Draft version March 20, 2017
Preprint typeset using LaTeX style AASTeX6 v. 1.0

RECOMMENDATIONS FOR LSST DATA PRODUCT: PHOTOMETRIC REDSHIFTS

Melissa L. Graham[1]

[1]University of Washington

ABSTRACT

This work is preliminary, and only in paper format because that's easy for me.
Bold fonts are used to highlight points and also make comments for future revisions.

Keywords: LSST; photometric redshifts

1. INTRODUCTION

The purpose of this document is to assemble the diversity of motivations driving the inclusion of photometric redshifts in the LSST Level 2 Object Catalog, evaluate the options for the algorithm to be used and the format in which the results will be presented in the catalog, and arrive at a recommendation.

The initial impression is that DM/QA might require a template-fitting estimator and that the early science goals do not impose a clear preference on the type of estimator, but that machine-learning codes return more accurate and more precise photo-z. So there is some contention that we need to work out.

Initial recommendations:
1) The DM/QA pipeline builders should please write down their potential use cases for Level 2 photo-z (Section 1.1).
2) Consider including a single "best" photo-z estimate and its error in addition to the full posterior PDF and, if a template-fitting algorithm is chosen, an identifier for the best fit SED (Section 1.2).

Contents:
Section 1.1 describes the internal and external motivations for why photo-z are being included in the Level 2 database and how they influence our choice of photo-z algorithm.
Section 1.2 discusses the format of PHOTOZ in the Level 2 catalog, two potential alterations that we might want to make, and options for compression.
Section 1.3 reviews the existing work of others who evaluated photo-z algorithms for similar purposes.
Section 2 uses mock galaxy catalogs that are simulated to resemble subsets of LSST data at 1, 2, 5, and 10 years, runs them through one template-fitting and one machine-learning photo-z estimator, and compares the results.

1.1. *Motivation for Choice of Photo-z Estimator*

Typically, photometric redshift algorithms work by either fitting template spectra to the observed photometry or by matching photometry to a training set of galaxies with spectroscopic redshifts. The method of "matching photometry" is often done with machine learning codes. Hybrid photo-z estimators also exist, attempting to mitigate the flaws of either process (e.g., the SDSS DR12 photo-z estimator by Beck et al. 2016, or the Gaussian Processes estimator described by Leistedt & Hogg 2016). To know which kind of photo-z estimator we should use, we start by collecting the various motivations for including a photo-z estimate in the Level 2 catalogs.

1.1.1. *Internal Requirements*

The relevant requirement is DMS-REQ-0046, "The DMS shall compute a photometric redshift for all detected Objects". From the Jira conversation on this issue we've managed to cobble together some potential internal uses for a Level 2 photo-z: **A)** "DM would like to be able to run some sort of photo-z code for QA purposes ... good enough to validate LSST's galaxy photometry". **B)** DM "should definitely also run some sort of photometric cluster finder for QA purposes, which provides much better photo-zs for red cluster galaxies and hence gives us much better photometric quality tests for a small subset of the population ... idea is that the width of the red sequence in clusters is really the only simple metric we have for galaxy photometry colors that doesn't rely on simulations". However, photo-z might not actually be involved in this metric, as the code uses "the colors of the galaxies directly to estimate the red sequence, rather than relying on photo-zs (which have a lot of assumptions built in already)". **C)** "DM is already on the hook for some sort of photo-z in the form of SED fitting (and I would assume templates/redshifts) for chromatic corrections".

While vague, all of these seem to suggest at least implicitly that a template-fitting code would be used to generate the photo-z. **A)** Using observed SED of real galaxies seems like the best choice for validating photometry, e.g., if the question is whether the photometric outputs match the colors of real galaxies. **B)** If photo-z for red sequence galaxies are being prioritized, perhaps the best results would be generated by fitting elliptical galaxy spectra only. Also, we'd need to test whether template-fitting or machine-learning is best for that particular scenario. **C)** In this case, it sounds like SED fitting is the only option that would satisfy the use case of chromatic corrections.

Before we can move forward, we need the DM/QA pipeline builders to explicitly write down the variety of ways that they might make use of photometric redshifts and, if there are many possible use cases, some estimated of the priority or importance of each.

1.1.2. *External Requirements*

Extragalactic astrophysics such as weak lensing, baryon acoustic oscillations, and Type Ia supernova cosmology are all main science drivers for the LSST, and all require catalogs of galaxies with photometric redshifts. However, the photo-z algorithms for precision cosmology will be custom-tailored to the needs of each domain and we should consider those applications as Level 3 photometric redshifts. Instead, the main consumers of the Level 2 photo-z product will probably be users wanting to:

- evaluate the cluster membership status of galaxies (e.g., for spectroscopic follow-up campaigns)
- reject compact extragalactic objects from stellar samples, or vice versa (e.g., for spectroscopic follow-up campaigns)
- identify and/or characterize extragalactic transient host galaxies (sometimes the transient redshift is known and used to identify the host, and sometimes the most probable host is used to constrain the transient redshift)
- begin their cosmological studies as soon as possible, before the Level 3 photometric redshift routines are perfected

The algorithm that we choose should be easily understandable and characterizable, and the output photometric redshifts and their uncertainties should be strictly defined. Furthermore I suspect that it will be more important for the uncertainties to be reliable than for the photo-z to be more accurate (to a point, of course). Additionally, to maximize early science capabilities of the scientific community we should choose an algorithm that will return the most accurate photo-z as early in the survey as possible – e.g., one that does not require a deep spectroscopic training set of galaxies with LSST photometry. For this reason, in Section 2 we focus the analysis on performance with galaxy catalogs simulated to resemble LSST data at 1, 2, 5 and 10 years.

So far, considerations for the external uses of photo-z in the Level 2 catalogs are not forming any clear preference for a type of photo-z estimator.

We might decide it is useful to open this up to all of the extragalactic LSST Science Collaborations for input. I could do this by contacting the chairs. That would be the following SC: Galaxies, Dark Energy, Active Galactic Nuclei, Transients, and Strong Lensing (unless the latter has become the DESC SL WG). We may also want to include the Stars, Milky Way, and Local Volume SC, to see if they rely on photo-z to reject objects from their sample. One option would be to identify a point of contact that I can converse with, or another option would be to poll the general community. If we want to do a poll, perhaps questions like this would be useful.

- 1. Please rate how important LSST-provided photometric redshifts are to your science goals, e.g., on a scale of 1 to 5 (not important, insignificant, useful, important, essential).
- 2. Please rate how well the proposed LSST-provided photo-z meet your science needs, e.g., on a scale from 1 to 5 (not met, poorly met, adequate, well met, perfectly met).
- 3. Regardless of your answer to 2, please rate how often you might use the proposed LSST-provided photometric redshifts, e.g., on a scale of 1 to 5 (never, rarely, occasionally, regularly, always).

1.2. *Format of Photo-z in Level 2 Catalog*

The photometric redshift products that the LSST DM team will provide in the Level 2 object catalogs are defined in DPDD, Table 4, "Level 2 Catalog Object Table" as:

- STDCOLOR = "standard color", color of the object measured in "standard seeing", suitable for photo-z
- STDCOLORERR = uncertainty on stdColor
- PHOTOZ (float[2x100]) = photometric redshift likelihood samples, pairs of redshift and likelihood $(z, \log L)$, computed using a published and widely accepted algorithm at the time of LSST

This desired format means that we need to choose an algorithm that returns $z, logL$, as opposed to a single "best" value and error, but as most estimators do return a posterior probability density function this is not a restrictive constraint. Both of the estimators we investigate in Section 2 return a PDF with a specified redshift range and resolution.

Proposed alterations – I have two low-priority suggestions for minor changes to the photo-z content in the Level 2 database. The first is that I suspect that many science users will simply want the photo-z and its uncertainty, and so I recommend making the first element of PHOTOZ the peak of the PDF and its 68% confidence interval. These values are natural outcomes of most algorithms anyway, so storing them will not result in more processing by DM and will save science users a step. Having 100 redshift bins (maximum) for the PDF means that no bin has $< 1\%$ of the probability, which is a nice, round number and easy to understand. Although not exactly a science-driven need, perhaps we could simply increase the storage space allotted to PHOTOZ to FLOAT[2X101] to accommodate the first elements being used to store the photo-z and error. The second suggestion is related only to template-fitting estimators, because in addition to a photo-z PDF they also identify the best-fit SED for the photometry. If we decide to use a template-fitting algorithm we should include information about the best fit template in the Level 2 catalog as an additional database entry (e.g., as a keyword or identifier code). We will also have to make the templates publicly available. This could help the user to evaluate whether the photo-z is sensible; e.g., allow them to weed out instances when an elliptical SED is best fit to a morphologically spiral object, or similar scenarios.

Compression – The posterior PDF will need to be compressed for storage within 100 array elements. From our analysis in Section 2, we look at the output from the BPZ code with a galaxy catalog simulated for a 1 year LSST survey to get an idea of the amount of information in a PDF. Approximately 50% of all test galaxies had a $P(z)$ with > 100 elements with $P > 0.000$ when a resolution of $\delta z = 0.01$ was used. This fraction drops to 6% of galaxies with > 100 elements with $P > 0.005$. This suggests it might be possible to do a really simple compression down to 100 elements by, e.g., combining adjacent low-probability bins in some efficient fashion until the number of bins is $= 100$. No bins with $> 1\%$ of the probability would need to be combined, and while some information would be lost, the benefit for the science user is that the posterior PDF is a simple machine-readable ascii table. Carrasco Kind & Brunner (2014) present an alternative method for storing photo-z PDFs that requires less storage space. With their algorithm for sparse representation, "an entire PDF can be stored by using a 4-byte integer per basis function". As they describe, "only ten to twenty points per galaxy are sufficient to reconstruct both the individual PDFs and the ensemble redshift distribution, $N(z)$, to an accuracy of 99.9% when compared to the one built using the original PDFs computed with a resolution of $\delta z = 0.01$, reducing the required storage of two hundred original values by a factor of ten to twenty". The advantage in this method seems to be the accuracy, and the disadvantage is that the science user would need to convert the stored format into a $P(z)$ before they can use it. It is unclear to me how much additional processing is required for this conversion, on the DM side and the user's side.

1.3. *Relevant Existing Work*

Relevant photo-z testing papers – Hildebrandt et al. (2010) tested 18 different photo-z codes on the same sets of simulated and real data and found no significantly outstanding method. Dahlen et al. (2013) test 11 different photo-z codes on the CANDLES data set (U-band through infrared) and also find that no method stands out as the "best", and that there is a strong dependence of photo-z accuracy on the SNR of the photometry (relevant for our tests at 1 year). They also found that most of the photo-z codes underestimate their redshift errors, which is important because we do want accurate errors.

Lessons from DES – Sánchez et al. (2014) use the science verification data (200 square degrees of $grizY$ photometry to a depth of $i_{AB} = 24$ magnitudes) of the Dark Energy Survey (DES) to evaluate several photometric redshift estimators. They found that the Trees for Photo-z code (TPZ; Carrasco Kind & Brunner 2013a) provided the most accurate results with the highest redshift resolution, and that template-fitting methods also performed well – especially with priors – but that in general there was no clear "winner".

Lessons from SDSS – Beck et al. (2016) describes the photo-z adopted for the SDSS DR12. As I understand it, they first use an empirical technique with a large training set to estimate the redshift and it's error, and then fit SED templates with that redshift in order to obtain additional galaxy information such as K-correction and spectral type. *They call it a hybrid technique but the photo-z sounds like it comes solely from the local linear regression, basically an interpolation in the color-redshift relation.*

Lessons from Lucy's work – Lucy Halperin used what we call "Brown" catalog[1] made by Sam Schmidt with simulated 10-year LSST-like magnitude uncertainties and ran it through 2 machine learning (ANNz and TPZ) and 2 template-fitting (LePhare and BPZ) photo-z codes. All four returned sets of photo-z with similar standard deviations and biases, but the template-fitting codes were more prone to failures and outliers. We found that for template-fitting photo-z codes, the choice of template SED set does make a significant difference in the results, particularly regarding photo-z outliers – however, this may have been particular to our use of the "Brown"-based galaxy catalog. We do not use this catalog in Section 2.

2. PRELIMINARY EVALUATION OF THE OPTIONS

Given the lack of a clear "winner" at the photo-z game (Section 1.3), here we examine the performance of two off-the-shelf photo-z codes, one template-fitting and one machine-learning. In Section 2.1 we describe the galaxy catalogs that we use. In Section 2.2 we describe the template-fitting photo-z estimator BPZ, in Section 2.3 the machine-learning code TPZ, and in Section 2.5 we compare and analyze their results. In Section 2.6 we write down some ideas for future tests of the photo-z estimators that would evaluate their performance in real world situations that are similar to the types of early science that might be done with LSST photo-z.

2.1. *Simulated Catalogs*

We use a randomly chosen 30000 galaxy test subset of the LC_DEEP_GONZALEZ2014A catalog, which is based on the Millennium simulation (Springel et al. 2005) and the galaxy formation models of Gonzalez-Perez et al. (2014) and constructed using the lightcone techniques described by Merson et al. (2013). We impose a limit on the true catalog redshift of $z < 3.5$, and a limit on the apparent i-band magnitude of $i < 25.5$, and furthermore require galaxies to be detected in gri. The latter requirement means that the test galaxies' apparent magnitude is brighter than a limit defined by a signal-to-noise ratio < 5 in all three filters gri. This limit depends on the number of years of survey elapsed, and since we want to use the same set of test galaxies to analyze the algorithms' results early in the survey, we require this gri non-detection with the expected limits after only 1 year of LSST. These restrictions are imposed prior to the random selection of 30000 test galaxies from the larger catalog. We then simulate 4 versions of the test galaxy catalog with errors appropriate for 1, 2, 5, and 10 years of LSST. We calculate galaxy magnitude uncertainties that are appropriate for the elapsed survey time, and observed photometry is simulated by adding a random scatter proportional to the uncertainties.

In addition to the test set, we need a training set of galaxies for the machine-learning algorithm to serve as a spectroscopic redshift catalog. Spectroscopic data sets containing tens of thousands of galaxies down to $i > 25$ and out to $z > 3$ are certainly possible, e.g., the VIMOS Ultra Deep Survey (VUDS; Le Fèvre et al. 2015). Assuming that the LSST will cover a spectroscopic field like the VUDS to the full 10-year depth during commissioning or with a first-year deep drilling field, we use as our training set a sample of 30000 catalog galaxies with photometric uncertainties equivalent to a 10-year LSST. This training set has the same redshift and magnitude distribution and limits as the galaxy catalogs, which may not be the case for a real spectroscopic set.

Needed: catalog characteristics like redshift, magnitude distributions etc.

2.2. *BPZ*

Bayesian Photometric Redshifts (BPZ[2]; Benítez 2000) is a template-fitting algorithm with a magnitude prior. We use all default parameters, including the i-band for the magnitude prior, except that we supply the CFHTLS set of SED templates. This set is 66 SEDs that were used for the CFHTLS photo-z paper and are from Ilbert et al. (2006), and they were interpolated from the CWW and Kinney models. The photo-z results are shown in Figure 1. Overall the results are quite poor even with the LSST 10-year predicted photometry, especially the level of quantization at

[1] We call it the Brown catalog because it uses the SEDs from Brown et al. (2014)

[2] http://www.stsci.edu/~dcoe/BPZ/

$z_{\rm phot} > 1.5$. We find that the results are not improved if we remove the magnitude prior, or use a different SED template set such as those from Brown et al. (2014). We do know from Lucy's work that the choice of SED template set has a significant impact on the results – the best results were achieved when we used the Brown SEDs with a galaxy catalog for which the photometry was simulated using those same SEDs. However, it's less straightforward to identify the "best" template SEDs to use with the Euclid galaxy catalog. Gonzalez-Perez et al. (2014) describes the wide variety of stellar population spectral synthesis models they used, but it would take quite some work to get them all together into a single catalog to provide to BPZ. Further analysis of the results is covered in Section 2.5.

2.3. *TPZ*

Trees for Photometric Redshifts (TPZ; Carrasco Kind & Brunner 2013a,b) is a machine learning algorithm that uses prediction trees and a training set of galaxies with known redshifts. We use all the default parameters from the example, except we increase the number of trees from 4 to 10 (this was set low in the provided example to decrease run time). Since the number of realizations is 2, this is a total of 20 trees. As shown in Carrasco Kind & Brunner (2013b), the bias and scatter of the resulting photo-z improve the most as the number of trees is increased to 20, and continues to improve more mildly to 100, and then are not much improved beyond 100 trees (i.e., their Figure 9). We also set the maximum redshift to 3.5 and the number of redshift bins to 350. We include both magnitudes and colors and their uncertainties as attributes to be used in the prediction trees, as Lucy's work found that this led to better results. From the TPZ output files, we take as $z_{\rm phot}$ the MODE of the redshift distribution instead of the MEAN, because this is the peak of the distribution (most likely redshift). The results are shown in Figure 1. They are quite poor 1 and 2 years, with a lot of quantization in the photo-z and many outliers at low and high redshift, but are significantly improved at 10 years.

In case a training set with LSST photometric uncertainties at the level of a 10-year survey is not available from commissioning or a dedicated deep drilling survey by the end of year 1, we also simulate the photo-z results with a training set that has the same level of photometric uncertainty as the test set. These results are shown in Figure 1. It is very interesting that there is actually a large **improvement** if the training set does **not** have better photometric errors than the test set (i.e., compare the 1 and 2 year results in the third row to the second row). **This is a concern:** I may have misunderstood how TPZ uses the photometric uncertainties. I thought it treated errors differently from other ATTRIBUTES, but perhaps it uses them just the same. That makes sense, but means it is inappropriate to include photometric errors as ATTRIBUTES if the train and test sets have different photometric precision. I'll have to figure this out.

Either way, TPZ is sensitive to the provided training set, so an extended investigation into what would truly be a realistic 1 year spectroscopic training set for LSST should be done (e.g., different redshift distributions, different magnitude limits). Although TPZ appears to give better accuracy, we also need to ensure that it gives realistic precision for it's photo-z results. Further analysis of the results is covered in Section 2.5.

2.4. *For Comparison, the "Color-Matching" Estimator*

As an additional comparison for those familiar with our other photo-z work, in Figure 1 we show the results of our "quick and simple" color-matching photo-z estimator at 1, 2, 5, and 10 years. That experiment uses a test set of 20000 and a training set of 60000, and the training set has the same photometric depth as the test set. We can see that the TPZ code does remarkably better at early times than this color-matching method.

2.5. *Comparing the Progressive Improvement of BPZ and TPZ*

We compare the BPZ and TPZ results over time in Figure 2 by plotting the following statistical measures of the photo-z precision: the standard deviation in $z_{\rm spec} - z_{\rm phot}$ as measured from the width of the intrerquartile range (IQR) and converted to 1σ assuming a Gaussian distribution; the bias as the mean value of $z_{\rm spec} - z_{\rm phot}$ for galaxies within the IQR; and the fraction of outliers, which is the fraction of galaxies with $|z_{\rm spec} - z_{\rm phot}| > 0.06$ and $> 3\sigma_{\rm IQR}$ (i.e., must be greater than whichever constraint is larger). For TPZ, we've used the trials in which the test and training sets have similar photometric uncertainties. From these statistical measures we can see that the photo-z from TPZ outperform those from BPZ at all years.

We take a closer look at the photo-z errors and posterior probability density functions of the BPZ and TPZ estimators. In Figure 3 we plot the photometric redshift as a function of its uncertainty as reported by BPZ and TPZ, along with the resulting distributions in photo-z and its error. We find a strict floor in the photo-z uncertainty that increases with redshift; I'm not sure why or how that would affect the use of these photo-z, but it seems unrealistic. For both BPZ and TPZ we can see that in some cases the clumps causing a quantization in photo-z also have high photo-z

errors, suggesting that a simple cut on photo-z error could return a sample for which the photo-z distribution matches the true distribution. However, there are other clumps in photo-z that have a relatively low errors. Overall, we find the TPZ algorithm returns a redshift distribution that is more similar to the true distribution. **Perhaps we should also plot accuracy (true-phot redshift) vs. precision (photo-z uncertainty)**.

In Figure 4 we plot examples of the posterior probability density functions output by the BPZ and TPZ algorithms for two test galaxies. One was chosen as a random representative of galaxies for which an inaccurate and imprecise photo-z was returned from both BPZ and TPZ for all years (top panel of Figure 4). The other was chosen as a random representative of galaxies which experienced a large and consistent improvement in both the accuracy and precision of its photo-z from year 1 to 10, for both BPZ and TPZ (bottom panel of Figure 4). In these plots, we can quite clearly see the quantization in the TPZ photo-z in the PDFs – **further investigation is needed to figure out if I've made a mistake in the TPZ inputs that has lead to this high level of quantization**. However, we can also see that it at least has non-zero probability at the true redshift of the galaxy in the top panel. Based on these plots **it is difficult to say whether either routine generates a better posterior PDF – further investigation and testing is needed**.

2.6. *Ideas for Real-World Science Tests*

Instead of comparing the general performance of photo-z estimators, we could compare them in specific real-world situations that represent some of the early science uses of Level 2 photo-z. Here is a short list of ideas.

Cluster Identification: take a line-of-sight from the simulated catalog with a known cluster, run the photometry through different algorithms, see which ones identify the cluster first/best.

Supernova Classification: Rahul is working on how including photo-z helps SN classification, so perhaps we could see which algorithm produces the most useful photo-z for likely SN hosts.

Star/Galaxy Photometric Separation: The Euclid catalog doesn't include $z = 0$ stars, but we could insert some and see how well the different algorithms are at picking them out, either by including stellar templates to actively identify them, or by passively identifying stars by how they fail to return a decent photo-z.

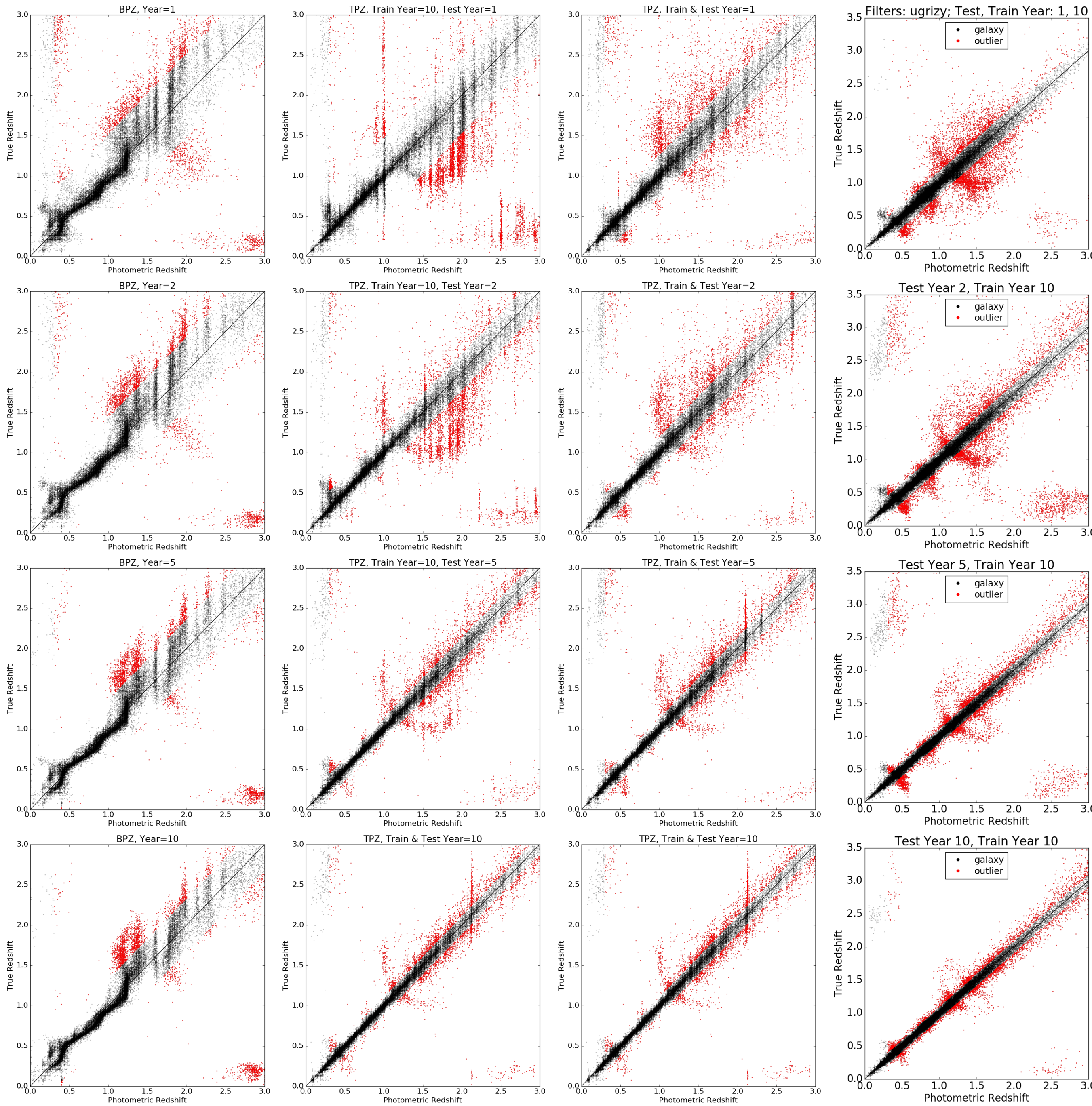

Figure 1. True *vs*. photometric redshifts for a variety of algorithms (by column), for 1, 2, 5, or 10 years of survey time elapsed (top to bottom), with outliers in red. **Left:** results for the BPZ algorithm. **Center-left:** results for the TPZ algorithm with a 10-year training set. **Center-right:** results for the TPZ algorithm with a co-evolving training set. **Right:** results for nearest-neighbors color-matching algorithm, with 50000 test galaxies and 10^6 training set galaxies with co-evolving photometric errors.

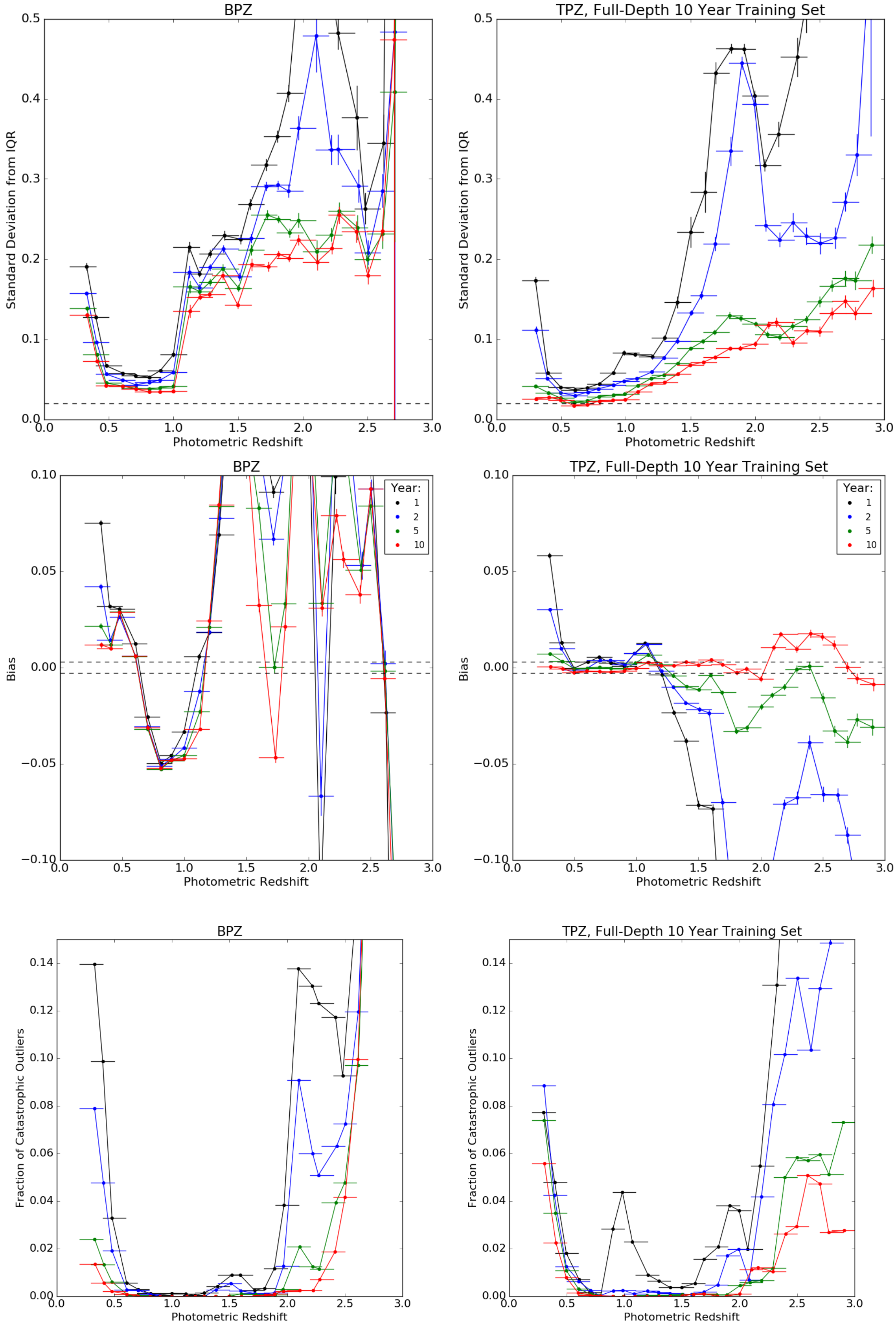

Figure 2. Statistical measures of the photo-z results from BPZ (left) and TPZ with an evolving training set (right) for simulated catalogs at 1 to 10 years (plot legends). From top to bottom we show the standard deviation from the IQR, the bias, and the fraction of outliers as a function of photo-z, with matched x- and y-axes to facilitate comparison.

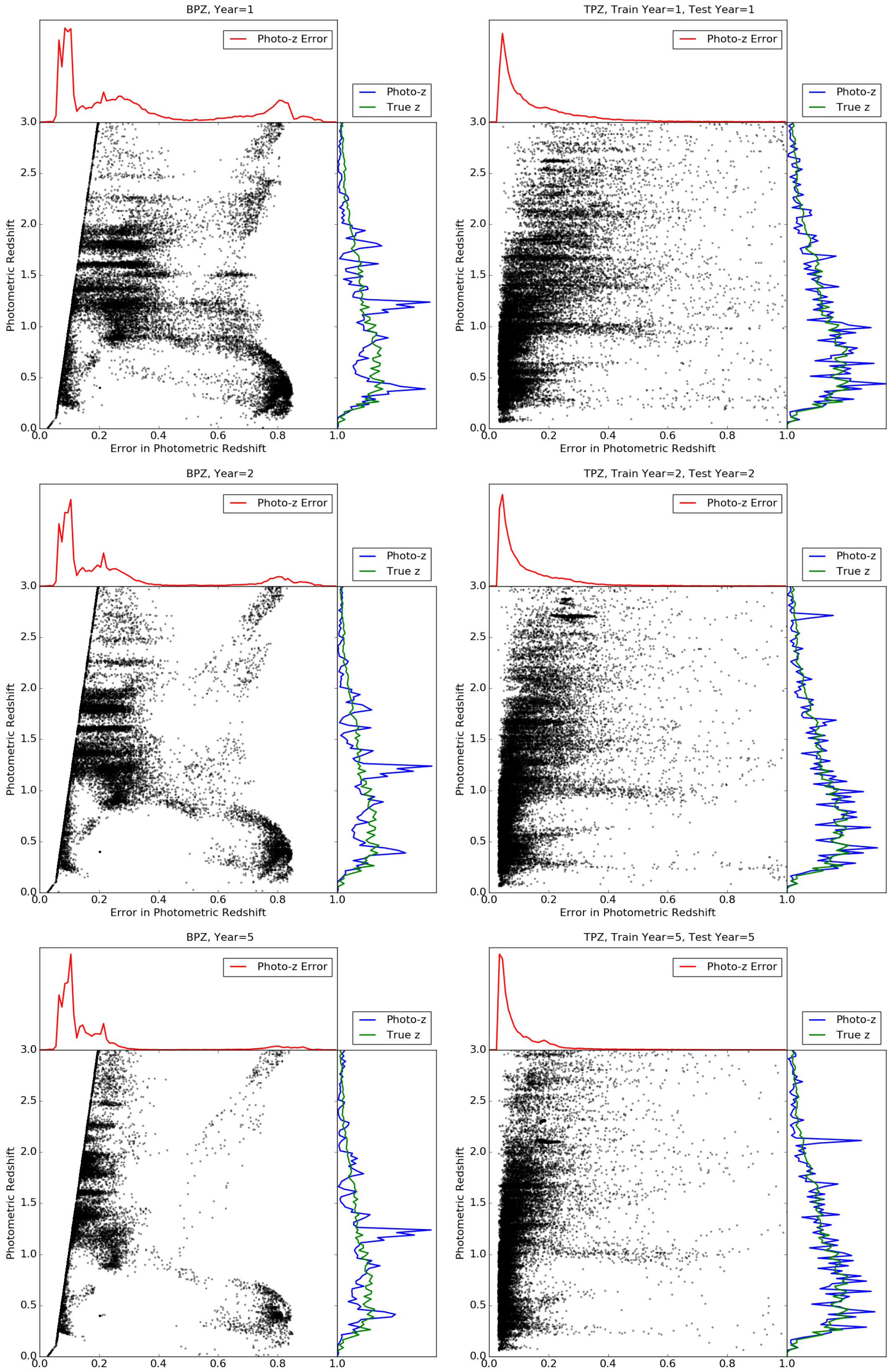

Figure 3. Photometric redshift *vs*. the error in photometric redshift output by the BPZ (left) and TPZ (right) estimators for simulated catalogs with photometric uncertainties at 1, 2, and 5 years of LSST (top to bottom). Red lines show the distribution of photo-z errors; blue and green lines compare the distributions of true and photometric redshifts.

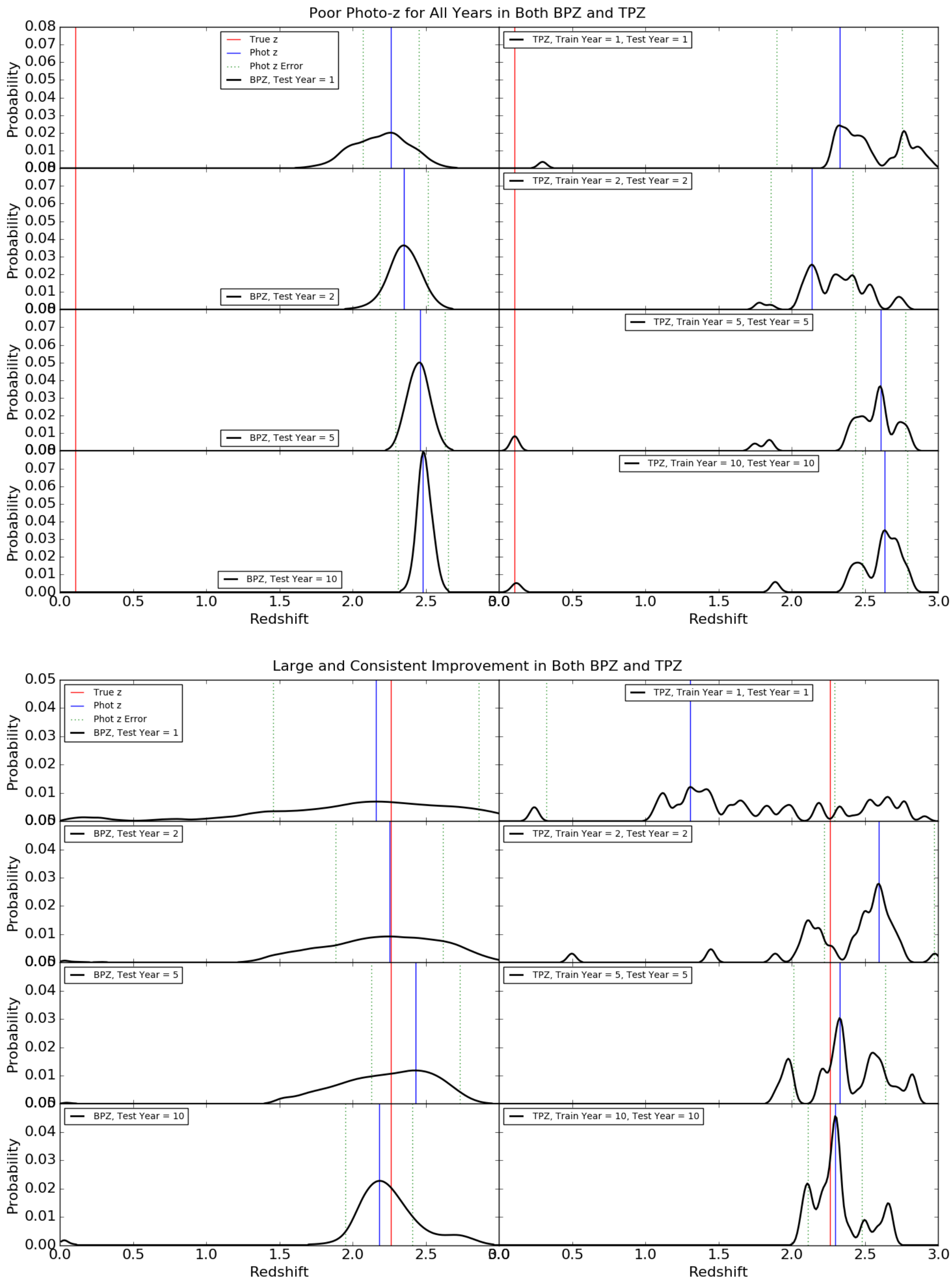

Figure 4. Posterior probability density functions for two test galaxies in all of our simulations: BPZ (left) and TPZ (right) for photometric uncertainties like 1, 2, 5, and 10 years of LSST (rows from top to bottom). In the top panel we choose a galaxy that return inaccurate and imprecise photo-z from all 8 trials, and in the bottom panel we choose a galaxy that experienced a large and consistent improvement in photo-z accuracy and precision from 1 to 10 years with both estimators.

REFERENCES

Beck, R., Dobos, L., Budavári, T., Szalay, A. S., & Csabai, I. 2016, MNRAS, 460, 1371

Benítez, N. 2000, ApJ, 536, 571

Brown, M. J. I., Moustakas, J., Smith, J.-D. T., et al. 2014, ApJS, 212, 18

Carrasco Kind, M., & Brunner, R. 2013a, TPZ: Trees for Photo-Z, Astrophysics Source Code Library, ascl:1304.011

Carrasco Kind, M., & Brunner, R. J. 2013b, MNRAS, 432, 1483

—. 2014, MNRAS, 441, 3550

Dahlen, T., Mobasher, B., Faber, S. M., et al. 2013, ApJ, 775, 93

Gonzalez-Perez, V., Lacey, C. G., Baugh, C. M., et al. 2014, MNRAS, 439, 264

Hildebrandt, H., Arnouts, S., Capak, P., et al. 2010, A&A, 523, A31

Ilbert, O., Arnouts, S., McCracken, H. J., et al. 2006, A&A, 457, 841

Le Fèvre, O., Tasca, L. A. M., Cassata, P., et al. 2015, A&A, 576, A79

Leistedt, B., & Hogg, D. W. 2016, ArXiv e-prints, arXiv:1612.00847

Merson, A. I., Baugh, C. M., Helly, J. C., et al. 2013, MNRAS, 429, 556

Sánchez, C., Carrasco Kind, M., Lin, H., et al. 2014, MNRAS, 445, 1482

Springel, V., White, S. D. M., Jenkins, A., et al. 2005, Nature, 435, 629