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5.1

$-(A-I)(A^T-I)=-(AA^T-AI-IA^T+I^2)$ Since A is orthogonal, So $AA^T=A^TA=I$

Hence $-(A-I)(A^T-I)=-(I-A-A^T+I)=A+A^T-2I$

5.2

Since $f(t)$ is an orthognal matrix, So from 5.1. $-(f(t)-I)(f(t)^T-I)=f(t)+f(t)^T-2I$.

$$\lim_{t\to 0}\frac{-(f(t)-I)(f(t)^T-I)}{t}=\lim_{t\to 0}\frac{(f(t)+f(t)^T-2I)}{t}=\lim_{t\to 0}\frac{(f(t)-I)+(f(t)^T-I)}{t}$$

And $f(0)=f(0)^T=I$ Then it equals $=\lim_{t\to 0}\frac{(f(t)-f(0))+(f(t)^T-f(0))}{t}$

We know that if $\lim_{t\to 0}A_t=M$ and $\lim_{t\to 0}B_t=N$. (M and N exist)

then $\lim_{t\to 0}(A_t+B_t)=\lim_{t\to 0}A_t+\lim_{t\to 0}B_t=M+N$

And we know $\lim_{dt\to 0}\frac{f(t+dt)-f(t)}{dt}$ exists at all t. Hence $\lim_{t\to 0}\frac{(f(t)-f(0))}{t}$ exists and equal to $f(0)$

and $\lim_{t\to 0}\frac{(f(t)^T-f(0))}{t}$ exists and equal to $f(0)^T$ Then $\lim_{t\to 0}\frac{(f(t)-f(0))+(f(t)^T-f(0))}{t}=f(0)+f(0)^T$

5.3

$$\lim_{t\to 0}\frac{-(f(t)-I)(f(t)^T-I)}{t}=\lim_{t\to 0}\frac{-(f(t)-I)(f(t)^T-I)\cdot t}{t\cdot t}=\lim_{t\to 0}\left(\frac{-(f(t)-I)}{t}\cdot\frac{(f(t)^T-I)}{t}\cdot t\right)$$

And we know if $\lim_{t\to 0}A_t=M$ and $\lim_{t\to 0}B_t=N$. (M and N exist)

then $\lim_{t\to 0}(A_t\cdot B_t)=\lim_{t\to 0}A_t\cdot\lim_{t\to 0}B_t=M\cdot N$ And $\lim_{t\to 0}\frac{f(t)^T-I}{t}=\lim_{t\to 0}\frac{f(t)^T-f(0)}{t}=f(0)^T$

$\lim_{t\to 0}t=0$ $\lim_{t\to 0}\frac{f(t)-I}{t}=\lim_{t\to 0}\frac{f(t)-f(0)}{t}=f(0)$ Then $\lim_{t\to 0}\left(\frac{f(t)^T-I}{t}\cdot t\right)=f(0)^T\cdot 0=0$

Then $\lim_{t\to 0}\left(\frac{-(f(t)-I)}{t}\cdot\frac{(f(t)^T-I)}{t}\cdot t\right)=\lim_{t\to 0}\frac{-(f(t)-I)}{t}\cdot\lim_{t\to 0}\left(\frac{f(t)^T-I}{t}\cdot t\right)=-f(0)\cdot 0=0$

And from 5.2. $\lim_{t\to 0}\frac{-(f(t)-I)(f(t)^T-I)}{t}=f(0)+f(0)^T$ So $f(0)+f(0)^T=0$

Hence $f(0)$ must be skew symmetric

5.4: we know that $f(t)$ is orthogonal Then $f(t) \cdot f(t)^T = I$
Suppose $F(t) = f(t) \cdot f(t)^T$ $G(t) = I$. We know $\lim_{dt \to 0} \frac{f(t+dt) - f(t)}{dt}$ exists at all t
then $F'(t) = f'(t) \cdot f(t)^T + f(t) \cdot f'(t)^T$ $G'(t) = 0$
Hence $f'(t) \cdot f(t)^T + f(t) \cdot f'(t)^T = 0$ $(f'(t) \cdot f(t)^T) + (f'(t) \cdot f(t)^T)^T = 0$ Hence $f'(t) \cdot f(t)^T$ is skew symmetric
Hence $f(t)^{-1} = f(t)^T$ Then $f'(t) \cdot f(t)^{-1} = f'(t) \cdot f(t)^T$, which is skew symmetric

5.5
$f'(t) \cdot f(t)^{-1} = A$ Then $f'(t) = A \cdot f(t)$. Note that this is a first-order Homogenous differential equation(一阶齐次微分方程)
$\frac{d f(A)}{dt} = A \cdot f(t)$ And e^{At} is a soultion to it. So then general solution to $f(t)$ is $e^{At} \cdot B$ where B is a constant matrix depending on other initial conditions
$f(0) = e^{A0} \cdot B = B$ And $f: R \to SO_n$ Then $B = f(0) \in SO_n$