2017-2018 秋离散数学期末试题

1、Simplify:

(a) $\sum_{k=1}^{n} \frac{1}{k(k+1)}$;

(b) $\sum_{k=m}^{n} \binom{n}{k}\binom{k}{m}$.

2、Every day Bob buys either a candy for $1 or a sundae for $2. There are two different flavors of sundae, but only one kind of candy. If he has n dollars, in how many ways can he spend the money?

3、Solve the congruence equation:

$x^2 \equiv 5x \pmod{6}$.

4、Prove the Weinstein Theorem: every set of n+1 Fibonacci numbers, selected from $F_1, F_2, \ldots, F_{2n}$, contains two elements so that one divides the other.

5、Show that a closed Eulerian walk admits a decomposition into cycles, namely, if a graph G has a closed Eulerian walk, then there are cycles $C_1, C_2, \ldots, C_k$, such that $E(G)=E(C_1)\cup E(C_2)\cup \ldots \cup E(C_k)$ and $E(C_i)\cap E(C_j)=\emptyset$.

6、Two people play a game on a graph by alternatively select distinct nodes v_0, v_1, v_2,… such that v_i is adjacent to v_{i-1} for i>0. The last player able to select a node wins. Show that the first player has a winning strategy if the graph has no perfect matching.

7、Show that if G is a simple graph on 11 nodes, then at least one of G & $\overline{G}$ is not planar.

8、There are nine schoolgirls in a school .Everyday all of them take a walk and walk in three-people groups. Is it possible to make a plan such that each girl walks with each other exactly once in four days, and why?

9、How less colors are enough to color a graph so that adjacent nodes get different colors if each node lies on at most k odd cycles?

（Copied by Mr. ZZ　　Typed by Mr. lllllllL）