LA SAPIENZA UNIVERSITÀ DI ROMA

DIPARTIMENTO DI INGEGNERIA INFORMATICA, AUTOMATICA E
GESTIONALE

TRAJECTORY ENCODING

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1 Numerical Methods

Hausdorff Distance

Hausdorff Distance (HD) has been widely used in image matching [1, 2] and trajectories comparison [3]. It is based between the maximum distance between each point from a set to all points in the other set, being the sets curves or images.

Fréchet Distance

Fréchet Distance (FD) [4] is similar to HD, but considers the directions of curves to compute the minimal trajectory.

Longest Common Subsequence

Longest Common Subsequence (LCSS) [5] considers as metric only the part of trajectories which match the most, given a tolerance parameter. This also makes it sensitive to the parameter selection and noise.

Dynamic Time Warp

Dynamic Time Warp (DTW) [6, 7] is based on aligning one or more adjacents points for computing a distance between trajectories. It handles trajectories of different lengths, however it causes distortions which might not have been in the trajectories.

Edit distance with Real Penalty

Edit distance with Real Penalty (ERP) [8] it is an "edit-based distance" meaning that this metric takes in account a cost which is based in how many edits one trajectory needs to match the other.

In this distance the points of a trajectory are taken as reference and the points and edits are performed on the other trajectory for reducing the distance.

Edit Distance on Real Sequence

Edit Distance on Real Sequence (EDR) [9] considers a tolerance parameter like LCSS and adds a cost value on its evaluation. This cost increases with the number of editions that are necessary to match both trajectories within the tolerance error. This makes EDR more robust to gaps in the trajectory and outliers.

Edit Distance with Projections

Edit Distance with Projections (EDwP) [10] performs interpolations between points and tries to match segments of trajectories.

Locality In-between Polylines

Locality In-between Polylines (LIP) [11] computes distance based on the area of polygons made between the two curves. It does not consider warping, but instead it is sort of an integral of the area between curves.

Variations are introduced in [11] to support time and direction.

One-Way Distance

One-Way Distance (OWD) [12] is a measure based on the spatial shapes of moving objects. It is based on the average distance of each point of one trajectory and the other trajectory.

2 Trajectory Learning

2014 - Sequence to Sequence Learning with Neural Networks

Name: seq2seq

[13]

2017 - Identifying Human Mobility via Trajectory Embeddings

Name: TULER
    Code on: https://github.com/gcooq/TUL

[14] classifies users based on trajectory data. The problem is hard because there are many more trajectories than users.

Recurrent Neural Network (RNN) is used, and said to be good for classification when the number of labels is small. In particular uses a Long Short-Term Memory (LSTM) for processing sub-trajectories. In particular, using a bi-directions LSTM yields best results.

There is a location embedding, not sure how that is computed. But the trajectories are points on google maps, so there maybe be semantic information in there. The sequence of location embedding is passed onto the LSTM, not sure how they handle different trajectory lengths.

2018 - Self-Consistent Trajectory Autoencoder: Hierarchical Reinforcement Learning with Trajectory Embeddings

Name: SeCTAR
    Code on: https://github.com/wyndwarrior/Sectar

[15] learns an embedding for trajectories with one encoder and two decoders: a state decoder to decode from the latent space back into trajectories, and a policy decoder, which generates the trajectory in the environment. As such the state decoder predict the trajectory of the policy. The encoder is used in a hierarchical Reinforcement Learning (RL) setup.

The state encoder and decoder are RNNs and the policy decoder is a feed-forward Neural Network (NN).

note: if the encoder is trained with trajectories from different tasks, the policy will be conditioned to each task, what is sort of parameterzing the policy to tasks.

In the paper, the policy has unknown dynamics, and hence the RL setup. Trajectories are continuos poses of joints over time. Tested in simulation.

2018 - Anomalous Trajectory Detection Using Recurrent Neural Network

Name: ATD-RNN
    Code on: https://github.com/LeeSongt/ATD-RNN

[16] proposes anomalous trajectory detection using RNN.

The trajectories are discretized, using a grid, and feed to a stacked RNN for learning the embedding, then a multi-layered perceptron and a soft-max layer detects if the trajectory is anomalous. The stacked RNN is made by feeding the hidden states od the previous to the next RNN.

The trajectories are padded in order to get trajectories of the same length.

LSTM and Gated Recurrent Unit (GRU) are two special types of RNNs are tested. GRUs seems to work better.

2018 - Deep Representation Learning for Trajectory Similarity Computation

Name: t2vec Code on: https://github.com/boathit/t2vec

[17] presents t2vec. A Deep Learning (DL) approach for trajectory similarity. States that using RNN is not a very good idea because you cannot reconstruct the trajectory and it fails to consider spatial proximity, which is inherited in trajectory data.

called in the paper as t2vec or seq2seq?

The approach is based on the encoder-decoder framework. Handling varying sampling rates is done by augmenting the training data creating sub-trajectories by sub-sampling and noise addition. They also propose a spatial-aware loss, and pre-train the ??cells?? and let them to be optimized during training.

Notes: The paper is very confusing. I do not really know that are the inputs and outputs or how the sequences are fed in the RNN inside the encoder.

2020 - Trembr: Exploring Road Networks for Trajectory Representation Learning

Name: Trembr

[18] uses two embedding one for trajectories traj2vec and another for the road network road2vec. It preprocesses the trajectories by projecting them in a road network and the trajectory is a sequence of road segments and travel time.

The RNN decoder is conditioned to the road network, and the training is made by optimising a loss for the trajectory and another for time.

Notes: Maybe the secret for velocities profiles is in the addition of time to the loss.

2019 - Computing Trajectory Similarity in Linear Time: A Generic Seed-Guided Neural Metric Learning Approach

Name: NeuTraj

Code on: https://github.com/yaodi833/NeuTraj

[19] proposes a method for accelerating trajectory similarity computation by sampling seeds of trajectories, computing their similarity, and approximating them with a neural metric.

States that RNNs, LSTMs, and GRUs can only model one sequence without considering the between-sequence correlation.

Does not consider time in the trajectory. Starts sampling from the trajectories and computes a distance matrix between the samples using a given trajectory distance metric which is then normalized.

The RNN is augmented with a memory, which is created by dividing the space into a grid, and for each grid slot, the memory stores the hidden vector of the RNN. This memory is used to extend the RNN cell, sort of like an LSTM.

The loss for training is $\mathcal{L}\tau_i, \tau_j = \sum_k w_k(f(\tau_i, \tau_j) - \exp(-||e_i - e_j||)$, a weighted difference between the similarity metric f and the distance in the embedding space $(e_i - e_j)$. The weight w_k is obtained using the normalized distance matrix, computing pairs of similar and dissimilar trajectories and more fancy stuff.

Notes: map is like google map.

2020 - Trajectory similarity learning with auxiliary supervision and optimal matching

Name:Traj2SimVec

[20] follows the same idea as in [19] which selects some trajectories for pre-training [something], the training samples are divided in three sub-trajectories [because it seems to help learning].

A distance matrix is computed which is used as supervision signal, similar to [19].

2020 - MARC: a robust method for multiple-aspect trajectory classification via space, time, and semantic embeddings

Name: MARC

[21] Embeds semantics on the trajectories. Each semantic information (weather, time, type of place) has an encoding, and a weight matrix which transform them into a fixed size vector. The semantic trajectory is fed to an LSTM, which encodes the trajectories, having the hidden states used for classification.

2021 - Embedding-Based Similarity Computation for Massive Vehicle Trajectory Data

Name: L2R*

[22] seems to propose the exact same thing as [19], but with interpolation for de-noising.

Adds a ranking learning loss on [19]

2021 - STENet: A hybrid spatio-temporal embedding network for human trajectory forecasting

Name: STENet

[23] Focuses on predicting pedestrian trajectories. Uses a LSTM with Convolutional Neural Networks (CNNs) to embed position features in multiple temporal time-scales. The encoder-decoder structure stack a CNN and a graph attention model. The decodes stacks many LSTMs.

They give related works on social trajectory learning.

Notes: They point to Variational Auto-Encoders (VAEs) for modelling multi-modality and for the generative capabilities.

2021 - A Graph-Based Approach for Trajectory Similarity Computation in Spatial Networks

Name: GTS

[24] Propose a Graph Neural Network (GNN)-based trajectory embedding. The framework measures trajectory similarities, learns Points of Interest (PoIs), and leans a trajectory embedding.

A trajectory is encoded as the points in a graph map. Then they define a trajectory similarity metric on the PoI graph, based on the graph distance between the points and trajectories. An embedding capturing the neighbours and graph trajectory is learned. The PoI embeddings and their neighbours are used to learn another embedding using its neighbours information. Finally, LSTMs are used to learn the trajectory over the graph embeddings. The loss function minimizes the above defined distance between trajectories and the distance between the two closest trajectories.

2021 - T3S: Effective Representation Learning for Trajectory Similarity Computation

Name: T3S

[25] combines LSTMs and attention NNs over the grid graph for learning the embedding. Close to [19, 20, 22].

2021 - How meaningful are similarities in deep trajectory representations?

Code on: https://dbis.ipd.kit.edu/2652.php

[26] presents a survey and evaluation of `t2vec` [17] and other methods. Seems like `t2vec` with some variations outperform the rest. `t2vec` seems to be stacked LSTMs.

Evaluate how changing `t2vec` parameters affect similarity values. `t2vec` seems robust to parameters.

Evaluate `t2vec` against non learning metrics. Seems like associating them lead to better results.

[They DO ignore the whole literature on learning methods?]

Concludes that using LCSS and `t2vec` leads to a better trajectory similarity, covering overlap, shape, direction and distance.

Notes: Maybe that should be 4 characteristics to consider for explainability.

2022 - Spatio-Temporal Trajectory Similarity Learning in Road Networks

Name: ST2vec

[27] learns a spatio-temporal representation. Two steps, which is based on learning a spatial model, a temporal model and a co-attention fusion module. It is based on a road network, trajectories are sequences of vertex on the road network.

Define the distance of a spatio-temporal trajectory as a weighted sum for a spatio-distance (d_s) and a temporal distance (d_t):

$$d(\tau_i, \tau_j) = \alpha d_s(\tau_i, \tau_j) + (1 - \alpha)d_t(\tau_i, \tau_j) | \alpha \in [0, 1]$$

Later uses LSTMs to learn using two strategies, using one LSTM for space and another for time, or using one for both.

2022 - Deep Fuzzy Contrast-Set Deviation Point Representation and Trajectory Detection

[28] Grid-map based, contrastive learning.

notes: hard to understand what they are doing here.

2022 - Contrastive Pre-training of Spatial-Temporal Trajectory Embeddings

Name: CSTTE

[29] employs contrastive learning for learning an embedding which retains high-level travel semantics.

Recovering the original trajectory is not a good approach when learning representations with RNNs since it fails to capture the high-level information of trajectories. Contrastive learning with noisy augmentation can handle the high-level information while being robust to noise. However data augmentation needs to be well designed.

The positive samples are created with subsampling the query trajectory, while the negative samples come from different trajectories.

Notes: Not sure this is correct, I think the "different trajectories" should be far enough from the query trajectory to be a negative sample.

The encoder stacks a spatio-temporal encoding layer and attention layers. For the first, a learnable encoding of locations is learned (each location leads to a vector) and location and time are passed to a trigonometric vector transformation to compute features which can capture periodic information; those vectors are then summed up. The attention layer is actually 2 stacked attention layers.

2022 - TMN: Trajectory Matching Networks for Predicting Similarity

Name: TMN

[30] uses attention to compute intra-trajectory similarities, and then uses a LSTM.

Notes: Comparison ignores many methods.

2022 - TSNE: Trajectory Similarity Network Embedding

Name: TSNE

[31] uses a pre-defined trajectory measure function to construct a k-NNG (K nearest neighbours graph) and computes the embedding based on the graph.

Notes: Not sure how they compute the embedding from the graph. Seems like the graph representation allows to handle partial similarity and unordered similarity.

2022 - Towards robust trajectory similarity computation: Representation-based spatio-temporal similarity quantification

Name: RSTS

[32] splits the spatio-temporal trajectories into cells, and uses a triplet loss for the learning. It enforces that if the time and space similarities are higher, then the distance in the encoded space must be smaller, and that, in the encoded space, the distance between two trajectories variations (noise and downsampling) must obey the distance of the trajectories.

An embedding is used for the tokens, which are then passed to a RNN encoder-decoder. The tokens for the embedded are an ID computed by splitting the space-time into cells. The input is grid-cells (gps + time).

Notes: Analysis is poor. Ignores all other works on learning. Seems like there is little innovation besides the loss.

2023 - Spatial-temporal fusion graph framework for trajectory similarity computation

Name: GTS

Sort of the same thing as [24]. Name is also the same. Even results are the same.

[33] first learns a point of interest representation on the road network, which is passed to a GNN for learning neighbours information as embeddings, and then a LSTM for learning the sequencing.

A symmetric distance between trajectories is defined based on the distance between each point of the trajectories and the other trajectory:

$$d(\tau_1, \tau_2) = \sum_{v \in \tau_1} e^{-d(v, \tau_2)} + \sum_{v \in \tau_2} e^{-d(v, \tau_1)}$$

The time is considered in an extension called `ST-LSTM`, which adds a time one-hot encoding into the gating functions of the LSTM.

Notes: Comparisons goes as far as `traj2SimVec` [20].

2023 - GRLSTM: Trajectory Similarity Computation with Graph-Based Residual LSTM

Name: GRLSTM

[34] combines Knowledge Graph Embedding (KGE), GNN and a multi-layer residual-LSTM. KGE is used to learn point and relation embeddings for constructing a graph, which is passed to the GNN for learning the

topology in the point-structure graph. Then the LSTM is used to learn the embeddings trajectories. Uses two losses: a graph-based loss and a trajectory-base loss.

The input is trajectories in a graph road network. The interesting thing here is that adjacent points in the trajectory may not be adjacent in the graph (due to data loss or lower sample rate).

The stacked LSTM is augmented with a residual layer for handling the gradient forgetting of traditional LSTM. It is stated that it does not add parameters so it does not affect training time considerably.

Notes: does not really say how the residual function is computed. Similarly to [24] they implement point and trajectory distances.

2023 - Contrastive Trajectory Similarity Learning with Dual-Feature Attention

Name: TrajCL

[35] introduces four trajectory augmentation and a dual feature self-attention encoder, for learning structural and spatial patterns of trajectories. It does not involve any recurrent structure. Instead, it uses a dual self-attention-based trajectory encoder.

Augmentations:

point shifting: adds an offset to the points

point masking randomly removes points from the trajectory

tuncation cuts a prefix, suffix, or both from the trajectory

simplification uses the Douglas–Peucker algorithm which removes non critical points from the trajectories (like points in a straight line).

The augmented trajectories are used to create two trajectory views to learn structural and spatial features. The augmented trajectories are used to compute two trajectory views. The structural features, the map is converted into a grid, and used to create a graph in which the grid locations are the vertices and the trajectory transitions the edges. Then a graph embedding (`node2vec`) is used to learn an embedding. For the spatial features, the angle and length of trajectory segments is computed. Both views are augmented by adding a [sketchy] sine and cosine value to the points to capture position information.

Finally the two views are passed to a two-head self attention module to learn the embeddings.

2023 - Spatio-Temporal Trajectory Similarity Measures: A Comprehensive Survey and Quantitative Study

Code on: https://github.com/ZJU-DAILY/TSM

[36] presents a survey with several methods, and benchmark for evaluating them. Apparently `Traj2SimVec` [20] is the learning method, which is not grid-based that handles our problem.

3 Trajectory Learning on Robotics

2020 - Controlling Assistive Robots with Learned Latent Actions

[37] Use encoders to learn latent task representations for assistive robot remote controlling. In this setup, VAEs are used, encoding states into a task representation, the user gives input from a joystick which are decoded together with the latent space representation.

The latent representation is encoded (ϕ) from state (s) and action (a), and decode from latent state (z) and state into action.

There is an emphasis on desired characteristics of the latent representation:

controlability: $s_i = T(s_{i-1}, \phi(s_{i-1}, z_{i-1}))$, where T is the transition function. Meaning that using the latent states should lead to the same actions as the ones in the training set.

consistency: $|T(s_i, \phi(s_i, z_i)) - T(s_j, \phi(s_j, z_j))| \leq \epsilon$ if $|s_i - s_j| \leq \delta$.

scalability: $|T(s_i, \phi(s_i, z_i)) - T(s_j, \phi(s_j, z_j))| \to \inf$ if $|s_i - s_j| \to \inf$

Notes: Seems like VAE is used straight up with the trajectories. But it is a bit blurry how the actions are being defined or learned (seem pre defined). conditional Variational Auto-Encoder (cVAE) seems to outperform other encoders.

2020 - DiversityGAN: Diversity-Aware Vehicle Motion Prediction via Latent Semantic Sampling

Name: DiversityGAN

[38] extends Generative Adversarial Network (GAN) using a low-dimensional approximate semantic (encoding) which is shaped to capture semantics. Sampling from this space allows to cover semantically distinguish outcomes. The work focuses on predicting vehicle trajectories.

An intermediate layer avoids the need of taxonomy [?] by using metric learning, in which a latent representation is trained to match annotations of high-level labels, and forcing the distance to be large if they represent two distinguish semantic labels. The latent space is trained to match human similarity measures.

Past trajectories and map information are embedded, and their embeddings are passed to an LSTM whose latent space is divided intro a high- and low-level parts. The decoder takes both parts to produce trajectory samples. The trajectory network is a series of fully connected layers that embed a trajectory into a vector [39] [seems this work uses LSTMs for the embeddings]. The map embedding is a fully connected network that maps polynomial coefficients (quadratic) into an embedding. The encoder is a LSTM, whose hidden states are added a Gaussian noise and passed to a non-linear fully-connected network to compute the high and low-level embedding representation. The high-level embedding part is not correlated with the low-level one, and is trained for learning semantic similarities from the human teacher (they use a hand coded oracle though). The decoder is a LSTM. There is also a discriminator trained for identifying if samples are generated by the architecture or if they are real data.

The loss design incorporates minimal and final displacement losses, a term to enforce the non-correlation between the high and low-level embeddings, and another to enforce that semantically related pairs should also be close in the encoding space.

Sampling is performed using Farthest Point Sampling.

Notes: It is interesting that they added semantics to the network.

2022 - Controlling Assistive Robots with Learned Latent Actions

[40] expands on [37] for assistive robotics. Here the latent space takes in consideration human input from a low-dimensional joystick, allowing to encode actions (e.g., cut, pour, dance) using buttons.

A Multi-Layer Perceptron (MLP) is used to learn an alignment between the user's intention/preferences and the context.

2022 - Promoting Quality and Diversity in Population-based Reinforcement Learning via Hierarchical Trajectory Space Exploration

Name: HTSE

[41] propose a trajectory embedding using VAE and LSTM with similarity constrains, which is used with a hierarchical trajectory space exploration to generate diverse samples in a reinforcement learning framework.

The encoder is a double layer bi-directional LSTM, and the hidden state is formed by the last state of both encoding-LSTMs. The decoder is an one layer LSTM which take as input the first trajectory state and the hidden variable. The constraint is computed by sampling a batch of trajectories and ordering them according to [point location distance?], the closest one in the batch is the positive sample and the bottom half are negative samples, and a loss function is computed using the encodings of the anchor, positive and negative samples. A hidden-state conditioned policy is added, learning $\pi : z, s \rightarrow a$, which is trained together with the encoder-decoder.

2023 - SIRL: Similarity-Based Implicit Representation Learning

Name: SIRL

[42] propose to ask humans what are similar trajectories (robotics manipulation), allowing to distinguish high- and low-level features for learning tasks. [sort of evolution of "learning one feature at a time".] A trajectory query is a triplet of trajectories which are presented to the user, who is asked which are the two most similar, forming a tripled (anchor, positive and negative) [vae?].

The triples are used to learn am embedding space such similar trajectories are close in the representation space, and dissimilar ones are far apart. The features are learned using a fully connected NNs, which are trained based on the distance in the embedding space using a contrastive loss based on the human triplet selection.

4 Trajectory Prediction

2018 - Convolutional Social Pooling for Vehicle Trajectory Prediction

Name: CS-LSTM

[43] Convolution Social LSTM (CS-LSTM) uses an LSTM encoder-decoder together with a convolutional social pooling for learning interdependencies between vehicles in the street, outputting multi-modal predictions of trajectories based on maneuver classes. The interdependencies are important because the decision possible trajectory of one car depends on the predicted trajectory of the other ones.

Tests are a simple simulator, maneuvers are `keep lane`, `change to right`, `change to left`, so it is a very simple test case.

The encoder is one LSTM for each vehicle, but all LSTMs have the same weights. The encoded state for the target vehicle is concatenated with an encoded state for all other vehicles, which is obtained by passing their LSTM states into the convolutional maxpooling. There is one decode for each maneuver for enabling multi-modality.

Notes: This interdependency is similar to the idea of the robot-human case.

2018 - 3DOF Pedestrian Trajectory Prediction Learned from Long-Term Autonomous Mobile Robot Deployment Data

[44]

2020 - CNN, Segmentation or Semantic Embeddings: Evaluating Scene Context for Trajectory Prediction

[45]

5 Social Robots

2016 - Learning Social Etiquette: Human Trajectory Understanding In Crowded Scenes

[46] provide a benchmark for social navigation with interactions between pedestrians, skaters, bikers, and small vehicles. The benchmark contain images (videos) of these interactions in a campus.

A feature called social sensitivity is proposed, which incorporates a distance which the target prefers for avoiding collision and another in which the targets starts to deviate from its trajectory to avoid collision. These parameters are learned with an energy-like minimization. When plotting these parameters, clusters emerge with different navigation styles.

Notes: No encoding here.

2018 - Social GAN: Socially Acceptable Trajectories With Generative Adversarial Networks

Social GAN

[47] proposed an LSTM encoder-decoder with max-pooling layer for handling interdependencies (the trajectory of a person depends on the trajectory of others). The encoder decoder is trained in a GAN fashion, in which uses an LSTM-based discriminator.

There is one LSTM encoder, decoder and discriminator for each tracked individual. On the encoder, first the individual's position is passed through an 1 layer MLP to be transformed into a fixed-sized vector, which is then passed to the LSTM. The weights of the encoders are shared among all individuals. The pooling module converts the hidden-state of every encoding LSTM into tensor for each individual. The decoder is a straight-forward LSTM and the discriminator takes the predicted trajectories and classifies encodes them using into "good" or "bad".

The pooling mechanism is what handles multiple people.

2018 - Soft + Hardwired attention: An LSTM framework for human trajectory prediction and abnormal event detection

[48]

2019 - Pedestrian Trajectory Prediction Using RNN Encoder-Decoder with Spatio-Temporal Attentions

[49]

2019 - GD-GAN: Generative Adversarial Networks for Trajectory Prediction and Group Detection in Crowds

[50]

2020 - Trajectron++: Dynamically-Feasible Trajectory Forecasting with Heterogeneous Data

[51]

2020 - It Is Not the Journey But the Destination: Endpoint Conditioned Trajectory Prediction

[52]

2020 - A Generative Approach for Socially Compliant Navigation

[53]

2021 - Learning World Transition Model for Socially Aware Robot Navigation

[54]

2021 - Probabilistic Dynamic Crowd Prediction for Social Navigation

[55]

2021 - Tra2Tra: Trajectory-to-Trajectory Prediction With a Global Social Spatial-Temporal Attentive Neural Network

[56]

2021 - Human Trajectory Forecasting in Crowds: A Deep Learning Perspective

[57]

2021 - Trajectory Prediction for Autonomous Driving based on Multi-Head Attention with Joint Agent-Map Representation

[58]

2022 - Social-PatteRNN: Socially-Aware Trajectory Prediction Guided by Motion Patterns

[59]

2023 - CSR: Cascade Conditional Variational Auto Encoder with Socially-aware Regression for Pedestrian Trajectory Prediction

[60]

2023 - MRGTraj: A Novel Non-Autoregressive Approach for Human Trajectory Prediction

[61]

2023 - EWareNet: Emotion-Aware Pedestrian Intent Prediction and Adaptive Spatial Profile Fusion for Social Robot Navigation

[62]

6 Comparison

7 Challenges

References

[1] D.P. Huttenlocher, G.A. Klanderman, and W.J. Rucklidge. Comparing images using the Hausdorff distance. *IEEE Transactions on Pattern Analysis and Machine Intelligence*, 15(9):850–863, 1993.

[2] Oliver Jesorsky, Klaus J. Kirchberg, and Robert W. Frischholz. Robust Face Detection Using the Hausdorff Distance. In Josef Bigun and Fabrizio Smeraldi, editors, *Audio- and Video-Based Biometric Person Authentication*, pages 90–95, Berlin, Heidelberg, 2001. Springer Berlin Heidelberg.

[3] E Belogay, C Cabrelli, U Molter, and R Shonkwiler. Calculating the Hausdorff distance between curves. *Information Processing Letters*, 64(1):17–22, 1997.

[4] Computing discrete Fréchet distance.

[5] David Maier. The Complexity of Some Problems on Subsequences and Supersequences. *J. ACM*, 25(2):322–336, apr 1978.

[6] David Sankoff and Joseph B Kruskal. Time warps, string edits, and macromolecules: the theory and practice of sequence comparison. *Reading: Addison-Wesley Publication*, 1983.

[7] Byoung-Kee Yi, H.V. Jagadish, and C. Faloutsos. Efficient retrieval of similar time sequences under time warping. In *Proceedings 14th International Conference on Data Engineering*, pages 201–208, 1998.

[8] Lei Chen and Raymond Ng. On the marriage of lp-norms and edit distance. In *Proceedings of the Thirtieth international conference on Very large data bases-Volume 30*, pages 792–803, 2004.

[9] Lei Chen, M. Tamer Özsu, and Vincent Oria. Robust and fast similarity search for moving object trajectories. In *Proceedings of the 2005 ACM SIGMOD International Conference on Management of Data*, SIGMOD '05, page 491–502, New York, NY, USA, 2005. Association for Computing Machinery.

[10] Sayan Ranu, Deepak P, Aditya D. Telang, Prasad Deshpande, and Sriram Raghavan. Indexing and matching trajectories under inconsistent sampling rates. In *2015 IEEE 31st International Conference on Data Engineering*, pages 999–1010, 2015.

[11] Nikos Pelekis, Ioannis Kopanakis, Gerasimos Marketos, Irene Ntoutsi, Gennady Andrienko, and Yannis Theodoridis. Similarity Search in Trajectory Databases. In *14th International Symposium on Temporal Representation and Reasoning (TIME'07)*, pages 129–140, 2007.

[12] Bin Lin and Jianwen Su. Shapes based trajectory queries for moving objects. In *Proceedings of the 13th Annual ACM International Workshop on Geographic Information Systems*, GIS '05, page 21–30, New York, NY, USA, 2005. Association for Computing Machinery.

[13] Ilya Sutskever, Oriol Vinyals, and Quoc V Le. Sequence to Sequence Learning with Neural Networks. In Z. Ghahramani, M. Welling, C. Cortes, N. Lawrence, and K.Q. Weinberger, editors, *Advances in Neural Information Processing Systems*, volume 27. Curran Associates, Inc., 2014.

[14] Qiang Gao, Fan Zhou, Kunpeng Zhang, Goce Trajcevski, Xucheng Luo, and Fengli Zhang. Identifying Human Mobility via Trajectory Embeddings. In *Proceedings of the 26th International Joint Conference on Artificial Intelligence*, IJCAI'17, page 1689–1695. AAAI Press, 2017.

[15] John Co-Reyes, YuXuan Liu, Abhishek Gupta, Benjamin Eysenbach, Pieter Abbeel, and Sergey Levine. Self-Consistent Trajectory Autoencoder: Hierarchical Reinforcement Learning with Trajectory Embeddings. In Jennifer Dy and Andreas Krause, editors, *Proceedings of the 35th International Conference on Machine Learning*, volume 80 of *Proceedings of Machine Learning Research*, pages 1009–1018. PMLR, 10–15 Jul 2018.

[16] Li Song, Ruijia Wang, Ding Xiao, Xiaotian Han, Yanan Cai, and Chuan Shi. Anomalous Trajectory Detection Using Recurrent Neural Network. In Guojun Gan, Bohan Li, Xue Li, and Shuliang Wang, editors, *Advanced Data Mining and Applications*, pages 263–277, Cham, 2018. Springer International Publishing.

[17] Xiucheng Li, Kaiqi Zhao, Gao Cong, Christian S. Jensen, and Wei Wei. Deep Representation Learning for Trajectory Similarity Computation. In *2018 IEEE 34th International Conference on Data Engineering (ICDE)*, pages 617–628, 2018.

[18] Tao-Yang Fu and Wang-Chien Lee. Trembr: Exploring Road Networks for Trajectory Representation Learning. *ACM Trans. Intell. Syst. Technol.*, 11(1), feb 2020.

[19] Di Yao, Gao Cong, Chao Zhang, and Jingping Bi. Computing Trajectory Similarity in Linear Time: A Generic Seed-Guided Neural Metric Learning Approach. In *2019 IEEE 35th International Conference on Data Engineering (ICDE)*, pages 1358–1369, 2019.

[20] Hanyuan ZHANG, Xingyu ZHANG, Qize JIANG, Baihua ZHENG, Zhenbang SUN, Weiwei SUN, and Changhu WANG. Trajectory similarity learning with auxiliary supervision and optimal matching. In *Proceedings of the Twenty-Ninth International Joint Conference on Artificial Intelligence,*, pages 3209–3215. Research Collection School Of Information Systems, 11–17 Jul 2020.

[21] Lucas May Petry, Camila Leite Da Silva, Andrea Esuli, Chiara Renso, and Vania Bogorny. MARC: a robust method for multiple-aspect trajectory classification via space, time, and semantic embeddings. *International Journal of Geographical Information Science*, 34(7):1428–1450, 2020.

[22] Yuanyi Chen, Peng Yu, Wenwang Chen, Zengwei Zheng, and Minyi Guo. Embedding-Based Similarity Computation for Massive Vehicle Trajectory Data. *IEEE Internet of Things Journal*, 9(6):4650–4660, 2022.

[23] Bo Zhang, Chengzhi Yuan, Tao Wang, and Hongbo Liu. STENet: A hybrid spatio-temporal embedding network for human trajectory forecasting. *Engineering Applications of Artificial Intelligence*, 106:104487, 2021.

[24] Peng Han, Jin Wang, Di Yao, Shuo Shang, and Xiangliang Zhang. A Graph-Based Approach for Trajectory Similarity Computation in Spatial Networks. In *Proceedings of the 27th ACM SIGKDD Conference on Knowledge Discovery & Data Mining*, KDD '21, page 556–564, New York, NY, USA, 2021. Association for Computing Machinery.

[25] Peilun Yang, Hanchen Wang, Ying Zhang, Lu Qin, Wenjie Zhang, and Xuemin Lin. T3S: Effective Representation Learning for Trajectory Similarity Computation. In *2021 IEEE 37th International Conference on Data Engineering (ICDE)*, pages 2183–2188, 2021.

[26] Saeed Taghizadeh, Abel Elekes, Martin Schäler, and Klemens Böhm. How meaningful are similarities in deep trajectory representations? *Information Systems*, 98:101452, 2021.

[27] Ziquan Fang, Yuntao Du, Xinjun Zhu, Danlei Hu, Lu Chen, Yunjun Gao, and Christian S. Jensen. Spatio-Temporal Trajectory Similarity Learning in Road Networks. In *Proceedings of the 28th ACM SIGKDD Conference on Knowledge Discovery and Data Mining*, KDD '22, page 347–356, New York, NY, USA, 2022. Association for Computing Machinery.

[28] Usman Ahmed, Jerry Chun-Wei Lin, and Gautam Srivastava. Deep Fuzzy Contrast-Set Deviation Point Representation and Trajectory Detection. *IEEE Transactions on Fuzzy Systems*, 31(2):571–581, 2023.

[29] Yan Lin, Huaiyu Wan, Shengnan Guo, and Youfang Lin. Contrastive Pre-training of Spatial-Temporal Trajectory Embeddings, 2022.

[30] Peilun Yang, Hanchen Wang, Defu Lian, Ying Zhang, Lu Qin, and Wenjie Zhang. TMN: Trajectory Matching Networks for Predicting Similarity. In *2022 IEEE 38th International Conference on Data Engineering (ICDE)*, pages 1700–1713, 2022.

[31] Jiaxin Ding, Bowen Zhang, Xinbing Wang, and Chenghu Zhou. TSNE: Trajectory Similarity Network Embedding. In *Proceedings of the 30th International Conference on Advances in Geographic Information Systems*, SIGSPATIAL '22, New York, NY, USA, 2022. Association for Computing Machinery.

[32] Ziwen Chen, Ke Li, Silin Zhou, Lisi Chen, and Shuo Shang. Towards robust trajectory similarity computation: Representation-based spatio-temporal similarity quantification. *World Wide Web*, pages 1–24, 2022.

[33] Silin Zhou, Peng Han, Di Yao, Lisi Chen, and Xiangliang Zhang. Spatial-temporal fusion graph framework for trajectory similarity computation. *World Wide Web*, 26(4):1501–1523, 2023.

[34] Silin Zhou, Jing Li, Hao Wang, Shuo Shang, and Peng Han. GRLSTM: Trajectory Similarity Computation with Graph-Based Residual LSTM. *Proceedings of the AAAI Conference on Artificial Intelligence*, 37(4):4972–4980, Jun. 2023.

[35] Yanchuan Chang, Jianzhong Qi, Yuxuan Liang, and Egemen Tanin. Contrastive Trajectory Similarity Learning with Dual-Feature Attention. In *2023 IEEE 39th International Conference on Data Engineering (ICDE)*, pages 2933–2945, 2023.

[36] Danlei Hu, Lu Chen, Hanxi Fang, Ziquan Fang, Tianyi Li, and Yunjun Gao. Spatio-Temporal Trajectory Similarity Measures: A Comprehensive Survey and Quantitative Study, 2023.

[37] Dylan P. Losey, Krishnan Srinivasan, Ajay Mandlekar, Animesh Garg, and Dorsa Sadigh. Controlling Assistive Robots with Learned Latent Actions. In *2020 IEEE International Conference on Robotics and Automation (ICRA)*, pages 378–384, 2020.

[38] Xin Huang, Stephen G. McGill, Jonathan A. DeCastro, Luke Fletcher, John J. Leonard, Brian C. Williams, and Guy Rosman. DiversityGAN: Diversity-Aware Vehicle Motion Prediction via Latent Semantic Sampling. *IEEE Robotics and Automation Letters*, 5(4):5089–5096, 2020.

[39] Alexandre Alahi, Kratarth Goel, Vignesh Ramanathan, Alexandre Robicquet, Li Fei-Fei, and Silvio Savarese. Social lstm: Human trajectory prediction in crowded spaces. In *Proceedings of the IEEE Conference on Computer Vision and Pattern Recognition (CVPR)*, June 2016.

[40] Dylan P Losey, Hong Jun Jeon, Mengxi Li, Krishnan Srinivasan, Ajay Mandlekar, Animesh Garg, Jeannette Bohg, and Dorsa Sadigh. Learning latent actions to control assistive robots. *Autonomous robots*, 46(1):115–147, 2022.

[41] Jiayu Miao, Tianze Zhou, Kun Shao, Ming Zhou, Weinan Zhang, Jianye Hao, Yong Yu, and Jun Wang. Promoting Quality and Diversity in Population-based Reinforcement Learning via Hierarchical Trajectory Space Exploration. In *2022 International Conference on Robotics and Automation (ICRA)*, pages 7544–7550, 2022.

[42] Andreea Bobu, Yi Liu, Rohin Shah, Daniel S. Brown, and Anca D. Dragan. SIRL: Similarity-Based Implicit Representation Learning. In *Proceedings of the 2023 ACM/IEEE International Conference on Human-Robot Interaction*, HRI '23, page 565–574, New York, NY, USA, 2023. Association for Computing Machinery.

[43] Nachiket Deo and Mohan M. Trivedi. Convolutional Social Pooling for Vehicle Trajectory Prediction. In *Proceedings of the IEEE Conference on Computer Vision and Pattern Recognition (CVPR) Workshops*, June 2018.

[44] Li Sun, Zhi Yan, Sergi Molina Mellado, Marc Hanheide, and Tom Duckett. 3DOF Pedestrian Trajectory Prediction Learned from Long-Term Autonomous Mobile Robot Deployment Data. In *2018 IEEE International Conference on Robotics and Automation (ICRA)*, pages 5942–5948, 2018.

[45] Arsal Syed and Brendan Tran Morris. CNN, Segmentation or Semantic Embeddings: Evaluating Scene Context for Trajectory Prediction. In George Bebis, Zhaozheng Yin, Edward Kim, Jan Bender, Kartic Subr, Bum Chul Kwon, Jian Zhao, Denis Kalkofen, and George Baciu, editors, *Advances in Visual Computing*, pages 706–717, Cham, 2020. Springer International Publishing.

[46] Alexandre Robicquet, Amir Sadeghian, Alexandre Alahi, and Silvio Savarese. Learning Social Etiquette: Human Trajectory Understanding In Crowded Scenes. In Bastian Leibe, Jiri Matas, Nicu Sebe, and Max Welling, editors, *Computer Vision – ECCV 2016*, pages 549–565, Cham, 2016. Springer International Publishing.

[47] Agrim Gupta, Justin Johnson, Li Fei-Fei, Silvio Savarese, and Alexandre Alahi. Social GAN: Socially Acceptable Trajectories With Generative Adversarial Networks. In *Proceedings of the IEEE Conference on Computer Vision and Pattern Recognition (CVPR)*, June 2018.

[48] Tharindu Fernando, Simon Denman, Sridha Sridharan, and Clinton Fookes. Soft + Hardwired attention: An LSTM framework for human trajectory prediction and abnormal event detection. *Neural Networks*, 108:466–478, 2018.

[49] Niraj Bhujel, Eam Khwang Teoh, and Wei-Yun Yau. Pedestrian Trajectory Prediction Using RNN Encoder-Decoder with Spatio-Temporal Attentions. In *2019 IEEE 5th International Conference on Mechatronics System and Robots (ICMSR)*, pages 110–114, 2019.

[50] Tharindu Fernando, Simon Denman, Sridha Sridharan, and Clinton Fookes. GD-GAN: Generative Adversarial Networks for Trajectory Prediction and Group Detection in Crowds. In C. V. Jawahar, Hongdong Li, Greg Mori, and Konrad Schindler, editors, *Computer Vision – ACCV 2018*, pages 314–330, Cham, 2019. Springer International Publishing.

[51] Tim Salzmann, Boris Ivanovic, Punarjay Chakravarty, and Marco Pavone. Trajectron++: Dynamically-Feasible Trajectory Forecasting with Heterogeneous Data. In Andrea Vedaldi, Horst Bischof, Thomas Brox, and Jan-Michael Frahm, editors, *Computer Vision – ECCV 2020*, pages 683–700, Cham, 2020. Springer International Publishing.

[52] Karttikeya Mangalam, Harshayu Girase, Shreyas Agarwal, Kuan-Hui Lee, Ehsan Adeli, Jitendra Malik, and Adrien Gaidon. It Is Not the Journey But the Destination: Endpoint Conditioned Trajectory Prediction. In Andrea Vedaldi, Horst Bischof, Thomas Brox, and Jan-Michael Frahm, editors, *Computer Vision – ECCV 2020*, pages 759–776, Cham, 2020. Springer International Publishing.

[53] Chieh-En Tsai and Jean Oh. A Generative Approach for Socially Compliant Navigation. In *2020 IEEE International Conference on Robotics and Automation (ICRA)*, pages 2160–2166, 2020.

[54] Yuxiang Cui, Haodong Zhang, Yue Wang, and Rong Xiong. Learning World Transition Model for Socially Aware Robot Navigation. In *2021 IEEE International Conference on Robotics and Automation (ICRA)*, pages 9262–9268, 2021.

[55] Stefan H. Kiss, Kavindie Katuwandeniya, Alen Alempijevic, and Teresa Vidal-Calleja. Probabilistic Dynamic Crowd Prediction for Social Navigation. In *2021 IEEE International Conference on Robotics and Automation (ICRA)*, pages 9269–9275, 2021.

[56] Yi Xu, Dongchun Ren, Mingxia Li, Yuehai Chen, Mingyu Fan, and Huaxia Xia. Tra2Tra: Trajectory-to-Trajectory Prediction With a Global Social Spatial-Temporal Attentive Neural Network. *IEEE Robotics and Automation Letters*, 6(2):1574–1581, 2021.

[57] Parth Kothari, Sven Kreiss, and Alexandre Alahi. Human Trajectory Forecasting in Crowds: A Deep Learning Perspective. *IEEE Transactions on Intelligent Transportation Systems*, 23(7):7386–7400, 2022.

[58] Kaouther Messaoud, Nachiket Deo, Mohan M. Trivedi, and Fawzi Nashashibi. Trajectory Prediction for Autonomous Driving based on Multi-Head Attention with Joint Agent-Map Representation. In *2021 IEEE Intelligent Vehicles Symposium (IV)*, pages 165–170, 2021.

[59] Ingrid Navarro and Jean Oh. Social-PatteRNN: Socially-Aware Trajectory Prediction Guided by Motion Patterns. In *2022 IEEE/RSJ International Conference on Intelligent Robots and Systems (IROS)*, pages 9859–9864, 2022.

[60] Hao Zhou, Dongchun Ren, Xu Yang, Mingyu Fan, and Hai Huang. CSR: Cascade Conditional Variational Auto Encoder with Socially-aware Regression for Pedestrian Trajectory Prediction. *Pattern Recognition*, 133:109030, 2023.

[61] Yusheng Peng, Gaofeng Zhang, Jun Shi, Xiangyu Li, and Liping Zheng. MRGTraj: A Novel Non-Autoregressive Approach for Human Trajectory Prediction. *IEEE Transactions on Circuits and Systems for Video Technology*, pages 1–1, 2023.

[62] Venkatraman Narayanan, Bala Murali Manoghar, Rama Prashanth RV, and Aniket Bera. EWareNet: Emotion-Aware Pedestrian Intent Prediction and Adaptive Spatial Profile Fusion for Social Robot Navigation. In *2023 IEEE International Conference on Robotics and Automation (ICRA)*, pages 7569–7575, 2023.