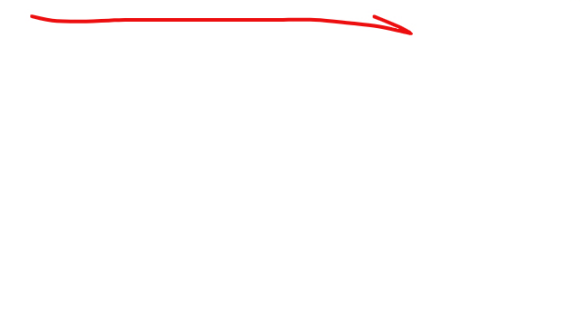

15
5
10
2
4
3
12

```
                          ↓
 0    1   2   3   4   5   6
[15,  5,  10, 2,  4,  3,  12]
          ‾
parent = (6-1)/2 = 5/2 = 2
```


15
5
12
2
4
3
10

```
          ↓
 0    1   2   3   4   5   6
[15,  5,  12, 2,  4,  3,  10]
parent: (2-1)/2 = 1/2 = 0
```

TC: O(ht) = O(log N)
Aux. space = O(log N)


k
k
k
vector<int> temp (k)
O(log N)

Aux. space = O(k * log N)