HW1 Linear Regression

1/29/2022

3.1

Describe the null hypotheses to which the p-values given in Table 3.4 correspond. Explain what conclusions you can draw based on these p-values. Your explanation should be phrased in terms of sales, TV, radio, and newspaper, rather than in terms of the coefficients of the linear model.

Answer:

H0: all coefficients are equal to zero.

We can see all factors' p values are small except newspaper, which means we can not believe advertising on newspapers has an impact on product sales in specific confidence level, while TV and radio have significant relationship with sales.

3.2

Carefully explain the differences between the KNN classifier and KNN regression methods.

Answer:

KNN classifier estimates the most possible class x_0 will be by detecting K observations closest to x_0, KNN regression estimates the value of response by computing average responses of K observations.

3.5

Consider the fitted values that result from performing linear regres- sion without an intercept. In this setting, the ith fitted value takes the form

$$\hat{y}_i = x_i\hat{\beta},$$

where

$$\hat{\beta} = (\sum_{i=1}^{n} x_i y_i)/(\sum_{i'=1}^{n} x_{i'}^2)$$

Show that we can write

$$\hat{y}_i = \sum_{i'=1}^{n} a_{i'} y_{i'}.$$

what is $a_{i'}$

Note: We interpret this result by saying that the fitted values from linear regression are linear combinations of the response values.

Answer:

$$\hat{y}_i = x_i(\sum_{i=1}^{n} x_i y_i)/(\sum_{i'=1}^{n} x_{i'}^2) = x_i \cdot \sum_{i''=1}^{n}(x_{i''} y_{i''}/\sum_{i'=1}^{n} x_{i'}^2) = \sum_{i''=1}^{n}(x_{i''} x_i/\sum_{i'=1}^{n} x_{i'}^2) y_{i''}$$

so,

$$a_{i'} = (x_{i''}/\sum_{i'=1}^{n} x_{i'}^2) \cdot x_i$$

.

3.6

Using (3.4), argue that in the case of simple linear regression, the least squares line always passes through the point $(\bar{x}, \bar{y})$.

Answer:

From (3.4), we know for

$$\hat{y_i} = \hat{\beta}_0 + \hat{\beta}_1 x$$

,

$$\hat{\beta}_0 = \bar{y} - \hat{\beta}_1 \bar{x}$$

, so when x_i takes $\bar{x}$,

$$\hat{y_i} = \bar{y} - \hat{\beta}_1 \bar{x} + \hat{\beta}_1 x = \bar{y}$$

.

So in the case of simple linear regression, the least squares line always passes through the point $(\bar{x}, \bar{y})$.

3.11

In this problem we will investigate the t-statistic for the null hypothesis H0 : = 0 in simple linear regression without an intercept. To begin, we generate a predictor x and a response y as follows.

```
set.seed(1)
x <- rnorm(100)
y <- 2 * x + rnorm(100)
```

(a)

Perform a simple linear regression of y onto x, without an intercept. Report the coefficient estimate ˆ, the standard error of this coefficient estimate, and the t-statistic and p-value associated with the null hypothesis H0 : = 0. Comment on these results. (You can perform regression without an intercept using the command lm(y x+0).)

```
##
## Call:
## lm(formula = y ~ x + 0)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -1.9154 -0.6472 -0.1771  0.5056  2.3109
##
## Coefficients:
##   Estimate Std. Error t value Pr(>|t|)
## x   1.9939     0.1065   18.73   <2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.9586 on 99 degrees of freedom
## Multiple R-squared:  0.7798, Adjusted R-squared:  0.7776
## F-statistic: 350.7 on 1 and 99 DF,  p-value: < 2.2e-16
```

Answer: The coefficient estimate $\hat{\beta}$ is 1.99, its standard error is 0.1, t value is 18.73, P value is <2e-16, which means this coefficient is significant differ from 0.

(b)

Now perform a simple linear regression of x onto y without an intercept, and report the coefficient estimate, its standard error, and the corresponding t-statistic and p-values associated with the null hypothesis H0 : = 0. Comment on these results.

Answer:

```
##
## Call:
## lm(formula = x ~ y + 0)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -0.8699 -0.2368  0.1030  0.2858  0.8938
##
## Coefficients:
##   Estimate Std. Error t value Pr(>|t|)
## y  0.39111    0.02089   18.73   <2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.4246 on 99 degrees of freedom
## Multiple R-squared:  0.7798, Adjusted R-squared:  0.7776
## F-statistic: 350.7 on 1 and 99 DF,  p-value: < 2.2e-16
```

The coefficient estimate $\hat{\beta}$ is 0.39, its standard error is 0.02, t value is 18.73, P value is <2e-16, which means this coefficient is significant differ from 0.

###(c)

What is the relationship between the results obtained in (a) and (b)?

Answer: Their t value are same, so

$$\beta_x \cdot \sum_{i=1}^{n} x_i^2 = \beta_y \cdot \sum_{i=1}^{n} y_i^2$$

(d)

Answer: From OLS, we know

$$\hat{\beta} = \sum_{i=1}^{n}(x_i - \bar{x})(y_i - \bar{y}) / \sum_{i=1}^{n}(x_i - \bar{x})^2$$

, here $\bar{x}, \bar{y}$ equal to zero.

So

$$t = \hat{\beta}/se(\hat{\beta})$$

,

$$t = (\sum_{i=1}^{n} x_i y_i / \sum_{i=1}^{n} x_i^2) \cdot \frac{\sqrt{(n-1) \cdot \sum_{i=1}^{n} x_i^2}}{\sqrt{\sum_{i=1}^{n}(y_i - x_i \cdot \frac{\sum_{i'=1}^{n} x_{i'} y_{i'}}{\sum_{i'=1}^{n} x_{i'}^2})}}$$

$$t = \frac{\sqrt{n-1} \cdot \sum_{i=1}^{n} x_i y_i}{\sqrt{\sum_{i=1}^{n} x_i^2 \cdot \sum_{i'=1}^{n} y_{i'}^2 - 2(\sum_{i'=1}^{n} x_{i'} y_{i'})^2 + (\sum_{i'=1}^{n} x_{i'} y_{i'})^2}} = \frac{\sqrt{n-1} \cdot \sum_{i=1}^{n} x_i y_i}{\sqrt{\sum_{i=1}^{n} x_i^2 \cdot \sum_{i'=1}^{n} y_{i'}^2 - (\sum_{i'=1}^{n} x_{i'} y_{i'})^2}}$$

```r
t <- (sqrt(100-1)*sum(x*y))/(sqrt(sum(x^2)*sum(y^2)-(sum(x*y))^2))
t
```

```
## [1] 18.72593
```

(e)

Answer: Because x,y have same position in the function from (d), so the t-statistic will be same.

(f)

```r
t_x <- (sqrt(100-1)*sum(x*y))/(sqrt(sum(x^2)*sum(y^2)-(sum(x*y))^2))
t_y <- (sqrt(100-1)*sum(x*y))/(sqrt(sum(y^2)*sum(x^2)-(sum(x*y))^2))
t_x == t_y
```

Answer:

```
## [1] TRUE
```

3.12

(a)

Recall that the coefficient estimate ˆ for the linear regression of Y onto X without an intercept is given by (3.38). Under what circumstance is the coefficient estimate for the regression of X onto Y the same as the coefficient estimate for the regression of Y onto X?

Answer:

$$\sum_{i=1}^{n} x_i^2 = \sum_{i=1}^{n} y_i^2$$

(b)

Generate an example in R with n = 100 observations in which the coefficient estimate for the regression of X onto Y is different from the coefficient estimate for the regression of Y onto X.

```r
set.seed(2022)
x <- rnorm(100,0,1)
y <- rnorm(100,0,2)
summary(lm(y~x+0))
```

Answer:

```
##
## Call:
## lm(formula = y ~ x + 0)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -3.9629 -2.1556 -0.4842  1.2437  3.7254
##
## Coefficients:
##   Estimate Std. Error t value Pr(>|t|)
## x  0.05827    0.20242   0.288    0.774
##
```

```
## Residual standard error: 2.074 on 99 degrees of freedom
## Multiple R-squared:  0.0008364,  Adjusted R-squared:  -0.009256
## F-statistic: 0.08287 on 1 and 99 DF,  p-value: 0.774
```

```
summary(lm(x~y+0))
```

```
##
## Call:
## lm(formula = x ~ y + 0)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -2.9319 -0.4335  0.2018  0.8647  2.9047
##
## Coefficients:
##   Estimate Std. Error t value Pr(>|t|)
## y  0.01435    0.04986   0.288    0.774
##
## Residual standard error: 1.029 on 99 degrees of freedom
## Multiple R-squared:  0.0008364,  Adjusted R-squared:  -0.009256
## F-statistic: 0.08287 on 1 and 99 DF,  p-value: 0.774
```

(c)

Generate an example in R with n = 100 observations in which the coefficient estimate for the regression of X onto Y is the same as the coefficient estimate for the regression of Y onto X.

```
set.seed(2022)
x <- rnorm(100,0,1)
y <- rnorm(100,0,1)
summary(lm(y~x+0))
```

Answer:

```
##
## Call:
## lm(formula = y ~ x + 0)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -1.9814 -1.0778 -0.2421  0.6218  1.8627
##
## Coefficients:
##   Estimate Std. Error t value Pr(>|t|)
## x  0.02914    0.10121   0.288    0.774
##
## Residual standard error: 1.037 on 99 degrees of freedom
## Multiple R-squared:  0.0008364,  Adjusted R-squared:  -0.009256
## F-statistic: 0.08287 on 1 and 99 DF,  p-value: 0.774
```

```
summary(lm(x~y+0))
```

```
##
## Call:
## lm(formula = x ~ y + 0)
```

```
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -2.9319 -0.4335  0.2018  0.8647  2.9047
##
## Coefficients:
##   Estimate Std. Error t value Pr(>|t|)
## y  0.02871    0.09972   0.288    0.774
##
## Residual standard error: 1.029 on 99 degrees of freedom
## Multiple R-squared:  0.0008364,  Adjusted R-squared:  -0.009256
## F-statistic: 0.08287 on 1 and 99 DF,  p-value: 0.774
```

3.13

(a)

```r
set.seed(1)
x <- rnorm(100,0,1)
```

(b)

```r
set.seed(100)
eps <- rnorm(100,0,0.5)
```

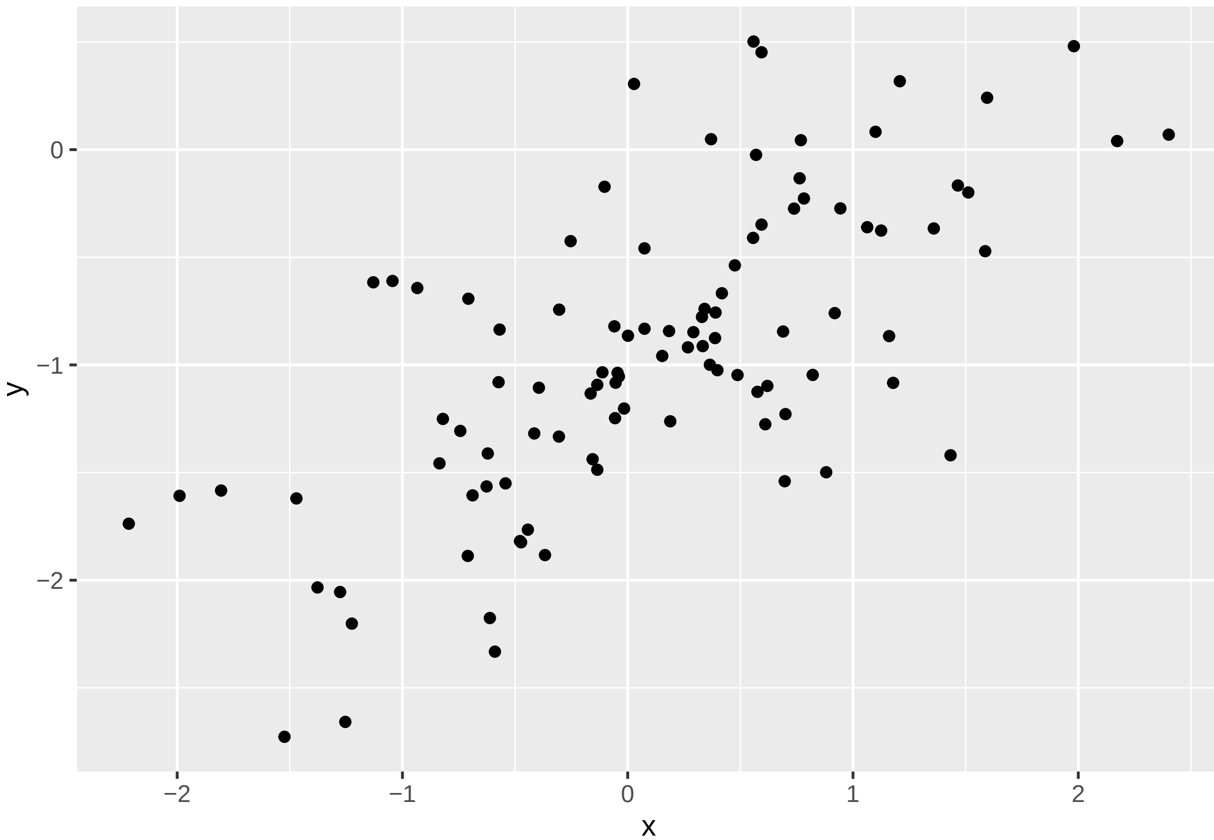
(c)

```r
y <- -1+0.5*x+eps
```

The length od y is 100, the β_0 is -1, β_1 is 0.5.

(d)

```r
ggplot(data.frame(x,y),aes(x=x,y=y))+
  geom_point(position = "jitter")
```



(e)

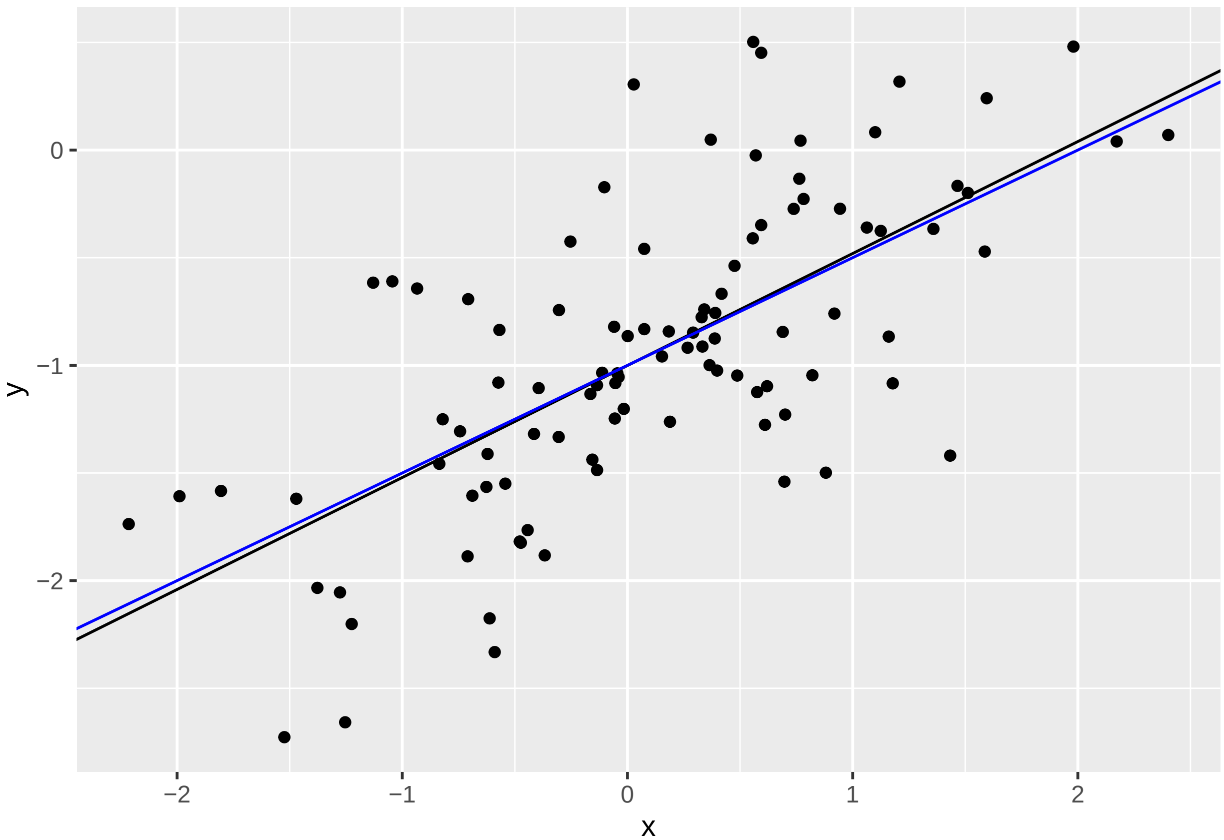
```r
lm_12 <- lm(y~x)
summary(lm_12)
```

```
##
## Call:
## lm(formula = y ~ x)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -1.16410 -0.30412 -0.02171  0.34324  1.29115
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept) -1.00074    0.05164 -19.378  < 2e-16 ***
## x            0.52015    0.05736   9.068 1.27e-14 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.5126 on 98 degrees of freedom
## Multiple R-squared:  0.4562, Adjusted R-squared:  0.4507
## F-statistic: 82.23 on 1 and 98 DF,  p-value: 1.274e-14
```

$\hat{\beta}_0$ is -1, $\hat{\beta}_1$ is 0.52, they are very close to real β_0 and β_1.

(f)

```
ggplot(data.frame(x,y),aes(x=x,y=y))+
  geom_point(position = "jitter")+
  geom_abline(intercept = coef(lm_12)[1], slope = coef(lm_12)[2])+
  geom_abline(intercept = -1, slope = 0.5, color="blue")
```




```
#legend(x=-1,legend=c("least sqares line", "population regression line"),col=c("black", "blue"))
```

(g)

```
x2 <- x^2
lm_12pol <- lm(y~x+x2)
summary(lm_12pol)
```

```
##
## Call:
## lm(formula = y ~ x + x2)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -1.16255 -0.30433 -0.02225  0.34264  1.28997
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept) -0.999556   0.063279 -15.796  < 2e-16 ***
## x            0.520375   0.058077   8.960 2.36e-14 ***
```

```
## x2          -0.001488   0.045590  -0.033    0.974
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.5153 on 97 degrees of freedom
## Multiple R-squared:  0.4563, Adjusted R-squared:  0.445
## F-statistic:  40.7 on 2 and 97 DF,  p-value: 1.468e-13
```

The R-squared is 0.445, the original regression's R-squared is 0.45, so the quadratic term does not improve the model fit.

(h)

```r
set.seed(2)
xl <- rnorm(100,0,1)
epsl <- rnorm(100,0,0.01)
yl <- -1+0.5*xl+epsl

lm_12l <- lm(yl~xl)
summary(lm_12l)
```

```
##
## Call:
## lm(formula = yl ~ xl)
##
## Residuals:
##       Min        1Q    Median        3Q       Max
## -0.021496 -0.008196  0.001370  0.007195  0.020560
##
## Coefficients:
##                Estimate Std. Error t value Pr(>|t|)
## (Intercept) -0.9997223  0.0009892 -1010.7   <2e-16 ***
## xl           0.4995290  0.0008566   583.2   <2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.009888 on 98 degrees of freedom
## Multiple R-squared:  0.9997, Adjusted R-squared:  0.9997
## F-statistic: 3.401e+05 on 1 and 98 DF,  p-value: < 2.2e-16
```

```r
x2l <- xl^2
lm_12poll <- lm(yl~xl+x2l)
summary(lm_12poll)
```

```
##
## Call:
## lm(formula = yl ~ xl + x2l)
##
## Residuals:
##        Min         1Q     Median         3Q        Max
## -0.0209221 -0.0078648  0.0009461  0.0075301  0.0210861
##
## Coefficients:
##                Estimate Std. Error  t value Pr(>|t|)
## (Intercept) -1.0004359  0.0013351 -749.315   <2e-16 ***
```

```
## xl           0.4994929  0.0008594  581.223   <2e-16 ***
## x2l          0.0005343  0.0006699    0.798    0.427
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.009907 on 97 degrees of freedom
## Multiple R-squared:  0.9997, Adjusted R-squared:  0.9997
## F-statistic: 1.694e+05 on 2 and 97 DF,  p-value: < 2.2e-16
```

When decreasing the noise, the R square in two regressions are improved a lot.

(i)

```r
set.seed(3)
xh <- rnorm(100,0,1)
epsh <- rnorm(100,0,1)
yh <- -1+0.5*xh+epsh

lm_12h <- lm(yh~xh)
summary(lm_12h)
```

```
##
## Call:
## lm(formula = yh ~ xh)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -2.35998 -0.81570  0.03617  0.61823  2.62146
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept)  -0.9801     0.1102  -8.897 2.99e-14 ***
## xh            0.4098     0.1293   3.169  0.00204 **
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 1.101 on 98 degrees of freedom
## Multiple R-squared:  0.09297,    Adjusted R-squared:  0.08371
## F-statistic: 10.04 on 1 and 98 DF,  p-value: 0.002038
```

```r
x2h <- xh^2
lm_12polh <- lm(yh~xh+x2h)
summary(lm_12polh)
```

```
##
## Call:
## lm(formula = yh ~ xh + x2h)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -2.34685 -0.82511  0.01994  0.64897  2.66907
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept) -1.03699    0.14810  -7.002 3.31e-10 ***
```

```
## xh           0.41664    0.13029   3.198  0.00187 **
## x2h          0.07834    0.13567   0.577  0.56497
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 1.105 on 97 degrees of freedom
## Multiple R-squared:  0.09608,    Adjusted R-squared:  0.07744
## F-statistic: 5.155 on 2 and 97 DF,  p-value: 0.007453
```

When increasing the noise, the R square in two regressions are decreased a lot, regressions become worse.

(j)

```
confint(lm_12)
```

```
##                  2.5 %     97.5 %
## (Intercept) -1.1032195 -0.8982555
## x            0.4063173  0.6339777
```

```
confint(lm_12l)
```

```
##                  2.5 %     97.5 %
## (Intercept) -1.0016853 -0.9977593
## xl           0.4978291  0.5012289
```

```
confint(lm_12h)
```

```
##                  2.5 %     97.5 %
## (Intercept) -1.1986672 -0.7614723
## xh           0.1532218  0.6664473
```

The confidence interval are smaller in lower noise model, higher in higher noise model. So data with less noise can be estimated more precisely because it has less uncertainty.

3.14

(a)

```
set.seed(1)
x1 <- runif(100)
x2 <- 0.5 * x1 + rnorm(100) / 10
y <- 2 + 2 * x1 + 0.3 * x2 + rnorm(100)
```

$$y = 2 + 2x_1 + 0.3x_2 + \epsilon$$

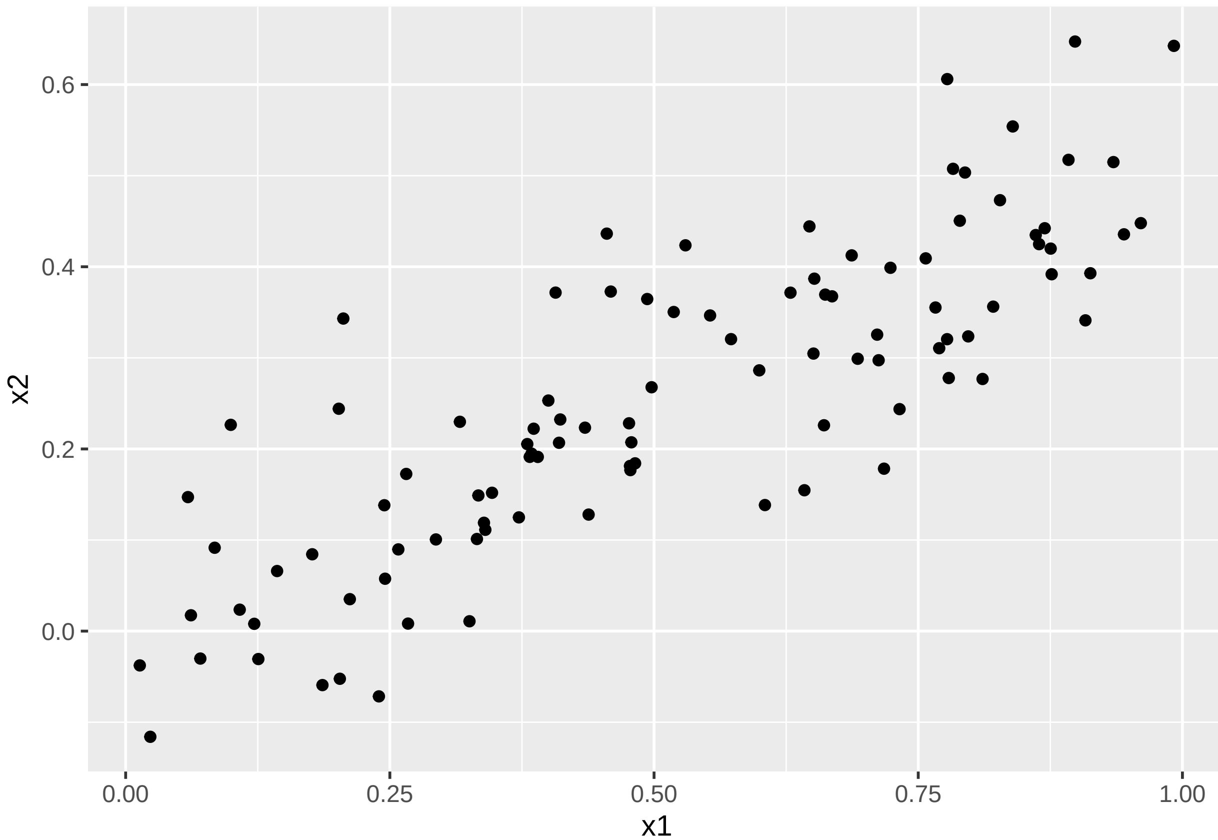
, the β_0 is 2, β_1 is 2, β_2 is 0.3.

(b)

```
cor(x1,x2)
```

```
## [1] 0.8351212
```

```
ggplot(data.frame(x1,x2),aes(x=x1,y=x2))+
  geom_point()
```



The correlation between x1 and x2 is 0.835.

(c)

```
lm_13 <- lm(y~x1+x2)
summary(lm_13)
```

```
##
## Call:
## lm(formula = y ~ x1 + x2)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -2.8311 -0.7273 -0.0537  0.6338  2.3359
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept)   2.1305     0.2319   9.188 7.61e-15 ***
## x1            1.4396     0.7212   1.996   0.0487 *
## x2            1.0097     1.1337   0.891   0.3754
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 1.056 on 97 degrees of freedom
## Multiple R-squared:  0.2088, Adjusted R-squared:  0.1925
## F-statistic:  12.8 on 2 and 97 DF,  p-value: 1.164e-05
```

The estimated $\beta_0, \beta_1, \beta_2$ are 2.13, 1.44, 1. They are very different from real coefficients.

The p value of β_1 is low, which means the estimate of β_1 is significant, so we can reject null hypothesis H0 : 1 = 0, but p value shows the estimate of β_2 is not significant, so we can not reject null hypothesis H0 : 2 = 0.

(d)

```
lm_131 <- lm(y~x1)
summary(lm_131)
```

```
##
## Call:
## lm(formula = y ~ x1)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -2.89495 -0.66874 -0.07785  0.59221  2.45560
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept)   2.1124     0.2307   9.155 8.27e-15 ***
## x1            1.9759     0.3963   4.986 2.66e-06 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 1.055 on 98 degrees of freedom
## Multiple R-squared:  0.2024, Adjusted R-squared:  0.1942
## F-statistic: 24.86 on 1 and 98 DF,  p-value: 2.661e-06
```

We can reject null hypothesis H0 : 1 =0 because p value of ß1 is small.

(e)

```
lm_132 <- lm(y~x2)
summary(lm_132)
```

```
##
## Call:
## lm(formula = y ~ x2)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -2.62687 -0.75156 -0.03598  0.72383  2.44890
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept)   2.3899     0.1949   12.26  < 2e-16 ***
## x2            2.8996     0.6330    4.58 1.37e-05 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 1.072 on 98 degrees of freedom
## Multiple R-squared:  0.1763, Adjusted R-squared:  0.1679
## F-statistic: 20.98 on 1 and 98 DF,  p-value: 1.366e-05
```

We can reject null hypothesis H0 : 2 =0 because p value of ß2 is small.

(f)

The results from (d) and (e) are not contradict, because x2 is generated from x1, it has all information from x1, so x2 can also interpret y as what x1 does.

(g)

```r
x1 <- c(x1, 0.1)
x2 <- c(x2, 0.8)
y <- c(y, 6)
```

```r
lm_13g <- lm(y~x1+x2)
summary(lm_13g)
```

```
##
## Call:
## lm(formula = y ~ x1 + x2)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -2.73348 -0.69318 -0.05263  0.66385  2.30619
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept)   2.2267     0.2314   9.624 7.91e-16 ***
## x1            0.5394     0.5922   0.911  0.36458
## x2            2.5146     0.8977   2.801  0.00614 **
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 1.075 on 98 degrees of freedom
## Multiple R-squared:  0.2188, Adjusted R-squared:  0.2029
## F-statistic: 13.72 on 2 and 98 DF,  p-value: 5.564e-06
```

```r
lm_131g <- lm(y~x1)
summary(lm_131g)
```

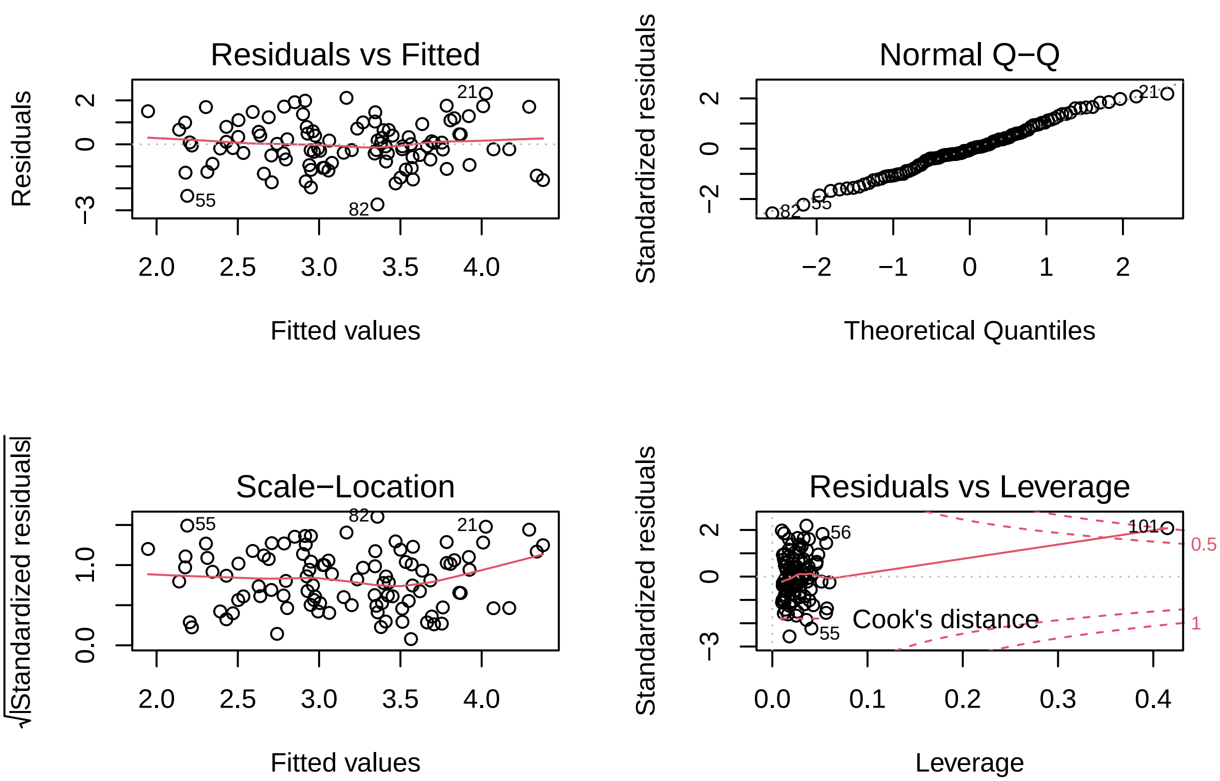
```
##
## Call:
## lm(formula = y ~ x1)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -2.8897 -0.6556 -0.0909  0.5682  3.5665
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept)   2.2569     0.2390   9.445 1.78e-15 ***
## x1            1.7657     0.4124   4.282 4.29e-05 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 1.111 on 99 degrees of freedom
## Multiple R-squared:  0.1562, Adjusted R-squared:  0.1477
## F-statistic: 18.33 on 1 and 99 DF,  p-value: 4.295e-05
```

```r
lm_132g <- lm(y~x2)
summary(lm_132g)
```

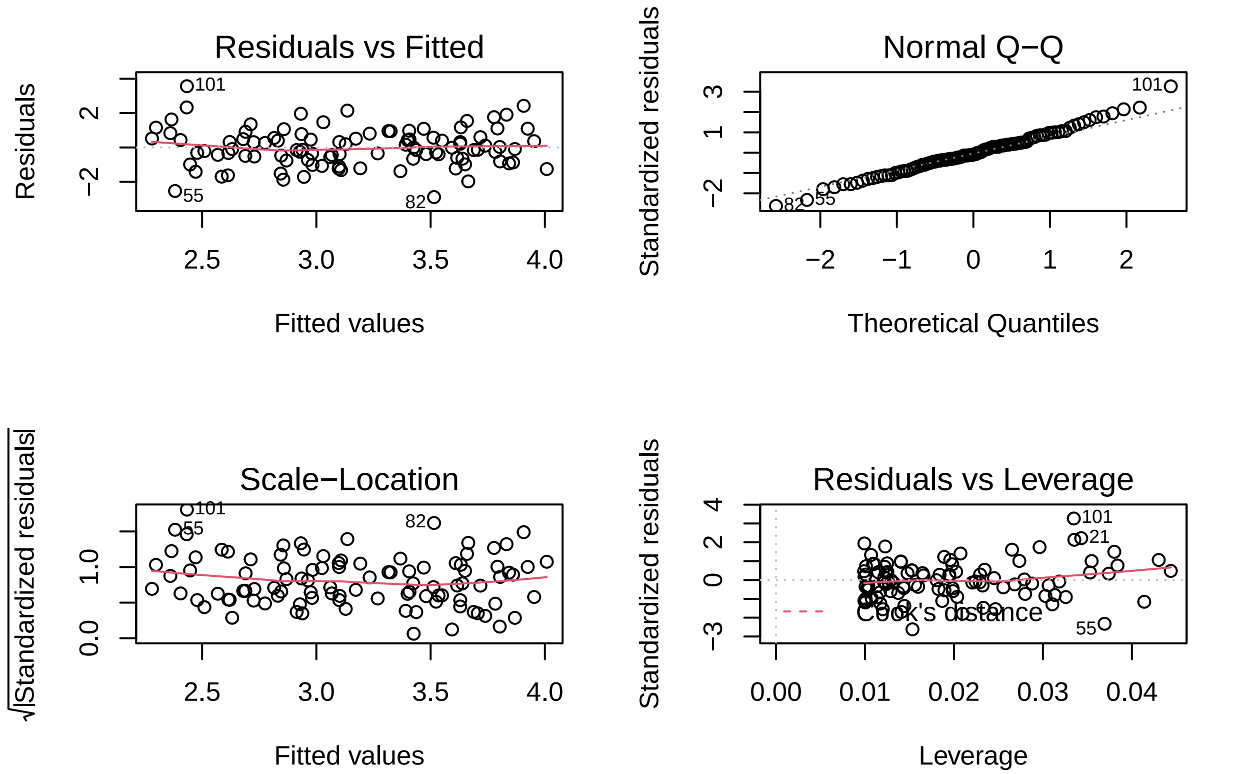
```
##
## Call:
## lm(formula = y ~ x2)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -2.64729 -0.71021 -0.06899  0.72699  2.38074
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept)   2.3451     0.1912  12.264  < 2e-16 ***
## x2            3.1190     0.6040   5.164 1.25e-06 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 1.074 on 99 degrees of freedom
## Multiple R-squared:  0.2122, Adjusted R-squared:  0.2042
## F-statistic: 26.66 on 1 and 99 DF,  p-value: 1.253e-06
```

The coefficient estimates are not changed a lot, so the new observation has little influence on regressions.

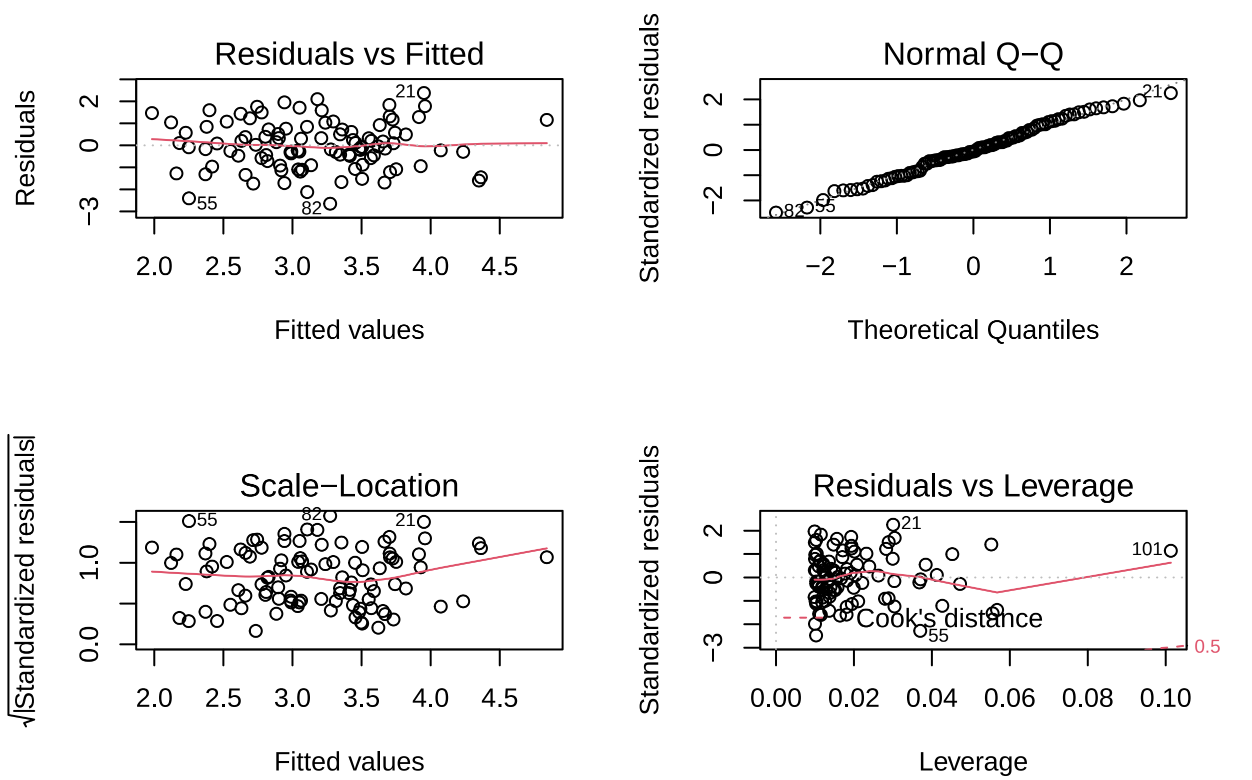
```r
par(mfrow=c(2,2))
plot(lm_13g)
```




```r
par(mfrow=c(2,2))
plot(lm_131g)
```




```r
par(mfrow=c(2,2))
plot(lm_132g)
```



The new observation is a high leverage point for three regressions, but it is not an outlier for three regressions except y~x1.