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DIPARTIMENTO DI INGEGNERIA INFORMATICA, AUTOMATICA E  
GESTIONALE

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# TRAJECTORY ENCODING

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# 1 Trajectory Learning

## 2014 - Sequence to Sequence Learning with Neural Networks

Name: seq2seq

[1]

## 2017 - Identifying Human Mobility via Trajectory Embeddings

[2] classifies users based on trajectory data. The problem is hard because there are many more trajectories than users.

Recurrent Neural Network (RNN) is used, and said to be good for classification when the number of labels is small. In particular uses a Long Short-Term Memory (LSTM) for processing sub-trajectories.

There is a location embedding, not sure how that is computed. But the trajectories are points on google maps, so there maybe be semantic information in there. The sequence of location embedding is passed onto the LSTM, not sure how they handle different trajectory lengths.

## 2018 - Self-Consistent Trajectory Autoencoder: Hierarchical Reinforcement Learning with Trajectory Embeddings

Code on: <https://github.com/wyndwarrior/Sectar>

[3] learns an embedding for trajectories with one encoder and two decoders: a state decoder to decode from the latent space back into trajectories, and a policy decoder, which generates the trajectory in the environment. As such the state decoder predict the trajectory of the policy. The encoder is used in a hierarchical Reinforcement Learning (RL) setup.

The state encoder and decoder are RNNs and the policy decoder is a feed-forward Neural Network (NN).

**note:** if the encoder is trained with trajectories from different tasks, the policy will be conditioned to each task, what is sort of parameterizing the policy to tasks.

In the paper, the policy has unknown dynamics, and hence the RL setup. Trajectories are continuous poses of joints over time. Tested in simulation.

## 2018 - Anomalous Trajectory Detection Using Recurrent Neural Network

Code on: <https://github.com/LeeSongt/ATD-RNN>

[4] proposes anomalous trajectory detection using RNN.

The trajectories are discretized, using a grid, and feed to a stacked RNN for learning the embedding, then a multi-layered perceptron and a soft-max layer detects if the trajectory is anomalous. The stacked RNN is made by feeding the hidden states of the previous to the next RNN.

The trajectories are padded in order to get trajectories of the same length.

LSTM and Gated Recurrent Unit (GRU) are two special types of RNNs are tested. GRUs seems to work better.

## 2018 - Deep Representation Learning for Trajectory Similarity Computation

Name: t2vec Code on: <https://github.com/boathit/t2vec>

[5] presents t2vec. A Deep Learning (DL) approach for trajectory similarity. States that using RNN is not a very good idea because you cannot reconstruct the trajectory and it fails to consider spatial proximity, which is inherited in trajectory data.

called in the paper as t2vec or seq2seq?

The approach is based on the encoder-decoder framework. Handling varying sampling rates is done by augmenting the training data creating sub-trajectories by sub-sampling and noise addition. They also propose a spatial-aware loss, and pre-train the ??cells?? and let them to be optimized during training.

**Notes:** The paper is very confusing. I do not really know that are the inputs and outputs or how the sequences are fed in the RNN inside the encoder.

## 2020 - Trembr: Exploring Road Networks for Trajectory Representation Learning

[6] presents `traj2vec`. The paper focus on trajectories on roads. It preprocesses the trajectories by projecting them in a road network and the trajectory is a sequence of road segments and travel time.

The `RNN` decoder is conditioned to the road network, and the training is made by optimising a loss for the trajectory and another for time.

**Notes:** Maybe the secret for velocities profiles is in the addition of time to the loss.

## 2019 - Computing Trajectory Similarity in Linear Time: A Generic Seed-Guided Neural Metric Learning Approach

**Name:** NeuTraj

**Code on:** <https://github.com/yaodi833/NeuTraj>

[7] proposes a method for accelerating trajectory similarity computation by sampling seeds of trajectories, computing their similarity, and approximating them with a neural metric.

States that `RNNs`, `LSTMs`, and `GRUs` can only model one sequence without considering the between-sequence correlation.

Does not consider time in the trajectory. Starts sampling from the trajectories and computes a distance matrix between the samples using a given trajectory distance metric which is then normalized.

The `RNN` is augmented with a memory, which is created by dividing the space into a grid, and for each grid slot, the memory stores the hidden vector of the `RNN`. This memory is used to extend the `RNN` cell, sort of like an `LSTM`.

The loss for training is  $\mathcal{L}_{\tau_i, \tau_j} = \sum_k w_k (f(\tau_i, \tau_j) - \exp(-||e_i - e_j||))$ , a weighted difference between the similarity metric  $f$  and the distance in the embedding space  $(e_i - e_j)$ . The weight  $w_k$  is obtained using the normalized distance matrix, computing pairs of similar and dissimilar trajectories and more fancy stuff.

**Notes:** map is like google map.

## 2020 - Trajectory similarity learning with auxiliary supervision and optimal matching

**Name:** Traj2SimVec [8] follows the same idea as in [7] which selects some trajectories for pre-training [something], the training samples are divided in three sub-trajectories [because it seems to help learning].

A distance matrix is computed which is used as supervision signal, similar to [7].

## 2020 - MARC: a robust method for multiple-aspect trajectory classification via space, time, and semantic embeddings

[9] Embeds semantics on the trajectories. Each semantic information (weather, time, type of place) has an encoding, and a weight matrix which transform them into a fixed size vector. The semantic trajectory is fed to an `LSTM`, which encodes the trajectories, having the hidden states used for classification.

## 2021 - Embedding-Based Similarity Computation for Massive Vehicle Trajectory Data

[10] seems to propose the exact same thing as [7], but with interpolation for de-noising.

## 2021 - STENet: A hybrid spatio-temporal embedding network for human trajectory forecasting

[11] Focuses on predicting pedestrian trajectories. Uses a `LSTM` with `Convolutional Neural Networks (CNNs)` to embed position features in multiple temporal time-scales. The encoder-decoder structure stack a `CNN` and a graph attention model. The decodes stacks many `LSTMs`.

They give related works on social trajectory learning.

**Notes:** They point to `Variational Auto-Encoders (VAEs)` for modelling multi-modality and for the generative capabilities.

## 2021 - A Graph-Based Approach for Trajectory Similarity Computation in Spatial Networks

[12] Propose a **Graph Neural Network (GNN)**-based trajectory embedding. The framework measures trajectory similarities, learns **Points of Interest (PoIs)**, and learns a trajectory embedding.

A trajectory is encoded as the points in a graph map. Then they define a trajectory similarity metric on the **PoI** graph, based on the graph distance between the points and trajectories. An embedding capturing the neighbours and graph trajectory is learned. The **PoI** embeddings and their neighbours are used to learn another embedding using its neighbours information. Finally, **LSTMs** are used to learn the trajectory over the graph embeddings. The loss function minimizes the above defined distance between trajectories and the distance between the two closest trajectories.

## 2021 - T3S: Effective Representation Learning for Trajectory Similarity Computation

**Name: T3S**

[13] combines **LSTMs** and attention **NNs** over the grid graph for learning the embedding. Close to [7, 8, 10].

## 2021 - How meaningful are similarities in deep trajectory representations?

**Code on:** <https://dbis.ipd.kit.edu/2652.php>

[14] presents a survey and evaluation of **t2vec** [5] and other methods. Seems like **t2vec** with some variations outperform the rest. **t2vec** seems to be stacked **LSTMs**.

Evaluate how changing **t2vec** parameters affect similarity values. **t2vec** seems robust to parameters.

Evaluate **t2vec** against non learning metrics. Seems like associating them lead to better results.

**[They DO ignore the whole literature on learning methods?]**

Concludes that using **Longest Common SubSequence (LCSS)** and **t2vec** leads to a better trajectory similarity, covering overlap, shape, direction and distance.

**Notes:** Maybe that should be 4 characteristics to consider.

## 2022 - Spatio-Temporal Trajectory Similarity Learning in Road Networks

**Name: ST2vec**

[15] learns a spatio-temporal representation. Two steps, which is based on learning a spatial model, a temporal model and a co-attention fusion module. It is based on a road network, trajectories are sequences of vertex on the road network.

Define the distance of a spatio-temporal trajectory as a weighted sum for a spatio-distance ( $d_s$ ) and a temporal distance ( $d_t$ ):

$$d(\tau_i, \tau_j) = \alpha d_s(\tau_i, \tau_j) + (1 - \alpha) d_t(\tau_i, \tau_j) | \alpha \in [0, 1]$$

Later uses **LSTMs** to learn using two strategies, using one **LSTM** for space and another for time, or using one for both.

## 2022 - Deep Fuzzy Contrast-Set Deviation Point Representation and Trajectory Detection

[16] Grid-map based, contrastive learning.

**notes:** hard to understand what they are doing here.

## 2022 - Contrastive Pre-training of Spatial-Temporal Trajectory Embeddings

[17] employs contrastive learning for learning an embedding which retains high-level travel semantics.

Recovering the original trajectory is not a good approach when learning representations with **RNNs** since it fails to capture the high-level information of trajectories. Contrastive learning with noisy augmentation can handle the high-level information while being robust to noise. However data augmentation needs to be well designed.

The positive samples are created with subsampling the query trajectory, while the negative samples come from different trajectories.

**Notes:** Not sure this is correct, I think the “different trajectories” should be far enough from the query trajectory to be a negative sample.

The encoder stacks a spatio-temporal encoding layer and attention layers. For the first, a learnable encoding of locations is learned (each location leads to a vector) and location and time are passed to a trigonometric vector transformation to compute features which can capture periodic information; those vectors are then summed up. The attention layer is actually 2 stacked attention layers.

## **2022 - TMN: Trajectory Matching Networks for Predicting Similarity**

[18] uses attention to compute intra-trajectory similarities, and then uses a [LSTM](#).

**Notes:** Comparison ignores many methods.

## **2022 - TSNE: Trajectory Similarity Network Embedding**

[19] uses a pre-defined trajectory measure function to construct a k-NNG (K nearest neighbours graph) and computes the embedding based on the graph.

**Notes:** Not sure how they compute the embedding from the graph. Seems like the graph representation allows to handle partial similarity and unordered similarity.

## **2022 - Towards robust trajectory similarity computation: Representation-based spatio-temporal similarity quantification**

[20]

## **2023 - GRLSTM: Trajectory Similarity Computation with Graph-Based Residual LSTM**

[21]

## **2023 - Spatial-temporal fusion graph framework for trajectory similarity computation**

[22]

## **2023 - Contrastive Trajectory Similarity Learning with Dual-Feature Attention**

[23]

## **2023 - Spatio-Temporal Trajectory Similarity Measures: A Comprehensive Survey and Quantitative Study**

[24]

**Code on:** <https://github.com/ZJU-DAILY/TSM>

Present a survey with several methods, and benchmark for evaluating them. Apparently Traj2SimVec [8] is the learning method, which is not grid-based that handles our problem.

## **2 Trajectory Learning on Robotics**

### **1991 - Solving the moving obstacle path planning problem using embedded variational methods**

[25]

### **2016 - Robobarista: Learning to Manipulate Novel Objects via Deep Multimodal Embedding**

[26]

**2020 - Controlling Assistive Robots with Learned Latent Actions**

[\[27\]](#)

**2020 - DiversityGAN: Diversity-Aware Vehicle Motion Prediction via Latent Semantic Sampling**

[\[28\]](#)

**2021 - Use of Embedding Spaces for Transferring Robot Skills in Diverse Sensor Settings**

[\[29\]](#)

**2022 - STL2vec: Signal Temporal Logic Embeddings for Control Synthesis With Recurrent Neural Networks**

[\[30\]](#)

**2022 - Learning latent actions to control assistive robots**

[\[31\]](#)

**2022 - Promoting Quality and Diversity in Population-based Reinforcement Learning via Hierarchical Trajectory Space Exploration**

[\[32\]](#)

**2023 - SIRL: Similarity-Based Implicit Representation Learning**

[\[33\]](#)

### **3 Comparison**

### **4 Challenges**



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