Solution 0

Euler Cat

July 17, 2023

1 Solutions to exercises in chapter 0

1.1 0.0.3

1.1.1 (a)

Note that they are independent, so

$$E[X_iX_j] = E[X_i]E[X_j] = 0, \forall i \neq j \tag{1}$$

As a result of this, $E||\sum_{j=1}^{k} Z_j||_2^2$ will only have terms like $E||Z_j||_2^2$ so the conclusion follows

1.1.2 (b)

Note that

$$E||Z - EZ||_2^2 \tag{2}$$
$$= E[(Z - EZ)(Z - EZ)] \tag{3}$$
$$= E[Z^2 - 2 * Z * EZ + (EZ)^2] \tag{4}$$
$$= E[Z^2] - [EZ]^2 \tag{5}$$

1.2 0.0.5

The second inequality is trivial, so we will only focus on the first one and last one.

For the first inequality, note that $\frac{n}{m} < \frac{n-k}{m-k}, \forall k > 0, n > m$ and $C_n^m = \frac{n}{m} * \frac{n-1}{m-1} * ... * \frac{n-m+1}{m}$.

For the second inequality, note that

$$\sum_{k=0}^{m} C_n^k * (\frac{m}{n})^m \tag{6}$$

$$\leq \sum_{k=0}^{m} C_n^k * (\frac{m}{n})^k \tag{7}$$

$$\leq (1 + \frac{m}{n})^n \tag{8}$$

$$\leq e^m \tag{9}$$

1.3 0.0.6

By using the last inequality from previous exercise, we can have

$$C^k_{N+k-1} \tag{10}$$
$$\leq (\frac{e*(N+k-1)}{k})^{\frac{1}{\epsilon^2}} \tag{11}$$
$$= ((1-\frac{1}{k})*e+e*N*\epsilon^2)^{\frac{1}{\epsilon^2}} \tag{12}$$
$$\leq (C+C*N*\epsilon^2)^{\frac{1}{\epsilon^2}} \tag{13}$$