CSC 191B: Lab #2: Approximating the Golden Ratio

Learning Outcomes

- Write scripts and custom functions
- Use MATLAB's for/while loop syntax
- Use iterative technique to approximate a value

Background. The golden ratio is an interesting and famous mathematical constant. It's defined as the ratio between the length and width of a rectangle such that if you cut off the largest square that fits in the rectangle, you're left with a smaller rectangle with the same length:width ratio. It can be derived using the quadratic formula to be

$$\phi = \frac{1+\sqrt{5}}{2}.$$

However, in this lab we will use a connection to another famous mathematical entity to approximate its value. The Fibonacci sequence is a sequence of integers that are defined recursively as

$$f_n = f_{n-1} + f_{n-2}. \tag{1}$$

We start them off with $f_0 = 0$ and $f_1 = 1$. It turns out that the ratio between consecutive Fibonacci numbers $r_n = f_n/f_{n-1}$ is an approximation of ϕ (that is, the sequence of ratios converges to the golden ratio).

While the recursive formula for f_n is very simple, it feels inefficient to use it to compute the nth Fibonacci number when n is huge, as it requires computing all the Fibonacci numbers between 2 and n. It turns out there is also an explicit formula for the nth Fibonacci number:

$$f_n = \frac{1}{\sqrt{5}}\left(\left(\frac{1+\sqrt{5}}{2}\right)^n - \left(\frac{1-\sqrt{5}}{2}\right)^n\right). \tag{2}$$

The formula is counter-intuitive – it's not clear why this expression even simplifies to an integer for all n.

Problem Statement. In this lab, we'll use both strategies to compute Fibonacci numbers and approximate ϕ. We'll use both `for` and `while` loops, and we'll write both scripts and custom functions.

1. Write a function called `recFib` with a single input n that uses a `for` loop to compute and return the nth Fibonacci number using the recursive formula (1). At every iteration, the function should print out the Fibonacci number f_n and the ratio $r_n = f_n/f_{n-1}$. For example, here is what the output should look like for $n = 6$ (Hint: use `fprintf` with `%d` and `%f` and appropriate arguments):

```
>> f6 = recFib(6)
   n           f_n              r_n
-------------------------------------------
   2              1     1.000000000000000
   3              2     2.000000000000000
   4              3     1.500000000000000
   5              5     1.666666666666667
   6              8     1.600000000000000

f6 =

     8
```

2. Write a function called `expFib` with a single input n that uses the explicit formula (2) to compute and return the nth Fibonacci number as output.

3. Write a script that calls the `expFib` function to find the smallest n such that $|r_n - r_{n-1}| < 10^{-15}$. The scripts should display the values of n, f_n, r_n, and $|r_n - r_{n-1}|$. For example, if 10^{-15} were replaced by 10^{-5}, your output should look like

```
>> approxPhi
Approximating the golden ratio...
        ...required the 16th Fibonacci number

Golden ratio is approximately 1.6180327868852458
Last change in approximation was 4.3e-06
Computed using ratio of 987 to 610
```

Discussion. It feels silly to compute an approximation to $\phi = (1 + \sqrt{5})/2$ using an iterative scheme this way. Why not just plug it into MATLAB? Have you ever wondered how MATLAB (or other languages, or your high school calculator) computes the square root function? If you're interested, Google around and see what you can find. (You do not have to answer this question to complete the lab.)

What to turn in.

- Three MATLAB files (`recFib.m`, `expFib.m`, and `approxPhi.m`) that are well commented to explain what your overall script/function does and what each section of your code does. You should also copy the output for `recFib(20)` into commented lines of your `recFib.m` file, and copy the output of your `approxPhi` script into commented lines of your `approxPhi.m` file.

Grading rubric

- Code: 20 points each for three MATLAB files, which must be well commented
- Results: 20 points each for results of `recFib(20)` and `approxPhi` in comments of MATLAB files