UNIVERSITY OF CALGARY

Title of Thesis

Second Thesis Title Line

by

Student's name

A THESIS

SUBMITTED TO THE FACULTY OF GRADUATE STUDIES

IN PARTIAL FULFILLMENT OF THE REQUIREMENTS FOR THE

DEGREE OF NAME OF DEGREE IN FULL

DEPARTMENT OF NAME OF DEPARTMENT

CALGARY, ALBERTA

monthname, year

Abstract

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Introduction

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Model of Computation and Definitions

In this chapter, we will describe our model of computation and give the definitions, which are based on Herlihy and Wing's [2] and Golab, Hadzilacos and Woelfel's [1].

The computational model we consider is the standard asynchronous shared memory model with a set of n processes, denoted as $\{p_1, p_2, ..., p_n\}$, where up to $n-1$ processes may fail by crashing.

Type and Object. A *type* τ could be defined as an automaton as follows [1],

$$\tau = (\mathcal{S}, s_{init}, \mathcal{O}, \mathcal{R}, \delta)$$

where $\mathcal{S}$ is a set of states, $s_{init} \in \mathcal{S}$ is the initial state, $\mathcal{O}$ is a set of operation types, $\mathcal{R}$ is the set of responses, and $\delta : \mathcal{S} \times \mathcal{O} \to \mathcal{S} \times \mathcal{R}$ is a one-to-many state transition mapping.

An *object* is an implementation of a type. For each type τ, the transition mapping δ captures the behaviour of objects of type τ, in the absence of concurreny, as follows: if a process applies an operation of type *opt* to an object of type τ which is in state s, the object may return to the process a response RSP and change its states to s' if and only if $(s', rsp) \in \delta(s, opt)$.

An object supports only `read()` and `write(x)` operations is called a *read/write register* (or just *register*). Operation `read()` returns the current state of register and leaves the state unchanged. Operation `write(x)` changes the state of register to be x.

An object supports `read()` and `CAS(x,y)` operations is called *compare-and-swap* (CAS) object. Operation `CAS(x,y)` changes the state of the obejct if and only if the current state is equal to the given state x, i.e, if current state equals x, then operation `CAS(x,y)` succeeds, and the state is changed to be y and *true* is returned. Otherwise, operation `CAS(x,y)` fails,

the current state remains unchanged and $false$ is returned.

History A *history* H, obtained by processes executing operations on objects, is a sequence of operation *invocation* and *response* events. An invocation event INV of operation type *opt* where process p invokes operation on object *obj* is defined as a 5-tuple

$$INV = (invocation, p, obj, opt, t)$$

where *invocation* is the event type and t is the order of event INV in history H. We also call t the *time* when INV happens.

A response event RSP where process p receives response rsp from an operation execution on object *obj* is defined as

$$RSP = (response, p, obj, rsp, t)$$

where *response* is the event type, t is the order of event RSP in history H. We also call t the time when RSP happens.

Response event $(response, p_j, obj_q, rsp, t_1)$ *matches* invocation event $(invocation, p_i, obj_p, opt, t_0)$ in history H, if the two events are applied by the same process to the same object, i.e, $i = j$ and $p = q$. In this case, the response event is also called the *matching* response of the invocation event.

An *operation execution* in H is a pair oe = (INV, RSP) consisting of an invocation event INV and the next matching response event RSP, or just an invocation event INV, denoted as oe = (INV, *null*). For the latter case, we call the operation execution is *pending*. For the former case, we call the operation execution is *complete*. A history H is *complete* if all operation executions in H are *complete*, otherwise, it is *incomplete*.

History H' is an extension of history H if H is the prefix of H'. History H' is called the *completion* of incomplete history H if H', containing the same events as H, is an extension of H and each operation in H' is complete.

$H|obj$ of history H is the subsequence of all invocation and response events in H on object obj. If all invocation and response events in a history H have the same object name obj, then the $H|obj = H$. Thus, in the following discussion, when we discuss the concurrency behavior of a specific objet obj, the history H and $H|obj$ are the same.

Let H be a complete history, we could associate a time interval $I_{(INV,RSP)} = [t_0, t_1]$ with each operation execution (INV, RSP) in H, where t_0 and t_1 is the time point when INV and RSP happen respectively. Similarly, for an uncomplete history, we denote the time interval $I_{(INV,RSP)}$ with respect to the pending operation execution by $I_{(INV,RSP)} = [t_0, \infty]$.

Operation execution oe_0 *precedes* operation execution oe_1 in H if the response event of oe_0 happens before the invocation event of oe_1 in H. We say that oe_0 and oe_1 are *concurrent* in H if neither precedes the other.

A history is *sequential* if the first event is an invocation event, and each invocation event, except possibly the last one, is immediately followed by a matching response event.

Linearization. A history H *linearizes* to a sequential history S, if and only if S satisfies the following conditions: (1) S and any completion of H have the same operations, (2) sequential history S is valid, and (3) there is a mapping from each time interval $I_{(INV,RSP)}$ to a time point $t_{(INV,RSP)} \in I_{(INV,RSP)}$, such that the sequential history S is obtained by sorting the operations in H based on their $t_{(INV,RSP)}$ values.

A history is *linearizable* if and only if H linearizes to some sequential history S. In this case, S is called the *linearization* of H. For each operation *opt* in history H, we call the time point $t_{(INV,RSP)}$ the *linearization point* of *opt*. An object *obj* is linearizable if every history H on *obj* is linearizable.

Randomness. A process can execute local coin flip operation that returns an integer value distributed uniformly at random from an arbitrary finite set of integers. In the following discussion, we use method `random(s)` to return a value which is distributed uniformly at

random from set $\{0, 1, 2, ..., s-1\}$.

Adversary. We analyze our algorithm under the assumption of a strong adaptive adversary. At any point of time, it can see the entire past history and know the states of all processes.

(*What a process do to execute a program? What is an adversary? What is the functionality of an adversary? (to schedule processes)? How it schedules processes?*)

Chapter 3

Dynamic Task Allocation Object

Our dynamic task allocation (DTA) type supports two operations, `DoTask()` and `InsertTask(`ℓ`)`, where ℓ is the task identifier that is unique for each task.

Now we formalize the notion of type DTA by specifying the above two operations. We assume that there exists an atomic operation `PutTask(`M, ℓ`)`, and a process associates task ℓ with memory location M by calling `PutTask(`M, ℓ`)`. It returns *success* if task l is associated with location M, and returns $failure$ if location M was already associated with another task. We say task ℓ is inserted if it is associated with a memory location M.

Similarly, we assume there exists an atomic operation `TryTask(`M`)`, and task ℓ associated with memory location M could be performed atomically by calling `TryTask(`M`)`. Out of several processes calling `TryTask(`M`)`, only one receives *success* and the index ℓ of that task, while all the others receive $failure$.

A task is *done* or $performed$ if its index has been returned by a process after calling `DoTask()`. A task is *available* at location M if it has been inserted to M and *success* is returned by a process after calling `InsertTask(`ℓ`)`, but is not done yet. A task is *available*, if it is *available* at some memory location.

The aim of operation `DoTask()` is to perform an available task on location M by calling `TryTask(`M`)`. Every `DoTask()` may perform several `TryTask(`M`)` operations. However, only one of them will succeed. Once one `TryTask(`M`)` succeeds, then there is no available task on M and the task index ℓ will be returned by `DoTask()`. Additionally, if there is no available task, then operation `DoTask()` returns $\perp$.

The goal of `InsertTask(`ℓ`)` operation is to find a free memory location M and insert task ℓ atomically by calling `PutTask(`M, ℓ`)`. `PutTask(`M, ℓ`)` fails if location M has been associated with another task, so each `InsertTask(`ℓ`)` operation may perform several `PutTask(`M, ℓ`)` operations, but only one of them will succeed. Once one `TryTask(`M`)` succeeds, then task ℓ is available on location M and *success* notification is returned by `InsertTask(`ℓ`)` operation.

Type DTA is required to satisfy: (Validity) If a `DoTask()` operation returns ℓ, then before the `DoTask()` operation, an `InsertTask(`ℓ`)` was executed and returned *success*. (Uniqueness) Each task is performed at most once, i.e, for each task ℓ, at most one `DoTask()` operation returns ℓ.

In addition, the property that every inserted task is eventually done is also a desired progress property of the implementation of type DTA.

Implementation of DTA

(under work...)

Method 1: `DoTask()`

```
1  while true do
2      v ← root;
3      if v.surplus() ≤ 0 then
4          return ⊥;
5      end
6      /* Descent */;
7      while v is not a leaf do
8          (x_L, y_L) ← v.left.read();
9          (x_R, y_R) ← v.right.read();
10         s_L ← min(x_L − y_L, 2^height(v));
11         s_R ← min(x_R − y_R, 2^height(v));
12         r ← random(0, 1);
13         if (s_L + s_R) = 0 then
14             Mark-up(v);
15         else if r < s_L/(s_L + s_R) then
16             v ← v.left;
17         else
18             v ← v.rght;
19         end
20     end
21     /* v is a leaf */;
22     (x, y) ← v.read();
23     (flag, l) ← v.TryTask(task[y + 1]);
24     /* Update Insertion Count */;
25     v.CAS((x, y), (x, y + 1));
26     v ← v.parent;
27     Mark-up(v);
28     if flag = success then
29         return ℓ
30     end
31 end
```

Method 2: `InsertTask(`ℓ`)`

```
32 while true do
33     v ← root;
34     /* Descent */;
35     while v is not a leaf do
36         (x_L, y_L) ← v.left.read();
37         (x_R, y_R) ← v.right.read();
38         s_L ← 2^height(v) − min(x_L − y_L, 2^height(v));
39         s_R ← 2^height(v) − min(x_R − y_R, 2^height(v));
40         r ← random(0, 1);
41         if (s_L + s_R) = 0 then
42             Mark-up(v);
43         else if r < s_L/(s_L + s_R) then
44             v ← v.left;
45         else
46             v ← v.rght;
47         end
48     end
49     /* v is a leaf */;
50     (x, y) ← v.read();
51     flag ← v.PutTask(task[x + 1]);
52     /* Update Insertion Count */;
53     v.CAS((x, y), (x + 1, y));
54     v ← v.parent;
55     Mark-up(v);
56     if flag = success then
57         return success
58     end
59 end
```

Method 3: `Mark-up(`v`)`

```
60 if v is not null then
61     for (i = 0; i < 2; i++) do
62         (x, y) ← v.read();
63         (x_L, y_L) ← v.left.read();
64         (x_R, y_R) ← v.right.read();
65         v.CAS((x, y), (max(x, x_L + x_R), max(y, y_L + y_R));
66     end
67 end
```

Chapter 4

Correctness Proof

4.1 Correctness

The standard correctness condition for shared memory algorithms is linearizability, which was introduced by Herlihy and Wing in 1990 [2]. The intuition of linearizability is that real-time behavior of method calls must be preserved, i.e, if one method call precedes another, then the earlier call must have taken effect before the later one. By contrast, if two method calls overlap, we are free to order them in any convenient way since the order is ambiguous. Informally, a concurrent object is linearizable if each method call appears to take effect instantaneously at some moment between its invocation and response.

4.1.1 Analysis and Proofs

By the definitions in Subsection 3.1.1, one way to show an object *obj* is linearizable is to prove every history H of *obj* is linearizable. Thus, we need to identify for each `DoTask` and `InsertTask` operation *op* (i.e, interval $I_H(op)$) in H a linearization point $t_H(op)$, and prove that the sequential history S obtained by sorting these operations according to their $t_H(op)$ satisfies the sequential specification S_{obj} of *obj*.

We notice that each complete `DoTask` or `InsertTask` operation can be associated with a unique task array slot based on the task it removed or inserted. Additionally, the removal and insertion count are both monotonically increasing. Thus, we could associate the node counts with operations which have been propagated to that node.

Now we define "an operation is counted at a node" recursively to formalize the operation

propagation.

A `DoTask` operation is counted at leaf v when the removal count of v is updated with the index of the task array slot where the performed task is located. Symmetrically, an `InsertTask` operation is counted at v when the insertion count of v is updated with the index of the task array slot where the inserted task is located.

Now we only define `DoTask` operation is counted at an inner node v because counting an `InsertTask` operation is symmetric as well.

Recall that the removal count of v is updated though CAS operation (line 6, method 3). Actually there could be more than one operations updating the count with the same value. We linearize all such CAS operations, which update the removal count of v with the same value y. We say for all these operations, only the first one in the linearization order counts the corresponding `DoTask` operation. In another word, a `DoTask` operation is counted at an inner node v as soon as the CAS updating operation that counts the `DoTask` is linearized. Based this definition, no operation will be counted twice at a node.

Please note that, the CAS operation counting the `DoTask` at node v is not necessary performed by the `DoTask` operation itself, i.e, suppose process p executes a `DoTask` operation and has successfully performed task ℓ at certain leaf. Then the CAS operation counting this $p.DoTask()$ at node v could be a different process q as long as q updates the removal count first in the linearization order.

Given the above concepts and properties, we could prove the following result: (under work...almost done)

4.2 Performance

4.2.1 DoTask Analysis

4.2.2 InsertTask Analysis

Bibliography

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[2] Maurice P. Herlihy and Jeannette M. Wing. Linearizability: A correctness condition for concurrent objects. *ACM Trans. Program. Lang. Syst.*, 12(3):463–492, July 1990.