homework 8. 习题三.

10. (1) 设基本回路矩阵 $C_f = [I \quad C_{f12}]$

将 B_5 各列重新排列，使树边排列在前，得 B_5'

$$B_5' = \begin{array}{c}\left[\begin{array}{cccc:cccc} -1 & 1 & 0 & 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & -1 & 0 & 0 & -1 & 0 & 0 & -1 \\ 0 & 0 & 0 & -1 & 1 & -1 & 0 & 0 \end{array}\right] = [B_{11} \quad B_{12}] \\ e_1\ e_2\ e_5\ e_8\ e_3\ e_4\ e_6\ e_7\end{array}, \quad B_{12}^{-1} = \begin{bmatrix} 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & -1 & -1 & -1 \end{bmatrix}$$

由 $C_{f12} = -B_{11}^T B_{12}^{-T}$

$$= \ominus \begin{bmatrix} -1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix} \begin{bmatrix} 0 & 0 & 1 & 0 \\ 1 & 1 & 0 & -1 \\ 0 & 0 & 0 & -1 \\ 1 & 0 & 0 & -1 \end{bmatrix} = \begin{bmatrix} -1 & -1 & 1 & 1 \\ 0 & 0 & -1 & -1 \\ -1 & -1 & 0 & 1 \\ 1 & 0 & 0 & -1 \end{bmatrix}$$

$$\therefore C_f = [I \quad C_{f12}] = \begin{bmatrix} 1 & 0 & 0 & 0 & -1 & -1 & 1 & 1 \\ 0 & 1 & 0 & 0 & 0 & 0 & -1 & -1 \\ 0 & 0 & 1 & 0 & -1 & -1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 1 & 0 & 0 & -1 \end{bmatrix}$$

(2) 将 B_5 的边重新排列，使树边在后：

$$B_5' = \begin{array}{c}\left[\begin{array}{cccc:cccc} -1 & 0 & 0 & 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & -1 & 0 & -1 & -1 & 0 & 0 & 0 \\ 0 & 1 & -1 & 0 & 0 & 0 & 0 & -1 \end{array}\right] = [B_{11} \quad B_{12}] \\ e_1\ e_3\ e_4\ e_7\ e_2\ e_5\ e_6\ e_8\end{array}$$

设基本割集矩阵 $S_f = [S_{f11} \quad I]$ 则 $S_{f11} = B_{12}^{-1} B_{11}$

$$S_{f11} = \begin{bmatrix} 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix} \begin{bmatrix} -1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 0 & -1 & 0 & -1 \\ 0 & 1 & -1 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ -1 & -1 & 0 & -1 \\ 0 & -1 & 1 & 0 \end{bmatrix}$$

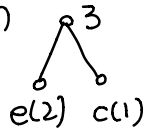
$$\therefore\ S_f=\begin{bmatrix}0&1&0&1&1&0&0&0\\1&0&1&0&0&1&0&0\\-1&-1&0&-1&0&0&1&0\\0&-1&1&0&0&0&0&1\end{bmatrix}$$

14. 出现"a" 5次，"t"4次，"s"3次，"e"2次，"c"1次，空格4次.

分别给节点a，t，空格，s，e，c 赋权 5，4，4，3，2，1.

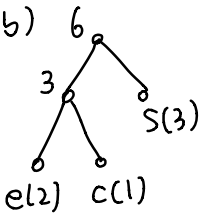
构造 Huffman树：

(a)



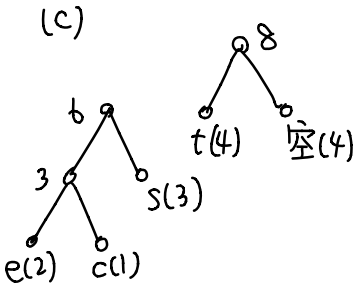
1，2，3，4，4，5

(b)



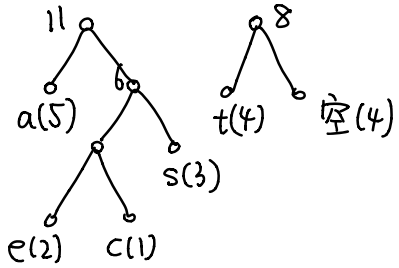
3，3，4，4，5

(c)



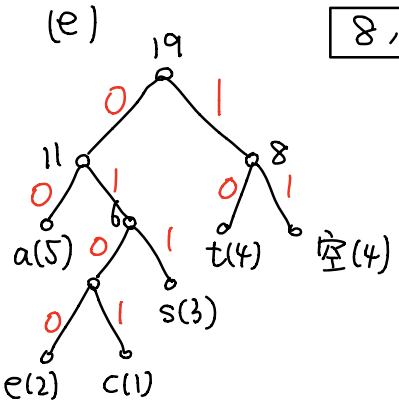
4，4，5，6

(d)



5，6，8.

(e)



8，11

得到编码：

a = 00

t = 10

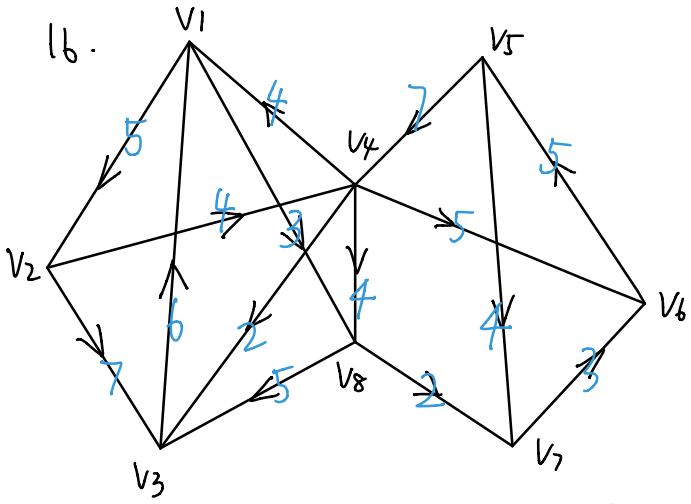
空格：11

S = 011

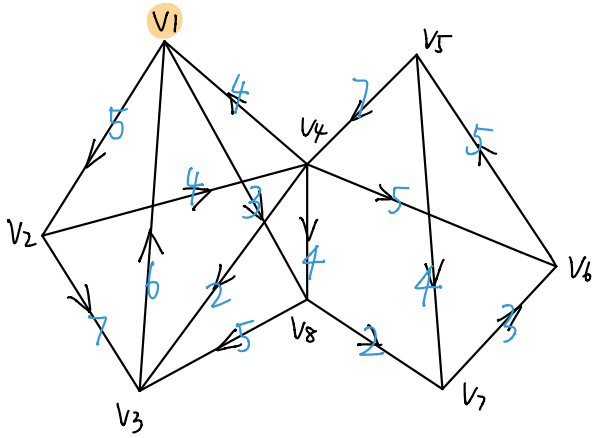
e = 0100

c = 0101

16.



用 Prim 算法：首先将 V1 选为第一个点。



$\pi(V_2) = 5$

$\pi(V_3) = 6$

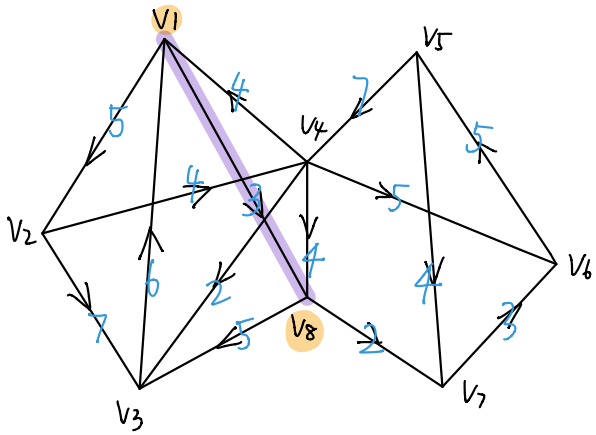
$\pi(V_4) = 4$

$\pi(V_5) = \infty$

$\pi(V_6) = \infty$

$\pi(V_7) = \infty$

$\pi(V_8) = 3$



$\pi(V_2) = 5$

$\pi(V_3) = 5$

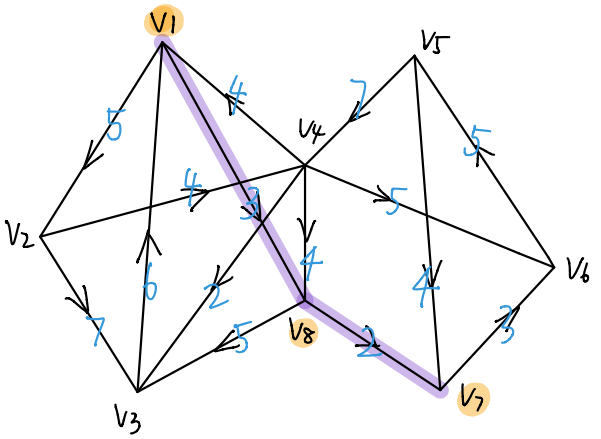
$\pi(V_4) = 4$

$\pi(V_5) = \infty$

$\pi(V_6) = \infty$

$\pi(V_7) = 2$

~~$\pi(V_8) = 3$~~



$\pi(V_2) = 5$

$\pi(V_3) = 5$

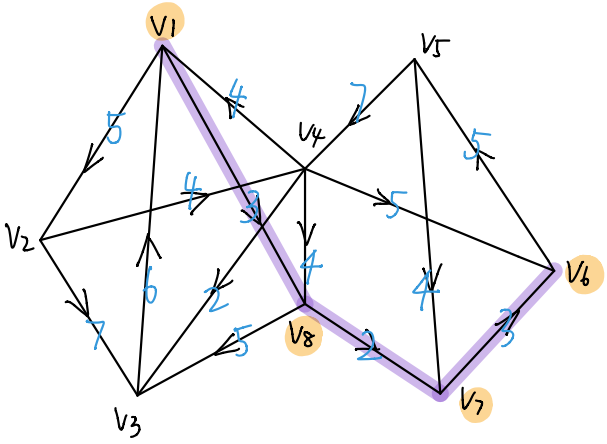
$\pi(V_4) = 4$

$\pi(V_5) = 4$

$\pi(V_6) = 3$

~~$\pi(V_7) = 2$~~

~~$\pi(V_8) = 3$~~



$\pi(V_2) = 5$

$\pi(V_3) = 5$

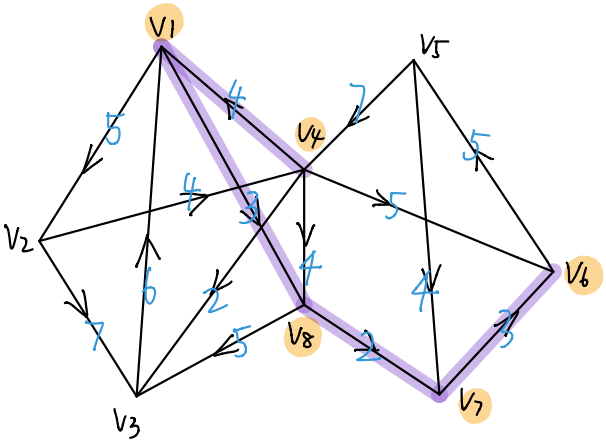
$\pi(V_4) = 4$

$\pi(V_5) = 4$

~~$\pi(V_6) = 3$~~

~~$\pi(V_7) = 2$~~

~~$\pi(V_8) = 3$~~



$\pi(V_2) = 4$

$\pi(V_3) = 2$

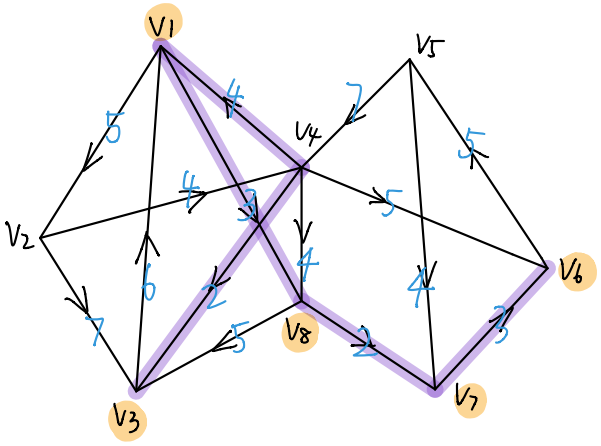
~~$\pi(V_4) = 4$~~

$\pi(V_5) = 4$

~~$\pi(V_6) = 3$~~

~~$\pi(V_7) = 2$~~

~~$\pi(V_8) = 3$~~



$\pi(V_2)=4$

~~$\pi(V_3)=2$~~

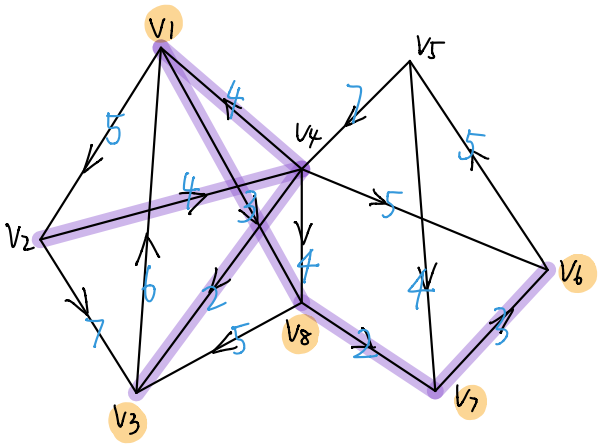
~~$\pi(V_4)=4$~~

$\pi(V_5)=4$

~~$\pi(V_6)=3$~~

~~$\pi(V_7)=2$~~

~~$\pi(V_8)=3$~~



~~$\pi(V_2)=4$~~

~~$\pi(V_3)=2$~~

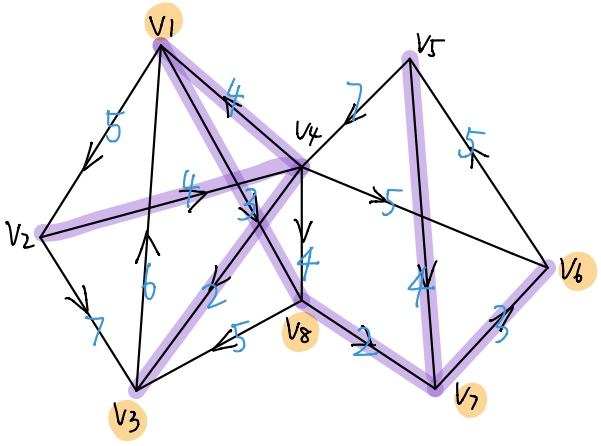
~~$\pi(V_4)=4$~~

$\pi(V_5)=4$

~~$\pi(V_6)=3$~~

~~$\pi(V_7)=2$~~

~~$\pi(V_8)=3$~~



~~$\pi(V_2)=4$~~

~~$\pi(V_3)=2$~~

~~$\pi(V_4)=4$~~

~~$\pi(V_5)=4$~~

~~$\pi(V_6)=3$~~

~~$\pi(V_7)=2$~~

~~$\pi(V_8)=3$~~

∴ 上图中边为最短树。