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# Abstract

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## Introduction

Asynchronous task allocation problem, also called do-all problem[6], is defined informally as the problem of n processes in the network, cooperatively performing m independent tasks, in the presence of adversity.

Such cooperation problems consisting of large numbers of tasks by multiple processes is highly related to a broad range of distributed computing problems, such as mutual exclusion [5], consensus problem [12] distributed clocks [4], and shared-memory collect [1].

In shared memory models, the task allocation problem is known as Write-All problem, introduced and studied by Kanellakis and Shvartsman [11] and defined as follows: Given a zero-valued array of m elements and n processors, write value 1 into each array location in the presence of adversity.

Following the initial work [11], the task allocation problem was studied in a variety of shared memory settings e.g., [5, 7, 14, 51, 65, 68, 69, 82, 87, 88, 89].

#### 1.1 Related Work

#### 1.2 Statement of Results

# Model of Computation and Definitions

#### 2.1 Asynchronous Shared Memory Model

In this chapter, we will describe our model of computation and give the definitions, which are based on Herlihy and Wing's[9] and Golab, Higham and Woelfel's[8].

The computational model we consider is the standard asynchronous shared memory model with a set  $\mathcal{P}$  of n processes, denoted as  $\mathcal{P} = [p] = \{p_0, p_2, ..., p_{n-1}\}$ , where up to n-1 processes may fail by crashing. A process may crash at any moment during the computation and once crashed it does not restart, and does not perform any further actions.

**Type and Object**. A type  $\tau$  is defined as an automaton as follows[7],

$$\tau = (\mathcal{S}, s_{init}, \mathcal{O}, \mathcal{R}, \delta)$$

where  $\mathcal{S}$  is a set of states,  $s_{init} \in \mathcal{S}$  is the initial state,  $\mathcal{O}$  is a set of operations,  $\mathcal{R}$  is the set of responses, and  $\delta : \mathcal{S} \times \mathcal{O} \to \mathcal{S} \times \mathcal{R}$  is a state transition mapping.

An object is an implementation of a type. For each type  $\tau$ , the transition mapping  $\delta$  captures the behaviour of objects of type  $\tau$ , in the absence of concurreny, as follows: if a process applies an operation opt to an object of type  $\tau$  which is in state s, the object may return to the process a response rsp and change its states to s' if and only if  $(s', rsp) \in \delta(s, opt)$ .

**History**. A *history* H, obtained by processes executing operations on objects, is a sequence of invocation and response events.

An invocation event is a 5-tuple,

$$INV = (invocation, p, obj, opt, t)$$

where *invocation* is the event type, p is the process executing the operation, obj is the object on which the operation is executed, opt is the operation and t is the time when INV happens which is defined as the position of event INV in history H. We also say the event INV is the invocation event of operation opt.

A response event is also a 5-tuple,

$$RSP = (response, p, obj, rsp, t)$$

where response is the event type, p is the process receiving response rsp from an oeration on object obj and t is the time when RSP happens which is defined as the position of event RSP in history H.

In the following discussion, we suppose in a history H, the situation that an invocation event  $(invocation, p_i, obj_p, opt_0, t_0)$  is followed immediately by another invocation event  $(invocation, p_j, obj_q, opt_1, t_1)$  where i = j and p = q will not happen.

Response event  $(response, p_j, obj_q, rsp, t_1)$  matches invocation event  $(invocation, p_i, obj_p, opt, t_0)$  in history H, if the two events are applied by the same process to the same object, i.e, i = j and p = q. In this case, the response event is also called the matching response of the invocation event.

An operation execution in H is a pair oe = (INV, RSP) consisting of an invocation event INV and its matching response event RSP, or just an invocation event INV with no matching response event, denoted as oe = (INV, null). In the latter case, we say the operation execution is pending. In the former case, we say the operation execution is complete. A history H is complete if all operation executions in H are complete, otherwise, it is incomplete. If events INV and RSP are applied by process p, then we say operation execution oe = (INV, RSP) is performed by process p. Thus, two operation executions performed by the same

process on the same project will not interleave in a history H.

We say that an operation opt is atomic in history H, if opt's invocation event is either the last event in H, or else is followed immediately in H by a matching response event.

History H' is an extension of history H if H is a prefix of H'. History H' is a completion of history H if H' contains all the events in H and H' is an extension of H, and each operation execution in H' is complete.

H|obj of history H is the subsequence of all invocation and response events in H on object obj. If all invocation and response events in a history H have the same object name obj, then the H|obj = H.

Let H be a complete history. We associate a time interval  $I_{oe} = [t_0, t_1]$  with each operation execution oe = (INV, RSP) in H, where  $t_0$  and  $t_1$  are the points in time when INV and RSP happen. Similarly, for an incomplete history, we denote the time interval  $I_{oe}$  with respect to a pending operation execution oe = (INV, null) by  $I_{oe} = [t_0, \infty]$ .

Operation execution  $oe_0$  precedes operation execution  $oe_1$  in H if the response event of  $oe_0$  happens before the invocation event of  $oe_1$  in H. We say that  $oe_0$  and  $oe_1$  are concurrent in H if neither precedes the other.

A history is *sequential* if its first event is an invocation event, and each invocation event, except possibly the last one, is immediately followed by a matching response event.

A sequential specification of an object is the set of all possible sequential histories for that object.

A sequential history S is *valid*, if for each object obj, S|obj is in the sequential specification of obj.

**Linearization**. A history H linearizes to a sequential history S, if and only if S satisfies the following conditions: (1) S and any completion of H have the same operation executions,

(2) sequential history S is valid, and (3) there is a mapping from each time interval  $I_{oe}$  to a time point  $t_{oe} \in I_{oe}$ , such that the sequential history S is obtained by sorting the operations in H based on their  $t_{oe}$  values.

A history is linearizable if and only if H linearizes to some sequential history S. In this case, S is called the linearization of H. For each operation opt in history H, we call time point  $t_{oe}$ , which is defined as above, the linearization point of opt. An object obj is linearizable if every history H on obj is linearizable.

#### 2.2 Base Objects

In this section, we describe the two base objects, i.e, read-write register and compare-and-swap (CAS) objects, which will be used in our following discussion. Most implementations of more sophisticated objects use them as the base objects in their implementations and most modern architectures support either read-write registers and CAS objects[10][13].

Read-Write Register. An object that supports only read() and write(x) operations is called a read-write register (or just register). Operation read() returns the current state of register and leaves the state unchanged. Operation write(x) changes the state of the register to x and returns nothing. If the set of states that can be stored in the register is unbounded then we say the register is unbounded register; otherwise the register is bounded register.

**CAS Object.** An object that supports read() and CAS(x,y) operations is called compare-and-swap (CAS) object. Operation read() is the same as defined above. Operation CAS(x,y) changes the state of the object if and only if the current state is equal to x and then operation CAS(x,y) succeeds, and the state is changed to y and true is returned. Otherwise, operation CAS(x,y) fails, the current state remains unchanged and false is returned.

### 2.3 Adversary Models for Randomized Algorithms

Randomness. A randomized algorithm is an algorithm where processes are allowed to make random decisions for the next step by calling a special operation called *coin-flip operation*. We also say a process *flips a coin* when it calls this operation.

When a process flips a coin, it receives a random value c from some arbitrary set  $\Omega$  which is calle the *coin-flip domain*. The process can then use this random value c in its program for future decisions.

A vector  $\overrightarrow{c} = (c_0, c_1, c_2, ...) \in \Omega^{\infty}$  is called a *coin-flip vector*. A history H is said to *observe* the coin-flip vector  $\overrightarrow{c}$  if for an arbitrary integer  $i \in [0, \infty)$ , the i-th coin-flip operation in H returns value  $c_i \in \Omega$ .

For a history H that contains k coin-flip operations, we use H[k] to denote the prefix of H that ends with the k-th invocation of a coin-flip operation. If fewer than k coin-flip operation executions occur during H, then H[k] denotes H.

In the following discussion, we use operation  $random(\Omega)$ , which is assumed to be atomic, to return a value which is distributed uniformly at random over coin-flip domain  $\Omega = \{0, 1, 2, ..., s-1\}$ .

Schedule. In the standard shared memory model, each process executes its program by applying shared memory operations (read(), write(x), CAS(x,y), etc) on objects, as determined by their program. Operation executions of concurrent processes can be interleaved arbitrarily.

A schedule with length k is represented by a sequence of process ids

$$p = (p_0, p_1, p_2, ..., p_{k-1})$$

where  $k \in [0, \infty)$  and for each  $i \in [0, k-1], p_i \in \mathcal{P}$ .

Let a schedule  $p = (p_0, p_1, p_2, ..., p_{k-1})$ . A history H is said to *observe* schedule p if the number of events in H is k, and for each integer  $i \in [0, k-1]$ , the i-th event is applied by process  $p_i$ .

**Adversary**. In randomized algorithm, the random choices processes make can influence the schedule. To model the worst-case possible way that the system can be influenced by the random choices, schedules are assumed to be generated by an adversarial scheduler, called the *adversary*.

Mathematically, an adversary is defined as a mapping [8]:

$$\mathcal{A}:\Omega^{\infty}\to\mathcal{P}^{\infty}$$

Given an algorithm  $\mathcal{M}$ , an adversary  $\mathcal{A}$ , and a coin-flip vector  $\overrightarrow{c} \in \Omega^{\infty}$ , a unique history  $H_{\mathcal{M},\mathcal{A},\overrightarrow{c}}$  is generated, such that all processes apply events as dictated by algorithm  $\mathcal{M}$ , and history  $H_{\mathcal{M},\mathcal{A},\overrightarrow{c}}$  observes the schedule  $\mathcal{A}(\overrightarrow{c})$  and the coin flip vector  $\overrightarrow{c}$ .

There are several adversary models with different strengths[3]. In our thesis, we only conside the *adaptive adversary*.

Informally, the adaptive adversary makes scheduling decisions as follows: At any point, it can see the entire history up to that point. This includes all coin-flip operations and their return values up to that point. Depending on this, the adversary decides which process takes the next step.

Adversary  $\mathcal{A}$  is adaptive for algorithm  $\mathcal{M}[8]$  if, for any two coin-flip vectors  $\overrightarrow{c} \in \Omega^{\infty}$  and  $\overrightarrow{d} \in \Omega^{\infty}$  that have a common prefix of length k (i.e, the first k elements of  $\overrightarrow{c}$  and  $\overrightarrow{d}$  are the same), then we have

$$H_{\mathcal{M},\mathcal{A},\overrightarrow{c}}[k+1] = H_{\mathcal{M},\mathcal{A},\overrightarrow{d}}[k+1]$$

In this case, we say adversary A is an adaptive adversary.

From the above definition, we can see an adaptive adversary can not use future coin flips to make current scheduling decisions.

#### 2.4 The Dynamic Task Alloction Problem

#### 2.4.1 Task

Performing a task can be be any execution performed by a single process in constant time and the tasks are assumed to satisfy the following three properties[6]: *Similarity*: The execution consume equal or comparable resources. *Independence*: One execution is independent of any of the others. *Idempotence*: Tasks could be performed more than one time.

For simplicity, in our thesis, *tasks* are abstracted and defined as zero-valued shared registers in an array, denoted as tasks[].

The state of each register is either 0 or 1. When a register is in state 0, then we say a task is *available*. When the state of that register is flipped to 1, we say that task is *performed*.

Each array cell tasks[i], where integer  $i \in [0, \infty)$  is defined as a memory location (or just location) and denoted as M. We say there is an available task associated with location M (or at M) if the register at location M is in state 0. Otherwise, we say there is no task at M if the register is in state 1.

Each task has a unique identifier from set  $\mathcal{L} = [m] = \{\ell_0, \ell_1, \ell_2, ..., \ell_m\}$ , where  $m \in [0, \infty)$ . A simple unique task identifier scheme is provided by [2].

Informally, performing a task is the execution that flipping the state of a register at some location from 0 to 1. If the above value m is fixed and the m tasks are available initially, and the execution ends when all tasks are performed, then we get the so-called static version of task allocation problem, i.e, write-all problem as follws: Given a zero-valued array of m elements and n processes, write value 1 into each array cell in the presence of adversity[11].

In this thesis, we consider the dynamic version of task allocation, i.e, Given n asynchronous processes that cooperate to perform tasks, while tasks are inserted dynamically by the adversary during the execution, where inserting a task is definied to be the execution that flipping the state of a register from state 1 back to 0. When the state of the register at location M is flipped from 1 to 0, we also say a new task is inserted to locaton M.

Since the state of the register at location M can be flipped from 0 to 1 and from 1 back to 0 several times, thus we say one location M can be associated with multiple tasks over time.

Now we specify the dynamic task alloction problem in terms of a type DTA, and the properties that an implementation of type DTA must satisfy.

#### 2.4.2 The Type DTA

The type DTA supports two types of operations, DoTask() and InsertTask( $\ell$ ), where  $\ell$  is the task identifier.

Operation DoTask(). The aim of operation DoTask() is to perform a task associated with location M by calling a special atomic operation TryTask(M) which is assumed to exist.

Task  $\ell$  can be performed atomically by a process by calling  $\mathsf{TryTask}(M)$  operation with argument M which is the memory location associated with task  $\ell$ . Out of several processes calling  $\mathsf{TryTask}(M)$ , only the first one receives notification success and the identifier  $\ell$  of that task, while all the others only receive failure.

Every DoTask() operation may perform several TryTask(M) operations. However, only one of them will succeed. Once one TryTask(M) succeeds, then task identifier  $\ell$  will be returned by DoTask(). If there is no task could be performed, i.e, the state of each task is 1, then operation DoTask() returns  $\bot$ .

Operation InsertTask( $\ell$ ). The goal of InsertTask( $\ell$ ) operation is to insert task  $\ell$  to

location M also by calling a special atomic operation  $PutTask(M, \ell)$ .

Task  $\ell$  can be associated with memory location M atomically by calling PutTask $(M, \ell)$  operation. PutTask $(M, \ell)$  returns success if task  $\ell$  is associated with M, and returns failure if location M was already associated with another task.

Several PutTask( $M, \ell$ ) operations may be executed but only one of them will succeed. PutTask( $M, \ell$ ) fails if location M has been associated with another task in state 1. Once one TryTask(M) succeeds, then task  $\ell$  is available on location M and success notification is returned by InsertTask( $\ell$ ) operation.

Similarly, atomic operations  $PutTask(M, \ell)$  it assumed to be contained in  $InsertTask(\ell)$  operation.

A task is performed if its identifier has been returned by a process after calling DoTask(). A task  $\ell$  is available at location M if success is returned by a process after calling  $InsertTask(\ell)$ , but is not performed yet.

**Properties**. An algorithm that accesses an instance of an object of type DTA must satisfy the following:

Validity. If a DoTask() operation returns  $\ell$ , then before the DoTask() operation, an InsertTask( $\ell$ ) was executed and returned success.

Uniqueness. Each task is performed at most once, i.e, for each task  $\ell$ , at most one DoTask() operation returns  $\ell$ .

In addition, the property that every inserted task is eventually done is also a desired progress property of the implementation of type DTA.

# Dynamic Task Allocation Object

Implementation of DTA

(under work...)

#### Method 1: DoTask()

```
1 while true do
        v \leftarrow root;
 2
        if v.surplus() \leq 0 then
 3
         return \perp;
 4
        end
 5
        /* Descent */;
 6
        while v is not a leaf do
 7
            (x_L, y_L) \leftarrow v.left.read();
 8
            (x_R, y_R) \leftarrow v.right.read();
 9
            s_L \leftarrow min(x_L - y_L, 2^{height(v)});
10
            s_R \leftarrow min(x_R - y_R, 2^{height(v)});
11
            r \leftarrow random(0, 1);
12
            if (s_L + s_R) = 0 then
13
                Mark-up(v);
14
            else if r < s_L/(s_L + s_R) then
15
                v \leftarrow v.left;
16
            else
17
               v \leftarrow v.rght;
18
            end
19
        end
20
        /* v is a leaf */;
21
        (x,y) \leftarrow v.read();
22
        (flag, l) \leftarrow v.TryTask(task[y + 1]);
\mathbf{23}
        /* Update Insertion Count */;
\mathbf{24}
        v.CAS((x, y), (x, y + 1));
25
        v \leftarrow v.parent;
26
        Mark-up(v);
27
        if flag = success then
28
         return \ell
29
        end
30
31 end
```

#### Method 2: InsertTask( $\ell$ )

```
32 while true do
        v \leftarrow root:
33
        /* Descent */;
34
        while v is not a leaf do
35
            (x_L, y_L) \leftarrow v.left.read();
36
            (x_R, y_R) \leftarrow v.right.read();
37
            s_L \leftarrow 2^{height(v)} - min(x_L - y_L, 2^{height(v)});
38
            s_R \leftarrow 2^{height(v)} - min(x_R - y_R, 2^{height(v)});
39
            r \leftarrow random(0, 1);
40
            if (s_L + s_R) = 0 then
41
                Mark-up(v);
42
            else if r < s_L/(s_L + s_R) then
43
                v \leftarrow v.left;
44
            else
45
                v \leftarrow v.rght;
46
            end
47
        end
48
        /* v is a leaf */;
49
        (x,y) \leftarrow v.read();
50
        flaq \leftarrow v. PutTask(task[x+1]);
51
        /* Update Insertion Count */;
52
        v.CAS((x, y), (x + 1, y));
53
        v \leftarrow v.parent;
54
        Mark-up(v);
55
        if flaq = success then
56
            return success
57
        end
58
59 end
```

#### Method 3: Mark-up(v)

```
60 if v is not null then
61 | for (i = 0; i < 2; i + +) do
62 | (x, y) \leftarrow v.read();
63 | (x_L, y_L) \leftarrow v.left.read();
64 | (x_R, y_R) \leftarrow v.right.read();
65 | v.CAS((x, y), (max(x, x_L + x_R), max(y, y_L + y_R));
66 | end
67 end
```

#### Correctness Proof

#### 4.1 Correctness

The standard correctness condition for shared memory algorithms is linearizability, which was introduced by Herlihy and Wing in 1990 [9]. The intuition of linearizability is that real-time behavior of method calls must be preserved, i.e, if one method call precedes another, then the earlier call must have taken effect before the later one. By contrast, if two method calls overlap, we are free to order them in any convenient way since the order is ambiguous. Informally, a concurrent object is linearizable if each method call appears to take effect instantaneously at some moment between its invocation and response.

#### 4.1.1 Analysis and Proofs

By the definitions in Subsection 3.1.1, one way to show an object obj is linearizable is to prove every history H of obj is linearizable. Thus, we need to identify for each DoTask and InsertTask operation op (i.e, interval  $I_H(op)$ ) in H a linearization point  $t_H(op)$ , and prove that the sequential history S obtained by sorting these operations according to their  $t_H(op)$  satisfies the sequential specification  $S_{obj}$  of obj.

We notice that each complete DoTask or InsertTask operation can be associated with a unique task array slot based on the task it removed or inserted. Additionally, the removal and insertion count are both monotonically increasing. Thus, we could associate the node counts with operations which have been propagated to that node.

Now we define "an operation is counted at a node" recursively to formalize the operation

propagation.

A DoTask operation is counted at leaf v when the removal count of v is updated with the index of the task array slot where the performed task is located. Symmetrically, an InsertTask operation is counted at v when the insertion count of v is updated with the index of the task array slot where the inserted task is located.

Now we only define DoTask operation is counted at an inner node v because counting an InsertTask operation is symmetric as well.

Recall that the removal count of v is updated though CAS operation (line 6, method 3). Actually there could be more than one operations updating the count with the same value. We linearize all such CAS operations, which update the removal count of v with the same value y. We say for all these operations, only the first one in the linearization order counts the corresponding DoTask operation. In another word, a DoTask operation is counted at an inner node v as soon as the CAS updating operation that counts the DoTask is linearized. Based this definition, no operation will be counted twice at a node.

Please note that, the CAS operation counting the DoTask at node v is not necessary performed by the DoTask operation itself, i.e, suppose process p executes a DoTask operation and has successfully performed task  $\ell$  at certain leaf. Then the CAS operation counting this p.DoTask() at node v could be a different process q as long as q updates the removal count first in the linearization order.

Given the above concepts and properties, we could prove the following result: (under work...almost done)

- 4.2 Performance
- 4.2.1 DoTask Analysis
- 4.2.2 InsertTask Analysis

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