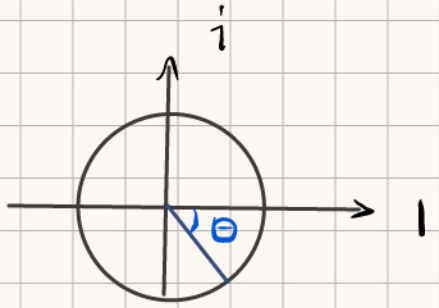
Problem set 9.1



3.

$|a|=10$ 转角为 θ

$|b|=100$ 转角为 2θ

① 转角相加，模长相乘

② $b=36-96i+64i^2=-28-96i$

$c=\frac{1}{6-8i}=\frac{6+8i}{100}=\frac{a}{|a|^2}$ ⇐ $|a|^2=a\cdot\bar{a}$

$=\frac{3}{50}+\frac{2}{25}i$ 转角为 $-\theta$ $|c|=\frac{1}{10}$

即 $z_1\cdot z_2=1$，则 $|z_1\cdot z_2|=1$，$|z_1|\cdot|z_2|=1$

$|d|=100$ 转角为 -2θ

10.

纯虚数：$z=bi,\ b\in\mathbb{R}\setminus\{0\}$

$z+\bar{z}$ 恒为实数；$z-\bar{z}$ 恒为纯虚数或零

$z\cdot\bar{z}$ 恒为正实数；$\left|\frac{z}{\bar{z}}\right|$ 恒为 1.

注：$|z|^2\neq z^2$. $\left|\frac{a+bi}{a-bi}\right|=\left|\frac{a^2+b^2+2abi}{a^2+b^2}\right|\neq\sqrt{\frac{(a^2-b^2+2abi)^2}{(a^2+b^2)^2}}$

↳ 表示取模运算，不是绝对值

$\frac{\sqrt{(a-b)^2+4a^2b^2}}{\sqrt{(a^2+b^2)^2}}$

15. $e^{i\theta}=\cos\theta+i\sin\theta$

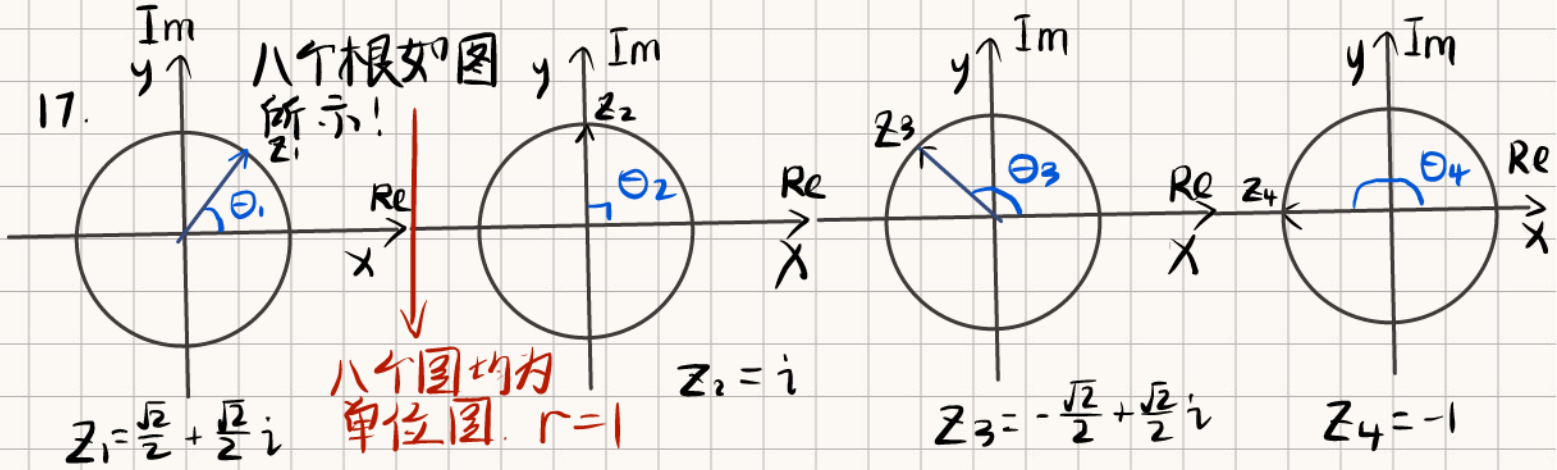
(b) $\cos2\theta+i\sin2\theta=e^{i2\theta}$

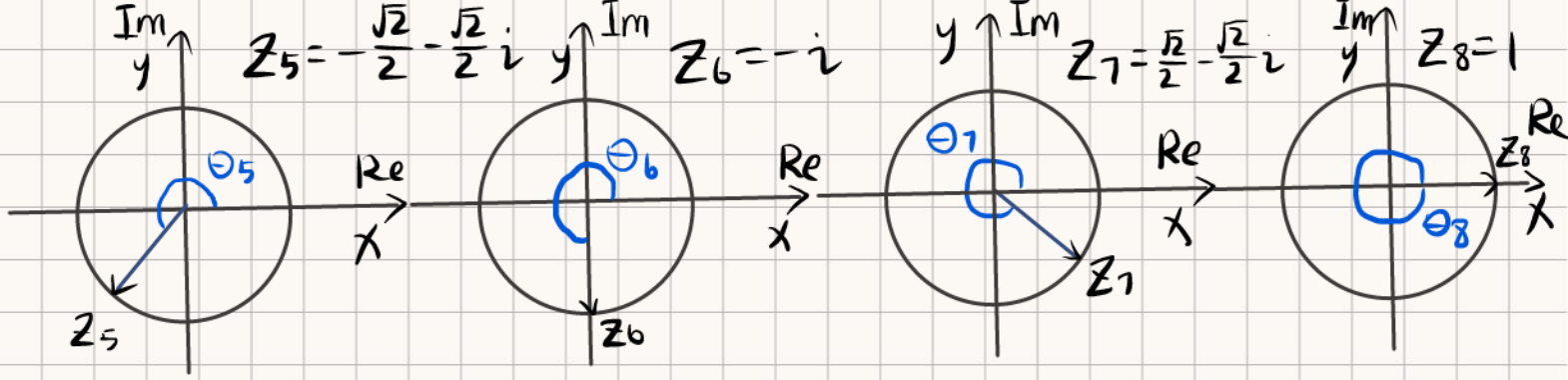
$(e^{i2\theta})^2=e^{i4\theta}$

(d) $5-5i=5\sqrt{2}\left(\frac{\sqrt{2}}{2}-\frac{\sqrt{2}}{2}i\right)=5\sqrt{2}\,e^{-\frac{\pi}{4}i}$

$(5\sqrt{2}\,e^{-\frac{\pi}{4}i})^2=50e^{-\frac{\pi}{2}i}=-50i$

17.





$$z=\bar{\omega}=e^{-\frac{\pi}{4}i}=\left(\cos-\frac{\pi}{4}+i\sin-\frac{\pi}{4}\right)=\frac{\sqrt{2}}{2}-\frac{\sqrt{2}}{2}i$$

19. $e^{3i\theta}=\cos3\theta+i\sin3\theta$

$$(e^{i\theta})^3=(\cos\theta+i\sin\theta)^3$$
$$=\cos^3\theta+\cos^2\theta\cdot i\sin\theta+3\cos\theta(i\sin\theta)^2+\sin^3\theta\cdot i^3$$
$$=(\cos^3\theta-3\cos\theta\cdot\sin^2\theta)+(-\sin^3\theta+3\cos^2\theta\sin\theta)i$$

$\therefore$ $\cos3\theta=\cos^3\theta-3\cos\theta\cdot\sin^2\theta$

$\sin3\theta=-\sin^3\theta+3\cos^2\theta\cdot\sin\theta$

22.