清华大学数学作业纸

（科目离散7）

班级 软01　姓名 赵晨阳　编号 2020012363　第 1 页

4.(4) $((\exists x)P(x)\land(\exists y)\neg Q(y))\lor(\forall z)R(z)$

$=(\exists x)(\exists y)(\forall z)((P(x)\land\neg Q(y))\lor R(z))$

(8). $\neg(\forall x)(\neg P(x)\lor Q(x))\lor((\forall x)\neg P(x)\lor(\exists x)Q(x))$

$=(\exists x)(P(x)\land\neg Q(x))\lor((\forall y)\neg P(y)\lor(\exists z)Q(z))$

$=(\exists x)(\exists z)(\forall y)(P(x)\land\neg Q(x))\lor\neg P(y)\lor Q(z))$

(9) $(\forall x)(\neg P(x)\lor(\exists y)Q(x,y))\lor(\forall z)R(z)$

$=(\forall x)(\exists y)(\forall z)(\neg P(x)\lor Q(x,y)\lor R(z))$

$=(\forall x)(\forall z)(\neg P(x)\lor Q(x,f(x))\lor R(z))$

(10) $(\forall x)(\forall z)(\forall v)P(x,a,z,f(x,z),v)$

5.(1) $((\forall x)(P(x)\lor Q(x))\land(\forall x)(Q(x)\to\neg R(x)))\to(\exists x)(R(x)\to P(x))$

$=(\exists x)(\neg P(x)\land\neg Q(x))\lor(\exists x)(Q(x)\land R(x))\lor(\exists x)(\neg R(x)\lor P(x))$

$=(\exists x)((\neg P(x)\land\neg Q(x))\lor(Q(x)\land R(x))\lor\neg R(x)\lor P(x))$

$=(\exists x)((\neg P(x)\land\neg Q(x))\lor(Q(x)\lor\neg R(x))\lor P(x))$

$=(\exists x)((\neg Q(x)\lor P(x))\lor Q(x)\lor\neg R(x))$

$=T$

归结法：

$(\forall x)(P(x)\lor Q(x))\land(\forall x)(\neg Q(x)\lor\neg R(x))\land(\forall x)(R(x)\land\neg P(x))$

子句集：$\{P(x)\lor Q(x),\ \neg Q(x)\lor\neg R(x),\ R(x),\ \neg P(x)\}$

① $\neg P(x)$	前提	
② $P(x)\lor Q(x)$	前提	
③ $Q(x)$	①②作结	
④ $\neg Q(x)\lor\neg R(x)$	前提	
⑤ $\neg R(x)$	③④作结	作结⇒归结
⑥ $R(x)$	前提	
⑦ □	⑤⑥作结	

归结法：陈列子句，最后作结□

① $P(x)\lor Q(x)$ ② $\neg Q(x)\lor\neg R(x)$ ③ $R(x)$ ④ $\neg P(x)$ ⑤ $Q(x)$ [①④归结] ⑥ $\neg Q(x)$ [②③归结] ⑦ □ [⑤⑥归结]

推理规则法：

① $(\forall x)(P(x)\lor Q(x))$	前提
② $(\forall x)(Q(x)\to\neg R(x))$	前提
③ $P(x)\lor Q(x)$	消去全称量词
④ $Q(x)\to\neg R(x)$	②全称量词消去
⑤ $R(x)\to\neg Q(x)$	④置换
⑥ $\neg Q(x)\to P(x)$	③置换
⑦ $R(x)\to P(x)$	⑤⑥三段论
⑧ $(\exists x)(R(x)\to P(x))$	⑦存在量词引入

5.(4) $P(x)$：x为学生；$Q(x)$：x为本科生，$R(x)$ x为研究生；$S(x)$：x为高材生．设 John$=a$．则：

$(\forall x)(P(x)\to((R(x)\land\neg Q(x))\lor(\neg R(x)\land Q(x))))\land(\exists x)(P(x)\land S(x))\land(\neg R(a)\land S(a))\Rightarrow(P(a)\to Q(a))$

推理规则法：

① $(\forall x)(P(x)\to(R(x)\bar{\lor}Q(x)))$	前提
② $\neg R(a)$	前提
③ $P(x)\to R(x)\bar{\lor}Q(x)$	①全称量词消去
④ $P(a)$	附加前提引入
⑤ $R(a)\bar{\lor}Q(a)$	④③分离
⑥ $Q(a)$	②⑤分离
⑦ $P(a)\to Q(a)$	条件证明规则

日期:　　/

(4) 建立子句集:

$\{\neg P(x) \vee (Q(x) \bar{\vee} R(x)), P(b), S(b), \neg R(a), S(a), P(a), \neg Q(a)\}$

① $\neg P(x) \vee (Q(x) \bar{\vee} R(x))$

② $P(b)$

③ $S(b)$

④ $\neg R(a)$

⑤ $S(a)$

⑥ $P(a)$

⑦ $\neg Q(a)$

⑧ $Q(a) \bar{\vee} R(a)$　　①⑥归结

⑨ $Q(a)$　　④⑧归结

⑩ $\square$　　⑦⑨归结