FINA4354 Final Project
Carry Trade Shield

Group 2

Lyu Xing u35xing@connect.hku.hk

Zheng Yiwen zyw666@connect.hku.hk

Tao Chenya u3630304@connect.hku.hk

He Zhiyue hezhiyue@connect.hku.hk

Liu Sitong lststar@connect.hku.hk

Background: Carry Trade

- Borrow at a low interest rate
- Reinvest in a financial product with higher return

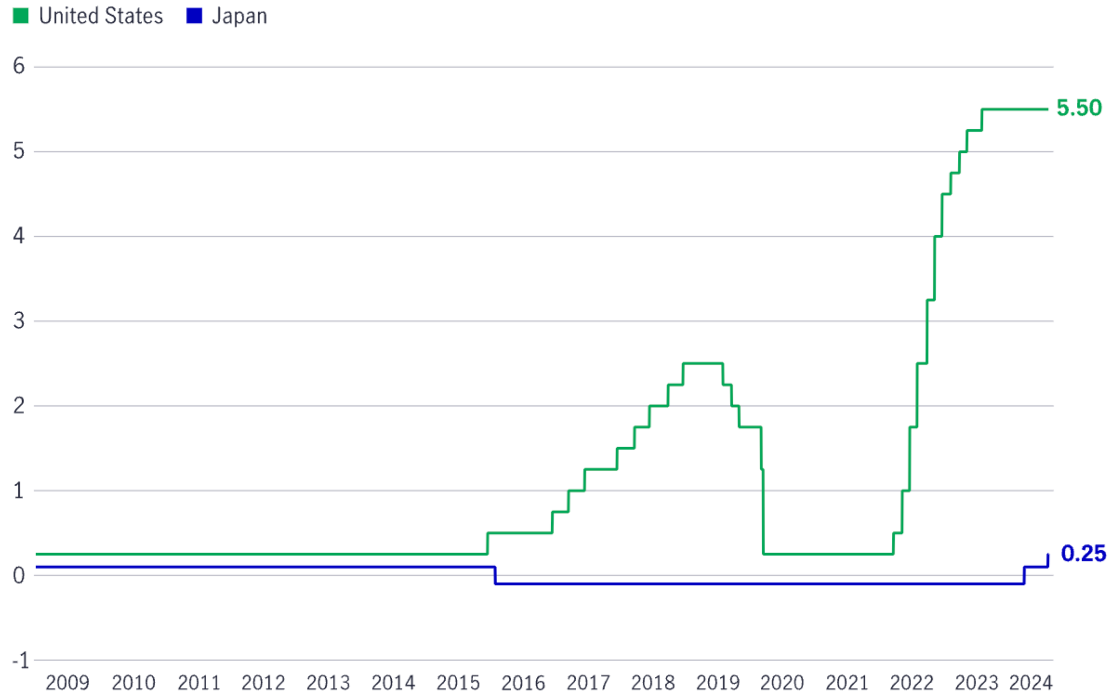
Risk

- American stock return may drop
- USD to JPY FX rate may drop

- Black Monday on Aug 5

Borrowing in JPY to invest in the USD has been a common carry trade

Central bank rates, 1/1/09–8/5/24 (%)



Source: Macrobond, 8/5/24.

Product Outline

- Underlying: from time 0 to time T, the return on carry trade derived from the return of Japanese bond and the return of NASDAQ

$$r_{S_T} \times \frac{Y_{\mathrm{T}}}{Y_0} - e^{r_{B_0} T}$$

where r_{B_0} denotes the prespecified interest rate when borrowing JPY

r_{S_T} denotes the return on NASDAQ index from time 0 to T

Y_0 denotes the USD to JPY FX rate at time 0

Y_T denotes the USD to JPY FX rate at time T

Product Target

- We try to help investors hedge the risk of a significant loss from carry trade between Japan and the US.
- At the same time, we shall also briefly compensate investors when there is a loss but not remarkable to improve the attraction of our product.
- When the loss is less than 5%, we compensate investors gradually by a proportion from 0 to 50%. With this feature, the price of the product could be significantly decreased.
- When the loss is larger than 5%, we will compensate with a proportion of 50%, which can provide more flexibility to investors.

Product Structure

- $Payoff$:

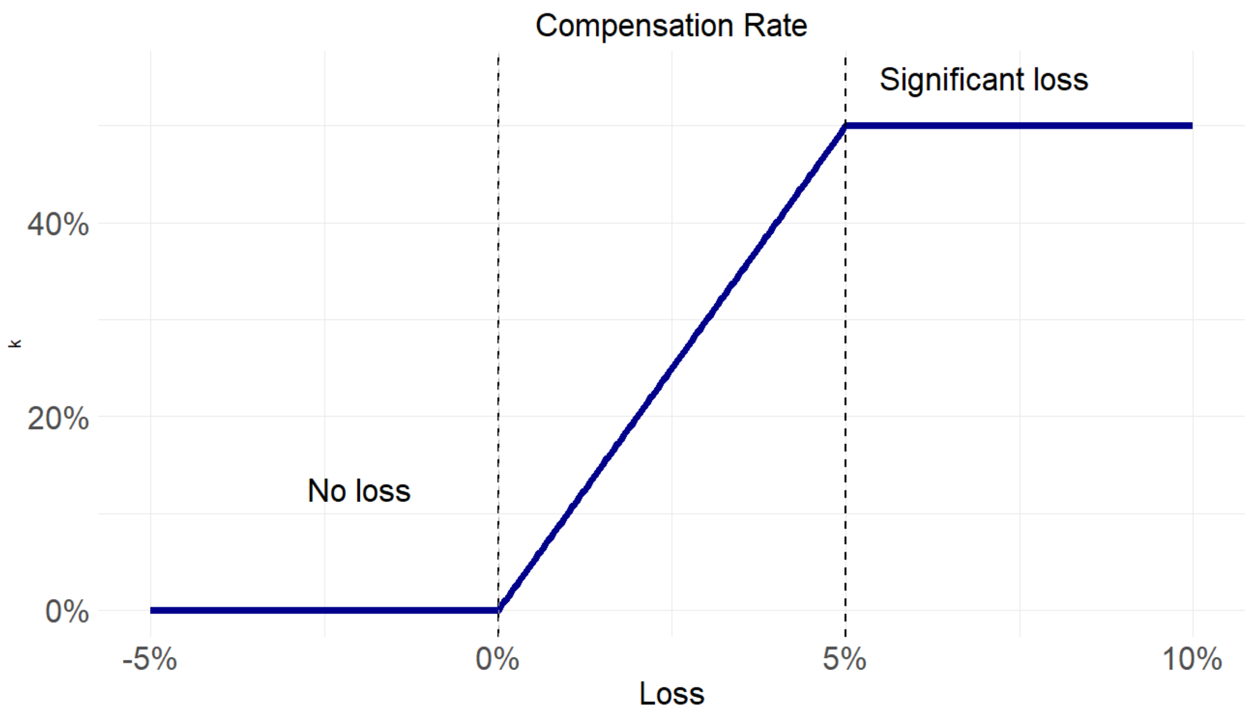
$$\Phi\left(r_{S_T}, Y_T\right) = \begin{cases} 0 & When\ r_{S_T} * \frac{Y_T}{Y_0} - e^{r_{B_0}T} > 0\ (No\ loss) \\ k * (-0.5) * \left(r_{S_T} * \frac{Y_T}{Y_0} - e^{r_{B_0}T}\right) & \text{When } -5\% < r_{S_T} * \frac{Y_T}{Y_0} - e^{r_{B_0}T} < 0\ (Loss\ but\ not\ significant) \\ -0.5\left(r_{S_T} * \frac{Y_T}{Y_0} - e^{r_{B_0}T}\right) & When\ r_{S_T} * \frac{Y_T}{Y_0} - e^{r_{B_0}T} < -5\%\ (Significant\ loss) \end{cases}$$

$k\ is\ the\ parameter\ that\ determine\ how\ many\ proportion\ will\ be\ conpensated.$

$$k = \frac{r_{S_T} * \frac{Y_T}{Y_0} - e^{r_{B_0}T}}{-5\%}$$

$Higher\ loss\ come\ with\ higher\ proportion\ to\ be\ compensated.$

Compensation Rate



The blue line shows how the compensate rate k changes with respect to loss proportion.

Compensation Rate

The **red line** shows the loss curve without our product.

The **green line** shows our compensation based on the loss rate.

Setting Parameters

- Foreign Exchange Process (USD to JPY)

$$dY_t = (r_{B_t} - r_{A_t})Y_t dt + \sigma_{\mathrm{Y}} Y_t dW_t^Y$$

- Vasicek

$$dr_{A_t} = \kappa_A(\theta_A - r_{A_t})dt + \sigma_A dW_t^A$$
$$dr_{B_t} = \kappa_B(\theta_B - r_{B_t})dt + \sigma_B dW_t^B$$

- NASDAQ

$$dS_t = r_{A_t} S_t dt + \sigma_S S_t dW_t^S$$

Thus, we need:

mean reversion (κ), long-term mean (θ), and volatility (σ)

Setting Parameters

- Prepare data: US 3-Month Bond Yields, Japanese 3-Month Bond Yields, and NASDAQ prices
- First, calculate DTB3 and logarithmic returns

$$Logarithmic\ Return\ = ln(\frac{Price_t}{Price_{t-1}})$$

- Second, Long-term Mean (θ) = mean of the series
- Mean Reversion (κ): Using a linear regression:

$$\Delta Y_t = \kappa(Y_{t-1} - \theta) + noise$$

- Volatility (σ): The standard deviation of the variable

κ_A	θ_A	σ_A	κ_B	θ_B	σ_B	$\sigma_{\mathbf{Y}}$	$\sigma_{\mathbf{S}}$
0.01299484	5.114246	0.01674026	-1.235734	-0.01198186	0.3558819	0.007230012	0.01098079

Monte Carlo Simulation

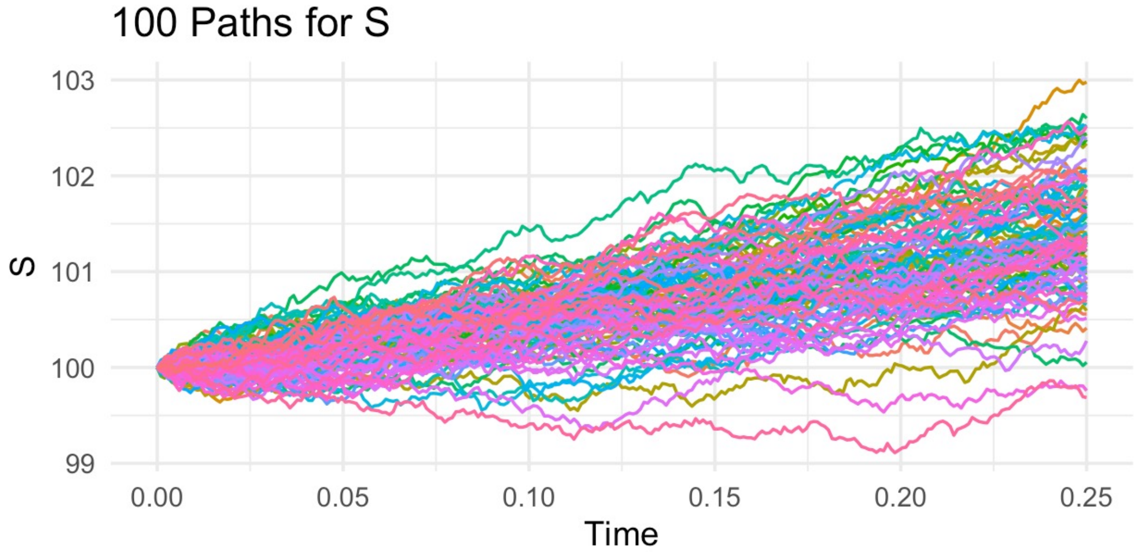
- Multiple possible future scenarios are simulated for the underlying assets to calculate the corresponding payoff.

- Why do we use Monte Carlo Simulation?
- The payoff function for this product cannot be solved analytically.
  - multiple factors and their processes, such as interest rates, volatility, and Nasdaq index

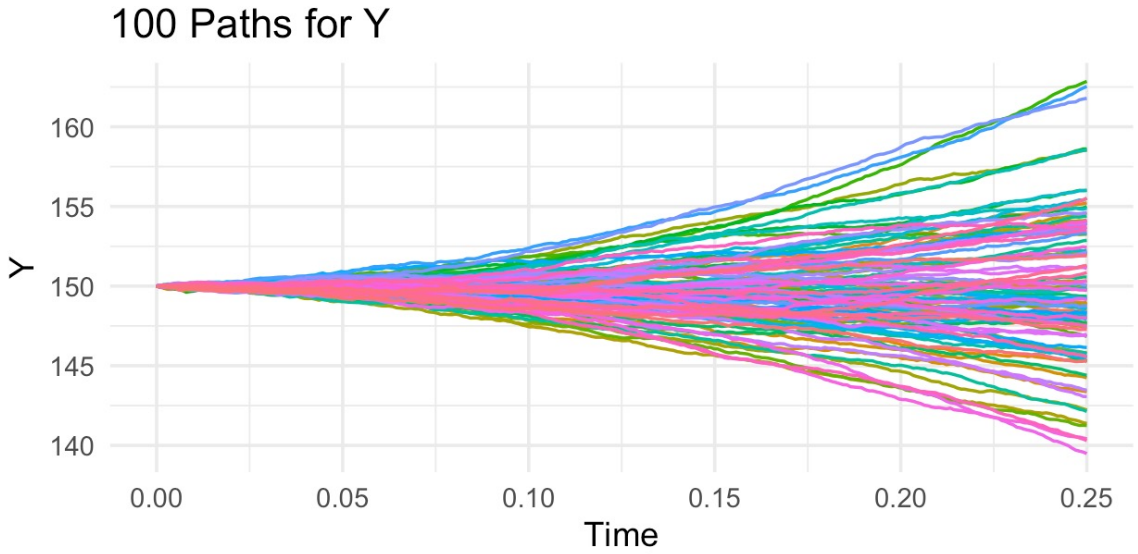
- A distribution of potential outcomes for the underlying is generated to show how the underlying price evolves under different market conditions.

Monte Carlo Results for the Random Terms

- NASDAQ return (%)

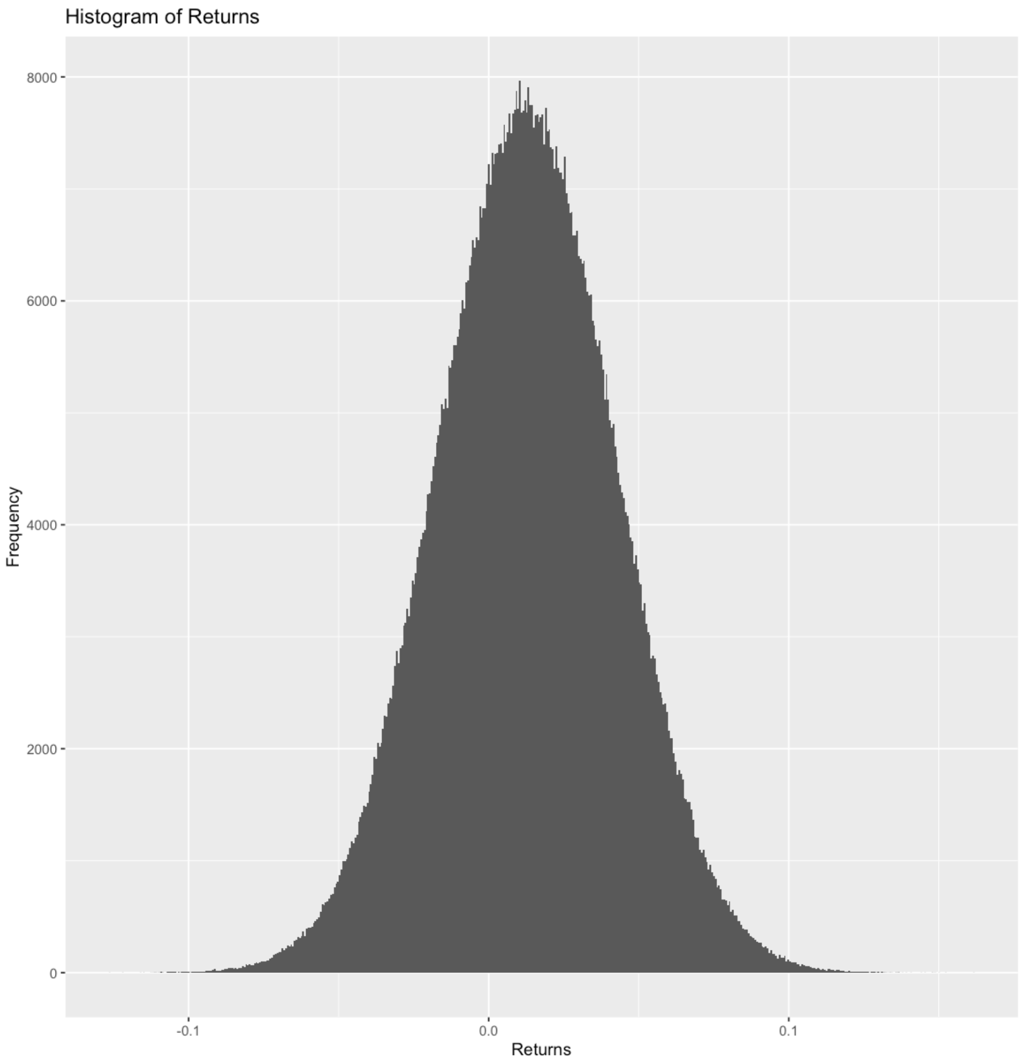


- USD to JPY FX rate



Potential for Out/Underperformance of NASDAQ relative to Bond

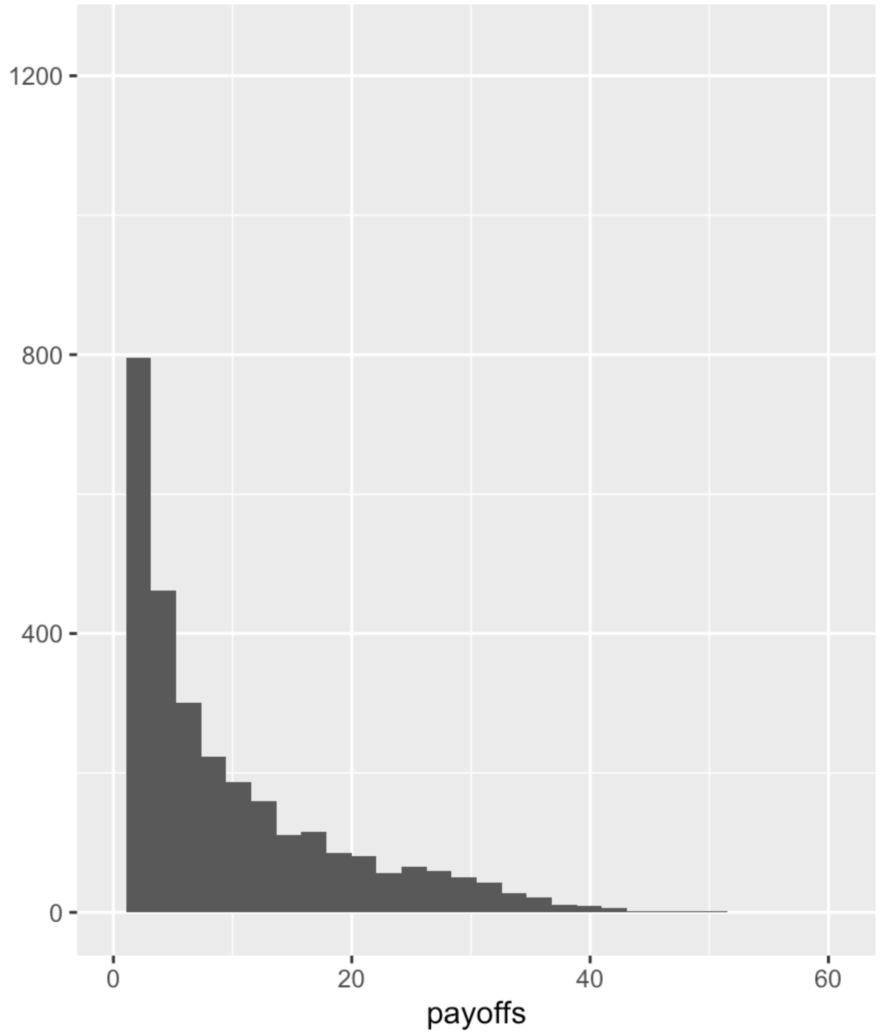
- Approximately symmetric
- Bell-shaped
- Centered slightly above 0
- No significant skewness



Payoff Distribution

(10000 iterations)

- Decreasing probability for larger payoffs
- No loss under all market scenarios



Product Pricing

- For this product, we set the maturity to 3 months ($T=0.25$) to provide investors with greater flexibility in hedging their investments.

- Following a simulation of 10,000,000 iterations, the product's value was calculated at 3.70543 (based on a principal of 1,000). This indicates that investors need to pay approximately 0.37% of the principal to hedge half of their risk (or 0.74% for a full hedge).

Risk Hedging

To hedge risks, we calculated **Delta Sensitivities** to
Stock Price (S)

Simulate paths with slight positive and negative shifts in the stock price.
Compute changes in average payoffs.
Derive the stock delta (at Stock = 100):

$$\Delta_{\text{Stock}} \approx \frac{\Pi(t,\ (1+0.5h)S, Y; \mathcal{X}) - \Pi(t,\ (1-0.5h)S, Y; \mathcal{X})}{hS} = \mathbf{-1.91601}$$

Risk Hedging

We also calculated **Delta Sensitivities** to
FX Rate (Y)

Simulate paths with slight positive and negative shifts in the FX rate.
Compute changes in average payoffs.
Derive the FX rate delta (at FX rate = 100):

$$\Delta_{FX} \approx \frac{\Pi(t, S, (1 + 0.5h)Y; \mathcal{X}) - \Pi(t, S, (1 - 0.5h)Y; \mathcal{X})}{hY} = \mathbf{-1.25157}$$

Risk Hedging

Delta Hedge Ratios:

The computed deltas provide a quantitative measure for hedging:

- Adjust stock holdings based on Δ_{Stock}.
- Use currency instruments to hedge against Δ_{FX}.

Risk Mitigation:

By implementing these deltas, we can construct a portfolio that neutralizes sensitivity to small market fluctuations, ensuring more stable profit.

Thank you

Group 2