UNIVERSITY OF CALGARY

Title of Thesis

Second Thesis Title Line

by

Student's name

A THESIS

SUBMITTED TO THE FACULTY OF GRADUATE STUDIES IN PARTIAL FULFILLMENT OF THE REQUIREMENTS FOR THE DEGREE OF NAME OF DEGREE IN FULL

DEPARTMENT OF NAME OF DEPARTMENT

CALGARY, ALBERTA

monthname, year

© Student's name year

Abstract

Table of Contents

Abs	stract	1
Tab	le of Contents	ii
1	Introduction	1
1.1	Related Work	1
1.2	Statement of Results	1
2	Model of Computation and Definitions	2
2.1	Asynchronous Shared Memory Model	2
2.2	Primitive Objects	5
2.3	Adversary Models for Randomized Algorithms	6
2.4	The Dynamic Task Alloction Problem	8
	2.4.1 The Type DTA	8
3	Dynamic Task Allocation Object	10
4	Correctness Proof	13
4.1	Correctness	13
	4.1.1 Analysis and Proofs	13
4.2	Performance	15
	4.2.1 DoTask Analysis	15
	4.2.2 InsertTask Analysis	15

Introduction

Asynchronous task allocation problem, also called do-all problem[5], is defined informally as the problem of n processes in the network, cooperatively performing m independent tasks, in the presence of adversity.

Such cooperation problems consisting of large numbers of tasks by multiple processes is highly related to a broad range of distributed computing problems, such as mutual exclusion [4], consensus problem [10] distributed clocks [3], and shared-memory collect [1].

In shared memory models, the task allocation problem is known as Write-All problem, introduced and studied by Kanellakis and Shvartsman [9] and defined as follows: Given a zero-valued array of m elements and n processors, write value 1 into each array location in the presence of adversity.

Following the initial work [9], the task allocation problem was studied in a variety of shared memory settings e.g., [5, 7, 14, 51, 65, 68, 69, 82, 87, 88, 89].

1.1 Related Work

1.2 Statement of Results

Model of Computation and Definitions

2.1 Asynchronous Shared Memory Model

In this chapter, we will describe our model of computation and give the definitions, which are based on Herlihy and Wing's [7] and Golab, Hadzilacos and Woelfel's [6].

The computational model we consider is the standard asynchronous shared memory model with a set \mathcal{P} of n processes, denoted as $\mathcal{P} = [p] = \{p_0, p_2, ..., p_{n-1}\}$, where up to n-1 processes may fail by crashing. A process may crash at any moment during the computation and once crashed it does not restart, and does not perform any further actions.

Type and Object. A type τ is defined as an automaton as follows [6],

$$\tau = (\mathcal{S}, s_{init}, \mathcal{O}, \mathcal{R}, \delta)$$

where S is a set of states, $s_{init} \in S$ is the initial state, O is a set of operation types, R is the set of responses, and $\delta: S \times O \to S \times R$ is a state transition mapping.

An object is an implementation of a type. For each type τ , the transition mapping δ captures the behaviour of objects of type τ , in the absence of concurreny, as follows: if a process applies an operation of type opt to an object of type τ which is in state s, the object may return to the process a response rsp and change its states to s' if and only if $(s', rsp) \in \delta(s, opt)$.

History A *history* H, obtained by processes executing operations on objects, is a sequence of invocation and response events.

An invocation event is a 5-tuple,

$$INV = (invocation, p, obj, opt, t)$$

where *invocation* is the event type, p is the process executing the operation, obj is the object on which the operation is executed, opt is the operation type and t is the time when INV happens which is defined as the position of event INV in history H.

A response event is also a 5-tuple,

$$RSP = (response, p, obj, rsp, t)$$

where response is the event type, p is the process receiving response rsp from an oeration on object obj and t is the time when RSP happens which is defined as the position of event RSP in history H.

For an event with respect to process p, we also say this event is applied by process p.

In the following discussion, we suppose in a history H, the situation that an invocation event $(invocation, p_i, obj_p, opt_0, t_0)$ is followed immediately by another invocation event $(invocation, p_j, obj_q, opt_1, t_1)$ where i = j and p = q will not happen.

Response event $(response, p_j, obj_q, rsp, t_1)$ matches invocation event $(invocation, p_i, obj_p, opt, t_0)$ in history H, if the two events are applied by the same process to the same object, i.e, i = j and p = q. In this case, the response event is also called the matching response of the invocation event.

An operation execution in H is a pair oe = (INV, RSP) consisting of an invocation event INV and its matching response event RSP, or just an invocation event INV with no matching response event, denoted as oe = (INV, null). In the latter case, we say the operation execution is pending. In the former case, we say the operation execution is complete. A history H is complete if all operation executions in H are complete, otherwise, it is incomplete. If events INV and RSP are applied by process p, then we say operation execution oe = (INV, null).

RSP) is *performed* by process p. Thus, two operation executions performed by the same process on the same project will not interleave in a history H.

History H' is an extension of history H if H is a prefix of H'. History H' is a completion of history H if H' contains all the events in H and H' is an extension of H, and each operation execution in H' is complete.

H|obj of history H is the subsequence of all invocation and response events in H on object obj. If all invocation and response events in a history H have the same object name obj, then the H|obj = H.

Let H be a complete history. We associate a time interval $I_{oe} = [t_0, t_1]$ with each operation execution oe = (INV, RSP) in H, where t_0 and t_1 are the points in time when INV and RSP happen. Similarly, for an incomplete history, we denote the time interval I_{oe} with respect to a pending operation execution oe = (INV, null) by $I_{oe} = [t_0, \infty]$.

Operation execution oe_0 precedes operation execution oe_1 in H if the response event of oe_0 happens before the invocation event of oe_1 in H. We say that oe_0 and oe_1 are concurrent in H if neither precedes the other.

A history is *sequential* if its first event is an invocation event, and each invocation event, except possibly the last one, is immediately followed by a matching response event.

A sequential specification of an object is the set of all possible sequential histories for that object.

A sequential history S is *valid*, if for each object obj, S|obj is in the sequential specification of obj.

Linearization. A history H linearizes to a sequential history S, if and only if S satisfies the following conditions: (1) S and any completion of H have the same operation executions, (2) sequential history S is valid, and (3) there is a mapping from each time interval I_{oe} to a

time point $t_{oe} \in I_{oe}$, such that the sequential history S is obtained by sorting the operations in H based on their t_{oe} values.

A history is *linearizable* if and only if H linearizes to some sequential history S. In this case, S is called the *linearization* of H. For each operation opt in history H, we call time point t_{oe} , which is defined as above, the *linearization point* of opt. An object obj is linearizable if every history H on obj is linearizable.

2.2 Primitive Objects

In this section, we describe the two primitive objects which will be used in our following discussion, namely read-write register and compare-and-swap (CAS) objects. Most implementations of more sophisticated objects use these objects as the base objects in their implementations, since most modern architectures support either read-write registers and CAS objects [8] [11].

Read-Write Register. An object that supports only read() and write(x) operations is called a read-write register (or just register). Operation read() returns the current state of register and leaves the state unchanged. Operation write(x) changes the state of the register to x and returns nothing. If the set of states that can be stored in the register is unbounded then we say the register is unbounded register; otherwise the register is bounded register.

CAS Object. An object that supports read() and CAS(x,y) operations is called compare-and-swap (CAS) object. Operation read() is the same as defined above. Operation CAS(x,y) changes the state of the object if and only if the current state is equal to x and then operation CAS(x,y) succeeds, and the state is changed to y and true is returned. Otherwise, operation CAS(x,y) fails, the current state remains unchanged and false is returned.

2.3 Adversary Models for Randomized Algorithms

Randomness. A randomized algorithm is an algorithm where processes are allowed to make random decisions for the next step. This is modelled by giving each process a special operation called *coin-flip operation*. We say a process can flip a coin when it calls this operation. When a process flips a coin, it receives a random value c from some arbitrary set Ω which is calle the *coin-flip domain*. The process can then use this random value c in its program for future decisions.

A vector $\overrightarrow{c} = (c_0, c_1, c_2, ...) \in \Omega^{\infty}$ is called a *coin-flip vector*. A history H is said to *observe* the coin-flip vector \overrightarrow{c} if for an arbitrary integer $i \in [0, \infty)$, the i-th coin-flip operation in H returns value c_i .

In the following discussion, we use method random(s), which is assumed to be linearizable, to return a value which is distributed uniformly at random over domain $\{0, 1, 2, ..., s - 1\}$.

Adversary. In the standard shared memory model, each process executes its program by applying shared memory operations (read(), write(x), CAS(x,y), etc) on objects, as determined by their program. Operation executions of concurrent processes can be interleaved arbitrarily.

A schedule with length k is represented by a sequence of process ids

$$p = (p_0, p_1, p_2, ..., p_{k-1})$$

where $k \in [0, \infty)$ and for each $i \in [0, k-1], p_i \in \mathcal{P}$.

Let a schedule $p = (p_0, p_1, p_2, ..., p_{k-1})$. A history H is said to *observe* schedule p if the number of events in H is k, and for each integer $i \in [0, k-1]$, the i-th event is applied by process p_i .

In randomized algorithm, the random choices processes make can influence the schedule. To model the worst-case possible way that the system can be influenced by the random choices, schedules are assumed to be generated by an adversarial scheduler, called the adversary.

Mathematically, an adversary is defined as a mapping:

$$A:\Omega^{\infty}\to\mathcal{P}^{\infty}$$

Given an algorithm \mathcal{M} , an adversary \mathcal{A} , and a coin-flip vector $\overrightarrow{c} \in \Omega^{\infty}$, a history $H_{\mathcal{M},\mathcal{A},\overrightarrow{c}}$ is generated, such that all processes apply events as dictated by algorithm \mathcal{M} , and history $H_{\mathcal{M},\mathcal{A},\overrightarrow{c}}$ observes the schedule $\mathcal{A}(\overrightarrow{c})$.

There are several adversary models with different strengths [2]. In our thesis, we only conside the *adaptive adversary*.

Informally, the adaptive adversary makes scheduling decisions as follows: At any point, it can see the entire history up to that point. This includes all coin-flip operations and their return values up to that point. Depending on this, the adversary decides which process takes the next step.

For a history H that contains k coin-flip operations, we use H[k] to denote the subsequence of H that contains all events up to the k-th invocation event of a coin-flip operation.

Adversary \mathcal{A} is adaptive for algorithm \mathcal{M} if, for any two coin-flip vectors $\overrightarrow{c} \in \Omega^{\infty}$ and $\overrightarrow{d} \in \Omega^{\infty}$ that have a common prefix of length k (i.e, the first k elements of them are the same), then we have

$$H_{\mathcal{M},\mathcal{A},\overrightarrow{c}}[k+1] = H_{\mathcal{M},\mathcal{A},\overrightarrow{d}}[k+1]$$

In this case, we also say Adversary A is an adaptive adversary.

From the above definition, we can see an adaptive adversary cannot use future coin flips to make current scheduling decisions.

2.4 The Dynamic Task Alloction Problem

Performing a *task* can be be any computation which can be performed by a single process in constant time [5] and the tasks are assumed to satisfy the following three properties: *Similarity*: The computations consume equal or comparable resources. *Independence*: One computation is independent of any of the others. *Idempotence*: Tasks admit at-least-once execution semantics.

For simplicity, in our thesis a task is defined as a register initialized with state 0. Each task has a unique identifier from the set $\mathcal{L} = [m] = \{l_0, l_1, l_2, ..., l_m\}$, where

Now we specify the dynamic task alloction problem in terms of a type DTA, and the properties that an implementation of type DTA must satisfy.

2.4.1 The Type DTA

(define what is a "task" here by giving the domain, like the process (task identifier == process identifier))

The type DTA supports two operation types, DoTask() and InsertTask(ℓ), where ℓ is the identifier that is unique for each task. A process *attempts* to perform a task by executing DoTask(), and the process associates a task ℓ with a memory location by executing InsertTask(ℓ).

We assume that there exists an atomic operation $PutTask(M, \ell)$, and a process associates (what is associate?) task ℓ with memory location (what is memory location?) M by calling $PutTask(M, \ell)$. It returns success if task ℓ is associated with location M, and returns failure if location M was already associated with another task.

Similarly, we assume there exists an atomic operation TryTask(M), and task ℓ associated

with memory location M could be performed atomically by calling TryTask(M). Out of several processes calling TryTask(M), one receives success and the index ℓ of that task, while all the others receive failure. ($state\ transition\ for\ all\ initial\ states$, what happens next if there is no...)

A task is *done* or *performed* if its index has been returned by a process after calling DoTask(). A task is *available* at location M if it has been inserted to M and *success* is returned by a process after calling InsertTask(ℓ), but is not done yet. A task is *available*, if it is *available* at some memory location.

The aim of operation DoTask() is to perform an available task on location M by calling TryTask(M). Every DoTask() may perform several TryTask(M) operations. However, only one of them will succeed. Once one TryTask(M) succeeds, then there is no available task on M and the task index ℓ will be returned by DoTask(). Additionally, if there is no available task, then operation DoTask() returns \bot .

The goal of $InsertTask(\ell)$ operation is to find a free memory location M and insert task ℓ atomically by calling $PutTask(M,\ell)$. $PutTask(M,\ell)$ fails if location M has been associated with another task, so each $InsertTask(\ell)$ operation may perform several $PutTask(M,\ell)$ operations, but only one of them will succeed. Once one TryTask(M) succeeds, then task ℓ is available on location M and success notification is returned by $InsertTask(\ell)$ operation.

Type DTA is required to satisfy: (Validity) If a DoTask() operation returns ℓ , then before the DoTask() operation, an InsertTask(ℓ) was executed and returned success. (Uniqueness) Each task is performed at most once, i.e, for each task ℓ , at most one DoTask() operation returns ℓ .

In addition, the property that every inserted task is eventually done is also a desired progress property of the implementation of type DTA.

Dynamic Task Allocation Object

Implementation of DTA

(under work...)

Method 1: DoTask()

```
1 while true do
        v \leftarrow root;
 2
        if v.surplus() \leq 0 then
 3
         return \perp;
 4
        end
 5
        /* Descent */;
 6
        while v is not a leaf do
 7
            (x_L, y_L) \leftarrow v.left.read();
 8
            (x_R, y_R) \leftarrow v.right.read();
 9
            s_L \leftarrow min(x_L - y_L, 2^{height(v)});
10
            s_R \leftarrow min(x_R - y_R, 2^{height(v)});
11
            r \leftarrow random(0, 1);
12
            if (s_L + s_R) = 0 then
13
                Mark-up(v);
14
            else if r < s_L/(s_L + s_R) then
15
                v \leftarrow v.left;
16
            else
17
               v \leftarrow v.rght;
18
            end
19
        end
20
        /* v is a leaf */;
\mathbf{21}
        (x,y) \leftarrow v.read();
22
        (flag, l) \leftarrow v.TryTask(task[y + 1]);
\mathbf{23}
        /* Update Insertion Count */;
\mathbf{24}
        v.CAS((x, y), (x, y + 1));
25
        v \leftarrow v.parent;
26
        Mark-up(v);
27
        if flag = success then
28
         return \ell
29
        end
30
31 end
```

Method 2: InsertTask(ℓ)

```
32 while true do
        v \leftarrow root:
33
        /* Descent */;
34
        while v is not a leaf do
35
            (x_L, y_L) \leftarrow v.left.read();
36
            (x_R, y_R) \leftarrow v.right.read();
37
            s_L \leftarrow 2^{height(v)} - min(x_L - y_L, 2^{height(v)});
38
            s_R \leftarrow 2^{height(v)} - min(x_R - y_R, 2^{height(v)});
39
            r \leftarrow random(0, 1);
40
            if (s_L + s_R) = 0 then
41
                Mark-up(v);
42
            else if r < s_L/(s_L + s_R) then
43
                v \leftarrow v.left;
44
            else
45
                v \leftarrow v.rght;
46
            end
47
        end
48
        /* v is a leaf */;
49
        (x,y) \leftarrow v.read();
50
        flag \leftarrow v. PutTask(task[x+1]);
51
        /* Update Insertion Count */;
52
        v.CAS((x, y), (x + 1, y));
53
        v \leftarrow v.parent;
54
        Mark-up(v);
55
        if flaq = success then
56
            return success
57
        end
58
59 end
```

Method 3: Mark-up(v)

```
60 if v is not null then

61 | for (i = 0; i < 2; i + +) do

62 | (x, y) \leftarrow v.read();

63 | (x_L, y_L) \leftarrow v.left.read();

64 | (x_R, y_R) \leftarrow v.right.read();

65 | v.CAS((x, y), (max(x, x_L + x_R), max(y, y_L + y_R));

66 | end

67 end
```

Correctness Proof

4.1 Correctness

The standard correctness condition for shared memory algorithms is linearizability, which was introduced by Herlihy and Wing in 1990 [7]. The intuition of linearizability is that real-time behavior of method calls must be preserved, i.e, if one method call precedes another, then the earlier call must have taken effect before the later one. By contrast, if two method calls overlap, we are free to order them in any convenient way since the order is ambiguous. Informally, a concurrent object is linearizable if each method call appears to take effect instantaneously at some moment between its invocation and response.

4.1.1 Analysis and Proofs

By the definitions in Subsection 3.1.1, one way to show an object obj is linearizable is to prove every history H of obj is linearizable. Thus, we need to identify for each DoTask and InsertTask operation op (i.e, interval $I_H(op)$) in H a linearization point $t_H(op)$, and prove that the sequential history S obtained by sorting these operations according to their $t_H(op)$ satisfies the sequential specification S_{obj} of obj.

We notice that each complete DoTask or InsertTask operation can be associated with a unique task array slot based on the task it removed or inserted. Additionally, the removal and insertion count are both monotonically increasing. Thus, we could associate the node counts with operations which have been propagated to that node.

Now we define "an operation is counted at a node" recursively to formalize the operation

propagation.

A DoTask operation is counted at leaf v when the removal count of v is updated with the index of the task array slot where the performed task is located. Symmetrically, an InsertTask operation is counted at v when the insertion count of v is updated with the index of the task array slot where the inserted task is located.

Now we only define DoTask operation is counted at an inner node v because counting an InsertTask operation is symmetric as well.

Recall that the removal count of v is updated though CAS operation (line 6, method 3). Actually there could be more than one operations updating the count with the same value. We linearize all such CAS operations, which update the removal count of v with the same value y. We say for all these operations, only the first one in the linearization order counts the corresponding DoTask operation. In another word, a DoTask operation is counted at an inner node v as soon as the CAS updating operation that counts the DoTask is linearized. Based this definition, no operation will be counted twice at a node.

Please note that, the CAS operation counting the DoTask at node v is not necessary performed by the DoTask operation itself, i.e, suppose process p executes a DoTask operation and has successfully performed task ℓ at certain leaf. Then the CAS operation counting this p.DoTask() at node v could be a different process q as long as q updates the removal count first in the linearization order.

Given the above concepts and properties, we could prove the following result: (under work...almost done)

- 4.2 Performance
- 4.2.1 DoTask Analysis
- 4.2.2 InsertTask Analysis

Bibliography

- [1] Miklos Ajtai, James Aspnes, Cynthia Dwork, and Orli Waarts. A theory of competitive analysis for distributed algorithms. In Foundations of Computer Science, 1994

 Proceedings., 35th Annual Symposium on, pages 401–411. IEEE, 1994.
- [2] James Aspnes. Randomized protocols for asynchronous consensus. *CoRR*, cs.DS/0209014, 2002.
- [3] Yonatan Aumann and Michael O Rabin. Clock construction in fully asynchronous parallel systems and pram simulation. In Foundations of Computer Science, 1992.

 Proceedings., 33rd Annual Symposium on, pages 147–156. IEEE, 1992.
- [4] Michael Bender, Seth Gilbert, et al. Mutual exclusion with o (log² log n) amortized work. In Foundations of Computer Science (FOCS), 2011 IEEE 52nd Annual Symposium on, pages 728–737. IEEE, 2011.
- [5] Chryssis Georgiou. Do-All Computing in Distributed Systems: Cooperation in the Presence of Adversity. Springer Science & Business Media, 2007.
- [6] Wojciech Golab, Vassos Hadzilacos, Danny Hendler, and Philipp Woelfel. Constant-rmr implementations of cas and other synchronization primitives using read and write operations. In *Proceedings of the 26th Annual ACM Symposium on Principles of Distributed Computing (PODC)*, pages 3–12, 2007.
- [7] Maurice P. Herlihy and Jeannette M. Wing. Linearizability: A correctness condition for concurrent objects. *ACM Trans. Program. Lang. Syst.*, 12(3):463–492, July 1990.
- [8] Itanium. Intel Itanium Architecture Software Developers Manual Volume 1: Application Architecture Revision 2.1. Intel Corporation, October 2002.

- [9] Paris C Kanellakis and Alex A Shvartsman. Efficient parallel algorithms can be made robust. *Distributed Computing*, 5(4):201–217, 1992.
- [10] Nancy A Lynch. Distributed algorithms. Morgan Kaufmann, 1996.
- [11] David L Weaver and Tom Gremond. The SPARC architecture manual. PTR Prentice Hall Englewood Cliffs, NJ 07632, 1994.