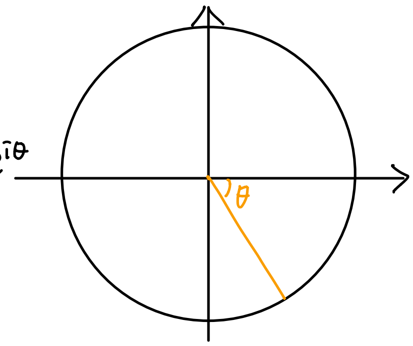
作业 1

Problem Set 9.1

3、(a) $|z|=\sqrt{6^2+8^2}=10$. $z=10\left(\frac{3}{5}-\frac{4}{5}i\right)=10e^{i\theta}$



(b) $z=10e^{i\theta}\cdot 10e^{i\theta}=100\cdot e^{i2\theta}$

$\therefore |z|=100$，角度 $2\theta+2k\pi$ $(k\in Z)$

(c) $z=\frac{1}{6-8i}=\frac{6+8i}{(6+8i)(6-8i)}=\frac{6+8i}{36+64}=\frac{1}{100}\cdot 10\left(\frac{3}{5}+\frac{4}{5}i\right)$
$=\frac{1}{10}e^{i(-\theta)}$

$\therefore |z|=\frac{1}{10}$，角度 $-\theta+2k\pi$ $(k\in Z)$.

(d) $z=(6+8i)^2=\left(10\cdot e^{i(-\theta)}\right)\left(10\cdot e^{i(-\theta)}\right)=100e^{i(2\theta)}$

$\therefore |z|=100$，角度 $-2\theta+2k\pi$ $(k\in Z)$.

10、 $z+\bar{z}$ 为实数

$z-\bar{z}$ 为0或纯虚数.

$z\times\bar{z}=(a+bi)(a-bi)=a^2+b^2$. 又 $\because z\neq 0$，$\therefore z\times\bar{z}$ 为正实数

$\frac{z}{\bar{z}}=\frac{z\times z}{z\times\bar{z}}=\frac{(a+bi)^2}{a^2+b^2}=\frac{1}{a^2+b^2}\left(a^2-b^2+2abi\right)$

$\therefore \left|\frac{z}{\bar{z}}\right|=\frac{1}{a^2+b^2}\sqrt{(a^2-b^2)^2+4a^2b^2}=\frac{1}{a^2+b^2}\sqrt{a^4+b^4+2a^2b^2}$
$=\frac{1}{a^2+b^2}\cdot a^2+b^2=1$

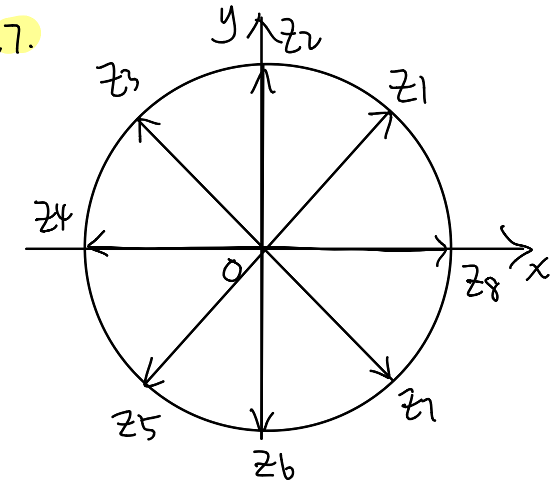
15. (b) $\cos 2\theta+i\sin 2\theta=e^{i2\theta}$.

平方：$(e^{i2\theta})^2=e^{i4\theta}=\cos 4\theta+i\sin 4\theta$.

(d) $5-5i=5\sqrt{2}\left(\frac{\sqrt{2}}{2}-\frac{\sqrt{2}}{2}i\right)=5\sqrt{2}e^{i(-\frac{\pi}{4})}$

平方：$\left(5\sqrt{2}e^{i(-\frac{\pi}{4})}\right)^2=50\cdot e^{i(-\frac{\pi}{2})}=50(-i)=-50i$

17.



$$z = e^{i(-\frac{2\pi}{8})}$$
$$= e^{i(-\frac{\pi}{4})}$$
$$= \cos(-\tfrac{\pi}{4}) + i\sin(-\tfrac{\pi}{4})$$
$$= \frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2}i$$

以注：$z^8=1$ 的解 = 几何上将圆8分的证明：

$$z^8 = 1 = e^{2\pi k i} \Rightarrow \quad z = e^{\frac{2\pi k i}{8}}, (k \in \mathbb{Z})$$

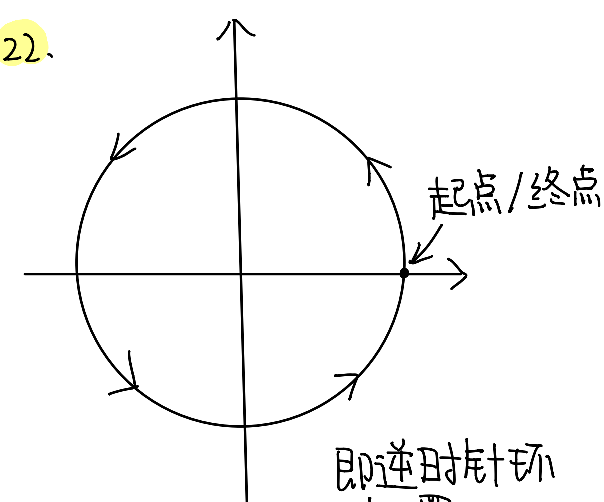
19. $\cos 3\theta + i\sin 3\theta = e^{3i\theta} = (e^{i\theta})^3 = (\cos\theta + i\sin\theta)^3$

$= \cos^3\theta - i\sin^3\theta + 3\cos^2\theta\, i\sin\theta - 3\cos\theta\sin^2\theta.$

$= (\cos^3\theta - 3\cos\theta\sin^2\theta) + i(-\sin^3\theta + 3\cos^2\theta\sin\theta).$

对比实部和虚部可得：
$$\begin{cases} \cos 3\theta = \cos^3\theta - 3\cos\theta\sin^2\theta. \\ \sin 3\theta = -\sin^3\theta + 3\cos^2\theta\sin\theta. \end{cases}$$

22.



即逆时针环绕一圈