

Typing Inference

$$\frac{}{\Theta; \Gamma \vdash * \Rightarrow ()} \text{STAR}$$

$$\frac{}{\Theta; \Gamma \vdash n \Rightarrow \mathbb{N}} \text{NUM}$$

$$\frac{}{\Theta; \Gamma \vdash c \Rightarrow \text{Char}} \text{CHAR}$$

$$\frac{\Theta; \Gamma \vdash t \Rightarrow A \rightarrow B \quad \Theta; \Gamma \vdash u \Leftarrow A}{\Theta; \Gamma \vdash t \ u \Rightarrow B} \text{APP}$$

$$\frac{\Theta; \Gamma \vdash C \text{ type} \quad \Theta; \Gamma \vdash n \Leftarrow \mathbb{N} \quad \Theta; \Gamma \vdash t_0 \Leftarrow C \quad \Theta; \Gamma \vdash t_s \Leftarrow C \rightarrow C}{\Theta; \Gamma \vdash \text{Iter} :: C \ (n, t_0, t_s) \Rightarrow C} \text{ITER}$$

$$\frac{\Theta; \Gamma \vdash t \Rightarrow A \times B}{\Theta; \Gamma \vdash \text{fst } t \Rightarrow A} \text{FST}$$

$$\frac{\Theta; \Gamma \vdash t \Rightarrow A \times B}{\Theta; \Gamma \vdash \text{snd } t \Rightarrow B} \text{SND}$$

$$\frac{\begin{array}{c} \Theta; \Gamma \vdash \text{ind } F, [\overrightarrow{C_i : B_i}] \rightarrow A \text{ type} \\ \{\Theta; \Gamma, f : \text{ind } F[\overrightarrow{C_i : B_i}] \rightarrow A, x_i : B_i \vdash t_i : A\} \end{array}}{\Theta; \Gamma \vdash \text{fix } f : \text{ind } F, [\overrightarrow{C_i : B_i}] \Rightarrow A \ \{ \overrightarrow{C_i \ x_i \rightarrow t_i} \} : \text{ind } F, [\overrightarrow{C_i : B_i}] \rightarrow A} \text{FIX}$$

$$\frac{\Theta \Gamma \vdash A \text{ type} \quad \Theta; \Gamma \vdash c \Leftarrow \text{Char} \quad \Theta \Gamma \vdash \text{Char}_i \Leftarrow A_i}{\Theta; \Gamma \vdash \text{matchChar} :: A \ c \ \text{chr}_i} \text{MATCH-CHAR}$$

$$\frac{\Theta; \Gamma \vdash A \text{ type} \quad \Theta; \Gamma \vdash t \Leftarrow IO \ A \quad \Theta; \Gamma, x : A \vdash u \Rightarrow IO \ B}{\Theta; \Gamma \vdash \text{do } \{x :: A \leftarrow t; u\} \Rightarrow IO \ B} \text{BINDINFER}$$

$$\frac{\Theta; \Gamma \vdash s \Leftarrow \text{String}}{\Theta; \Gamma \vdash \text{print } s \Rightarrow IO \ ()} \text{PRINT}$$

$$\frac{\Theta; \Gamma \vdash s \Leftarrow \text{String}}{\Theta; \Gamma \vdash \text{readFile } s \Rightarrow IO \ \text{String}} \text{READFILE}$$

$$\frac{\Gamma; \Theta \vdash A \text{ type} \quad \Gamma; \Theta \vdash t \Leftarrow A \quad \Theta; \Gamma, x : A \vdash u \Rightarrow B}{\Gamma; \Theta \vdash \text{let } x :: A = t \text{ in } u \Rightarrow B} \text{LETINFER}$$

$$\frac{\Theta; \Gamma \vdash B \text{ type} \quad \Theta, A : K; \Gamma \vdash t \Rightarrow C}{\Theta; \Gamma \vdash \text{let type } A :: K = B \text{ in } t \Rightarrow C} \text{LETTYPEINFER}$$

$$\frac{\Theta; \Gamma \vdash A \text{ type} \quad \Theta; \Gamma \vdash t \Rightarrow \forall B :: K . C}{\Theta; \Gamma \vdash t @A \Rightarrow C[A/B]} \text{TYPEAPP}$$

$$\frac{\Theta; \Gamma \vdash t \Leftarrow \mathbb{N} \quad \Theta; \Gamma \vdash u \Leftarrow \mathbb{N}}{\Theta; \Gamma \vdash t + u \Rightarrow \mathbb{N}} \text{ADD}$$

$$\frac{\Theta; \Gamma \vdash t \Leftarrow \mathbb{N} \quad \Theta; \Gamma \vdash u \Leftarrow \mathbb{N}}{\Theta; \Gamma \vdash t - u \Rightarrow \mathbb{N}} \text{MINUS}$$

$$\frac{\Theta; \Gamma \vdash t \Leftarrow \mathbb{N} \quad \Theta; \Gamma \vdash u \Leftarrow \mathbb{N}}{\Theta; \Gamma \vdash t \times u \Rightarrow \mathbb{N}} \text{TIMES}$$

$$\frac{\Theta; \Gamma \vdash t \Leftarrow \mathbb{N} \quad \Theta; \Gamma \vdash u \Leftarrow \mathbb{N}}{\Theta; \Gamma \vdash t < u \Rightarrow \mathbb{N}} \text{LESSTHAN}$$

$$\frac{\Theta; \Gamma \vdash t \Leftarrow \mathbb{N} \quad \Theta; \Gamma \vdash u \Leftarrow \mathbb{N}}{\Theta; \Gamma \vdash t \leq u \Rightarrow \mathbb{N}} \text{LESSTHANEQ}$$

$$\frac{\Theta; \Gamma \vdash t \Leftarrow \mathbb{N} \quad \Theta; \Gamma \vdash u \Leftarrow \mathbb{N}}{\Theta; \Gamma \vdash t \geq u \Rightarrow \mathbb{N}} \text{GREATERTHANEQ}$$

$$\frac{\Theta; \Gamma \vdash t \Leftarrow \mathbb{N} \quad \Theta; \Gamma \vdash u \Leftarrow \mathbb{N}}{\Theta; \Gamma \vdash t = u \Rightarrow \mathbb{N}} \text{ISEQ}$$

Typing Checking

$$\frac{\Theta; \Gamma \vdash t \Leftarrow A \quad \Theta; \Gamma \vdash u \Leftarrow B}{\Theta; \Gamma \vdash (t, u) \Leftarrow (A, B)} \text{PAIR}$$

$$\frac{\Theta; \Gamma, x : A \vdash t \Leftarrow B}{\Theta; \Gamma \vdash \lambda x. t \Leftarrow A \rightarrow B} \text{ABS}$$

$$\frac{\Theta; \Gamma \vdash t \Leftarrow \mathbb{N} \quad \Theta; \Gamma \vdash u \Leftarrow \mathbb{N}}{\Theta; \Gamma \vdash t > u \Rightarrow \mathbb{N}} \text{GREATERTHAN}$$

$$\frac{\Theta, X : K; \Gamma \vdash t \Leftarrow B}{\Theta; \Gamma \vdash \Lambda X. t \Leftarrow \forall A :: K . B} \text{TYPEABS}$$

$$\frac{\Theta; \Gamma \vdash t \Leftarrow B_i[\text{ind } F. \overrightarrow{[C_i : B_i]} / F]}{\Theta; \Gamma \vdash C_i t \Leftarrow \text{ind } F. \overrightarrow{[C_i : B_i]}} \text{CONS}$$

$$\frac{\Theta; \Gamma \vdash A \text{ type} \quad \Theta; \Gamma \vdash t \Leftarrow IO A \quad \Theta; \Gamma, x :: A \vdash u \Leftarrow IO B}{\Theta; \Gamma \vdash \text{do } \{x :: A \leftarrow t; u\} \Leftarrow IO B} \text{BINDCHECK}$$

$$\frac{\Theta; \Gamma \vdash t \Leftarrow A}{\Theta; \Gamma \vdash \text{return } t \Leftarrow IO A} \text{RETURNCHECK}$$

$$\frac{\Theta; \Gamma \vdash A \text{ type} \quad \Theta; \Gamma \vdash t \Leftarrow A \quad \Theta; \Gamma, x : A \vdash u \Leftarrow B}{\Theta; \Gamma \vdash \text{let } x :: A \leftarrow t \text{ in } u \Leftarrow B} \text{LETCHECK}$$

$$\frac{\Theta; \Gamma \vdash B \text{ type} \quad \Theta, A : K; \Gamma \vdash t \Leftarrow C}{\Theta; \Gamma \vdash \text{let type } A :: K = B \text{ in } t \Leftarrow C} \text{LETTYPECHECK}$$

$$\frac{\Theta; \Gamma \vdash t \Rightarrow A}{\Theta; \Gamma \vdash t \Leftarrow A} \text{CONV}$$