A Linear Panel Model with Heterogeneous Coefficients and Variation in Exposure

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Introduction

- Social scientists often seek to evaluate the effects of a certain event, such as the adoption of a national policy
- Running example

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- One heuristic model:

$$\begin{aligned}\text{Outcome} &= \text{Unit effect} + \text{Time effect} \\ &\quad + \text{Coefficient}\,(\text{Event} \times \text{Exposure}) + \text{Error}\end{aligned}$$

- Event $\times$ Exposure is the term of greatest interest, as it captures the fact that different units are affected differently by the event because of their different exposure to it
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- Dube and Vargas (ReStud 2013, equation 1): impact of income shocks on violence in Colombia
- Dafny et al. (AER 2012, equation 5): impact of a merger on health insurance premiums
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Outline of video series

1. Using the Medicare example to illustrate a model of treatment effects heterogeneity
   1.1 TWFE can fail to estimate even a weighted average of unit-specific effects
2. Discuss some identification challenges due to unmodeled heterogeneity when there is no group totally unaffected by the event
   2.1 there exists *no* estimator that is guaranteed to estimate an average of unit-specific effects
3. Solutions: with a group that is totally unaffected by the event
   3.1 de Chaisemartin and D'Haultfœuille (ReStud 2018): estimate an average effect by replacing the TWFE with an average of difference-in-differences estimators

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Outline

The effect of Medicare on health care expenditures

- Medicare is a US government program introduced in 1965 to provide health insurance to *all* the elderly
- Stylized version of Finkelstein's (2007) study
- We observe per capita health care expenditures y_{st} on the elderly for each US state s in each of two time periods t:
  - let $t = 0$ denote the period before the introduction of Medicare
  - let $t = 1$ denote the period after

Exposure measured by penetration of private insurance

TABLE I
SHARE OF ELDERLY WITHOUT HOSPITAL INSURANCE, 1963

	Blue Cross	Any insurance
New England (CT, ME, MA, NH, RI, VT)	0.49	0.37
Middle Atlantic (NJ, NY, PA)	0.60	0.41
East North Central, Eastern Part (MI, OH)	0.55	0.32
East North Central, Western Part (IL, IN, WI)	0.75	0.42
West North Central (IA, KS, MN, MO, NE, ND, SD)	0.81	0.47
South Atlantic, Upper Part (DE, DC, MD, VA, WV)	0.75	0.45
South Atlantic, Lower Part (FL, GA, NC, SC)	0.81	0.50
East South Central (AL, KY, MS, TN)	0.88	0.57
West South Central (AR, LA, OK, TX)	0.85	0.55
Mountain (AZ, CO, ID, MT, NV, NM, UT, WY)	0.78	0.50
Pacific (OR, WA, CA, AK, HI)	0.87	0.52
National Total	0.75	0.45

Data are from individuals aged 65 and over in the 1963 National Health Survey. Sample size is 12,757. Minimum sample size for a subregion is 377.

- Medicare had a relatively small effect on rates of insurance coverage for e.g. a New England state v.s. a Pacific state

Linear panel data model

- Formally, let x_{st} be the fraction of elderly with health insurance in a given state s at time t
  - x_{s0} measures the fraction of elderly with private insurance in state s prior to Medicare
  - x_{s1} as being equal to 1 for all states s due to the universal coverage afforded by Medicare
- A linear panel data model of health care expenditures – what we will refer to as the linear model – might then take the form

$$y_{st} = \alpha_s + \delta_t + \beta x_{st} + \varepsilon_{st} \qquad \text{(linear model)}$$

  - The parameter β measures the causal effect of going from no coverage ($x_{st} = 0$) to full coverage ($x_{st} = 1$)

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Exposure model

- We can rewrite the linear panel data model closer to the heuristic model
- Since $x_{s1} = 1$ for all states, the linear panel data model implies that

  $$y_{st} = \tilde{\alpha}_s + \delta_t + \beta\,(1 - x_{s0})\,t + \varepsilon_{st} \qquad \text{(exposure model)}$$

  where we have redefined the state fixed effect as $\tilde{\alpha}_s = \alpha_s + \beta x_{s0}$
- Here $(1 - x_{s0})$ is the observed exposure variable and the term t is an indicator for whether the observation is from the post-Medicare period

TWFE

- The exposure model is

$$y_{st} = \tilde{\alpha}_s + \delta_t + \beta\,(1 - x_{s0})\,t + \varepsilon_{st}$$

- We can estimate the unknown coefficient β by a two-way fixed effects (TWFE) estimator $\hat{\beta}$
- Appealing properties:
  - if the exposure model holds, and ε_{st} is unrelated to x_{st}, then $\hat{\beta}$ is unbiased for β
  - if further ε_{st} are homoskedastic and not clustered / serially correlated, then $\hat{\beta}$ is also efficient
- The exposure model implies that the effect of Medicare on expenditures is $\beta\,(1 - x_{s0})$
  - The per-unit effect of insurance on expenditures is the same across states
  - Effects differ across states only due to different effect of Medicare on insurance rate: $(1 - x_{s0})$

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Outline

Heterogeneous coefficients

- Imagine that each state s has its own coefficient β_s describing the effect of insurance on expenditures in the state:

$$y_{st} = \alpha_s + \delta_t + \beta_s x_{st} + \varepsilon_{st} \qquad \text{(heterogeneous model)}$$

- For example, a state with a less healthy uninsured population may see expenditures rise more in response to a given expansion in insurance
  - Only the least healthy elderly remain uninsured so that the uninsured population is less healthy in states with greater insurance penetration prior to Medicare (high x_{s0} and high β_s)

Behavior of the TWFE estimator

- We are still maintaining that the error term ε_{st} is unrelated to x_{st} as before, so absent changes in the insurance levels x_{st}, all states would follow identical average trends over time
- How reasonable would the TWFE estimator $\hat{\beta}$ be, which is based on the exposure model that assumes all states have the same β?
- Recent literature has investigated the expected value of the TWFE estimator $\hat{\beta}$ under common trends assumptions

Expected value of the TWFE estimator

- Under the heterogeneous model, the expected value of the two-way fixed effects (TWFE) estimator of the exposure model, given the data $x = \{x_{10}, ..., x_{S0}\}$ for states $s \in \{1, ..., S\}$, is given by

$$\mathrm{E}\left(\hat{\beta}|x\right) = \frac{\mathrm{Cov}\left(\beta_s\left(1 - x_{s0}\right), \left(1 - x_{s0}\right)\right)}{\mathrm{Var}\left(1 - x_{s0}\right)}$$

claim

- In certain situations, $\hat{\beta}$ is still centered on an average of the true state-level coefficients β_s.
  - One situation is where β_s is unrelated to (i.e., statistically independent of) $(1 - x_{s0})$
- Otherwise $\hat{\beta}$ is no longer centered around the effect in a "typical" state

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A numerical example

- Let the coefficient β_s vary across states according to the equation

$$\beta_s = 1 + 0.5\lambda - \lambda x_{s0} \qquad \text{(numerical example)}$$

  - λ is a parameter that governs how the state-level coefficient β_s is related to the fraction of elderly with insurance before Medicare
  - When $\lambda = 0$, the coefficient β_s is equal to 1 in all states regardless of prior insurance penetration
  - When $\lambda < 0$, states with greater insurance penetration prior to Medicare have a larger coefficient β_s
  - Set $x_{s0} = 0.245 + s/100$ so that no matter the value of λ, the average value of β_s across all states is always 1

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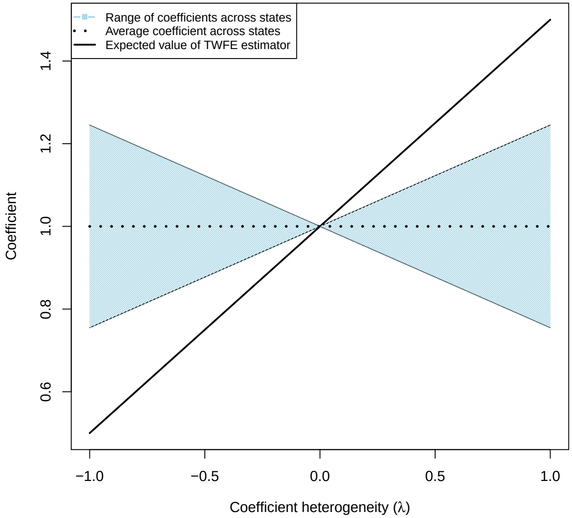
Illustration

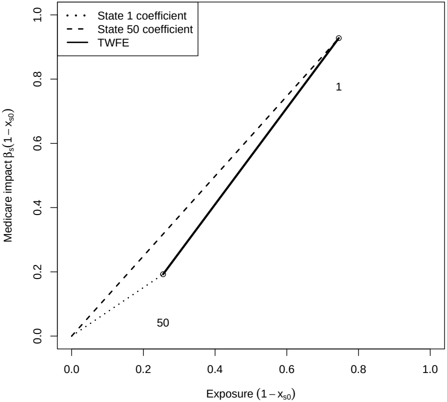


identification

Intuition

- When $\lambda > 0$, states with a larger increase in insurance coverage, $(1 - x_{s0})$, also have larger coefficients β_s
- Following Medicare's introduction, expenditure therefore grows more in states with larger $(1 - x_{s0})$ because
  - these states experience a larger increase in insurance coverage
  - these states experience a larger change in expenditure for a given change in insurance coverage

Intuition



- TWFE estimator $\hat{\beta}$ conflates them, thus overstating the effect of insurance on expenditure.

Outline

Identification challenge

- The TWFE estimator $\hat{\beta}$ cannot, in general, be guaranteed
  - to center around the average β_s across the states
  - to center around a value inside the range of the true coefficients $[\min_s \beta_s, \max_s \beta_s]$
- This phenomenon is not specific to the TWFE estimator
- Without any restriction on coefficients β_s and if $x_{s0} \in (0, 1)$ for all states, then no estimator is guaranteed to center around a value inside $[\min_s \beta_s, \max_s \beta_s]$ claim

Proof:1/2

- Consider the special case with $S = 2$, some x_{s0}'s with $0 < x_{20} \leq x_{10} < 1$, $\beta_1 < \beta_2$, and δ_0 known to be zero
- The model for the data is then

$$y_{s0} = \alpha_s + \beta_s \cdot x_{s0} + \varepsilon_{s0}$$
$$y_{s1} = \alpha_s + \delta_1 + \beta_s + \varepsilon_{s1}$$

  with parameters $\theta = \left(\{(\alpha_s, \beta_s)\}_{s=1}^{2}, \delta_1, F_{\varepsilon|X}\right)$, for $F_{\varepsilon|X}$ the distribution of $(\varepsilon_{s0}, \varepsilon_{s1})$ conditional on x_{s0}
- The distribution of the data we observe is then $F_{Y_0,Y_1|X}\left(y_0, y_1 \mid x_{s0} = x; \theta\right)$

Proof:1/2

- Given any parameter θ, define the distinct parameter $\theta' = \left(\left\{\left(\alpha_s', \beta_s'\right)\right\}_{s=1}^{2}, \delta_1', F_{\varepsilon|X}\right)$ given by

$$\theta' = \left(\left\{\left(\alpha_s + \frac{\Delta \cdot x_{s0}}{1 - x_{s0}}, \beta_s - \frac{\Delta}{1 - x_{s0}}\right)\right\}_{s=1}^{2}, \delta_1 + \Delta, F_{\varepsilon|X}\right)$$

for some $\Delta > (\beta_2 - \beta_1) \cdot (1 - x_{20}) > 0$.

- Parameter θ and θ' are observationally equivalent: $F_{Y_0,Y_1|X}\left(y_0, y_1 \mid x_{s0} = x; \theta'\right) = F_{Y_0,Y_1|X}\left(y_0, y_1 \mid x_{s0} = x; \theta\right)$

calculations

- For any estimator $\hat{\beta}'$ that depends on the data, the expected value must be the same under θ and θ'
- However, the Δ is chosen such that $\beta_1' = \beta_1 - \frac{\Delta}{1-x_{10}} < \beta_2 - \frac{\Delta}{1-x_{20}} = \beta_2' < \beta_1 < \beta_2$ TWFE

Outline

A Difference-in-Differences Perspective

- This identification challenge applies to all estimators
- Consider an exposure-adjusted difference-in-differences estimator, which provides one possible way to account for changes in insurance rates and addresses the conflation problem of TWFE

$$\hat{\beta}^{DID}_{s,s'} = \frac{(y_{s1} - y_{s0}) - (y_{s'1} - y_{s'0})}{(1 - x_{s0}) - (1 - x_{s'0})}$$

A Difference-in-Differences Perspective

- de Chaisemartin and D'Haultfœuille (2018) call $\hat{\beta}^{DID}_{s,s'}$ a Wald-difference-in-differences estimator because it consists of the ratio of the difference-in-differences estimator for the outcome (in our case, expenditures) to the one for exposure (insurance)

$$\hat{\beta}^{DID}_{s,s'} = \frac{(y_{s1} - y_{s0}) - (y_{s'1} - y_{s'0})}{(1 - x_{s0}) - (1 - x_{s'0})}$$

- As with the TWFE estimator, this estimator can be centered around a value outside the range of coefficients, including in our numerical example if $x_{s0}, x_{s'0} \in (0, 1)$

Solutions

- Impose further structure on the coefficients β_s
  - For example, suppose that a researcher is willing to posit a linear relationship between β_s and x_{s0}, but does not know the value of the parameter λ that governs this relationship
  - Then substitute the expression for β_s to arrive at a linear panel model whose unknown parameter, λ, can be estimated by a two-way fixed effects estimator
- Bounds on variation in coefficients and mean of the error term (Manski and Pepper, 2018)
- More data: a "close to" totally unaffected state ($x_{s'0} = 1$) and/or a control state ($x_{s'0} = x_{s'1} = 0$)

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- Suppose that in state s' Medicare had no effect on insurance rates, for example because all elderly in the state were insured prior to Medicare, $x_{s'0} = 1$
  - Then this $\hat{\beta}^{DID}_{s,s'}$ is unbiased for β_s, the true coefficient for the affected state s

$$\hat{\beta}^{DID}_{s,s'} = \frac{(y_{s1} - y_{s0}) - (y_{s'1} - y_{s'0})}{(1 - x_{s0})}$$

  - The presence of an unaffected state brings it closer to the classical difference-in-differences setting of Card and Krueger, 1994)
- Average of $\hat{\beta}^{DID}_{s,s'}$ is centered around the average of true coefficients for all affected states $s \neq s'$.

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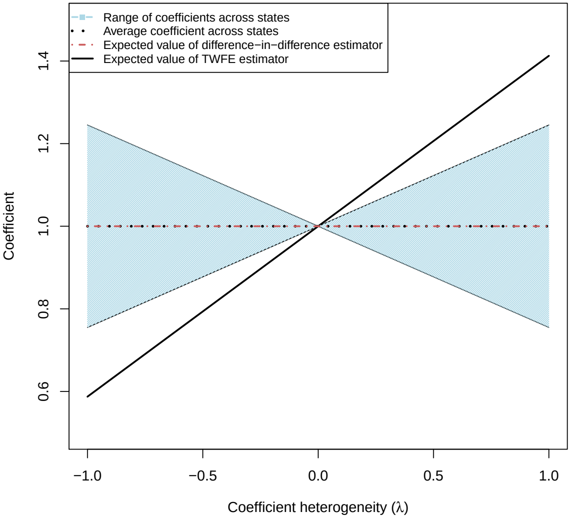
Solutions

- It does not repair the TWFE estimator $\hat{\beta}$



Solutions

- Recent papers propose alternative estimators in a range of settings
  - Callaway and Sant'Anna (2021), de Chaisemartin and D'Haultfœuille (2018, 2020), Sun and Abraham (2021) among others
  - Stata implementations: `csdid`, `fuzzydid`, `did_multipledgt`, and `eventstudyinteract`

Conclusion

- An active literature tries to interpret the two-way fixed effects (TWFE) estimator, in the presence of unmodeled coefficient heterogeneity
- We illustrate some implications for the case where the research design takes advantage of variation across units (say, US states) in exposure to some treatment
- TWFE can still fail to estimate the average of the units' coefficients
  - With unmodeled heterogeneity and without totally unaffected states, there exists *no* estimator that is guaranteed to estimate a value inside the true range
- Building on the literature, we note that when there is a totally unaffected unit, it is possible to estimate an average effect by an average of difference-in-differences estimators

References

- Callaway, Brantly, Andrew Goodman-Bacon, and Pedro Sant'Anna, "Difference-in-Differences with a Continuous Treatment," ArXiv:2107.02637 [Econ], 2021.
- Callaway, Brantly and Pedro H.C. Sant'Anna, "Difference-in-Differences with Multiple Time Periods," Journal of Econometrics, 2021, 225 (2), 200–230. Themed Issue: Treatment Effect 1.
- Card, David and Alan B. Krueger, "Minimum Wages and Employment: A Case Study of the Fast-Food Industry in New Jersey and Pennsylvania," American Economic Review, September 1994, 84 (4), 772–793.
- de Chaisemartin, Clément and Xavier D'Haultfœuille, "Fuzzy Differences-in- Differences," Review of Economic Studies, April 2018, 85 (2), 999–1028.
- _ and _, "Two-Way Fixed Effects Estimators with Heterogeneous Treatment Effects," American Economic Review, September 2020, 110 (9), 2964–2996.

References

- Finkelstein, Amy, "The Aggregate Effects of Health Insurance: Evidence from the Introduction of Medicare," Quarterly Journal of Economics, February 2007, 122 (1), 1–37.
- Sant'Anna, Pedro H. C. and Jun Zhao, "Doubly Robust Difference-in-differences Estimators," Journal of Econometrics, November 2020, 219 (1), 101–122.
- Sun, Liyang and Sarah Abraham, "Estimating Dynamic Treatment Effects in Event Studies with Heterogeneous Treatment Effects," Journal of Econometrics, 2021, 225 (2), 175–199. Themed Issue: Treatment Effect 1.

Expected value of the TWFE estimator

Claim

Under the heterogeneous model, the expected value of the two-way fixed effects (TWFE) estimator of the exposure model, given the data $x = \{x_{10}, ..., x_{S0}\}$ for states $s \in \{1, ..., S\}$, is given by

$$\mathrm{E}\left(\hat{\beta}|x\right) = \frac{\mathrm{Cov}\left(\beta_s\left(1 - x_{s0}\right), \left(1 - x_{s0}\right)\right)}{\mathrm{Var}\left(1 - x_{s0}\right)}$$

where $\mathrm{Cov}\left(\cdot, \cdot\right)$ and $\mathrm{Var}\left(\cdot\right)$ denote the sample covariance and variance, respectively, and the expectation $\mathrm{E}\left(\hat{\beta}|x\right)$ is taken with respect to the distribution of the errors ε_{st} conditional on the data $x = \{x_{10}, ..., x_{S0}\}$. back

Underidentification

Claim

There exists no estimator $\hat{\beta}'$ that can be expressed as a function of the data $\{(x_{s0}, y_{s0}, y_{s1})\}_{s=1}^{S}$ and whose expected value is guaranteed to be contained in $[\min_s \beta_s, \max_s \beta_s]$ for any heterogeneous model and any $\{x_{s0}\}_{s=1}^{S}$.

back

Details

We show that the two parameter values θ and θ' are observationally equivalent, which means the expected value of $\hat{\beta}'$ must be the same under θ and θ'. To see this, note that the distribution of (y_{s0}, y_{s1}) conditional on x_{s0} is the same under θ and θ':

$$
\begin{aligned}
& F_{Y_0,Y_1|X}\left(y_0, y_1 \mid x_{s0} = x; \theta\right) \\
=& \Pr\left\{\varepsilon_{s0} \leq y_0 - \alpha_s - \beta_s \cdot x,\ \varepsilon_{s1} \leq y_1 - \alpha_s - \delta_1 - \beta_s \mid x_{s0} = x; \theta\right\} \\
=& \Pr\left\{\varepsilon_{s0} \leq y_0 - \alpha_s - \beta_s \cdot x,\ \varepsilon_{s1} - \varepsilon_{s0} \leq y_1 - y_0 - \delta_1 - \beta_s\left(1 - x\right) \middle| x_{s0} = x; \theta\right\} \\
=& \Pr\left\{\begin{array}{c} \varepsilon_{s0} \leq y_0 - \left(\alpha_s + \frac{\Delta \cdot x}{1-x}\right) - \left(\beta_s - \frac{\Delta}{1-x}\right) \cdot x, \\ \varepsilon_{s1} - \varepsilon_{s0} \leq y_1 - y_0 - \left(\delta_1 + \Delta\right) - \left(\beta_s - \frac{\Delta}{1-x}\right)\left(1 - x\right) \end{array} \middle| x_{s0} = x; \theta\right\} \\
=& \Pr\left\{\begin{array}{c} \varepsilon_{s0} \leq y_0 - \alpha_s' - \beta_s' \cdot x, \\ \varepsilon_{s1} - \varepsilon_{s0} \leq y_1 - y_0 - \delta_1' - \beta_s'\left(1 - x\right) \end{array} \middle| x_{s0} = x; \theta'\right\} \\
=& F_{Y_0,Y_1|X}\left(y_0, y_1 \mid x_{s0} = x; \theta'\right).
\end{aligned}
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