Bayesian Stackelberg Game Model for Multi-Objective Inspection in the Los Angeles Metro Rail System

Siting Li
IIS, Tsinghua University
li-st19@mails.tsinghua.edu.cn

Abstract

Inspection strategies for public transportation systems are usually multi-objective in real-world scenarios, including detecting fare evasion and preventing crimes. In this work, we model the multi-objective inspectors, riders, and criminals in the Los Angeles Metro Rail System as leaders and followers in the Bayesian Stackelberg Game Model. Then we derive the Bayesian Stackelberg Equilibrium and the best strategy for inspectors by solving the corresponding linear program. Compared with previous work, our method succeeds in balancing different objectives. Further, by looking into the simulation on the real-world map of Los Angeles, we point out multiple factors affecting the probability of fare evasion and crime committing: traveling distance, regions, and the intensity of inspection.

1 Introduction

Public transportation systems are a vital component of modern urban infrastructure, providing millions of people with a reliable and affordable means of getting around cities every day. Metro systems, in particular, are an essential mode of transportation in many cities around the world, offering fast and efficient travel options for commuters and tourists alike.

However, metro systems face numerous challenges that can impact their efficiency and safety. One such challenge is fare evasion, which remains a persistent problem that can lead to significant revenue losses. According to a report by Los Angeles Times in 2014, in the Los Angeles Metro Rail System, half of the system's 80 stations remain ungated, providing the chance for free entry. As a result, only 70 million of the 115 million rides are legal in 2013. Another challenge is safety issues, such as crimes and terrorist attacks. In 2022, 1435 cases of the most serious offenses, such as assault and robbery, are recorded in the Los Angeles Metro Rail System's data sheet.

Traditional methods of fare inspection, such as random checks or station barriers, can be costly and inefficient. These methods may not be able to detect all instances of fare evasion or may result in long lines and delays for riders. As such, there is a need for more effective and efficient methods for fare inspection that take into account the strategic behavior of riders and inspectors. Previous works explored the Bayesian Stackelberg Game Model, which has emerged as a powerful tool for modeling strategic interactions between different agents in complex systems, to design fare inspection strategies for the metro system. However, as mentioned in their summary, their strategy did not consider objectives other than fare inspection. For example, inspectors should also patrol and perform random checks to ensure the security of riders and their belongings.

In this project, we propose a game theoretic approach to multi-objective inspection in metro systems that takes into account the strategic behavior of riders (including criminals) and inspectors. Specifically, we model the interaction between these two groups as a Bayesian Stackelberg game. In this game, the inspector (the leader) has more information about rider behavior than the riders (the followers). The inspector uses this information to strategically allocate resources to maximize security and revenue, and the riders and criminals respond to the inspection strategy in an optimal way. Using the standard modeling, the Bayesian Stackelberg Equilibrium of this game can be obtained by solving a linear program.

For experiments, we choose the Los Angeles Metro Rail System with four lines as an example, which does not have fare inspection or safety check at the entrance as mentioned in previous research [3]. Both the results of a toy example and the real-world map are well-aligned with our intuition: In the single-objective case, as the inspection becomes more intense, the revenue per rider increases and the percentage of fare evasion reduces monotonically. However, in the multi-objective case, the trade-off between objectives breaks the monotonicity in the single-objective case. Taking a closer look at the best responses of inspectors, riders, and criminals, we point out multiple factors affecting the probability of fare evasion and crime committing: traveling distance, regions, and the intensity of inspection.

The proposed approach has several advantages over methods in previous works. First, by taking into account the strategic behavior of both riders and inspectors, our approach leads to more efficient use of resources while reducing revenue losses and safety risks. Second, our modeling reflects a more realistic scenario involving transfer. Overall, we hope that the proposed approach will contribute to the development of methods for inspection in metro systems, ultimately leading to better transportation experiences for riders and improved security and revenue streams for transit agencies.

2 Related Work

From the theoretical side, [1] explored the concept of leadership models in multiagent systems and how they differ from the traditional simultaneous strategy selection model. The authors studied how to compute optimal strategies to commit to under both commitment to pure strategies and commitment to mixed strategies, in both normal-form and Bayesian games. They gave both positive results (efficient algorithms) and negative results (NP-hardness results). For normal-form games, they showed that the optimal pure strategy to commit to can be found efficiently for any number of players. Overall, the paper provided insights into computing game-theoretic solutions in leadership models.

From the empirical side, [2] proposed a game theoretic approach to improving security at the Los Angeles International Airport by modeling the security problem as a Bayesian Stackelberg game and applying an efficient algorithm named DOBSS to find the optimal security schedule. [3] presented a problem setup for patrolling transit systems to deter fare evasion and proposed two approaches to address this problem: a basic formulation and an extended formulation. The basic formulation uses marginal coverage to compactly represent the problem, but it has issues that make it difficult for end-users to deploy patrol strategies computed. The extended formulation addresses these issues by adding dummy edges with zero duration and effectiveness. However, they did not provide multi-objective optimization techniques that can be used to allocate resources among different objectives.

3 Problem Setup

Following previous work, we consider the four lines (Blue, Gold, Green, and Red) in the Los Angeles Metro Rail System. In this game, the metro system is represented by a directed graph $G = \langle V, E\rangle$, where each vertex $v = \langle s, t\rangle$ corresponds to a station s and a time point t. Each edge in G has a length of 1. The patrol effectiveness value f_e mainly depends on the ridership volume in the real world, and thus we set different f_e for edges in different regions. The graph G is called the *transition graph* of the original graph. An example of the original graph and its transition graph is shown in Figure 1 and 2.

There are many riders $\langle s, d, t\rangle$ and criminals (followers) with different source vertex and destination vertex in G, and they need a time interval for waiting for the train. Riders have two possible options (buying or not buying). Criminals also have two possible options (committing or not committing). For simplicity, we assume that the criminals will stay at the stations for one interval or only take the train to the next station (and then leave instantly), which indicates that the crimes happen at the edges of G and succeed if there is no patroller.

There are γ patrol units (leader). A patrol strategy is represented by a set of paths $P_1, \ldots, P_\gamma$ (each of length at most κ). The number of patrol units on the same train should not exceed μ to ensure patrol efficiency. Since there are exponentially many patrol strategies, we only need to design a *marginal strategy* following the previous work [3]. In other words, we only need to assign flow x_e to each edge while keeping the flow balance constraint held at each vertex.

We assume that all players are rational. The ticket price is ρ and the fine is τ. For criminals, the outcome of a successful crime is α and the cost of a failed one is β. Given a pure patrol strategy of the γ units, the inspection probability for a rider with route λ would be

$$\min\{1, \sum_{i=1}^{\theta} \sum_{e \in P_i \cap \lambda} f_e\}$$

Note that this is an overestimate of the inspection probability since an illegal ride may be fined several times on different edges in this scenario.

Hence, given a priori probabilities p_λ of each type of rider, and a priori probabilities p_e of each type of criminal, the leader solves the following Mixed-Integer Quadratic Program (MIQP):

$$\begin{aligned}
\max_{x,u} & \sum_{\lambda \in \Lambda} p_\lambda u_\lambda - \theta \sum_{e \in E} p_e u_e \\
\text{s.t. } & u_\lambda \leq \min\{\rho, \tau \sum_{e \in \lambda} x_e f_e\}, \forall \lambda \in \Lambda \\
& u_e \geq \max\{0, x_e f_e(-\beta - \alpha) + \alpha\} \\
& \sum_{v \in V^+} x_{(v^+, v)} = \sum_{v \in V^-} x_{(v, v^-)} \leq \gamma \\
& \sum_{(v', v) \in E} x_{(v', v)} = \sum_{(v, v^\dagger) \in E} x_{(v, v^\dagger)}, \forall v \in V \\
& \sum_{e \in E} l_e \cdot x_e \leq \gamma \cdot \kappa, 0 \leq x_e \leq \mu, \forall e \in E
\end{aligned}$$

where θ in the objective function controls the relative importance of different objectives. In Section 4, we test two settings: $\theta = 0$ and $\theta = 1$, corresponding to the single-objective and multi-objective patrol mode. The first two inequalities ensure the optimality of riders' response and criminals' response, where the u_λ represents the expected fare that a rider on route λ would pay, and the u_e represents the expected money that a criminal on edge e would earn. The following two equalities represent the total flow control of the graph and the flow balance at each vertex, respectively. The last two inequalities ensure the length limit and the capacity limit. From the objective function, it is observed that this game is zero-sum, so the solution of this linear program is a Stackelberg Equilibrium.

4 Experiments

In this section, we perform simulations on a toy example and the map of four lines in the Los Angeles Metro Rail System. We solve the linear programs using CPLEX 12.10 and MATLAB 2019. The results show that our method successfully increases the revenue per rider and reduces the crime rate. Further, the solution of the real-world map provides insight into the influential factors of fare evasion and crime: traveling distance, regions, and the intensity of inspection.

4.1 A Toy Example

4.1.1 Experiment Settings

We use the map in Figure 1 and the parameter setting in Table 1. The traveling routes of passengers are $\{e_1 \to e_5, e_2 \to e_4, e_3 \to e_6\}$ and the probabilities are $0.3, 0.4, 0.3$. The criminals are evenly distributed on the six blue edges.

4.1.2 Results

The revenue per rider, the percentage of fare evasion, and the crime rate for different γ and κ are shown in Figure 3, 4, and 5. From the results, it is observed that:

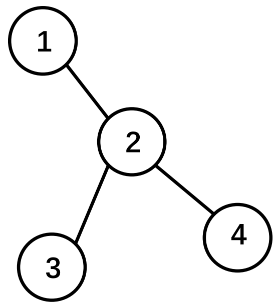

Figure 1: The original map of the toy example.

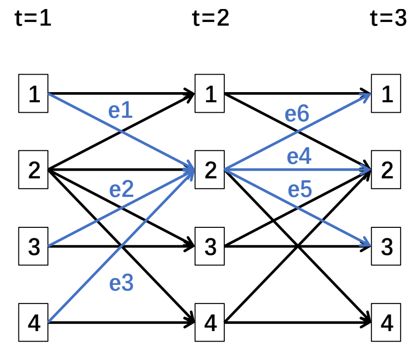

Figure 2: The transition map of the toy example.

Table 1: Default value and the description of parameters.

Parameters	Default Value	Description
ρ	1.5	Ticket price
τ	10	Fine for fare evasion
α	20	Outcome of a successful crime
β	200	Fine for a crime
f_e	0.2	Patrol efficiency
γ	1	Total amount of patrol units
κ	2	Length limit of a patrol
μ	2	Capacity limit of an edge

- For $\theta = 0$, the increase of γ and κ leads to the increase of the intensity of inspection and thus causes the growth of revenue per rider and reduction in fare evasion and crime rate. This verifies the effectiveness of our design.
- For $\theta = 1$, the curve of the percentage of fare evasion is above the curve of $\theta = 0$, and the curve of the crime rate is under the curve of $\theta = 0$, which suits our intuition.
- For $\theta = 1$, the change in the percentage of fare evasion and the crime rate is not monotone due to the trade-off between those two objectives.

4.2 Simulations on Real-World Map

4.2.1 Experiment Settings

We perform simulations on the real-world Los Angeles Metro Rail System. First, we label all the stations with the number $1, \dots, 73$ as shown in Figure 6. The metro police stations are near stations 11 and 49, so the start points of the patrol are set to these two stations.

To investigate how the regions affect fare evasion and crime rate, we divide the map into three regions: The downtown area, which is inside the red brackets, the urban area, which is between the red brackets and the green brackets, and the suburb, which is outside the green brackets.

The routes of riders are divided into four classes: Long ride at suburb, which includes $\{1-10, 15-24, 61-70\}$, short ride at suburb, which includes $\{39-36, 73-49, 60-55\}$, long ride at downtown, which includes $\{10-28, 45-8, 36-41\}$, and short ride at downtown, which includes $\{41-10, 31-13, 9-12\}$. The corresponding probabilities are $0.2, 0.1, 0.3, 0.4$.

The locations of criminals are divided into two classes: Downtown, which includes $\{11-12, 11-10, 11-40, 14-13, 14-33\}$, and suburb, which includes $\{61-62, 15-16, 60-59, 39-38, 73-72\}$. The criminals are evenly distributed on these edges.

We apply the parameters in Table 2.

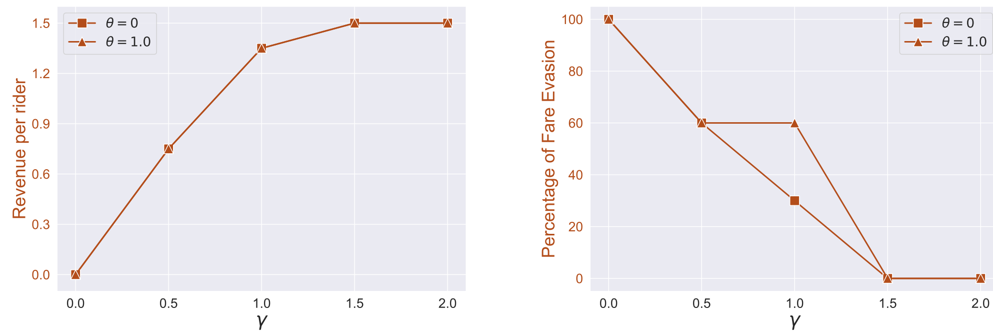

Figure 3: The change of revenue per rider and percentage of fare evasion with the increase of γ, the total amount of patrol units.

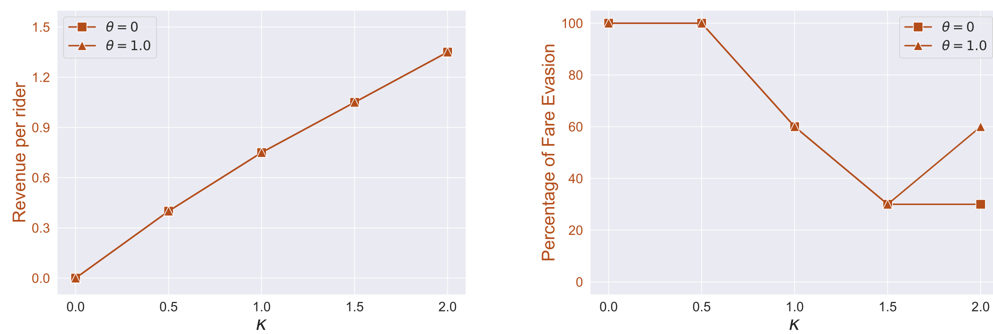

Figure 4: The change of revenue per rider and percentage of fare evasion with the increase of κ, the length limit of a patrol.

4.2.2 Results

The results are shown in Figure 7 and 8, from which we observe that:

- For both $\theta = 0$ and $\theta = 1$, the increase of γ causes the growth of revenue per rider and the reduction in crime rate. This verifies the effectiveness of our design on the real-world map.
- In the left figure of Figure 7, when the amount of patrol units is small, the distance of the ride is the main decisive factor for fare evasion: Riders with a short ride tend to evade fare. When the amount of patrol units is large, the region of the ride becomes more influential: Riders at suburb tend to evade fare. The increase in patrol units mainly improves the revenue per rider with a short ride at downtown.
- In the right figure of Figure 7, for $\theta = 1$, the trend is similar to that for $\theta = 0$. However, the revenue per rider is less due to the trade-off between multiple objectives. Particularly, the fare evasion rate of short rides in the suburb is 100%, indicating that the patrol units are all placed in the urban or downtown areas.
- In Figure 8, the crime rate at the suburb is higher than that at downtown in most settings. The increase of patrol units is mainly effective for crime prevention at downtown.

5 Conclusions

In this project, we model the multi-objective inspection in the Los Angeles Metro Rail System using the Bayesian Stackelberg Game Model. The results of the simulation prove the effectiveness of the solution and further provides insight into the influential factors for fare evasion and crime on trains. By considering multi-objective inspection, transfer, and the difference between regions, our

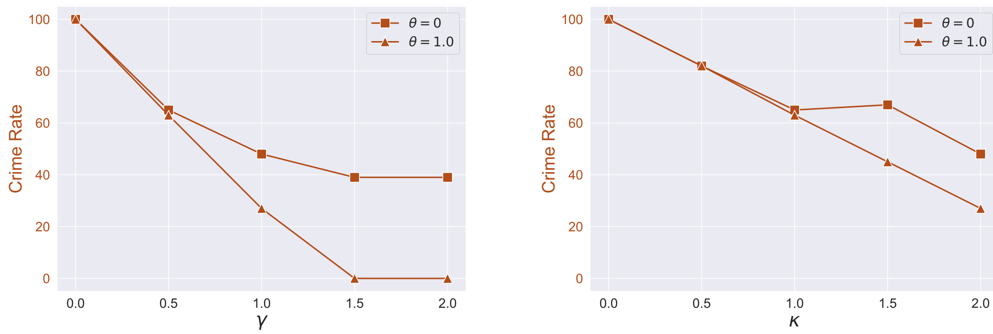

Figure 5: The change of crime rate with the increase of γ and κ.

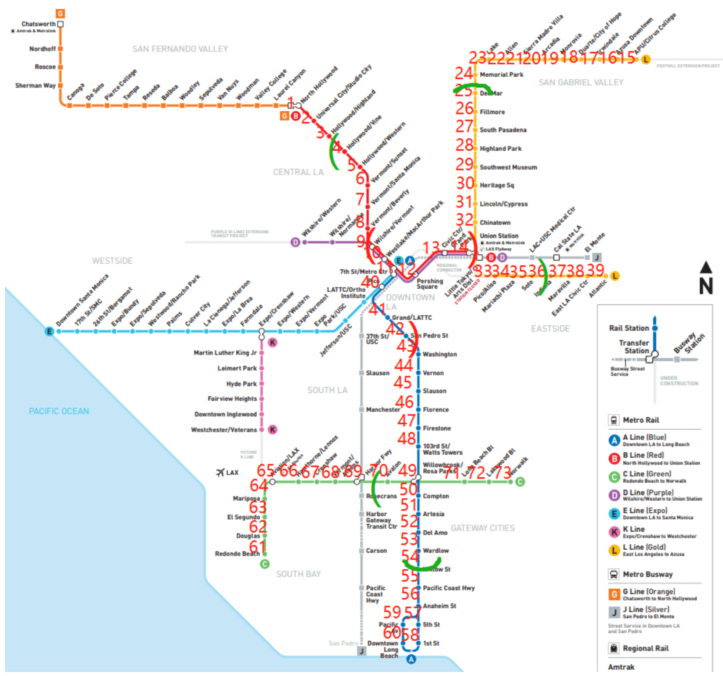

Figure 6: The line map of Los Angeles Metro Rail System.

modeling is more realistic compared with previous work and is valuable for the metro police and transit agencies.

Limitations. First, we use self-made data for testing since we do not have access to the data sheet of the Los Angeles Metro Rail System. Second, since riders and criminals are human players, bounded rationality is a more realistic assumption and can be considered in future work. Last, we do not consider the difficulty of patrol routes, such as the number of transfers. The trick named HDT from previous work [3] can be added to restrict the difficulty of patrol routes.

Work division. This work is completed by Siting Li.

Table 2: Default value and the description of parameters.

Parameters	Default value	Description
ρ	1.5	Ticket price
τ	10	Fine for fare evasion
α	20	Outcome of a successful crime
β	200	Fine for a crime
f_{e_1}	0.1	Patrol efficiency at downtown
f_{e_2}	0.8	Patrol efficiency at suburb area
f_{e_3}	0.5	Patrol efficiency at urban area
γ	1	Total amount of patrol units
κ	100	Length limit of a patrol (not used)
μ	5	Capacity limit of an edge (not used)

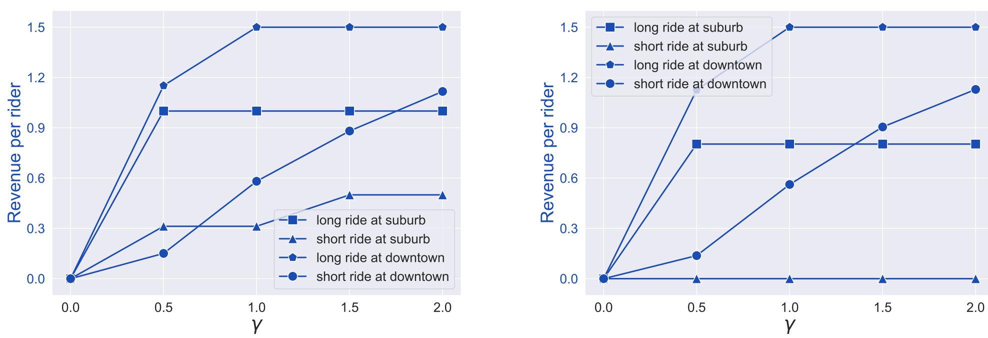

Figure 7: The change of revenue per rider of ride with different types with the increase of γ, the total amount of patrol units. The left figure shows the result of $\theta = 0$, while the right figure is for $\theta = 1$.

References

[1] Vincent Conitzer and Tuomas Sandholm. Computing the optimal strategy to commit to. In *Proceedings of the 7th ACM conference on Electronic commerce*, pages 82–90, 2006.

[2] James Pita, Manish Jain, Janusz Marecki, Fernando Ordóñez, Christopher Portway, Milind Tambe, Craig Western, Praveen Paruchuri, and Sarit Kraus. Deployed armor protection: the application of a game theoretic model for security at the los angeles international airport. In *Proceedings of the 7th international joint conference on Autonomous agents and multiagent systems: industrial track*, pages 125–132, 2008.

[3] Zhengyu Yin, Albert Jiang, Matthew Johnson, Milind Tambe, Christopher Kiekintveld, Kevin Leyton-Brown, Tuomas Sandholm, and John Sullivan. Trusts: Scheduling randomized patrols for fare inspection in transit systems. In *Proceedings of the AAAI Conference on Artificial Intelligence*, volume 26, pages 2348–2355, 2012.

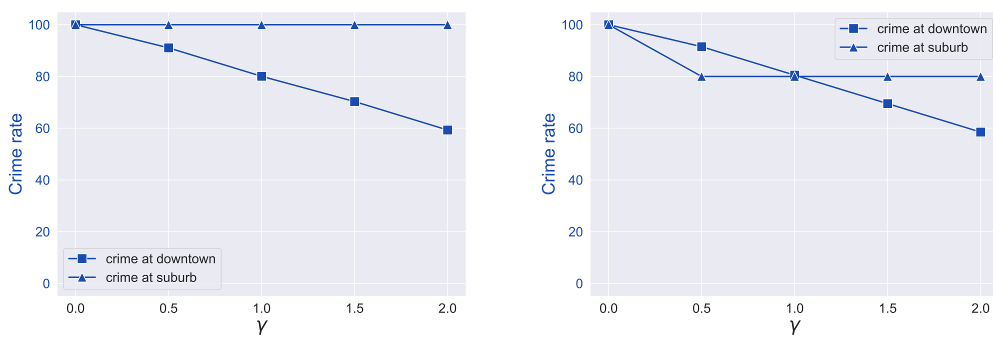

Figure 8: The change of crime rate of criminals in different regions with the increase of γ, the total amount of patrol units. The left figure shows the result of $\theta = 0$, while the right figure is for $\theta = 1$.