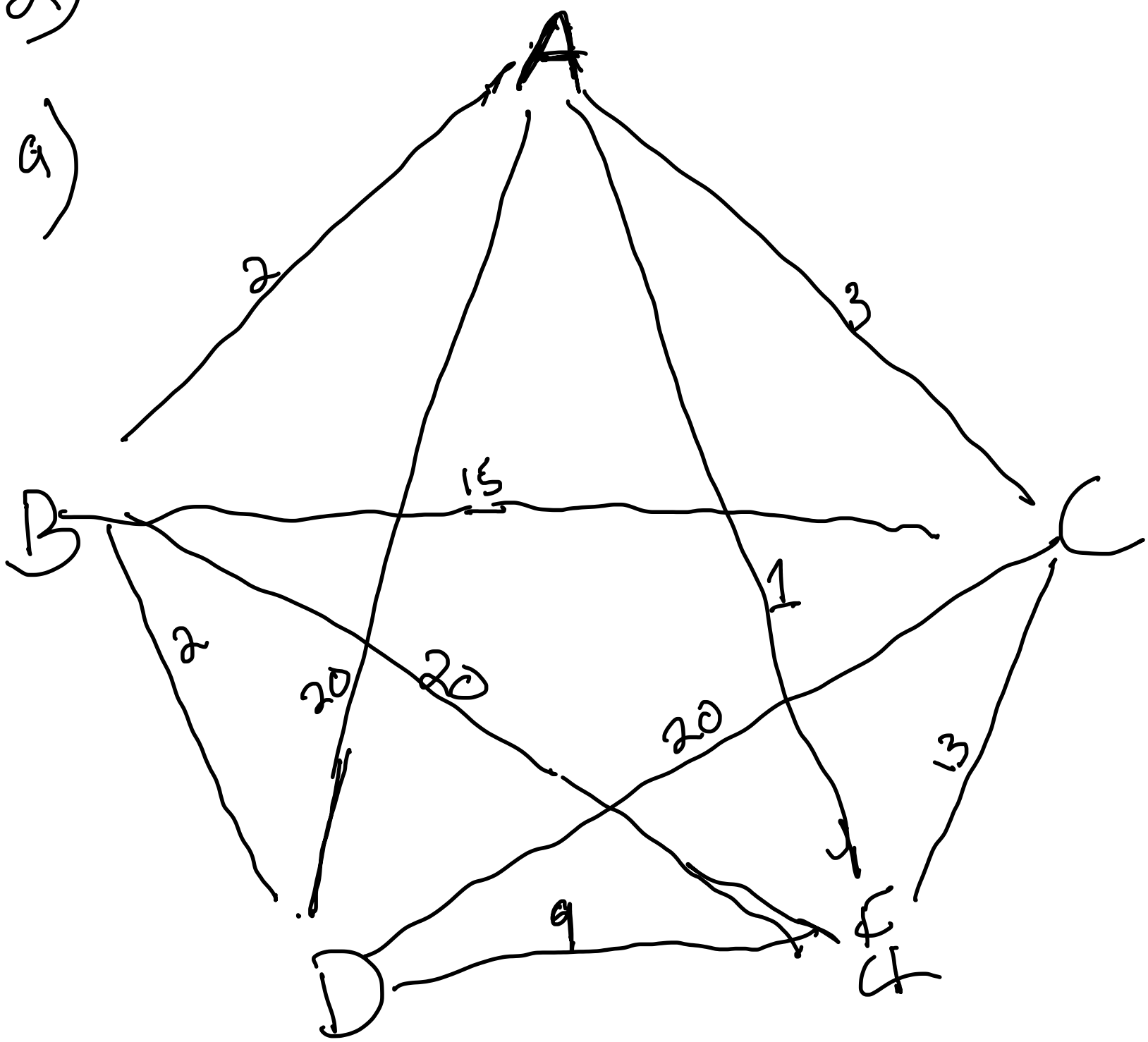
1) We proceed by reducing the hamilton cycle to TSP. To do this, we complete graph G by adding edges between vertices that are not connected.

The new graph $G' = (V', E')$ will consist of $V' = V$ and $E' = \{(u,v)\}$ for u,v in V'. For edges in G' that exist in G, we assign a weight of ϕ else weight 1. By doing these steps the graph is constructed in polynomial time.

If there exists a cycle in G' that passes through each vertex once and the weight is ϕ throughout, then the cycle contains the edges in graph G. Hence we have a hamilton cycle in graph G. Since there exists a hamilton cycle in graph G, it forms a cycle in graph G' of weight ϕ. Hence, there exists a path in graph G' of weight totaling ϕ so TSP is NP-complete.

2)

a)



b) Applying the nearest neighbor heuristic starting at node A, we get the following:

$$A \xrightarrow{1} E \xrightarrow{9} D \xrightarrow{2} B \xrightarrow{15} C \xrightarrow{3} A = 30$$

C) $\frac{\text{nearest neighbor}}{\text{optimal solution}} = \frac{30}{29} = 1.034$

D) see the python file