

CSC 191B: Lab #3: Growth Curves

Learning Outcomes

- Perform operations on 1D arrays
 - Plot functions and set plotting options and formatting
 - Use experiments to reason about asymptotic behavior of functions
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Background. A central question in computer science is: “How long will it take for my program to finish?” In particular, we’d like to know the time required as a function of the size of the input to the program. For example, we’d expect a text analysis algorithm to process one newspaper article a lot faster than it takes to analyze all the New York Times articles written in a given year. We usually call this function of the input size the *time complexity* of an algorithm.

Searching through an unsorted array of numbers requires accessing every element in the array, so the time complexity of that algorithm applied to an array of size n is approximately $f(n) = n$. If the array is sorted, then binary search can be done with time complexity approximately $f(n) = \log_2(n)$, which is much more faster than linear search when n is large.

A logarithmic time complexity is a desirable trait of an algorithm – it means the algorithm won’t take long to run even for big inputs because the function grows so slowly. A surprising fact is that $\log(n)$ grows more slowly than n^ϵ for *any* $\epsilon > 0$. That is, given any $\epsilon > 0$, such as $1/4$ or 0.000001 , there exists an N such that $\log(n) < n^\epsilon$ for all $n > N$. This is true of logarithms for any base, but for this lab, $\log n$ refers to the natural logarithm, $\ln n$ or $\log_e n$, which corresponds to MATLAB’s `log` function.

Problem Statement. In this lab, we’ll use MATLAB’s plotting functions to visualize growth curves and understand their behavior.

1. Plot the functions $f_1(n) = n$, $f_2(n) = n^2$, $f_3(n) = n^{1/4}$, and $f_4(n) = \log n$ on the same axes over the range $n \in [1, 7]$. Include a title, axes labels, and a legend with your plot, and find ways to make the plot presentable by adjusting `FontSize` or `legend Location`, for example. Useful MATLAB functions for this task include `linspace`, `.^`, `log`, `plot`, `hold on`, `legend`, `title`, `xlabel`, and `ylabel`.
2. Use MATLAB’s plotting tools to find when $n^{1/4}$ overtakes $\log n$. Plot the two functions in the range that shows the crossover point using `logspace` and `semilogx`. (Hint: it’s a big number.) Include a title, axes labels, and a legend with your plot, and find ways to make the plot presentable by adjusting `FontSize` or `legend Location`, for example. Use plot symbols (such as `'*'` and `'o'` to demonstrate that the logarithmically spaced n values look linearly spaced on a logarithmic x-axis.

Discussion.

1. What techniques did you use to find the crossover point between $\log n$ and $n^{1/4}$? What is the crossover point between $\log n$ and $n^{1/5}$? (You need only provide the number here.)
2. What plotting options did you use to make the plots presentable? Why did you choose those? What else did you try?

What to turn in.

- One MATLAB file `polylog.m` that contains the commands to generate the two plots and commented lines that answer the discussion questions
 - Two pdfs that contain the MATLAB-generated plots (don’t attached `.fig` files, use File/Save As on the MATLAB figure window)
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Grading rubric

- Code: 40 points each for MATLAB commands to generate the plot (they must be well commented) and the plots themselves
- Results: 10 points each for the discussion questions