UNIVERSITY OF CALGARY

Title of Thesis

Second Thesis Title Line

by

Student's name

A THESIS

SUBMITTED TO THE FACULTY OF GRADUATE STUDIES

IN PARTIAL FULFILLMENT OF THE REQUIREMENTS FOR THE

DEGREE OF NAME OF DEGREE IN FULL

DEPARTMENT OF NAME OF DEPARTMENT

CALGARY, ALBERTA

monthname, year

Abstract

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Introduction

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Chapter 2

Model of Computation and Definitions

In this chapter, we will describe our model of computation and give the definitions, which are based on Herlihy and Wing's [2] and Golab, Hadzilacos and Woelfel's [1].

The computational model we consider is the standard asynchronous shared memory model with a set of n processes which is denoted as $\{p_1, p_2, ..., p_n\}$ and up to $n-1$ processes may fail by crashing.

Type and Object. A *type* τ is defined as follows [1],

$$\tau = (\mathcal{S}, s_{init}, \mathcal{O}, \mathcal{R}, \delta)$$

where $\mathcal{S}$ is a set of states, $s_{init} \in \mathcal{S}$ is the initial state, $\mathcal{O}$ is a set of operation types, $\mathcal{R}$ is the set of responses, and $\delta : \mathcal{S} \times \mathcal{O} \rightarrow \mathcal{S} \times \mathcal{R}$ is a state transition mapping.

A compare-and-swap (CAS) type τ supports two operations: `read()` and `CAS(x,y)`. Operation `read()` returns the current value of τ and leaves the value unchanged. Operation `CAS(x,y)` writes new value y into it if and only if the current value of τ is equal to the given value x, i.e, if current value of τ equals x, then operation `CAS(x,y)` succeeds, and the value of τ is changed to be y and *true* is returned. Otherwise, operation `CAS(x,y)` fails, the current value remains unchanged and $false$ is returned.

An object is an implementation of a type. In our thesis, we consider the system that supports atomic CAS object and atomic read/write register. Processes interact with objects by applying operations on them.

History. A *history* H, obtained by processes executing operations on shared objects, is a sequence of method *invocation* and *response* events. $H|obj$ of history H is the subsequence of all invocation and response events in H whose object names are obj. If all invocation and response events in a history H have the same object name obj, then the $H|obj = H$. Thus, in the following discussion, when we discuss the concurrency behavior of a specific objet obj, the history H and $H|obj$ are the same.

We use $inv_H(op)$ to denote the invocation of operation op in history H and use $rep_H(op)$ to denote the matching response of op in H. For each invocation event $inv_H(op)$, it is not necessary to have a matching response. In this case, we call the operation op is *pending* and denote it as $rsp_H(op) = \infty$. Otherwise, we call operation op is *complete*.

A history H is *complete* if all operations in H are *complete*. A completion of an incomplete history H is an extension H' of H, such that H' contains exactly the same operations as H but the matching responses are added for all pending operations in H.

Let H be a complete history, we could associate a time interval $I_H(op) = [inv(op), rsp(op)]$ with each operation op in H. Similarly, for an uncomplete history, we denote the interval with respect to the pending operation op by $I_H(op) = [inv(op), \infty]$.

A history is *sequential* if the first event is an invocation, and each invocation, except possibly the last one, is immediately followed by a matching response.

Linearization. A history H linearizes to a sequential history S, if and only if S satisfies the following conditions: (1) S and the completion of H have the same operations, (2) sequential history S is valid, and (3) there is a mapping from each time interval $I_H(op)$ to a time point $t_H(op) \in I_H(op)$, such that the sequential history S could be obtained by sorting the operations in H by their $t_H(op)$ values.

A history is linearizable if and only if there exists a sequential history S that linearizes H. In this case, S is called the linearization of H. Each linearization of H defines a point $t_H(op)$. For each operation op in history H, we call the point $t_H(op)$ linearization point of op. A shared object is linearizable if every history H of that object is linearizable.

Randomness. A process can execute local coin flip operation that returns an integer value distributed uniformly at random from an arbitrary finite set of integers. In the following discussion, we use method `random(s)` to return a value which is distributed uniformly at random from set $\{0, 1, 2, ..., s-1\}$.

Adversary. We analyze our algorithm under the assumption of a strong adaptive adversary. At any point of time, it can see the entire past history and know the states of all processes.

Chapter 3

Dynamic Task Allocation Object

Our dynamic task allocation (DTA) type supports two operations, `DoTask()` and `InsertTask(`ℓ`)`, where ℓ is the task identifier that is unique for each task.

Now we formalize the notion of type DTA by specifying the above two operations. We assume that there exists an atomic operation `PutTask(`M, ℓ`)`, and a process associates task ℓ with memory location M by calling `PutTask(`M, ℓ`)`. It returns *success* if task l is associated with location M, and returns $failure$ if location M was already associated with another task. We say task ℓ is inserted if it is associated with a memory location M.

Similarly, we assume there exists an atomic operation `TryTask(`M`)`, and task ℓ associated with memory location M could be performed atomically by calling `TryTask(`M`)`. Out of several processes calling `TryTask(`M`)`, only one receives *success* and the index ℓ of that task, while all the others receive $failure$.

A task is *done* or $performed$ if its index has been returned by a process after calling `DoTask()`. A task is *available* at location M if it has been inserted to M and *success* is returned by a process after calling `InsertTask(`ℓ`)`, but is not done yet. A task is *available*, if it is *available* at some memory location.

The aim of operation `DoTask()` is to perform an available task on location M by calling `TryTask(`M`)`. Every `DoTask()` may perform several `TryTask(`M`)` operations. However, only one of them will succeed. Once one `TryTask(`M`)` succeeds, then there is no available task on M and the task index ℓ will be returned by `DoTask()`. Additionally, if there is no available task, then operation `DoTask()` returns $\perp$.

The goal of `InsertTask(`ℓ`)` operation is to find a free memory location M and insert task ℓ atomically by calling `PutTask(`M, ℓ`)`. `PutTask(`M, ℓ`)` fails if location M has been associated with another task, so each `InsertTask(`ℓ`)` operation may perform several `PutTask(`M, ℓ`)` operations, but only one of them will succeed. Once one `TryTask(`M`)` succeeds, then task ℓ is available on location M and *success* notification is returned by `InsertTask(`ℓ`)` operation.

Type DTA is required to satisfy: (Validity) If a `DoTask()` operation returns ℓ, then before the `DoTask()` operation, an `InsertTask(`ℓ`)` was executed and returned *success*. (Uniqueness) Each task is performed at most once, i.e, for each task ℓ, at most one `DoTask()` operation returns ℓ.

In addition, the property that every inserted task is eventually done is also a desired progress property of the implementation of type DTA.

Implementation of DTA

(under work...)

Method 1: `DoTask()`

```
while true do
    v ← root;
    if v.surplus() ≤ 0 then
        return ⊥;
    end
    /* Descent */;
    while v is not a leaf do
        (x_L, y_L) ← v.left.read();
        (x_R, y_R) ← v.right.read();
        s_L ← min(x_L − y_L, 2^{height(v)});
        s_R ← min(x_R − y_R, 2^{height(v)});
        r ← random(0, 1);
        if (s_L + s_R) = 0 then
            Mark-up(v);
        else if r < s_L/(s_L + s_R) then
            v ← v.left;
        else
            v ← v.rght;
        end
    end
    /* v is a leaf */;
    (x, y) ← v.read();
    (flag, l) ← v.TryTask(task[y + 1]);
    /* Update Insertion Count */;
    v.CAS((x, y), (x, y + 1));
    v ← v.parent;
    Mark-up(v);
    if flag = success then
        return ℓ
    end
end
```

Method 2: `InsertTask(ℓ)`

while *true* **do**
  $v \leftarrow root$;
  /* Descent */;
  while *v is not a leaf* **do**
    $(x_L, y_L) \leftarrow v.left.read()$;
    $(x_R, y_R) \leftarrow v.right.read()$;
    $s_L \leftarrow 2^{height(v)} - min(x_L - y_L, 2^{height(v)})$;
    $s_R \leftarrow 2^{height(v)} - min(x_R - y_R, 2^{height(v)})$;
    $r \leftarrow random(0, 1)$;
    if $(s_L + s_R) = 0$ **then**
      Mark-up(v);
    else if $r < s_L/(s_L + s_R)$ **then**
      $v \leftarrow v.left$;
    else
      $v \leftarrow v.rght$;
    end
  end
  /* v is a leaf */;
  $(x, y) \leftarrow v.read()$;
  $flag \leftarrow v.$`PutTask`(task[$x + 1$]);
  /* Update Insertion Count */;
  v.CAS($(x, y), (x + 1, y)$);
  $v \leftarrow v.parent$;
  Mark-up(v);
  if $flag$ = *success* **then**
    return *success*
  end
end

Method 3: `Mark-up`(v)

if *v is not null* **then**
  for $(i = 0; i < 2; i++)$ **do**
    $(x, y) \leftarrow v.read()$;
    $(x_L, y_L) \leftarrow v.left.read()$;
    $(x_R, y_R) \leftarrow v.right.read()$;
    v.CAS($(x, y), (max(x, x_L + x_R), max(y, y_L + y_R))$);
  end
end

Chapter 4

Correctness Proof

4.1 Correctness

The standard correctness condition for shared memory algorithms is linearizability, which was introduced by Herlihy and Wing in 1990 [2]. The intuition of linearizability is that real-time behavior of method calls must be preserved, i.e, if one method call precedes another, then the earlier call must have taken effect before the later one. By contrast, if two method calls overlap, we are free to order them in any convenient way since the order is ambiguous. Informally, a concurrent object is linearizable if each method call appears to take effect instantaneously at some moment between its invocation and response.

4.1.1 Analysis and Proofs

By the definitions in Subsection 3.1.1, one way to show an object obj is linearizable is to prove every history H of obj is linearizable. Thus, we need to identify for each `DoTask` and `InsertTask` operation op (i.e, interval $I_H(op)$) in H a linearization point $t_H(op)$, and prove that the sequential history S obtained by sorting these operations according to their $t_H(op)$ satisfies the sequential specification S_{obj} of obj.

We notice that each complete `DoTask` or `InsertTask` operation can be associated with a unique task array slot based on the task it removed or inserted. Additionally, the removal and insertion count are both monotonically increasing. Thus, we could associate the node counts with operations which have been propagated to that node.

Now we define "an operation is counted at a node" recursively to formalize the operation propagation.

A `DoTask` operation is counted at leaf v when the removal count of v is updated with the index of the task array slot where the performed task is located. Symmetrically, an `InsertTask` operation is counted at v when the insertion count of v is updated with the index of the task array slot where the inserted task is located.

Now we only define `DoTask` operation is counted at an inner node v because counting an `InsertTask` operation is symmetric as well.

Recall that the removal count of v is updated though CAS operation (line 6, method 3). Actually there could be more than one operations updating the count with the same value. We linearize all such CAS operations, which update the removal count of v with the same value y. We say for all these operations, only the first one in the linearization order counts the corresponding `DoTask` operation. In another word, a `DoTask` operation is counted at an inner node v as soon as the CAS updating operation that counts the `DoTask` is linearized. Based this definition, no operation will be counted twice at a node.

Please note that, the CAS operation counting the `DoTask` at node v is not necessary performed by the `DoTask` operation itself, i.e, suppose process p executes a `DoTask` operation and has successfully performed task ℓ at certain leaf. Then the CAS operation counting this $p.DoTask()$ at node v could be a different process q as long as q updates the removal count first in the linearization order.

Given the above concepts and properties, we could prove the following result: (under work...almost done)

4.2 Performance

4.2.1 DoTask Analysis

4.2.2 InsertTask Analysis

Bibliography

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[2] Maurice P. Herlihy and Jeannette M. Wing. Linearizability: A correctness condition for concurrent objects. *ACM Trans. Program. Lang. Syst.*, 12(3):463–492, July 1990.