习题 5.4

2.(1)(3)

(1) $x\le 0$时, $\int x^2+1\,dx=\frac{1}{3}x^3+x+C_1$

$x>0$时, $\int \cos x=\sin x+C_2$

原函数在0处连续:

$\therefore\ C_1=C_2.$

故 $F(x)=\begin{cases}\frac{1}{3}x^3+x+C & (x\le 0)\\ \sin x+C & (x>0)\end{cases}$

改正后

(3) $x\le 0$时, $\int -\sin x=\cos x+C_1$

$\int \frac{1}{\sqrt{x}}=2\sqrt{x}+C_2.$

$F(x)$在0处连续

$\therefore\ 1+C_1=C_2$. 故

$F(x)=\begin{cases}\cos x+C-1 & (x\le 0)\\ 2\sqrt{x}+C & (x>0)\end{cases}$

3.(1)(4)(9)

(1) 令 $x=t^4$, $t>0$, $\sqrt{x\sqrt{x}}=t^3$

$x-x^{-2}=t^4-\frac{1}{x^2}=t^4-t^{-8}$

$dx=4t^3dt$

$\int (t^4-t^{-8})\cdot t^3\cdot 4t^3dt$

$=\int 4t^{10}-4t^{-2}\,dt$

$=\frac{4}{11}t^{11}+4t^{-1}+C \qquad t=x^{\frac{1}{4}}$

$=\frac{4}{11}x^{\frac{11}{4}}+4x^{-\frac{1}{4}}+C$

(4) $(x-1)(3x-2)=3x^2-5x+2$

$\int(3x^2-5x+2)dx=x^3-\frac{5}{2}x^2+2x+C$

(9) $|(x-1)(3x-2)|=\begin{cases}3x^2-5x+2 & (-\infty,\frac{2}{3}]\\ -3x^2+5x-2 & [\frac{2}{3},1]\\ 3x^2-5x+2 & (1,+\infty)\end{cases}$

$\therefore$原函数 $F(x)=\begin{cases}x^3-\frac{5}{2}x^2+2x+C_1 & (-\infty,\frac{2}{3}]\\ -x^3+\frac{5}{2}x^2-2x+C_2 & (\frac{2}{3},1]\\ x^3-\frac{5}{2}x^2+2x+C_3 & (1,+\infty)\end{cases}$

$F(x)$连续.

$\begin{cases}\frac{14}{27}+C_1=-\frac{14}{27}+C_2\\ -\frac{1}{2}+C_2=\frac{1}{2}+C_3\end{cases}$

$\therefore F(x)=\begin{cases}x^3-\frac{5}{2}x^2+2x+C-\frac{28}{27} & (-\infty,\frac{2}{3}]\\ -x^3+\frac{5}{2}x^2-2x+C & [\frac{2}{3},1]\\ x^3-\frac{5}{2}x^2+2x+C-1 & [1,+\infty)\end{cases}$

4.(2)(11)(14)

(2) 设 $x=2\sin t$ 则 $|t|<\frac{\pi}{2}$

对应 $|x|<2$ 且 $dx=2\cos t\,dt$

$\sqrt{4-x^2}=2\cos t$ ($\cos t$ 当然为正)

$\int\frac{4\sin t\cos t}{2\cos t}dt=\int 2\sin t\,dt$

$=-2\cos t+C$

而 $\cos t=\sqrt{1-\frac{x^2}{4}}$

$\therefore$ 原式 $=-\sqrt{4-x^2}+C$

(11) $\int\frac{1}{e^x+e^{-x}}dx$

$=\int\frac{e^x}{e^{2x}+1}dx$，令 $e^x=t$ 则

$x=\ln t$. $\quad dx=\frac{1}{t}dt$

$\int\frac{t}{t^2+1}\cdot\frac{1}{t}dt=\int\frac{dt}{t^2+1}$

$=\arctan t+C$

$\therefore$ 原式 $=\arctan e^x+C$

(14) 令 $2^x=t$ 则 $\quad x=\log_2 t=\frac{\ln t}{\ln 2}$

$dx=\frac{1}{\ln 2}\cdot\frac{1}{t}dt$

$\therefore$ 原式 $=\int\frac{t}{\sqrt{4-t^2\cdot 4}}\cdot\frac{1}{t}\cdot\frac{1}{\ln 2}dt$

$=\frac{1}{2\ln 2}\int\frac{1}{\sqrt{1-t^2}}dt$

$=\frac{1}{2\ln 2}\cdot\arcsin t+C$

$=\frac{1}{2\ln 2}\cdot\arcsin 2^x+C$

5.(4)

$\int\frac{x-1}{x^2-4x+8}dx=\int\frac{(x-2)+1}{(x-2)^2+4}dx$

令 $u=x-2$. $\quad=\int\frac{u+1}{u^2+4}du$

$=\int\frac{u}{u^2+4}du+\int\frac{1}{u^2+4}du$

$\int\frac{u}{u^2+4}du=\frac{1}{2}\ln u^2+4+C$

$\int\frac{1}{u^2+4}du$. 令 $u=2w$. $\quad du=2dw$

$=\int\frac{2}{4w^2+4}dw=\frac{1}{2}\int\frac{1}{w^2+1}dw$

$=\frac{1}{2}\arctan w+C$

$\therefore$ 原式 $=\frac{1}{2}\ln x^2-4x+8+\frac{1}{2}\arctan\frac{x-2}{2}+C$

6.(5)(6)

(5) 令 $2x+1=t$. $\therefore x=\frac{t-1}{2}$

$dx=\frac{1}{2}dt$

原式 $=\int\frac{t-2}{\sqrt{t^2+4}}\cdot\frac{1}{2}dt$

$=\frac{1}{2}\int\frac{t}{\sqrt{t^2+4}}dt-\int\frac{1}{\sqrt{t^2+4}}dt$

$\int\frac{t}{\sqrt{t^2+4}}dt=\sqrt{t^2+4}+C$

$\int\frac{1}{\sqrt{t^2+4}}dt=\ln|t+\sqrt{t^2+4}|+C$

$\therefore$ 原式 $=\frac{1}{2}\sqrt{t^2+4}-\ln|t+\sqrt{t^2+4}|+C$

$=\frac{1}{2}\sqrt{4x^2+4x+5}-\ln|2x+1+\sqrt{4x^2+4x+5}|+C$

改正见后

(6) 令 $x-1=t$. 则 $x=t+1$ $dx=dt$

原式 $=\int\frac{t^2+2t+1}{\sqrt{4-t^2}}dt$

令 $t=2\sin\alpha$. 则 $|\alpha|<\frac{\pi}{2}$ 对应

$|t|<2$. 原式 $=\int\frac{4\sin^2\alpha+4\sin\alpha+1}{2\cos\alpha}d2\sin\alpha$

$=\int 4\sin^2\alpha+4\sin\alpha+1\,d\alpha$

$=\int(2\sin\alpha+1)^2d\alpha$ 这一步离谱

$=\frac{1}{3}\cdot(2\sin\alpha+1)^3\cdot\frac{1}{2\cos\alpha}+C$

又 $x=t+1$. $t=2\sin\alpha$

$\therefore \sin\alpha=\frac{x-1}{2}$ $\cos\alpha=\sqrt{1-\frac{(x-1)^2}{4}}$

$=\frac{1}{2}\sqrt{3+2x-x^2}$

原式 $=\frac{1}{3}\cdot(2\sin\alpha+1)^3\cdot\frac{1}{2\cos\alpha}+C$

$=\frac{1}{3}\cdot x^3\cdot\frac{1}{\sqrt{3+2x-x^2}}+C$

习题 5.4

7.(1)(2)(4)(5)(10)

(1) $\int x\cdot\cos 2x\,dx$

$\int t\cdot\cos t\cdot dt=t\cdot\sin t-\int\sin t\,dt$

$=t\sin t+\cos t+C$

$\therefore\int 2x\cdot\cos 2x\cdot d2x=2x\cdot\sin 2x+\cos 2x+C$

$\therefore\int x\cdot\cos 2x\,dx=\frac{1}{2}x\sin 2x+\frac{1}{4}\cos 2x+C$

(2) $\int t\cdot e^{-t}dt$

$= -t\cdot e^{-t} + \int e^{-t}dt$

$= -t\cdot e^{-t} - e^{-t} + C$

$\therefore \int 3x e^{-3x} d3x = -3x\cdot e^{-3x} - e^{-3x} + C$

$\therefore \int x\cdot e^{-3x}dx = -\frac{1}{3}x\cdot e^{-3x} - \frac{e^{-3x}}{9} + C$

(4) $\int x \arctan x\, dx$

$\int 2x\cdot \arctan x\, dx$

$= x^2\cdot \arctan x - \int \frac{x^2}{1+x^2}dx$

$= x^2\cdot \arctan x - \int (1 - \frac{1}{1+x^2})dx$

$= x^2\cdot \arctan x - x + \arctan x + C$

$\therefore$ 原式 $= \frac{x^2+1}{2}\cdot \arctan x - \frac{1}{2}x + C$

(5) $\int x \ln^{(x-1)} dx$

$\int 2x \ln^{(x-1)}dx = x^2 \ln^{x-1} - \int x^2\cdot\frac{1}{x-1}dx$

$\int x^2\cdot\frac{1}{x-1}dx = \int x+1\, dx + \int \frac{1}{x-1}dx$

$= \frac{1}{2}x^2 + x + \ln^{x-1} + C$

$\therefore$ 原式 $= \frac{1}{2}x^2 \ln^{x-1} - \frac{1}{4}x^2 - \frac{1}{2}x - \frac{1}{2}\ln^{x-1} + C$

10. $\int e^x\cdot \sin^2 x\, dx$

$= e^x\cdot \sin^2 x - \int e^x \sin 2x\, dx$

$= e^x\cdot \sin^2 x - e^x \sin 2x + \int e^x\cdot 2\cos 2x\, dx$

$= e^x(\sin^2 x - \sin 2x) + \int e^x \cancel{(2\sin^2 x)}\, dx$

$\therefore \int e^x\cdot \sin^2 x = \frac{e^x}{2}(\cancel{\sin^2 x - \sin 2x + 2})$

重大错误 $2\cos 2x = 2\cdot(1-2\sin^2 x)$

降幂升角？二倍角

$= e^x(\sin^2 x - \sin 2x + 2) - 4\int e^x \sin^2 x\, dx$

$\int e^x \sin^2 x\, dx = \frac{1}{5}\cdot e^x\cdot(\sin^2 x - \sin 2x + 2) + C$

习题 5.5

1. (1)(3)(7)

$\frac{1}{(x+1)(x+2)^2} = \frac{A}{x+1} + \frac{B}{x+2} + \frac{C}{(x+2)^2}$

$= \frac{Ax^2+4Ax+4A + Bx^2+3Bx+2B+Cx+C}{(x+1)(x+2)^2}$

$\therefore \begin{cases} A+B=0 \\ 4A+3B+C=0 \\ 4A+2B+C=1 \end{cases} \Rightarrow \begin{cases} A=1 \\ B=-1 \\ C=-1 \end{cases}$

$$\therefore \int \frac{1}{(x+1)(x+2)^2}dx$$
$$=\int\left(\frac{1}{x+1}-\frac{1}{x+2}-\frac{1}{(x+2)^2}\right)dx$$
$$=\ln|x+1|-\ln|x+2|+\frac{1}{x+2}+C$$

(3) $\int \frac{x^3+1}{x^3-5x^2+6x}dx$

$$\frac{x^3+1}{x^3-5x^2+6x}=1+\frac{5x^2-6x+1}{x^3-5x^2+6x}$$
$$\frac{5x^2-6x+1}{x(x-2)(x-3)}=\frac{A}{x}+\frac{B}{x-2}+\frac{C}{x-3}$$
$$=\frac{Ax^2-5Ax+6A+Bx^2-3Bx+Cx^2-2Cx}{x(x-2)(x-3)}$$
$$\therefore\begin{cases}5=A+B+C\\-6=-5A-3B-2C\\1=6A\end{cases}$$
$$\therefore A=\frac{1}{6}\quad B=-\frac{9}{2}\quad C=\frac{28}{3}$$
原式$=x+\frac{1}{6}\ln|x|-\frac{9}{2}\ln|x-2|+\frac{28}{3}\ln|x-3|+C$

(7) $\int\frac{x^7}{(1-x^2)^5}dx=\int\frac{\frac{1}{x^3}}{(\frac{1}{x^2}-1)^5}dx$

令 $\frac{1}{x^2}=t\quad dt=-2\cdot\frac{1}{x^3}dx$

$\therefore$ 原式$=\int\frac{1}{(t-1)^5}\cdot\frac{-dt}{2}$

$$=\frac{1}{8}\frac{1}{(t-1)^4}+C=\frac{1}{8}\cdot\frac{1}{(1-\frac{1}{x^2})^4}+C$$
$$=\frac{x^8}{8(1-x^2)^4}+C$$

2.(1)(2)(7)(9)

2.(1)

$$\int\frac{\sin^4x}{\cos^3x}dx=\int\frac{\sin^4x\,(\sin'x)}{\cos^4x}dx$$
令 $\sin x=t$ $==\int\frac{t^4}{(1-t^2)^2}dt$
$$=\int 1+\frac{2t^2-1}{(1-t)^2(1+t)^2}dt$$
$$=\int\left(1-\frac{3}{4}\frac{1}{1-t}+\frac{1}{4}\frac{1}{(1-t)^2}-\frac{3}{4}\frac{1}{1+t}+\frac{1}{4}\frac{1}{(1+t)^2}\right)dt$$
$$=t+\frac{3}{4}\ln|1-t|-\frac{3}{4}\ln|t+1|+\frac{1}{4}\frac{1}{(1-t)}-\frac{1}{4}\cdot\frac{1}{1+t}+C$$
$$=\sin x+\frac{3}{4}\ln\left(\frac{1-\sin x}{1+\sin x}\right)+\frac{\sin x}{2\cos^2x}+C$$

2.(2) $\int\frac{1}{\sin x\cdot\cos^4x}dx$

$$=-\int\frac{\cos'x}{\sin^2x\cdot\cos^4x}dx\ \underline{\underline{令\cos x=t}}$$
$$-\int\frac{dt}{(1-t^2)t^4}=-\int\left(\frac{1}{t^4}+\frac{1}{t^2}+\frac{1}{2(1-t)}+\frac{1}{2(1+t)}\right)dt$$
$$=\frac{1}{3}\cdot\frac{1}{t^3}+\frac{1}{t}+\frac{1}{2}\ln\left|\frac{1-t}{1+t}\right|+C$$
$$=\frac{1}{3}\cdot\frac{1}{\cos^3x}+\frac{1}{\cos x}+\frac{1}{2}\ln\left(\frac{1-\cos x}{1+\cos x}\right)+C$$

2.(7)

$$\int \frac{\sin x}{\sin x+\cos x}\,dx=\int \frac{\sin x\cos x-\sin^2 x}{\cos^2 x-\sin^2 x}\,dx$$

$$=\frac{1}{2}\int \frac{\sin 2x+\cos 2x-1}{\cos 2x}\,dx$$

令 $2x=t$. $\therefore dx=\frac{1}{2}dt$

原式 $=\frac{1}{4}\int(\tan t+1-\sec t)\,dt$

$$=\frac{1}{4}\ln|\sec t|+\frac{1}{4}t-\frac{1}{4}\ln|\sec t+\tan t|+C$$

$$=\frac{1}{4}\ln\left|\frac{1}{\cos 2x}\right|+\frac{x}{2}-\frac{1}{4}\ln|\sec 2x+\tan 2x|+C$$

$$=\frac{1}{2}x-\frac{1}{4}\ln|1+\sin 2x|+C$$

2.9 $\int \frac{\cos x}{\sin x+\cos x}\,dx$

$$=\int\left(1-\frac{\sin x}{\sin x+\cos x}\right)dx$$

$$=\frac{x}{2}+\frac{1}{4}\ln(1+\sin 2x)+C$$

改正：习题5.4.一2.3

Def: 原函数

设f在区间I上定义，而F是f在I上的一个原函数.则$\forall x\in I$有$F(x)=f(x)$

故$F(x)$可导.则$F'(0)=f(0)=0$. 即 $\lim\limits_{x\to0^+}F(x)=\lim\limits_{x\to0^-}F(x)$

但 $\lim\limits_{x\to0^+}f(x)\neq\lim\limits_{x\to0^-}f(x)$

$\therefore F(x)$不存在

5.4.6.(6) $\int \frac{x^2}{\sqrt{3+2x-x^2}}\,dx$

令 $x-1=t$. 则 $x=t+1$. $dx=dt$

原式 $=\int \frac{(t+1)^2}{\sqrt{4-t^2}}\,dt$. 令 $t=2\sin\theta$

则 $x=2\sin\theta+1$ $dt=2\cos\theta\,d\theta$

原式 $=\int \frac{(2\sin\theta+1)^2}{2\cos\theta}\cdot 2\cos\theta\,d\theta$

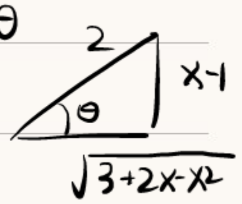
$$=\int 4\sin^2\theta+4\sin\theta+1\,d\theta$$

$$=2\int 1-\cos 2\theta\,d\theta+4\int\sin\theta\,d\theta+\theta$$

$$=3\theta-2\int\cos 2\theta\,d\theta+4\int\sin\theta\,d\theta$$

$$=3\theta-4\cos\theta-\sin 2\theta$$

$\theta=\arcsin\frac{x-1}{2}$



$\cos\theta=\frac{\sqrt{3+2x-x^2}}{2}$

$\sin 2\theta=2\cos\theta\cdot\sin\theta$

$$=\sqrt{3+2x-x^2}\cdot\frac{x-1}{2}$$

$\therefore$ 原式 $=3\arcsin\frac{x-1}{2}-2\sqrt{3+2x-x^2}-\frac{x-1}{2}\sqrt{3+2x-x^2}+C$

$$=3\arcsin\frac{x-1}{2}-\frac{x+3}{2}\sqrt{3+2x-x^2}+C$$