CSC 191B: Lab #5: Contours and Cross Sections

Learning Outcomes

- Manipulation of 2D arrays and functions of two variables
- Visualization of surfaces in 3D using MATLAB's tools

Background. In this lab we'll explore a particular function of two variables and experiment with MATLAB's tools for visualizing the function. We will model a rectangular plate that has been warmed by a pair of heat sources: the plate is 6 units wide and 4 units long, and the heat sources are at $(1, 3)$ and $(5, 1)$. The temperature drops exponentially away from these points and is modeled by the function

$$T(x, y) = 100e^{-0.4((x-1)^2+0.7(y-3)^2)} + 80e^{-0.2(2(x-5)^2+1.5(y-1)^2)}.$$

Note that a MATLAB file called `temperature.m` has been posted with this PDF that evaluates this function.

Problem Statement. To understand this function better, we'll visualize it in multiple ways. Write a single MATLAB script to accomplish the following tasks:

1. Use `linspace` and `meshgrid` to create a grid of (x, y) points. For debugging purposes, start with a grid such that the distance between adjacent points in each dimension is 0.5.
2. Use `mesh` to plot the function evaluated at the grid points. Use the "Rotate 3D" button in the figure window to estimate the maximum value and the minimum value.
3. Use `contour` to plot contour lines for the function at 20 different values, linearly spaced between the minimum and maximum values.
4. Plot 2D cross sections for selected values of x and y using `plot`: make a single plot showing cross sections for $x = \{1, 3, 5\}$ and a separate plot showing cross sections for $y = \{1, 2, 3\}$. Compare them with the surface and contour plots to make sure they're computed correctly.

Change the grid of points to have 100 linearly spaced points in each dimension (for a total of 10,000 grid points) and re-run your script, saving the generated images as PDFs. Be sure to label your axes and plot lines appropriately. You can use `saveas(gcf, 'output', 'pdf')` inside your script to save the current figure with filename output.pdf, for example, in the current directory to avoid having to do it manually.

Discussion.

1. What visualization is most helpful to you in understanding the function (and why)? Describe another MATLAB tool or option that you like for visualizing functions of two variables.
2. What do you predict will happen over time if the heat sources are turned off and the edges of the plate are held at temperature 0?

What to turn in.

- One MATLAB file that contains the script and the answers to the discussion questions.
- Four PDF files that contain the mesh, contour, and two cross section plots.

Grading rubric

- Code: 20 points for the script, which should be well-organized and well-documented
- Plots: 15 points each for the plots, which should be clearly labeled
- Discussion: 10 points each for the discussion questions