Modeling and Simulation in Python

Chapter 11

In [1]:
```
# Configure Jupyter so figures appear in the notebook
%matplotlib inline

# Configure Jupyter to display the assigned value after an assignment
%config InteractiveShell.ast_node_interactivity='last_expr_or_assign'

# import functions from the modsim.py module
from modsim import *
```

SIR implementation

We'll use a `State` object to represent the number (or fraction) of people in each compartment.

In [2]:
```
init = State(S=89, I=1, R=0)
```

Out[2]:

	values
S	89
I	1
R	0

To convert from number of people to fractions, we divide through by the total.

In [3]:
```
init /= sum(init)
```

Out[3]:

	values
S	0.988889
I	0.011111
R	0.000000

`make_system` creates a `System` object with the given parameters.

In [4]:
```
def make_system(beta, gamma):
    """Make a system object for the SIR model.

    beta: contact rate in days
    gamma: recovery rate in days

    returns: System object
    """
    init = State(S=89, I=1, R=0)
```

```
    init /= sum(init)

    t0 = 0
    t_end = 7 * 14

    return System(init=init, t0=t0, t_end=t_end,
                  beta=beta, gamma=gamma)
```

Here's an example with hypothetical values for `beta` and `gamma`.

In [5]:
```
tc = 3      # time between contacts in days
tr = 4      # recovery time in days

beta = 1 / tc      # contact rate in per day
gamma = 1 / tr     # recovery rate in per day

system = make_system(beta, gamma)
```

Out[5]:

	values
init	S 0.988889 I 0.011111 R 0.000000 dtyp...
t0	0
t_end	98
beta	0.333333
gamma	0.25

The update function takes the state during the current time step and returns the state during the next time step.

In [6]:
```
def update_func(state, t, system):
    """Update the SIR model.

    state: State with variables S, I, R
    t: time step
    system: System with beta and gamma

    returns: State object
    """
    s, i, r = state

    infected = system.beta * i * s
    recovered = system.gamma * i

    s -= infected
    i += infected - recovered
    r += recovered

    return State(S=s, I=i, R=r)
```

To run a single time step, we call it like this:

In [7]:
```
state = update_func(init, 0, system)
```

Out[7]:

	values

	values
S	0.985226
I	0.011996
R	0.002778

Now we can run a simulation by calling the update function for each time step.

In [8]:

```
def run_simulation(system, update_func):
    """Runs a simulation of the system.

    system: System object
    update_func: function that updates state

    returns: State object for final state
    """
    state = system.init

    for t in linrange(system.t0, system.t_end):
        state = update_func(state, t, system)

    return state
```

The result is the state of the system at `t_end`

In [9]:

```
run_simulation(system, update_func)
```

Out[9]:

	values
S	0.520568
I	0.000666
R	0.478766

Exercise Suppose the time between contacts is 4 days and the recovery time is 5 days. After 14 weeks, how many students, total, have been infected?

Hint: what is the change in `S` between the beginning and the end of the simulation?

In [10]:

```
# Solution goes here

tc = 4      # time between contacts in days
tr = 5      # recovery time in days

beta = 1 / tc      # contact rate in per day
gamma = 1 / tr     # recovery rate in per day

system = make_system(beta, gamma)
```

Out[10]:

	values
init	S 0.988889 I 0.011111 R 0.000000 dtyp...
t0	0
t_end	98

	values
beta	0.25
gamma	0.2

In [11]:

```
run_simulation(system, update_func)
final_state=run_simulation(system, update_func)
```

Out[11]:

	values
S	0.610171
I	0.004672
R	0.385157

In [12]:

```
# Solution goes here
# how many students, total, have been infected?

# Hint: what is the change in S between the beginning and the end of the simulation?
print(init.S)
print(final_state.S)
(init.S-final_state.S)*98
```

```
0.9888888888888889
0.6101711446474097
```

Out[12]: 37.11433893566496

Using TimeSeries objects

If we want to store the state of the system at each time step, we can use one `TimeSeries` object for each state variable.

In [13]:

```
def run_simulation(system, update_func):
    """Runs a simulation of the system.

    Add three Series objects to the System: S, I, R

    system: System object
    update_func: function that updates state
    """
    S = TimeSeries()
    I = TimeSeries()
    R = TimeSeries()

    state = system.init
    t0 = system.t0
    S[t0], I[t0], R[t0] = state

    for t in linrange(system.t0, system.t_end):
        state = update_func(state, t, system)
        S[t+1], I[t+1], R[t+1] = state

    return S, I, R
```

Here's how we call it.

In [14]:
```
tc = 3      # time between contacts in days
tr = 4      # recovery time in days

beta = 1 / tc      # contact rate in per day
gamma = 1 / tr     # recovery rate in per day

system = make_system(beta, gamma)
S, I, R = run_simulation(system, update_func)
```

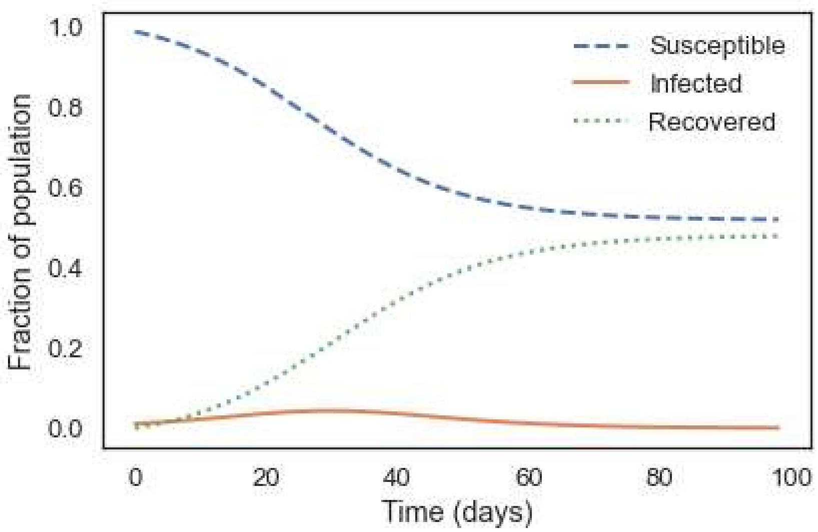
And then we can plot the results.

In [15]:
```
def plot_results(S, I, R):
    """Plot the results of a SIR model.

    S: TimeSeries
    I: TimeSeries
    R: TimeSeries
    """
    plot(S, '--', label='Susceptible')
    plot(I, '-', label='Infected')
    plot(R, ':', label='Recovered')
    decorate(xlabel='Time (days)',
             ylabel='Fraction of population')
```

Here's what they look like.

In [16]:
```
plot_results(S, I, R)
savefig('figs/chap11-fig01.pdf')
```

```
Saving figure to file figs/chap11-fig01.pdf
```



Using a DataFrame

Instead of making three `TimeSeries` objects, we can use one `DataFrame`.

We have to use `row` to selects rows, rather than columns. But then Pandas does the right thing, matching up the state variables with the columns of the `DataFrame`.

In [17]:
```
def run_simulation(system, update_func):
    """Runs a simulation of the system.

    system: System object
```

```
    update_func: function that updates state

    returns: TimeFrame
    """
    frame = TimeFrame(columns=system.init.index)
    frame.row[system.t0] = system.init

    for t in linrange(system.t0, system.t_end):
        frame.row[t+1] = update_func(frame.row[t], t, system)

    return frame
```

Here's how we run it, and what the result looks like.

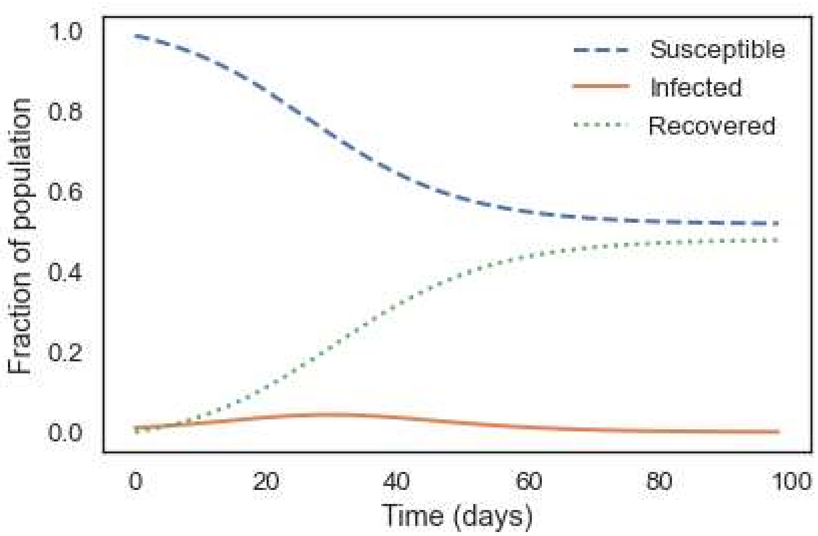
In [18]:
```
tc = 3      # time between contacts in days
tr = 4      # recovery time in days

beta = 1 / tc      # contact rate in per day
gamma = 1 / tr     # recovery rate in per day

system = make_system(beta, gamma)
results = run_simulation(system, update_func)
results.head()
```

Out[18]:

	S	I	R
0	0.988889	0.011111	0.000000
1	0.985226	0.011996	0.002778
2	0.981287	0.012936	0.005777
3	0.977055	0.013934	0.009011
4	0.972517	0.014988	0.012494

We can extract the results and plot them.

In [19]:
```
plot_results(results.S, results.I, results.R)
```



Exercises

Exercise Suppose the time between contacts is 4 days and the recovery time is 5 days. Simulate this scenario for 14 weeks and plot the results.

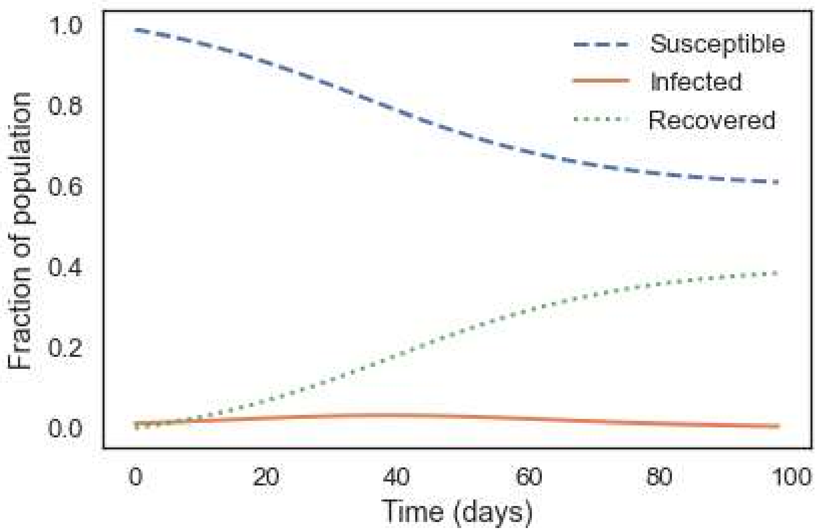
In [20]:
```
# Solution goes here

def run_simulation(system, update_func):
    """Runs a simulation of the system.

    system: System object
    update_func: function that updates state

    returns: TimeFrame
    """
    frame = TimeFrame(columns=system.init.index)
    frame.row[system.t0] = system.init

    for t in linrange(system.t0, system.t_end):
        frame.row[t+1] = update_func(frame.row[t], t, system)

    return frame
```

In [21]:
```
tc = 4      # time between contacts in days
tr = 5      # recovery time in days

beta = 1 / tc      # contact rate in per day
gamma = 1 / tr     # recovery rate in per day

system = make_system(beta, gamma)
results = run_simulation(system, update_func)
results.head()
```

Out[21]:

	S	I	R
0	0.988889	0.011111	0.000000
1	0.986142	0.011636	0.002222
2	0.983273	0.012177	0.004549
3	0.980280	0.012735	0.006985
4	0.977159	0.013309	0.009532

In [22]:
```
plot_results(results.S, results.I, results.R)
```



With the longer time between contacts and recovery time, the plot converges more slowly.