DATA 604 - FINAL PROJECT : SIMULATION OF DMV WAIT TIMES

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Problem:

Using SimPy, write a process simulation that includes waiting time (discrete event simulation). You may use any topic of interest to you. Write the simulation and all of the following in Jupyter.

Each element is worth 5 points and will be graded using the rubric shown here.

1. State the problem and its significance.

Waiting in line at the department of motor vehicles or motor vehicle registry is a necessity of modern day living which is unavoidable. Customers dread this situation in which you must appear in person because of the insanely long waits at the registry.

Simulate wait time at the Department of Motor Vehicles (DMV). This information can be helpful in reducing wait time by adding additional servers at busy times when wait times are expected to exceed a reasonable time.

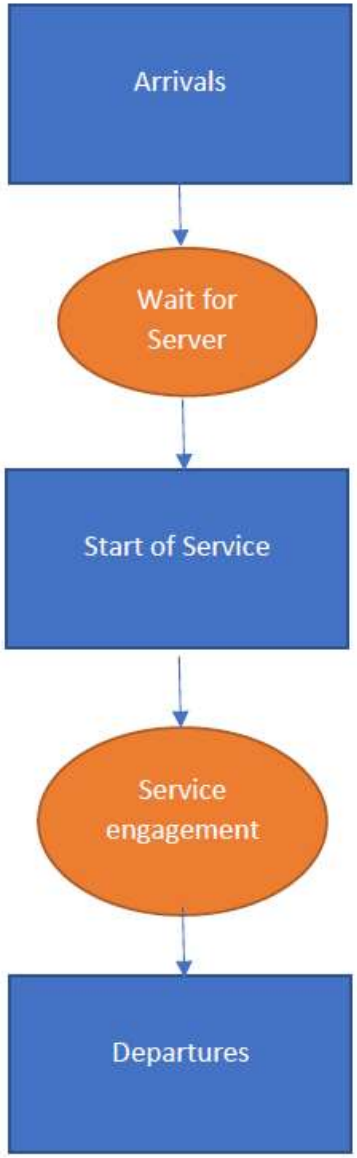
Let's take a look at a couple of simulations using 1,2,3 servers based on exponential customer arrival times and service times and plot the results.

2. Provide a flow-chart model.

The flow chart below depicts the flow of a customer at the DMV.

In [1]:
```
from IPython.display import Image
Image(filename = "flow1.png", width = 200, height = 100)
```

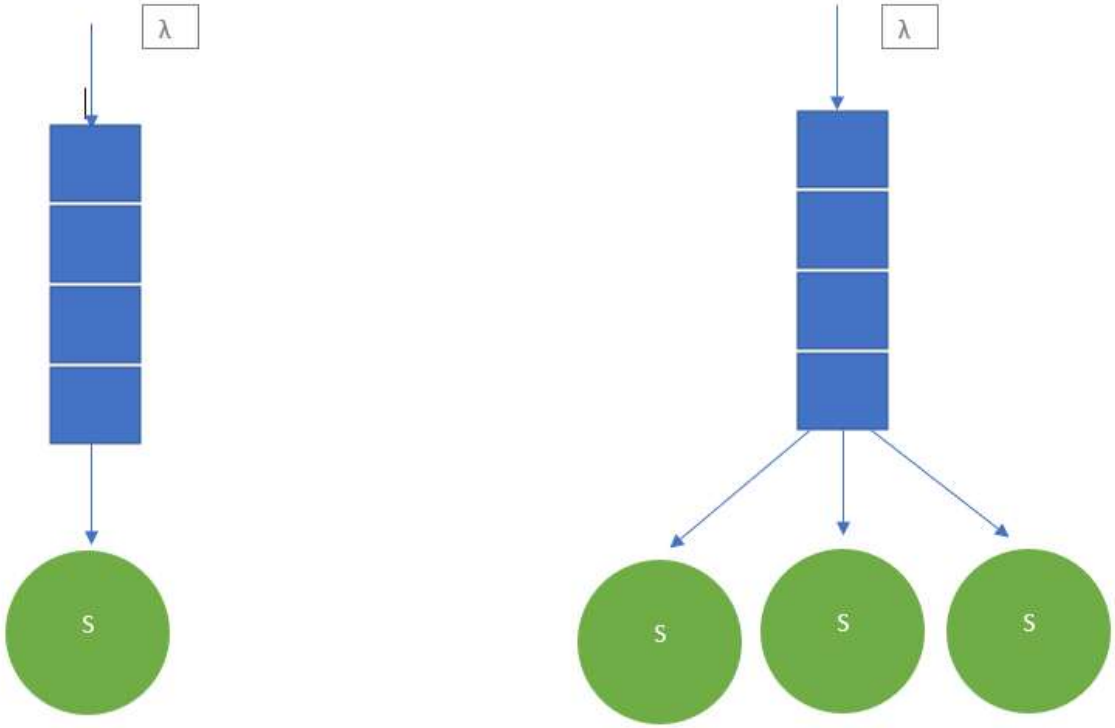
Out[1]:



In order to reduce the wait time in this model, adding more servers that service the single line will result in lower wait times and shorter lines. This can be seen in the two side by side flow charts. For example, the left chart is a single server and the right chart displays 3 servers.

In [2]:
```
Image(filename = "flow2a.png", width = 600, height = 300)
```

Out[2]:



Let's try to simulate the model with 1 server, then repeat with 2, 3 servers.

Suppose we are given the information:

```
The average time to serve a customer is 10 minutes, or in order words, .1 customers are served per
minute.

The average time between customers arriving is 3.3 minutes, or .3 customers arrive per minute.
```

Let's assume these times are exponentially distributed.

3. Simulate the process for the appropriate number of iterations (justify) Try the simulation with 1 server..... The simulation was run for n=120 (a random seed was set for descriptive purposes). This was chosen to look at a 2 hour time period from opening. We are looking to see if the queue becomes to grow unmanagable. If this occurs, we need more servers to accomodate the influx of customers.

In [3]:
```
import simpy
import numpy as np
import random
```

```
import matplotlib.pyplot as plt
import scipy.stats as stats
```

In [4]:

```
NUMBER_OF_EMPLOYEES1=1
CUSTOMERS_SERVED_PER_MINUTE1=.1
CUSTOMERS_ARRIVING_PER_MINUTE1=.3
SIMULATION_TIME1=120


customer_arrived=0
customer_with_server=0
customers_handled=0



def generate_interarrival():
    return np.random.exponential(1./CUSTOMERS_ARRIVING_PER_MINUTE1)

def generate_service():
    return np.random.exponential(1./CUSTOMERS_SERVED_PER_MINUTE1)

def registry(env,servers):
    i=0
    while True:
        i += 1
        yield env.timeout(generate_interarrival())
        env.process(customer(env,i, servers))



tot_t=[]
service_t=[]
queue_t=[]


def customer(env,customer, servers):
    global customers_handled
    global customers_arrived
    global customer_with_server
    with servers.request() as request:
        t_arrival=env.now
        print (env.now, 'customer {} arrives'.format(customer))
        customers_arrived=customer
        yield request
        print (env.now, 'customer {} is being served'.format(customer))
```

```
        customer_with_server=customer
        t_serve=env.now
        yield env.timeout(generate_service())
        print (env.now, 'customer {} departs'.format(customer))
        t_depart=env.now
        tot_t.append(t_depart-t_arrival)
        service_t.append(t_depart-t_serve)
        queue_t.append(t_serve-t_arrival)
        customers_handled=customer



obs_times=[]
q_length=[]

def observe(env, servers):
    while True:
        obs_times.append(env.now)
        q_length.append(len(servers.queue))
        yield env.timeout(0.5)

np.random.seed(24400)

env=simpy.Environment()

servers=simpy.Resource(env, capacity=NUMBER_OF_EMPLOYEES1)

env.process(registry(env,servers))
env.process(observe(env,servers))


env.run(until=SIMULATION_TIME1)
```

```
0.14624127844428425 customer 1 arrives
0.14624127844428425 customer 1 is being served
0.41854463050909413 customer 2 arrives
0.608589561988669 customer 3 arrives
1.5900317205020897 customer 4 arrives
1.93899649915988 customer 1 departs
1.93899649915988 customer 2 is being served
2.2802460271838054 customer 5 arrives
6.762223993436342 customer 2 departs
6.762223993436342 customer 3 is being served
```

```
6.776627034487789 customer 6 arrives
7.502117961346676 customer 7 arrives
26.989735817615298 customer 3 departs
26.989735817615298 customer 4 is being served
29.04117180233679 customer 4 departs
29.04117180233679 customer 5 is being served
31.50452355609548 customer 8 arrives
33.04986993117447 customer 9 arrives
34.46129862802505 customer 5 departs
34.46129862802505 customer 6 is being served
35.42479396021173 customer 6 departs
35.42479396021173 customer 7 is being served
39.6864574051166 customer 7 departs
39.6864574051166 customer 8 is being served
44.20849873861552 customer 10 arrives
44.6595635625325 customer 11 arrives
45.26653553658929 customer 8 departs
45.26653553658929 customer 9 is being served
45.479904263921895 customer 9 departs
45.479904263921895 customer 10 is being served
54.144634625081444 customer 12 arrives
54.39687504441497 customer 10 departs
54.39687504441497 customer 11 is being served
54.78210953463593 customer 13 arrives
55.52789946860937 customer 14 arrives
56.416449092339896 customer 15 arrives
64.96873108044686 customer 16 arrives
69.21144147409765 customer 17 arrives
70.06798199790241 customer 11 departs
70.06798199790241 customer 12 is being served
72.3191934452534 customer 18 arrives
73.50040895382507 customer 12 departs
73.50040895382507 customer 13 is being served
77.28178463630428 customer 13 departs
77.28178463630428 customer 14 is being served
88.98650316033552 customer 19 arrives
91.28792658832768 customer 20 arrives
91.69348312121289 customer 21 arrives
93.05738129968852 customer 22 arrives
99.99888360832541 customer 23 arrives
100.23089566428226 customer 14 departs
100.23089566428226 customer 15 is being served
108.1999123905905 customer 24 arrives
110.82533554171167 customer 25 arrives
113.08646792976603 customer 26 arrives
116.72446256592666 customer 27 arrives
```

In [5]:
```
print("Customers_handled (1 server): " +str(customers_handled))
print("Customers_arrived (1 server): " +str(customers_arrived))
```

```
print("CustomerID_with_service (1 server): " +str(customer_with_server))
print("Length of queue (1 server): " +str(customers_arrived-customer_with_server))
print("Average Customer Queue time (1 server): " +str(sum(queue_t)/customers_handled))
print("Average Customer Service time (1 server): " +str(sum(service_t)/customers_handled))
print("Average Customer Total Q+S time (1 server): " +str(sum(tot_t)/customers_handled))
print("Customer Arrival rate (1 server): " +str(customers_arrived/SIMULATION_TIME1))
print("Customer Service rate (1 server): " +str(customers_handled/sum(service_t)))

print("t-test: "+str(stats.ttest_1samp(a=service_t, popmean=10)))
```

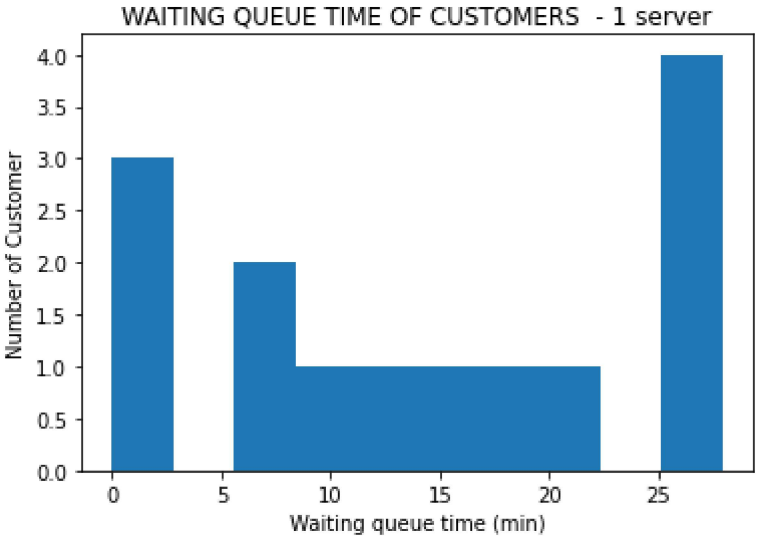
```
Customers_handled (1 server): 14
Customers_arrived (1 server): 27
CustomerID_with_service (1 server): 15
Length of queue (1 server): 12
Average Customer Queue time (1 server): 14.517493727578339
Average Customer Service time (1 server): 7.148903884702712
Average Customer Total Q+S time (1 server): 21.666397612281052
Customer Arrival rate (1 server): 0.225
Customer Service rate (1 server): 0.13988158410407628
t-test: Ttest_1sampResult(statistic=-1.4730215732887186, pvalue=0.16454125210818848)
```

In 120 minutes, 27 customers arrived, with 14 customers served and departed. The line at the end of the simulation is of length 12. Let's take a look at the mean time of those served. Although the wait time is 21.6, it is important to note that these are wait times for those customers already departed, and more importantly, 12 still remain in the queue.

Generate appropriate graphs (more than one) to illustrate the results

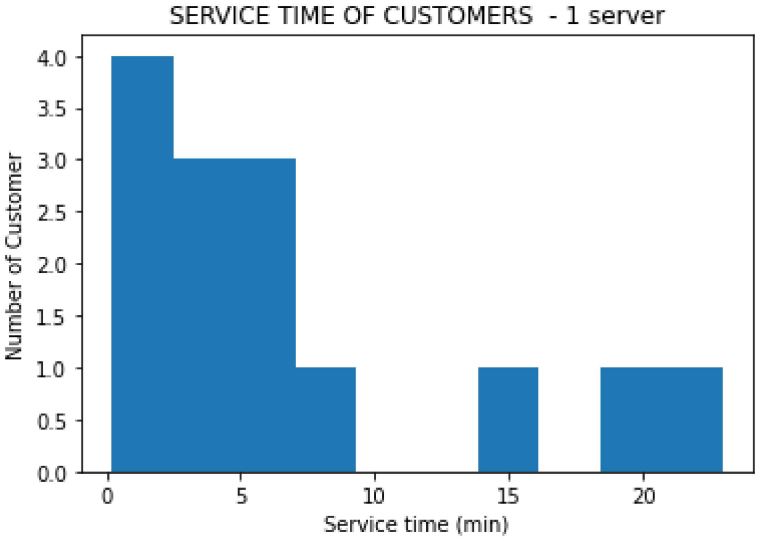
In [6]:
```
plt.figure()
plt.hist(queue_t)
plt.xlabel('Waiting queue time (min)')
plt.ylabel('Number of Customer')
plt.title("WAITING QUEUE TIME OF CUSTOMERS  - 1 server")
```

Out[6]:
```
Text(0.5, 1.0, 'WAITING QUEUE TIME OF CUSTOMERS  - 1 server')
```



In [7]:
```
plt.figure()
plt.hist(service_t)
plt.xlabel('Service time (min)')
plt.ylabel('Number of Customer')
plt.title("SERVICE TIME OF CUSTOMERS  - 1 server")
```

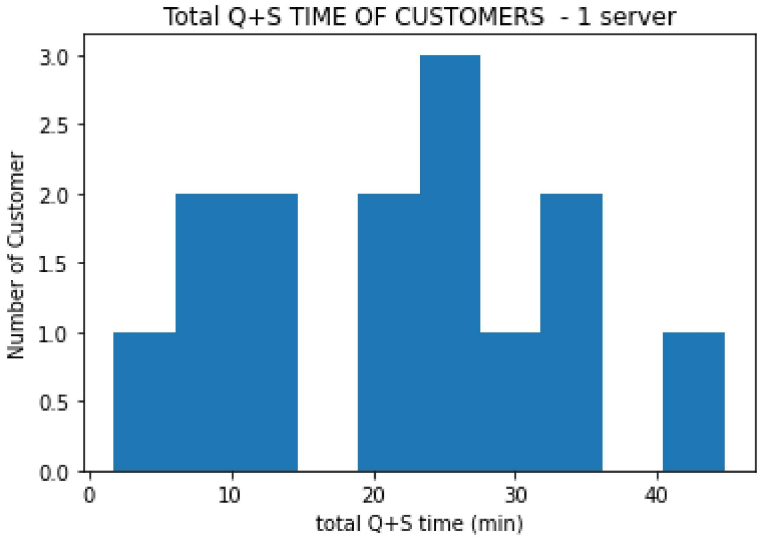
Out[7]:
```
Text(0.5, 1.0, 'SERVICE TIME OF CUSTOMERS  - 1 server')
```



In [8]:
```
plt.figure()
```

```
plt.hist(tot_t)
plt.xlabel('total Q+S time (min)')
plt.ylabel('Number of Customer')
plt.title("Total Q+S TIME OF CUSTOMERS  - 1 server")
```

Out[8]: Text(0.5, 1.0, 'Total Q+S TIME OF CUSTOMERS - 1 server')

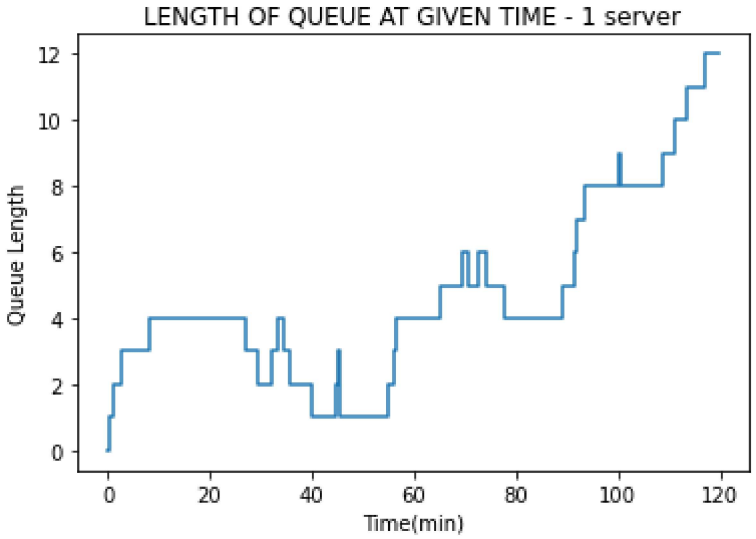


At the opening of the DMV, there is no line. In this system, the line and therefore the wait time will increase if customers are not serviced in an adequate amount of time.

With the notion in mind that the line is growing, let's take a look at the length of the line over time.

In [9]:
```
plt.figure()
plt.step(obs_times,q_length,where='post')
plt.xlabel('Time(min)')
plt.ylabel('Queue Length')
plt.title("LENGTH OF QUEUE AT GIVEN TIME - 1 server")
```

Out[9]: Text(0.5, 1.0, 'LENGTH OF QUEUE AT GIVEN TIME - 1 server')



The line starts to grow quite starkly.

This situation is not going to work, let's add another server to our simulation.

In [10]:
```
NUMBER_OF_EMPLOYEES2=2
CUSTOMERS_SERVED_PER_MINUTE2=.1
CUSTOMERS_ARRIVING_PER_MINUTE2=.3
SIMULATION_TIME2=120


customer_arrived2=0
customer_with_server2=0
customers_handled2=0


def generate_interarrival():
    return np.random.exponential(1./CUSTOMERS_ARRIVING_PER_MINUTE2)

def generate_service():
    return np.random.exponential(1./CUSTOMERS_SERVED_PER_MINUTE2)

def registry(env,servers):
    i=0
    while True:
        i += 1
        yield env.timeout(generate_interarrival())
        env.process(customer(env,i, servers))
```

```
tot_t=[]
service_t=[]
queue_t=[]


def customer(env,customer, servers):
    global customers_handled2
    global customers_arrived2
    global customer_with_server2
    with servers.request() as request:
        t_arrival=env.now
        print (env.now, 'customer {} arrives'.format(customer))
        customers_arrived2=customer
        yield request
        print (env.now, 'customer {} is being served'.format(customer))
        customer_with_server2=customer
        t_serve=env.now
        yield env.timeout(generate_service())
        print (env.now, 'customer {} departs'.format(customer))
        t_depart=env.now
        tot_t.append(t_depart-t_arrival)
        service_t.append(t_depart-t_serve)
        queue_t.append(t_serve-t_arrival)
        customers_handled2=customer



obs_times=[]
q_length=[]

def observe(env, servers):
    while True:
        obs_times.append(env.now)
        q_length.append(len(servers.queue))
        yield env.timeout(0.5)

np.random.seed(24400)

env=simpy.Environment()

servers=simpy.Resource(env, capacity=NUMBER_OF_EMPLOYEES2)

env.process(registry(env,servers))
```

```
env.process(observe(env,servers))
```

Out[10]: <Process(observe) object at 0x217bf2bb910>

In [11]:
```
env.run(until=SIMULATION_TIME2)
```

```
0.14624127844428425 customer 1 arrives
0.14624127844428425 customer 1 is being served
0.41854463050909413 customer 2 arrives
0.41854463050909413 customer 2 is being served
0.608589561988669 customer 3 arrives
1.2988038686703849 customer 4 arrives
1.93899649915988 customer 1 departs
1.93899649915988 customer 3 is being served
2.9065463667625386 customer 5 arrives
3.3628711060493566 customer 2 departs
3.3628711060493566 customer 4 is being served
5.539343886626019 customer 4 departs
5.539343886626019 customer 5 is being served
9.649050308155525 customer 6 arrives
10.33286230306269 customer 7 arrives
12.139571244958777 customer 8 arrives
13.684917620037764 customer 9 arrives
15.42813952107183 customer 3 departs
15.42813952107183 customer 6 is being served
16.391634853258516 customer 6 departs
16.391634853258516 customer 7 is being served
20.653298298163385 customer 7 departs
20.653298298163385 customer 8 is being served
24.843546427478813 customer 10 arrives
25.29461125139579 customer 11 arrives
26.233376429636074 customer 8 departs
26.233376429636074 customer 9 is being served
26.446745156968685 customer 9 departs
26.446745156968685 customer 10 is being served
34.77968231394473 customer 12 arrives
35.363715937461755 customer 10 departs
35.363715937461755 customer 11 is being served
35.41715722349922 customer 13 arrives
36.16294715747266 customer 14 arrives
37.051496781203184 customer 15 arrives
45.603778769310146 customer 16 arrives
49.846489162960935 customer 17 arrives
51.034822890949194 customer 11 departs
51.034822890949194 customer 12 is being served
52.954241134116685 customer 18 arrives
```

```
54.46724984687185 customer 12 departs
54.46724984687185 customer 13 is being served
58.24862552935107 customer 13 departs
58.24862552935107 customer 14 is being served
69.6215508491988 customer 19 arrives
71.92297427719096 customer 20 arrives
72.32853081007617 customer 21 arrives
73.6924289885518 customer 22 arrives
77.54656067087242 customer 5 departs
77.54656067087242 customer 15 is being served
80.6339312971887 customer 23 arrives
81.19773655732904 customer 14 departs
81.19773655732904 customer 16 is being served
88.43388983420304 customer 24 arrives
89.07400601069256 customer 16 departs
89.07400601069256 customer 17 is being served
90.6950222222574 customer 25 arrives
95.20512555927102 customer 26 arrives
99.98798991917444 customer 17 departs
99.98798991917444 customer 18 is being served
100.66155227587748 customer 18 departs
100.66155227587748 customer 19 is being served
101.27096925162506 customer 27 arrives
102.14964701766769 customer 15 departs
102.14964701766769 customer 20 is being served
102.95272232307781 customer 28 arrives
108.16026324413231 customer 19 departs
108.16026324413231 customer 21 is being served
109.42921380944175 customer 29 arrives
109.75600924290232 customer 30 arrives
118.33125833552839 customer 31 arrives
118.4838891609887 customer 21 departs
118.4838891609887 customer 22 is being served
```

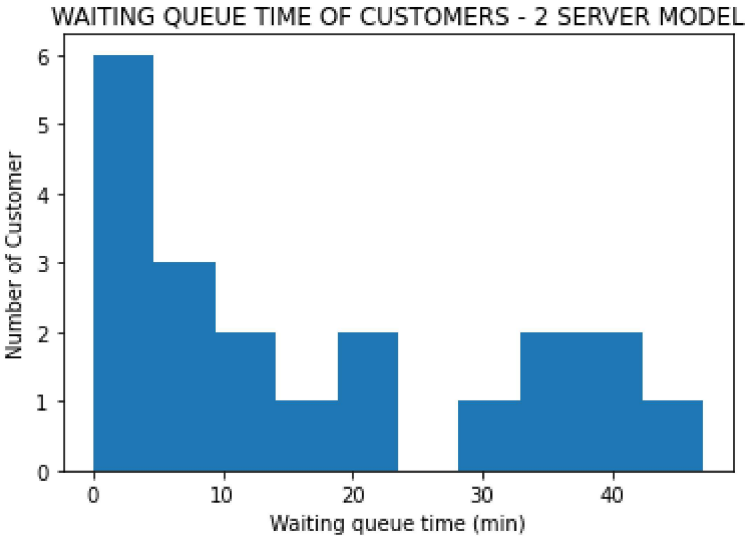
In [12]:

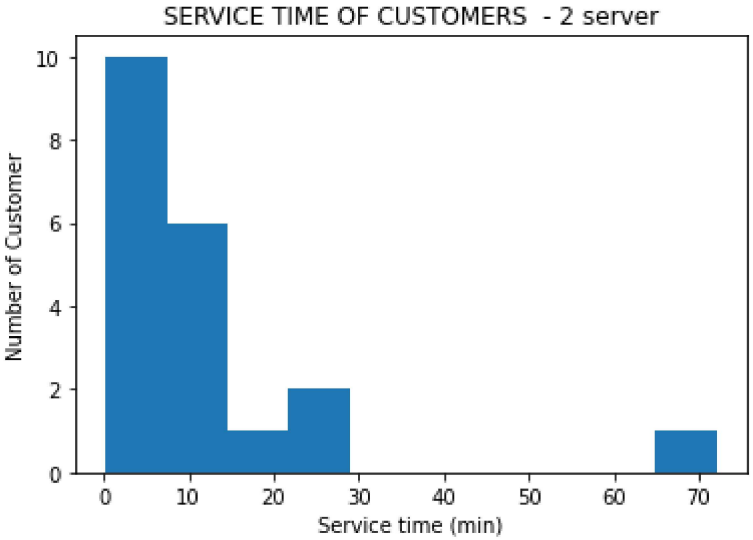
```
print("Customers_handled (2 server): " +str(customers_handled2))
print("Customers_arrived (2 server): " +str(customers_arrived2))
print("CustomerID_with_service (2 server): " +str(customer_with_server2))
print("Length of queue (2 server): " +str(customers_arrived2-customer_with_server2))
print("Average Customer Queue time (2 server): " +str(sum(queue_t)/customers_handled2))
print("Average Customer Service time (2 server): " +str(sum(service_t)/customers_handled2))
print("Average Customer Total Q+S time (2 server): " +str(sum(tot_t)/customers_handled2))
print("Customer Arrival rate (2 server): " +str(customers_arrived2/SIMULATION_TIME2))
print("Customer Service rate (2 server): " +str(customers_handled2/sum(service_t)))

print("t-test: "+str(stats.ttest_1samp(a=service_t, popmean=10)))


plt.figure()
```

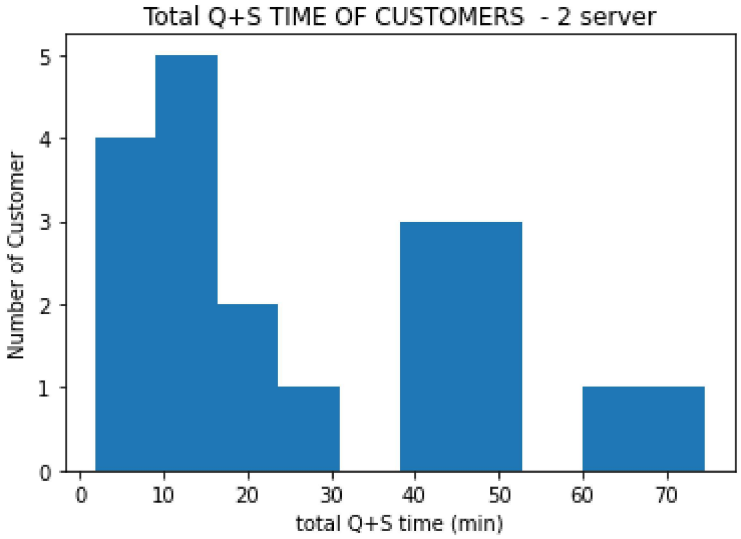
```
plt.hist(queue_t)
plt.xlabel('Waiting queue time (min)')
plt.ylabel('Number of Customer')
plt.title("WAITING QUEUE TIME OF CUSTOMERS - 2 SERVER MODEL")

plt.figure()
plt.hist(service_t)
plt.xlabel('Service time (min)')
plt.ylabel('Number of Customer')
plt.title("SERVICE TIME OF CUSTOMERS  - 2 server")

plt.figure()
plt.hist(tot_t)
plt.xlabel('total Q+S time (min)')
plt.ylabel('Number of Customer')
plt.title("Total Q+S TIME OF CUSTOMERS  - 2 server")

plt.figure()
plt.step(obs_times,q_length,where='post')
plt.xlabel('Time(min)')
plt.ylabel('Queue Length')
plt.title("LENGTH OF QUEUE AT GIVEN TIME - 2 SERVER MODEL")
```

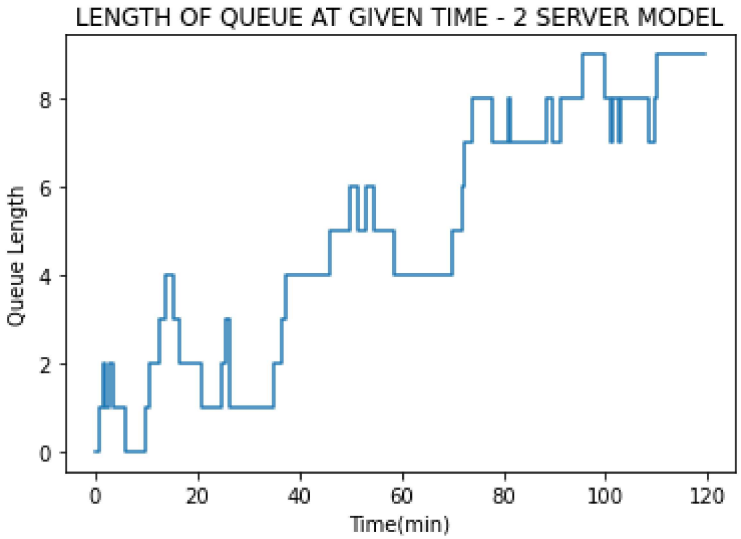
```
Customers_handled (2 server): 21
Customers_arrived (2 server): 31
CustomerID_with_service (2 server): 22
Length of queue (2 server): 9
Average Customer Queue time (2 server): 16.05774073711202
Average Customer Service time (2 server): 10.479464298557286
Average Customer Total Q+S time (2 server): 26.537205035669302
Customer Arrival rate (2 server): 0.25833333333333336
Customer Service rate (2 server): 0.09542472511096493
t-test: Ttest_1sampResult(statistic=0.2811656131408623, pvalue=0.7816224870976518)
```

Out[12]: Text(0.5, 1.0, 'LENGTH OF QUEUE AT GIVEN TIME - 2 SERVER MODEL')


WAITING QUEUE TIME OF CUSTOMERS - 2 SERVER MODEL
Number of Customer
Waiting queue time (min)
0
1
2
3
4
5
6
0
10
20
30
40


SERVICE TIME OF CUSTOMERS - 2 server
Number of Customer
Service time (min)
0
2
4
6
8
10
0
10
20
30
40
50
60
70





The 2 server model shown above resulted in the queue length growing markedly.

Let's try 3 server's.....

In [13]:
```
NUMBER_OF_EMPLOYEES3=3
CUSTOMERS_SERVED_PER_MINUTE3=.1
CUSTOMERS_ARRIVING_PER_MINUTE3=.3
SIMULATION_TIME3=120
```

```
customer_arrived3=0
customer_with_server3=0
customers_handled3=0


def generate_interarrival():
    return np.random.exponential(1./CUSTOMERS_ARRIVING_PER_MINUTE3)

def generate_service():
    return np.random.exponential(1./CUSTOMERS_SERVED_PER_MINUTE3)

def registry(env,servers):
    i=0
    while True:
        i += 1
        yield env.timeout(generate_interarrival())
        env.process(customer(env,i, servers))

tot_t=[]
service_t=[]
queue_t=[]


def customer(env,customer, servers):
    global customers_handled3
    global customers_arrived3
    global customer_with_server3
    with servers.request() as request:
        t_arrival=env.now
        print (env.now, 'customer {} arrives'.format(customer))
        customers_arrived3=customer
        yield request
        print (env.now, 'customer {} is being served'.format(customer))
        customer_with_server3=customer
        t_serve=env.now
        yield env.timeout(generate_service())
        print (env.now, 'customer {} departs'.format(customer))
        t_depart=env.now
        tot_t.append(t_depart-t_arrival)
        service_t.append(t_depart-t_serve)
        queue_t.append(t_serve-t_arrival)
        customers_handled3=customer
```

```
obs_times=[]
q_length=[]

def observe(env, servers):
    while True:
        obs_times.append(env.now)
        q_length.append(len(servers.queue))
        yield env.timeout(0.5)

np.random.seed(24400)

env=simpy.Environment()

servers=simpy.Resource(env, capacity=NUMBER_OF_EMPLOYEES3)

env.process(registry(env,servers))
env.process(observe(env,servers))




np.random.seed(24400)

env=simpy.Environment()

servers=simpy.Resource(env, capacity=NUMBER_OF_EMPLOYEES3)

env.process(registry(env,servers))
env.process(observe(env,servers))
```

Out[13]: <Process(observe) object at 0x217bf47d0a0>

In [14]:
```
env.run(until=SIMULATION_TIME3)
```

```
0.14624127844428425 customer 1 arrives
0.14624127844428425 customer 1 is being served
0.41854463050909413 customer 2 arrives
0.41854463050909413 customer 2 is being served
0.608589561988669 customer 3 arrives
0.608589561988669 customer 3 is being served
1.2988038686703849 customer 4 arrives
1.93899649915988 customer 1 departs
1.93899649915988 customer 4 is being served
```

```
3.3628711060493566 customer 2 departs
5.431817056265131 customer 3 departs
5.7951848759743685 customer 5 arrives
5.7951848759743685 customer 5 is being served
6.520675802833256 customer 6 arrives
6.520675802833256 customer 6 is being served
7.204487797740421 customer 7 arrives
8.749834172819408 customer 8 arrives
11.940802628521517 customer 6 departs
11.940802628521517 customer 7 is being served
12.904297960708202 customer 7 departs
12.904297960708202 customer 8 is being served
17.16596140561307 customer 8 departs
19.908462980260456 customer 9 arrives
19.908462980260456 customer 9 is being served
21.261657452011388 customer 9 departs
21.768489024084687 customer 10 arrives
21.768489024084687 customer 10 is being served
21.981857751417298 customer 10 departs
22.16650832333884 customer 4 departs
31.253560086633634 customer 11 arrives
31.253560086633634 customer 11 is being served
33.165984815297094 customer 11 departs
34.225883680131325 customer 12 arrives
34.225883680131325 customer 12 is being served
36.46325348205164 customer 12 departs
39.44958599796047 customer 13 arrives
39.44958599796047 customer 13 is being served
40.338135621690995 customer 14 arrives
40.338135621690995 customer 14 is being served
44.580846015341784 customer 15 arrives
45.72498833398267 customer 16 arrives
49.661391535158245 customer 14 departs
49.661391535158245 customer 15 is being served
53.44276721763746 customer 15 departs
53.44276721763746 customer 16 is being served
62.39229804906479 customer 17 arrives
64.69372147705695 customer 18 arrives
65.09927800994217 customer 19 arrives
65.10643196228136 customer 13 departs
65.10643196228136 customer 17 is being served
66.46317618841779 customer 20 arrives
74.66420497068287 customer 21 arrives
76.39187824561543 customer 16 departs
76.39187824561543 customer 18 is being served
77.80240166022078 customer 5 departs
77.80240166022078 customer 19 is being served
82.46416350769722 customer 22 arrives
84.26814769897895 customer 18 departs
```

```
84.26814769897895 customer 20 is being served
84.58579882438387 customer 19 departs
84.58579882438387 customer 21 is being served
85.93093888819207 customer 17 departs
85.93093888819207 customer 22 is being served
86.10215814385785 customer 23 arrives
86.60450124489512 customer 22 departs
86.60450124489512 customer 23 is being served
88.60172846660946 customer 24 arrives
91.64976045925336 customer 23 departs
91.64976045925336 customer 24 is being served
97.79845771001979 customer 20 departs
102.783329901446 customer 21 departs
103.24541641479152 customer 25 arrives
103.24541641479152 customer 25 is being served
104.22580271517323 customer 25 departs
106.68662505374365 customer 26 arrives
106.68662505374365 customer 26 is being served
111.07923491834518 customer 24 departs
115.26187414636973 customer 27 arrives
115.26187414636973 customer 27 is being served
116.46045501428995 customer 28 arrives
116.46045501428995 customer 28 is being served
117.92810882241346 customer 26 departs
118.34055788744348 customer 28 departs
119.04406824141707 customer 27 departs
```

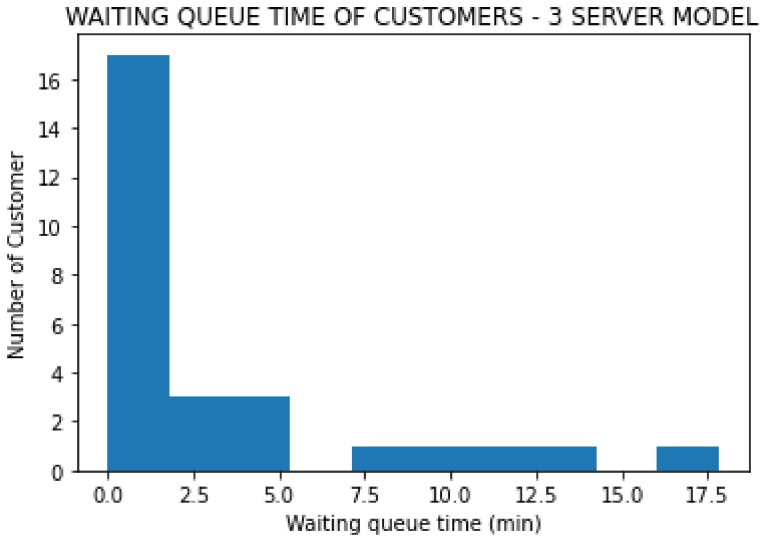
In [15]:

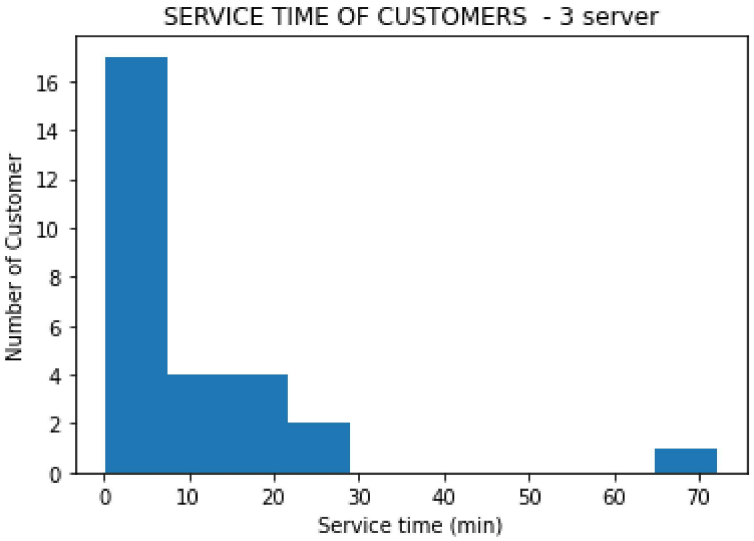
```
print("Customers_handled (3 server): " +str(customers_handled3))
print("Customers_arrived (3 server): " +str(customers_arrived3))
print("CustomerID_with_service (3 server): " +str(customer_with_server3))
print("Length of queue (3 server): " +str(customers_arrived3-customer_with_server3))
print("Average Customer Queue time (3 server): " +str(sum(queue_t)/customers_handled3))
print("Average Customer Service time (3 server): " +str(sum(service_t)/customers_handled3))
print("Average Customer Total Q+S time (3 server): " +str(sum(tot_t)/customers_handled3))
print("Customer Arrival rate (3 server): " +str(customers_arrived3/SIMULATION_TIME3))
print("Customer Service rate (3 server): " +str(customers_handled3/sum(service_t)))
print("t-test: "+str(stats.ttest_1samp(a=service_t, popmean=10)))


plt.figure()
plt.hist(queue_t)
plt.xlabel('Waiting queue time (min)')
plt.ylabel('Number of Customer')
plt.title("WAITING QUEUE TIME OF CUSTOMERS - 3 SERVER MODEL")

plt.figure()
plt.hist(service_t)
```

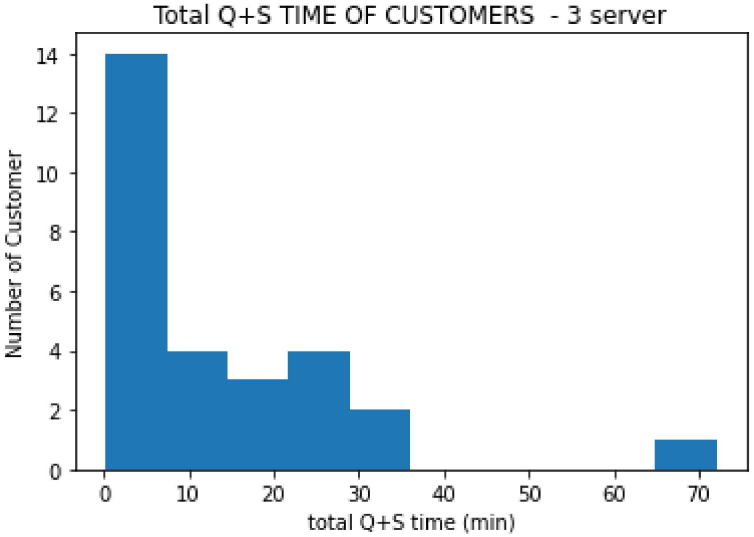
```
plt.xlabel('Service time (min)')
plt.ylabel('Number of Customer')
plt.title("SERVICE TIME OF CUSTOMERS  - 3 server")

plt.figure()
plt.hist(tot_t)
plt.xlabel('total Q+S time (min)')
plt.ylabel('Number of Customer')
plt.title("Total Q+S TIME OF CUSTOMERS  - 3 server")

plt.figure()
plt.step(obs_times,q_length,where='post')
plt.xlabel('Time(min)')
plt.ylabel('Queue Length')
plt.title("LENGTH OF QUEUE AT GIVEN TIME - 3 SERVER MODEL")
```

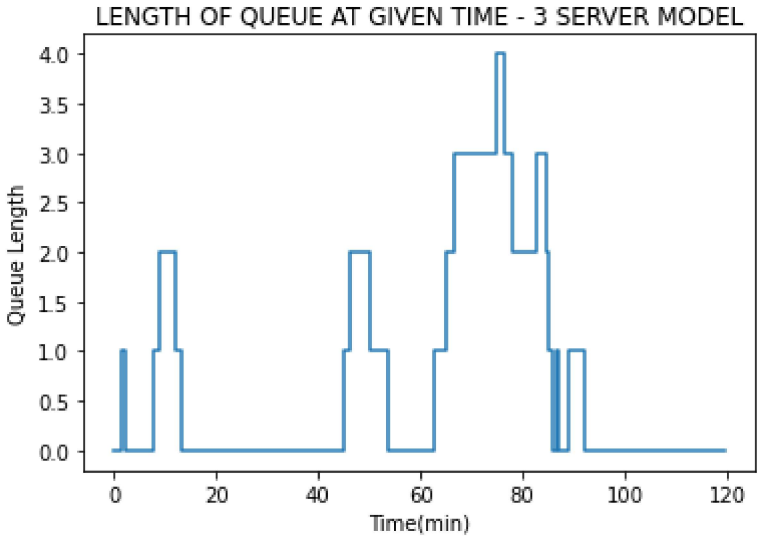
```
Customers_handled (3 server): 27
Customers_arrived (3 server): 28
CustomerID_with_service (3 server): 28
Length of queue (3 server): 0
Average Customer Queue time (3 server): 3.1180898453008323
Average Customer Service time (3 server): 10.744879534022072
Average Customer Total Q+S time (3 server): 13.862969379322907
Customer Arrival rate (3 server): 0.23333333333333334
Customer Service rate (3 server): 0.09306758599141553
t-test: Ttest_1sampResult(statistic=0.1330085201064279, pvalue=0.8951730101559207)
```

Out[15]: Text(0.5, 1.0, 'LENGTH OF QUEUE AT GIVEN TIME - 3 SERVER MODEL')




SERVICE TIME OF CUSTOMERS - 3 server
Number of Customer
Service time (min)
0
2
4
6
8
10
12
14
16
0
10
20
30
40
50
60
70


Total Q+S TIME OF CUSTOMERS - 3 server
Number of Customer
total Q+S time (min)
0
2
4
6
8
10
12
14
0
10
20
30
40
50
60
70



The model with 3 servers reduces the wait time and the queue length to a reasonable, comfortable amount.

Justify the validity of the model and discuss how you verified it.

The problem of understanding DMV wait times is a subject that has been carefully scrutinized. Does the conceptual model represent the real system accurately?

Although a random seed was set for descriptive and illustrative purposes, the simulations were run several times (n=120) with consistent results assuring verification and validation. Again, the model was built with the ability to change parameters: simulation time, servers, customer arrival time, service time.

Validation

ref: https://www.tutorialspoint.com/modelling_and_simulation/modelling_and_simulation_verification_validation.htm

"Validation is the process of comparing two results. In this process, we need to compare the representation of a conceptual model to the real system. If the comparison is true, then it is valid, else invalid."

The model's key assumptions lie in customers arriving and customers serviced. We have assumed these parameters based on educated guesses and research as part of the validation.

The model is built so that these parameters can be adjusted as well.

The average time to serve a customer is 10 minutes, or in order words, .1 customers are served per minute.

The average time between customers arriving is 3.3 minutes, or .3 customers arrive per minute.

Let's assume these times are exponentially distributed.

We ran 3 simulations (1, 2, 3 servers) with 120 iterations.

Here is a summary of the output values:

In [16]:
```
from IPython.display import Image
Image(filename = "table.png", width =4200, height = 400)
```

Out[16]:

	Pop mean	1 Server	2 Server	3 Server
Customers_handled		14	21	27
Customers_arrived		27	31	28
CustomerID_with_service		15	22	28
Length of queue		12	9	0
Average Customer Queue time		14.1	16.1	3.1
Average Customer Service time		7.2	10.5	10.7
Average Customer Total Q+S time		21.7	26.5	13.9
Customer Arrival rate	0.3	0.23	0.26	0.23
Customer Service rate	0.1	0.14	0.1	0.09
t-test Service time pop mean=10 p-value		0.165	0.782	0.895

From the table above, the assumption of customer arrival rate=.3 and the simulations produced similar results: .23, .26, .23.

Also, the assumption of customer service rate=.1 and the simulations produced: .14, .1, .09.

Furthermore, the assumption of average customer service time=10 and the simulations produced: 7.2, 10.5, 10.7

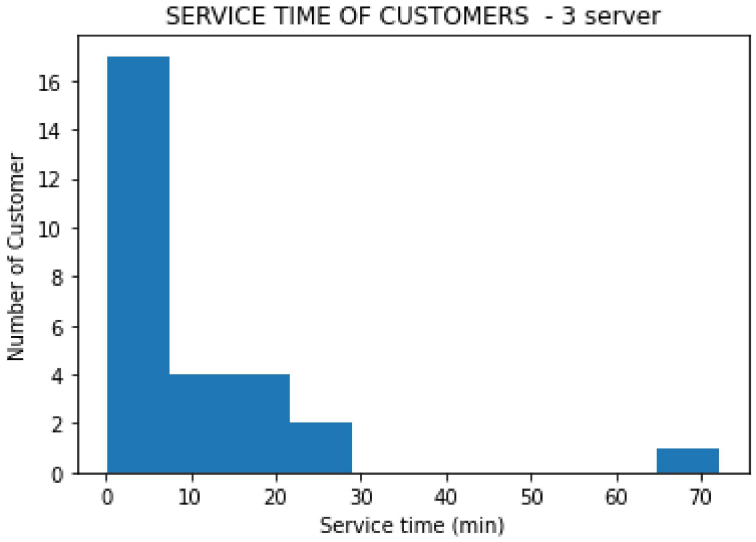
For illustrative purposes, a one sample t-test using population mean customer service time=10 resulted in no significant difference.

The simulation of the 3-server model has produced a service time plot depicted an exponential distribution as output below. This was the assumption on input.

The plots for the server2 and server1 shown previouslyis similar in shape.

In [17]:
```
plt.figure()
plt.hist(service_t)
plt.xlabel('Service time (min)')
plt.ylabel('Number of Customer')
plt.title("SERVICE TIME OF CUSTOMERS  - 3 server")
```

Out[17]:
```
Text(0.5, 1.0, 'SERVICE TIME OF CUSTOMERS  - 3 server')
```



"Verification is the process of comparing two or more results to ensure its accuracy. In this process, we have to compare the model's implementation and its associated data with the developer's conceptual description and specifications."

The flow charts depict the flow of customers in the actual system and the model follows this flow. The flow chart specifies 1) customer enters, 2) customers may wait, 3) customers are served, 4) customers depart.

Below we can trace the flow of customer 1 in the first simulation and confirm that the model is performing as conceptualized.

In [18]:
```
Image(filename = "excerpt_sim.png", width = 600, height = 300)
```

Out[18]:

```
0.14624127844428425 customer 1 arrives
0.14624127844428425 customer 1 is being served
0.41854463050909413 customer 2 arrives
0.608589561988669 customer 3 arrives
1.5900317205020897 customer 4 arrives
1.93899649915988 customer 1 departs
1.93899649915988 customer 2 is being served
2.2802460271838054 customer 5 arrives
6.762223993436342 customer 2 departs
6.762223993436342 customer 3 is being served
6.776627034487789 customer 6 arrives
7.502117961346676 customer 7 arrives
26.989735817615298 customer 3 departs
26.989735817615298 customer 4 is being served

...
```

State your conclusions/ findings from the model.

Conclusions:

This model does accomplish the goal of simulating the queue at a DMV. The model was run with 1,2 and 3 servers. Reviewing the results of the simulations, it can be concluded that a 3 server configuration adequately serves the customers.

From California's DMV blog: "DMV's existing goals are for wait times (amount of time in the queue) to not exceed 45 minutes for customers without an appointment or 15 minutes for customers with an appointment. "

In conclusion, it is apparent the 3 server configuration meets this goal. Future endeavors could improve this simpler model by adding servers periodically, modifying to allow for appointment/non-appointment queues, changing arrival time distribution throughout the day to name a few.

Be sure that your code works!