1、设度为1的结点数为n_1，则总结点数为$\sum n_i$，总度数为$2\sum n_i-2$。

$\therefore \sum i n_i = 2\sum n_i - 2$。

$n_1+\sum_{i=2}^{k} i n_i = 2n_1 + 2\sum_{i=2}^{k} n_i - 2$，$n_1 = 2+\sum_{i=2}^{k}(i-2)n_i$。

4.(a) 每条边任意定向后，得v_1的基本关联矩阵$B_1=$

$$\begin{array}{cccccccccc}
\left[\begin{matrix} -1 & 0 & 0 & 1 & 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 & 1 & 1 & 1 & 0 \\ 0 & -1 & 0 & 0 & 0 & 0 & -1 & -1 & 0 & 1 \\ 0 & 0 & -1 & 0 & -1 & -1 & 0 & 0 & -1 & -1 \end{matrix}\right] \\
\begin{matrix} e_1 & e_2 & e_3 & e_4 & e_5 & e_6 & e_7 & e_8 & e_9 & e_{10} \end{matrix}
\end{array}$$

$$B_1B_1^T=\begin{bmatrix} 4 & -1 & 0 & -2 \\ -1 & 4 & -2 & -1 \\ 0 & -2 & 4 & -1 \\ -2 & -1 & -1 & 5 \end{bmatrix}$$

$\det(B_1B_1^T)=4\times 48-(-1)\times(-31)-(-2)\times(-30)$
$=101$

(b) 去掉(v_1,v_5)，$B_1=$

$$\begin{array}{c}
\left[\begin{matrix} -1 & 0 & 1 & 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 1 & 1 & 1 & 0 \\ 0 & -1 & 0 & 0 & 0 & -1 & -1 & 0 & 1 \\ 0 & 0 & 0 & -1 & -1 & 0 & 0 & -1 & -1 \end{matrix}\right] \\
\begin{matrix} e_1 & e_2 & e_4 & e_5 & e_6 & e_7 & e_8 & e_9 & e_{10} \end{matrix}
\end{array}$$

$$B_1B_1^T=\begin{bmatrix} 4 & -1 & 0 & -2 \\ -1 & 4 & -2 & -1 \\ 0 & -2 & 4 & -1 \\ -2 & -1 & -1 & 4 \end{bmatrix}$$

$\det(B_1B_1^T)=101+(4-5)\times\begin{vmatrix} 4 & -1 & 0 \\ -1 & 4 & -2 \\ 0 & -2 & 4 \end{vmatrix}=57$

$\therefore$ 必含(v_1,v_5)的树有44种。

(c) 去掉(v_4,v_5)，$B_1=$

$$\begin{array}{c}
\left[\begin{matrix} -1 & 0 & 0 & 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 & 1 & 1 & 1 \\ 0 & -1 & 0 & 0 & 0 & 0 & -1 & -1 & 0 \\ 0 & 0 & -1 & 0 & -1 & -1 & 0 & 0 & -1 \end{matrix}\right] \\
\begin{matrix} e_1 & e_2 & e_3 & e_4 & e_5 & e_6 & e_7 & e_8 & e_9 \end{matrix}
\end{array}$$

$$B_1B_1^T=\begin{bmatrix} 4 & -1 & 0 & -2 \\ -1 & 4 & -2 & -1 \\ 0 & -2 & 3 & 0 \\ -2 & -1 & 0 & 4 \end{bmatrix}$$

$\det(B_1B_1^T)=2\times(-24)+3\times 36=60$。

8. 基本关联矩阵$B_1=\begin{bmatrix} & & & 1 & \cdots & 1 & \\ & & & 0 & & & \cdots \\ -1 & & & -1 & \ddots & & \\ \vdots & -1 & & 0 & & \ddots & \\ 0 & 0 & \cdots & -1 & \vdots & & -1 \end{bmatrix}$

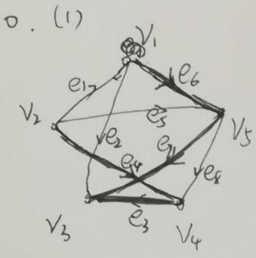
$\det(B_1B_1^T)=\det\begin{bmatrix} nI & -1_{(m-1)\times n} \\ -1_{n\times(m-1)} & mI \end{bmatrix}$

$=\det\begin{bmatrix} nI & -1 \\ 0 & mI-\frac{1}{n} \end{bmatrix}=\det\begin{bmatrix} nI & 0 \\ 0 & mI-\frac{1}{n} \end{bmatrix}=n^{m-1}\cdot\det\begin{bmatrix} m-\frac{m-1}{n} & -\frac{m-1}{n} & \cdots & -\frac{m-1}{n} \\ -\frac{m-1}{n} & m-\frac{m-1}{n} & & \\ \vdots & & \ddots & \\ -\frac{m-1}{n} & & & m-\frac{m-1}{n} \end{bmatrix}_{n\times n}$

$=\frac{n^{m-1}}{n^n}\cdot\det\begin{bmatrix} nm-1 & -1 & \cdots & -1 \\ -1 & nm-1 & & \\ -1 & & \ddots & \\ \vdots & & & \\ -1 & & & nm-1 \end{bmatrix}=\frac{n^{m-1}}{\left(\frac{n}{m-1}\right)^n}\left(\frac{nm}{m-1}\right)^{n-1}(-n+1)\left(\frac{nm}{m-1}\right)^{n-1}=n^{m-1}\cdot\frac{m-1}{n}\cdot m^{n-1}\cdot\frac{n}{m-1}$

$=n^{m-1}m^{n-1}$。

10.（1）



C_1: $e_1, e_6, e_7, \overline{e_3}, \overline{e_4}$ （$\overline{e_i}$表示反向e_i）

C_2: $e_2, \overline{e_7}, \overline{e_6}$

C_3: $e_5, e_7, \overline{e_3}, \overline{e_4}$

C_4: $e_8, e_3, \overline{e_7}$

$$C_f = \begin{array}{c} \begin{bmatrix} 1 & 0 & 0 & 0 & -1 & -1 & 1 & 1 \\ 0 & 1 & 0 & 0 & 0 & 0 & -1 & -1 \\ 0 & 0 & 1 & 0 & -1 & -1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 1 & 0 & 0 & -1 \end{bmatrix} \\ \begin{matrix} e_1 & e_2 & e_5 & e_8 & e_3 & e_4 & e_6 & e_7 \end{matrix} \end{array}.$$

5.(a)
$$\vec{B}_1 = \begin{array}{c} \begin{bmatrix} -1 & 0 & 0 & 1 & 1 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 1 & -1 & 0 & -1 & 0 \\ 0 & -1 & 0 & 0 & 0 & -1 & 1 & 0 & 0 & -1 \\ 0 & 0 & -1 & 0 & -1 & 0 & 0 & 1 & 1 & 1 \end{bmatrix} \\ \begin{matrix} e_1 & e_2 & e_3 & e_4 & e_5 & e_6 & e_7 & e_8 & e_9 & e_{10} \end{matrix} \end{array}$$

$$\vec{B}_1 B_1^T = \begin{bmatrix} 2 & 0 & 0 & -1 \\ -1 & 3 & -1 & -1 \\ 0 & -1 & 3 & -1 \\ -1 & 0 & 0 & 2 \end{bmatrix}$$

$$\det(\vec{B}_1 B_1^T) = 2\times 16 - (-1)\times(-8) = 24$$

(b)
$$\vec{B}_1 = \begin{array}{c} \begin{bmatrix} -1 & 0 & 1 & 1 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 & 1 & -1 & 0 & -1 & 0 \\ 0 & -1 & 0 & 0 & -1 & 1 & 0 & 0 & -1 \\ 0 & 0 & 0 & -1 & 0 & 0 & 1 & 1 & 1 \end{bmatrix} \\ \begin{matrix} e_1 & e_2 & e_4 & e_5 & e_6 & e_7 & e_8 & e_9 & e_{10} \end{matrix} \end{array}$$

$$\vec{B}_1 B_1^T = \begin{bmatrix} 2 & 0 & 0 & -1 \\ -1 & 3 & -1 & -1 \\ 0 & -1 & 3 & -1 \\ -1 & 0 & 0 & 1 \end{bmatrix}$$

$$\det(\vec{B}_1 B_1^T) = 2\times 8 - (-1)\times(-8) = 8$$

(c)
$$B_1 = \begin{array}{c} \begin{bmatrix} -1 & 0 & 0 & 1 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & -1 & 0 & -1 & 0 \\ 0 & -1 & 0 & 0 & -1 & 1 & 0 & 0 & -1 \\ 0 & 0 & -1 & -1 & 0 & 0 & 1 & 1 & 1 \end{bmatrix} \\ \begin{matrix} e_1 & e_2 & e_3 & e_5 & e_6 & e_7 & e_8 & e_9 & e_{10} \end{matrix} \end{array}$$

$$\vec{B}_1 B_1^T = \begin{bmatrix} 2 & 0 & 0 & -1 \\ 0 & 2 & -1 & -1 \\ 0 & -1 & 3 & -1 \\ -1 & 0 & 0 & 2 \end{bmatrix}$$

$$\det(\vec{B}_1 B_1^T) = 2\times 10 - (-1)\times(-5) = 15$$

数目为$24-15=9$.