13. (3).

$A=\{1,2,7,8\}$. $B=$ ~~$\{x \mid 0, \pm1, \pm2, \cdots \pm7\}$~~ 审题: N的子集:

B: $\{1,2,3,4,5,6,7\}$

C: $\{0,3,6,9,12,15,18\}$

D: $\{1,2,4,8,16,32\}$

$A\cup C=\{0,1,2,3,6,7,8,9,12,15,18\}$

$B-(A\cup C)=\{4,5\}$

14. (1) (2)

(1) $\{3,4,\{3\},\{4\},\}$

(2) $\{3\}$

15. (1) (2)

$P(\emptyset)=\{\emptyset\}$ $PP(\emptyset)=\{\emptyset,\{\emptyset\}\}$

$PPP(\emptyset)=\{\emptyset,\{\emptyset\},\{\{\emptyset\}\},\{\emptyset,\{\emptyset\}\}\}$

∴ 答案为

$\{\emptyset,\{\emptyset\},\{\{\emptyset\}\},\{\emptyset,\{\emptyset\}\}\}$

(2) 答案为 $\{\emptyset\}$

16. (1) (2)

(1) $P(A)=\{\emptyset,\{\{\emptyset\}\},\{\{\{\emptyset\}\}\},\{\{\emptyset\},\{\{\emptyset\}\}\}\}$

$\cup P(A)=\{\{\emptyset\},\{\{\emptyset\}\}\}$

(2)

$\cup A=\{\emptyset,\{\emptyset\}\}$

$P(\cup A)=\{\emptyset,\{\emptyset\},\{\{\emptyset\}\},\{\emptyset,\{\emptyset\}\}\}$

17. (1) (2)

① $(A-B)-C=(A\cap -B)\cap -C$

$= A\cap -(B\cup C) = A-(B\cup C)$

② 正难则反

$(A-C)-(B-C)=(A\cap -C)-(B\cap -C)$

$= A\cap -C \cap (-B\cup C)$

$=(A\cap -C\cap -B)\cup(A\cap -C\cap C)$

$=((A-B)-C)\cup\emptyset$

$=(A-B)-C$

18. (1)(2)

(1) $A=B=\phi$ 证明见后!!!

(2) $A=B$

19. (1)(2)

(1) 充要条件为: $A\cap B\cap C=\phi$

必要性:

设 $x\in A$. 则 $x\in A-B \vee x\in A-C$

$\therefore x\in A\cap -B \vee x\in A\cap -C$

又 $\because x\in A$. 故有:

$\therefore x\in -B \vee x\in -C$

$\therefore x\in(-B\cup -C)$

$\therefore x\in -(B\cap C)$

$\therefore x\notin B\cap C$

$\therefore A\cap B\cap C=\phi$

(2) 充分性:

若 $A\cap B\cap C=\phi$. 则若 $x\in A$, 有

$x\notin B\cap C$;

$\therefore x\in -B\cup -C$;

$\therefore x\in -B \vee x\in -C$

又 $\because x\in A$. $\therefore x\in A\cap -B \vee$

$x\in A\cap -C$

$\therefore x\in A-B \vee x\in A-C$

$\therefore x\in (A-B)\cup(A-C)$

$\therefore$ 即 $\forall x\in A$, 均有 $x\in(A-B)\cup(A-C)$

$\therefore A\subseteq(A-B)\cup(A-C)$

另一方面. $\forall x\in(A-B)\cup(A-C)$

有: $x\in(A-B) \vee x\in(A-C)$

$\therefore x\in A\cap -B \vee x\in(A\cap -C)$

$\therefore x\in A$

$\therefore (A-B)\cup(A-C)\subseteq A$

$\therefore A=(A-B)\cup(A-C)$

故充要条件为

$A\cap B\cap C=\phi$

(2) 充要条件为: $A \subseteq B \cap C$

① 必要性:

$(A-B) \cup (A-C) = \phi$

$\therefore A-B = \phi \wedge A-C = \phi$

$\therefore A \subseteq B \wedge A \subseteq C$

$\therefore A \subseteq B \cap C$

② 充分性:

若 $A \subseteq B \cap C$ 则 $A \subseteq B \cap A \subseteq C$

$\therefore A-B = \phi \cap A-C = \phi$

$\therefore (A-B) \cup (A-C) = \phi$

实际上: 在证必要性过程中两端等价. 即全为 $\Leftrightarrow$ 而非 $\Rightarrow$

26. (1)(2)

(1) $A \times B = \{\langle x, y\rangle \mid x \in A \wedge y \in B\}$

而 $A \times B = \phi$. 故 $A = \phi$ ~~且~~ $B = \phi$

注意到 x, y 中一者不存在时. $\langle x, y\rangle$ 便无法定义. 故 $A = \phi \vee B = \phi$

(2) $A = A \times A$ 当且仅当 $A = \phi$

即 $A = \phi$ 时. $A = A \times A$.

$A \neq \phi$ 时. $A \neq A \times A$.

28.

令 A. B. C 分别为 [1,250] 之间可被 2, 3, 5 整除的数的集合

$\therefore |A| = 125 \quad |B| = 83 \quad |C| = 50$

$|A \cap B| = 41 \quad |A \cap C| = 25 \quad |B \cap C| = 16$

$|A \cap B \cap C| = 8$

$\therefore |A \cup B \cup C| = |A| + |B| + |C| - |A \cap B| - |A \cap C| - |B \cap C| + |A \cap B \cap C|$

$= 125 + 83 + 50 - 41 - 25 - 16 + 8$

$= 184$

18. 证明如下:

① $A-B=B$.

$\Leftrightarrow (\forall x)(x\in A-B \leftrightarrow x\in B)$

$\Leftrightarrow (\forall x)((x\in A\wedge x\notin B \rightarrow x\in B)$

$\wedge (x\in B\rightarrow x\in A\wedge x\notin B))$

若$(x\in A\wedge x\notin B)\rightarrow x\in B$为真

则 $x\notin A$ 或 $x\in B$.

$\therefore \Leftrightarrow (\forall x)(x\notin A\vee x\in B)$

而$x\in B\rightarrow(x\in A\wedge x\notin B)$为真

即 $x\notin B$.

$(\forall x)(x\notin A\vee x\in B)\wedge(x\notin B)$

$\therefore (\forall x)(x\notin A\wedge x\notin B)$

即 $A=B=\phi$

② $A-B=B-A$

$\Leftrightarrow (\forall x)(x\in A\wedge x\notin B \leftrightarrow x\in B\wedge x\notin A)$

$x\in A\wedge x\notin B\leftrightarrow x\in B\wedge x\notin A$

为真则前后同真或同假

但不可能同真

$\therefore (\forall x)(\neg(x\in A\wedge x\notin B)\wedge\neg$

$(x\in B\wedge x\notin A))$

$\therefore (\forall x)((x\notin A\vee x\in B)\wedge(x\notin B$

$\vee x\in A))$

$\therefore (\forall x)((x\in A\rightarrow x\in B)\wedge(x\in B$

$\rightarrow x\in A))$

$\therefore A=B$