Likelyhood Of Shapes In Moment Space Documentation

Release

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CONTENTS

CHAPTER ONE

WEIGHTED MOMENTS

Define the weighted moments as measured from the image. Let r_i be a vector $[x_i, y_i]$.

$$R_0 = \sum_i z(r_i) w(r_i)$$

$$R_1 = \sum_i z(r_i) w(r_i) r_i \quad [1] \tag{1.1}$$

$$R_2 = \sum_i z(r_i) W(r_i) r_i r_i^T \quad (3) \tag{1.2}$$

CHAPTER

TWO

NORMALIZED WEIGHTED MOMENTS

The normalized weighted moments can be calculated as follows:

$$M_0 = R_0 \tag{2.1}$$

$$M_1 = \frac{R_1}{M_0} \quad (5) \tag{2.2}$$

$$M_2 = \frac{R_2}{M_0} - M_1 M_1^T \tag{2.3}$$

CHAPTER

THREE

WEIGHT FUNCTION

We will define the weight function used in calculating moments to be an elliptical Gaussian:

$$w(r_i) = \frac{e^{-\frac{1}{2} r_i C_2^{-1} r_i^T}}{w_0}$$
$$w_0 = \int_{-\infty}^{\infty} e^{-\frac{1}{2} r C_2^{-1} r^T} d^k r$$

3.1 Debiasing

The weight function used in calculating the moments is useful for suppressing extranious contributions from noise, but it also biases the measurement. To correct for this, we can calculate an approximate debiasing factor by assuming the source $z(r_i)$ is itself a Gaussian with moments Q_i of the form:

$$z(r_i) = \frac{e^{-\frac{1}{2}(r_i - Q_1) Q_2^{-1} (r_i - Q_1)^T}}{z_0}$$
$$z_0(Q_2) = \int_{-\infty}^{\infty} e^{-1\frac{1}{2} r Q_2^{-1} r^T} d^k r$$

3.1.1 Zeroth Moment

With this assumption the calculation of R_0 becomes:

$$R_0 = \frac{1}{W_0} \frac{Q_0}{z_0(Q_2)} \int_{-\infty}^{\infty} e^{-\frac{1}{2}(r - Q_1) Q_2^{-1} (r - Q_1)^T} e^{-\frac{1}{2}(r - C_1) C_2^{-1} (r - C_1)^T} d^k r$$

Using the Matrix cook book, we recognize the integral can be re-expressed as:

$$= \frac{1}{N} \int_{-\infty}^{\infty} e^{-\frac{1}{2}(r - \alpha) \beta^- 1 (r - \alpha)^T} d^k r$$
$$N = \sqrt{\det(2\pi (Q_2^{-1} + C_2^{-1}))}$$
$$\alpha = (Q_2^{-1} + C_2^{-1})^{-1} (Q_2^{-1} Q_1 + C_2^{-1} C_1)$$
$$\beta = (Q_2^{-1} + C_2^{-1})^{-1}$$

If we make the assumption that $\beta_x \beta_y - \beta_{xy}^2 > 0$ the above integral evaluates to:

$$= \frac{1}{N} \frac{2\sqrt{2}\sqrt{\frac{1}{2}}\pi}{\sqrt{\beta_x}\sqrt{-\frac{\beta_{xy}^2 - \beta_x \beta_y}{\beta_x}}}$$

3.1.2 First Moment

see *eq 1*