

HW5_Tree_Shuting_Li

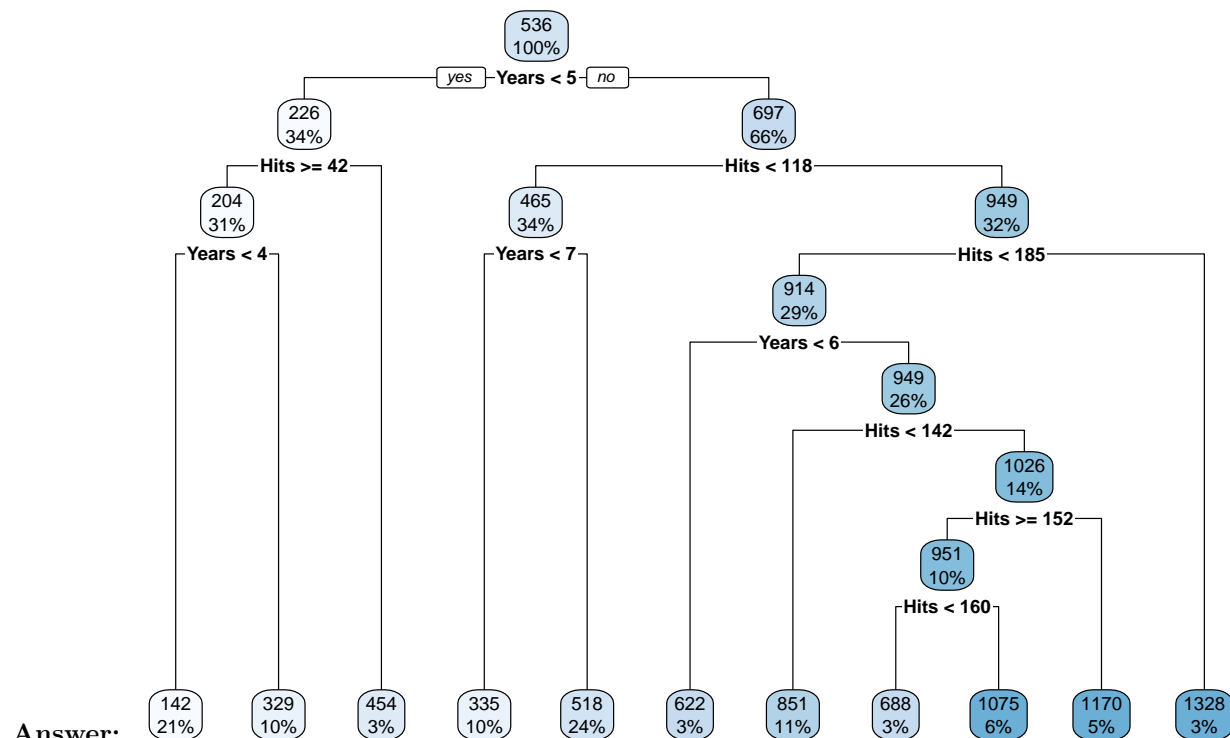
Shuting

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8.1

Draw an example (of your own invention) of a partition of two-dimensional feature space that could result from recursive binary splitting. Your example should contain at least six regions. Draw a decision tree corresponding to this partition. Be sure to label all aspects of your figures, including the regions R1, R2, . . ., the cutpoints t_1, t_2, \dots , and so forth.

```
attach(Hitters)
library(rpart)
library(rpart.plot)
fit <- rpart(Salary~Years+Hits, data=Hitters, method='anova')
rpart.plot(fit)
```



Answer:

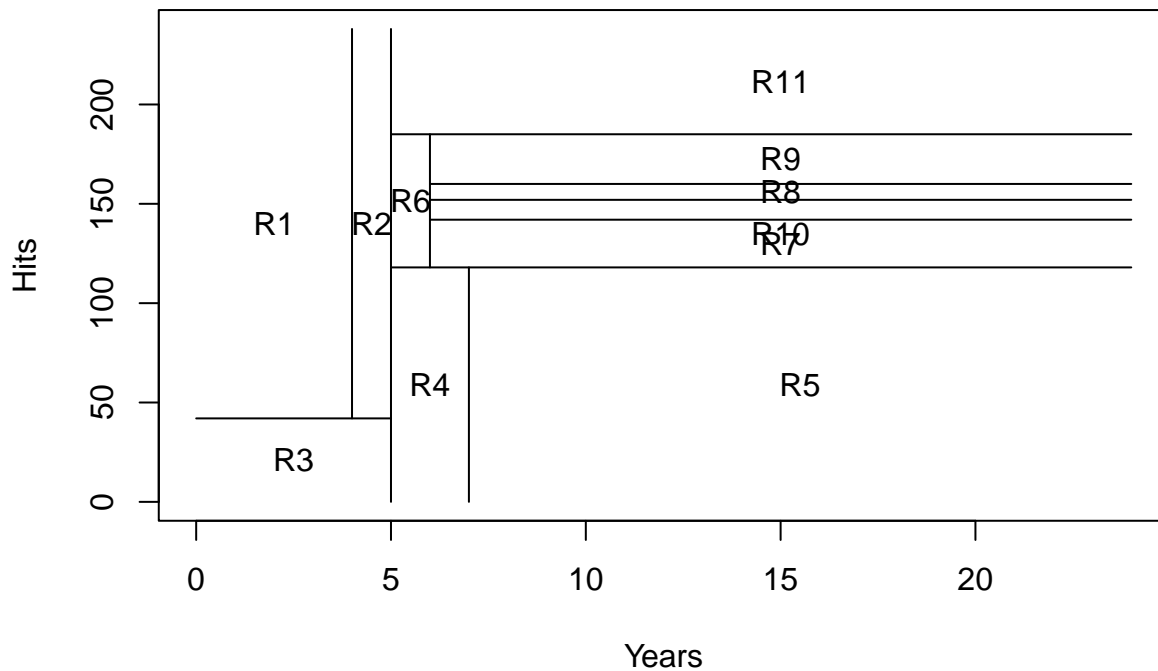
```
#####
#par(xpd = NA)
plot(NA, NA, type = "n", xlim = c(0, 24), ylim = c(0, 238), xlab = "Years", ylab = "Hits")
# t1: x = 5
lines(x = c(5, 5), y = c(0, 238))
```

```

# t2: y = 42
lines(x = c(0, 5), y = c(42, 42))
# t3: x = 4
lines(x = c(4, 4), y = c(42, 238))
# t4: y = 118
lines(x = c(5, 24), y = c(118, 118))
# t5: x = 7
lines(x = c(7, 7), y = c(0, 118))
# t6: y = 185
lines(x = c(5, 24), y = c(185, 185))
# t7: x = 6
lines(x = c(6, 6), y = c(118, 185))
# t8: y = 142
lines(x = c(6, 24), y = c(142, 142))
# t9: y = 152
lines(x = c(6, 24), y = c(152, 152))
# t8: y = 160
lines(x = c(6, 24), y = c(160, 160))

text(x = 2, y = (42 + 238)/2, labels = c("R1"))
text(x = 4.5, y = (42 + 238)/2, labels = c("R2"))
text(x = (0 + 5)/2, y = (42 + 0)/2, labels = c("R3"))
text(x = (5 + 7)/2, y = 118/2, labels = c("R4"))
text(x = (24 + 7)/2, y = 118/2, labels = c("R5"))
text(x = (5 + 6)/2, y = (118+185)/2, labels = c("R6"))
text(x = (24 + 6)/2, y = (118+142)/2, labels = c("R7"))
text(x = (24 + 6)/2, y = (152+160)/2, labels = c("R8"))
text(x = (24 + 6)/2, y = (160+185)/2, labels = c("R9"))
text(x = (24 + 6)/2, y = (118+152)/2, labels = c("R10"))
text(x = (24 + 6)/2, y = (185+238)/2, labels = c("R11"))

```

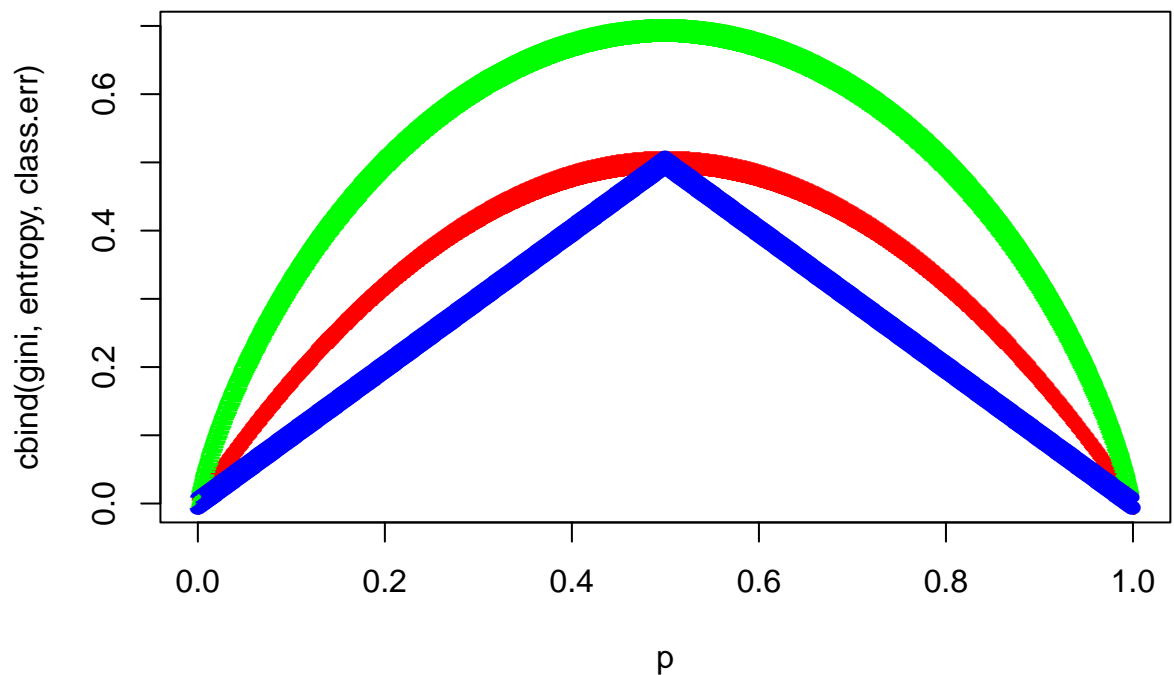


8.2

Answer: For the first stump $f_1(X)$, we denote it as $f_1(X) = \beta_1 * I(X_1 < t_1) + \beta_0$, Then calculate residual $r_1 = Y - \lambda f_1(X)$ and fit the second decision $f_2(X)$. Then model is $f(X) = f_1(X) + f_2(X)$. Repeat the stage for p times and get the final model $f(X) = f_1(X) + f_2(X) + \dots + f_p(X)$.

8.3

```
p = seq(0, 1, 0.001)
gini = p * (1 - p) * 2
entropy = -(p * log(p) + (1 - p) * log(1 - p))
class.err = 1 - pmax(p, 1 - p)
matplot(p, cbind(gini, entropy, class.err), col = c("red", "green", "blue"))
```



Answer:

8.5

```
p = c(0.1, 0.15, 0.2, 0.2, 0.55, 0.6, 0.6, 0.65, 0.7, 0.75)
mean(p)
```

Answer:

```
## [1] 0.45
```

For majority approach, the final class is red. For average approach, the final class is green.

8.7

In the lab, we applied random forests to the Boston data using $mtry = 6$ and using $ntree = 25$ and $ntree = 500$. Create a plot displaying the test error resulting from random forests on this data set for a more comprehensive range of values for $mtry$ and $ntree$. You can model your plot after Figure 8.10. Describe the results obtained.

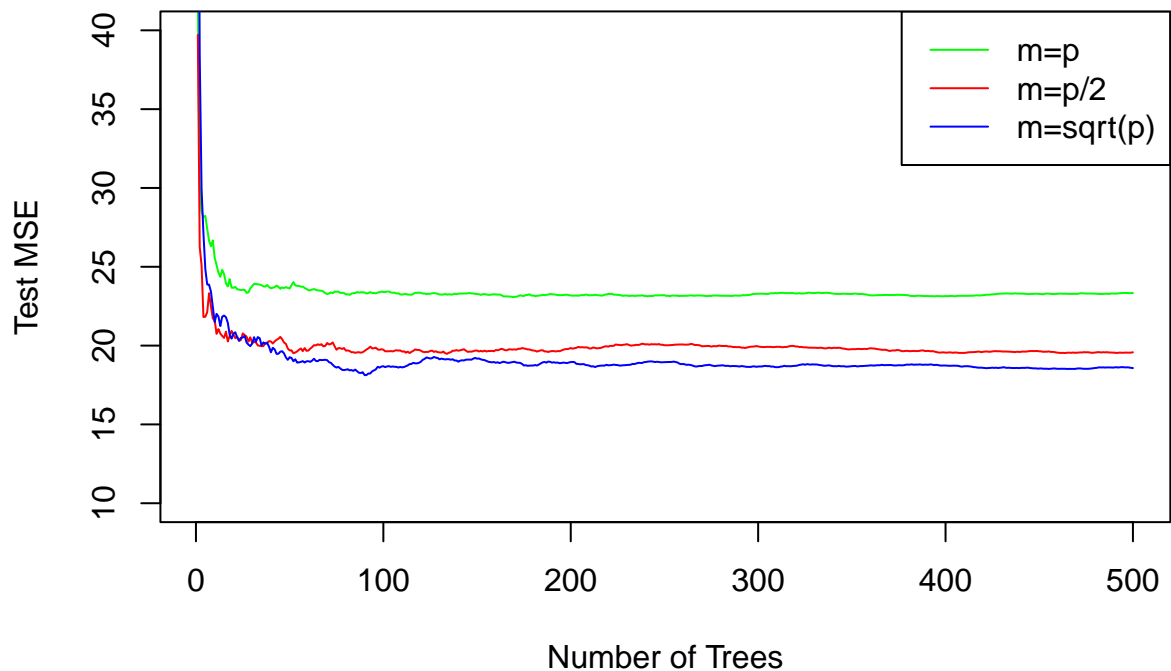
```

set.seed(1)
train = sample(dim(Boston)[1], dim(Boston)[1]/2)
X.train = Boston[train, -14]
X.test = Boston[-train, -14]
Y.train = Boston[train, 14]
Y.test = Boston[-train, 14]

p = dim(Boston)[2] - 1
rf.boston.p = randomForest(X.train, Y.train, xtest = X.test, ytest = Y.test,
  mtry = p, ntree = 500)
rf.boston.p.2 = randomForest(X.train, Y.train, xtest = X.test, ytest = Y.test,
  mtry = p/2, ntree = 500)
rf.boston.p.sq = randomForest(X.train, Y.train, xtest = X.test, ytest = Y.test,
  mtry = sqrt(p), ntree = 500)

plot(1:500, rf.boston.p$test$mse, col = "green", type = "l", xlab = "Number of Trees",
  ylab = "Test MSE", ylim = c(10, 40))
lines(1:500, rf.boston.p.2$test$mse, col = "red", type = "l")
lines(1:500, rf.boston.p.sq$test$mse, col = "blue", type = "l")
legend("topright", c("m=p", "m=p/2", "m=sqrt(p)"), col = c("green", "red", "blue"),
  cex = 1, lty = 1)

```



Answer:

When choosing sqrt-number of variables to construct random forest, model's testing error has good performance.

8.8

In the lab, a classification tree was applied to the Carseats data set after converting Sales into a qualitative response variable. Now we will seek to predict Sales using regression trees and related approaches, treating the response as a quantitative variable.

(a)

Split the data set into a training set and a test set.

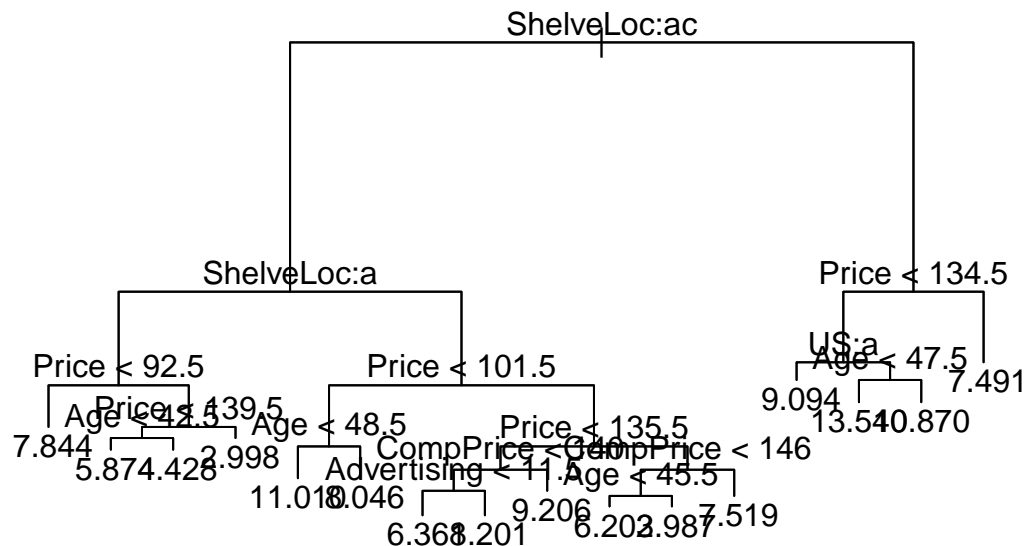
```
attach(Carseats)
set.seed(3)
train = sample(dim(Carseats)[1], dim(Carseats)[1]/2)
Carseats.train = Carseats[train, ]
Carseats.test = Carseats[-train, ]
```

Answer:

(b)

Fit a regression tree to the training set. Plot the tree, and interpret the results. What test MSE do you obtain?

```
library(tree)
fit.tree = tree(Sales ~ ., data = Carseats.train)
plot(fit.tree)
text(fit.tree)
```



Answer:

```
fit.pred = predict(fit.tree, Carseats.test)
testMSE = mean((Carseats.test$Sales - fit.pred)^2)
testMSE
```

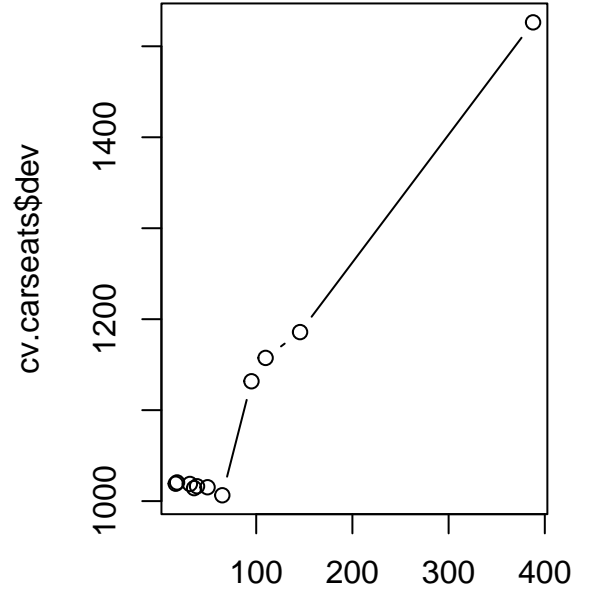
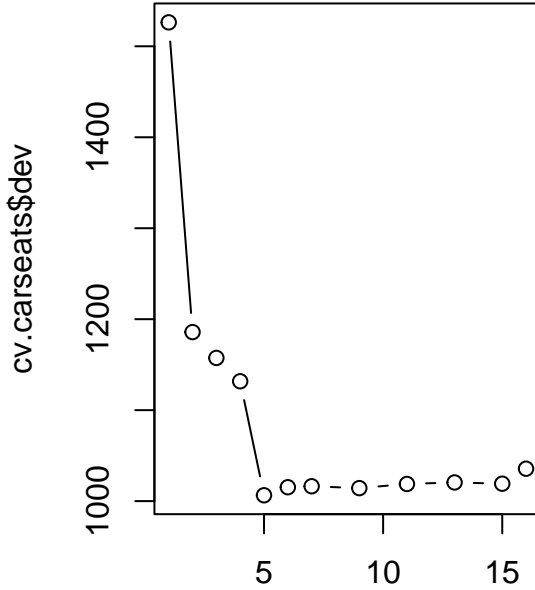
```
## [1] 4.784151
```

The test MSE is 4.78.

(c)

Use cross-validation in order to determine the optimal level of tree complexity. Does pruning the tree improve the test MSE?

```
cv.carseats = cv.tree(fit.tree, FUN = prune.tree)
par(mfrow = c(1, 2))
plot(cv.carseats$size, cv.carseats$dev, type = "b")
plot(cv.carseats$k, cv.carseats$dev, type = "b")
```



Answer:

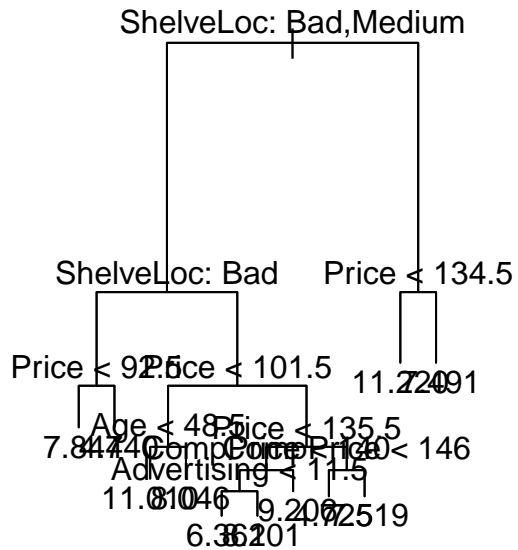
cv.carseats\$size

cv.carseats\$k

```
pruned.carseats = prune.tree(fit.tree, best = 11)
plot(pruned.carseats)
text(pruned.carseats, pretty = 0)

pred.pruned = predict(pruned.carseats, Carseats.test)
MSE.pruned = mean((Carseats.test$Sales - pred.pruned)^2)
MSE.pruned
```

```
## [1] 4.636068
```



After pruning, the test MSE is 4.63.

(d)

Use the bagging approach in order to analyze this data. What test MSE do you obtain? Use the importance() function to determine which variables are most important.

```
bag.carseats = randomForest(Sales ~ ., data = Carseats.train, mtry = 10, ntree = 500,
                             importance = T)
bag.pred = predict(bag.carseats, Carseats.test)
(MSE.bag = mean((Carseats.test$Sales - bag.pred)^2))
```

Answer:

```
## [1] 2.758793
```

```
importance(bag.carseats)
```

##		%IncMSE	IncNodePurity
##	CompPrice	20.724998	130.421567
##	Income	2.616103	66.153373
##	Advertising	14.214948	121.847519
##	Population	-1.433690	58.523885
##	Price	50.544432	408.079671
##	ShelveLoc	57.131042	495.464946
##	Age	14.442394	118.497755
##	Education	-1.849036	38.905898
##	Urban	-3.593040	7.957335
##	US	5.850580	11.115617

Test MSE is 2.82. The most important variables are Price, ShelveLoc.

(e)

Use random forests to analyze this data. What test MSE do you obtain? Use the importance() function to determine which variables are most important. Describe the effect of m, the number of variables considered at each split, on the error rate obtained.

```
rf.carseats = randomForest(Sales ~ ., data = Carseats.train, mtry = 5, ntree = 500,
                             importance = T)
rf.pred = predict(rf.carseats, Carseats.test)
(MSE.rf = mean((Carseats.test$Sales - rf.pred)^2))
```

Answer:

```
## [1] 2.944176
```

```
importance(rf.carseats)
```

##		%IncMSE	IncNodePurity
##	CompPrice	13.00451010	127.846808
##	Income	1.65976881	88.871227
##	Advertising	14.34590005	155.997084
##	Population	0.11699701	79.608572
##	Price	40.75107160	365.768662
##	ShelveLoc	45.93594836	423.846648
##	Age	12.31028105	128.337651
##	Education	-0.03831749	47.621583

```
## Urban      -0.14685440      9.706616
## US         6.44072844      25.417226
```

Test MSE is 2.93. The most important variables are Price, ShelfLoc.

(f)

Now analyze the data using BART, and report your results.

```
xtrain <- Carseats.train[,-1]
ytrain <- Carseats.train[,1]
xtest  <- Carseats.test[,-1]
ytest  <- Carseats.test[,1]
set.seed(3)
bart.carseats <- gbart(xtrain, ytrain, x.test = xtest)
```

Answer:

```
## *****Calling gbart: type=1
## *****Data:
## data:n,p,np: 200, 14, 200
## y1,yn: 0.331550, 0.851550
## x1,x[n*p]: 129.000000, 1.000000
## xp1,xp[np*p]: 138.000000, 1.000000
## *****Number of Trees: 200
## *****Number of Cut Points: 63 ... 1
## *****burn,nd,thin: 100,1000,1
## *****Prior:beta,alpha,tau,nu,lambda,offset: 2,0.95,0.281075,3,0.197675,7.33845
## *****sigma: 1.007374
## *****w (weights): 1.000000 ... 1.000000
## *****Dirichlet:sparse,theta,omega,a,b,rho,augment: 0,0,1,0.5,1,14,0
## *****printevery: 100
##
## MCMC
## done 0 (out of 1100)
## done 100 (out of 1100)
## done 200 (out of 1100)
## done 300 (out of 1100)
## done 400 (out of 1100)
## done 500 (out of 1100)
## done 600 (out of 1100)
## done 700 (out of 1100)
## done 800 (out of 1100)
## done 900 (out of 1100)
## done 1000 (out of 1100)
## time: 4s
## trcnt,tecnt: 1000,1000

yhat.bart <- bart.carseats$yhat.test.mean
(MSE.bart <- mean((ytest - yhat.bart)^2))

## [1] 1.649891

ord <- order(bart.carseats$varcount.mean, decreasing = T)
bart.carseats$varcount.mean[ord]
```



```
##      Price    CompPrice  ShelfLoc1      US2      Urban2  ShelfLoc3
##      23.428      18.811      17.477      17.381      17.153      16.617
##      Income  ShelfLoc2      Age      US1      Urban1  Population
##      16.466      16.441      16.227      15.989      15.947      15.255
## Education Advertising
##      14.968      14.753
```

Tset MSE is 1.59. The most important variables are Price, CompPrice.

8.11

(a)

Create a training set consisting of the first 1,000 observations, and a test set consisting of the remaining observations.

```
train = 1:1000
Caravan$Purchase = ifelse(Caravan$Purchase == "Yes", 1, 0)
Caravan.train = Caravan[train, ]
Caravan.test = Caravan[-train, ]
```

Answer:

(b)

Fit a boosting model to the training set with Purchase as the response and the other variables as predictors. Use 1,000 trees, and a shrinkage value of 0.01. Which predictors appear to be the most important?

Answer: PPERSON and MKOOPKLA are most important variables.

(c)

Use the boosting model to predict the response on the test data. Predict that a person will make a purchase if the estimated probability of purchase is greater than 20 %. Form a confusion matrix. What fraction of the people predicted to make a purchase do in fact make one? How does this compare with the results obtained from applying KNN or logistic regression to this data set?

```
boost.prob = predict(boost.caravan, Caravan.test, n.trees = 1000, type = "response")
boost.pred = ifelse(boost.prob > 0.2, 1, 0)
table(Caravan.test$Purchase, boost.pred)
```

Answer:

```
##      boost.pred
##      0      1
## 0 4410  123
## 1   256   33
```

```
33/(123+33)
```

```
## [1] 0.2115385
```

```
####
```

```
lm.caravan = glm(Purchase ~ ., data = Caravan.train, family = binomial)
lm.prob = predict(lm.caravan, Caravan.test, type = "response")
```

```
lm.pred = ifelse(lm.prob > 0.2, 1, 0)
table(Caravan.test$Purchase, lm.pred)
```

```
##      lm.pred
##           0      1
##    0 4183  350
##    1  231   58
```

```
58/(350+58)
```

```
## [1] 0.1421569
```

21.15% of people predicted to make purchase do in fact make one. 14.21% of people using logistic predicted to make purchase do in fact make one.